Name: ______________________________ (please print)

Signature: ___________________________

ECE 3455
Final Exam
April 30, 2008

Exam duration: 170 minutes

- You may have *two* 8 ½ x 11 in. "crib" sheets, written on both sides, during the exam. You may have any calculator you choose, but no computers. No other notes or materials will be allowed.
- Show all work necessary to complete the problem on these pages. A solution without the work shown will receive no credit.
- Show units in intermediate and final results, and in figures.
- If your work is sloppy or difficult to follow, points will be subtracted.

This exam has 11 pages, including the cover sheet. Raise your hand if you are missing a page.

1 _________________/20

2 _________________/20

3 _________________/25

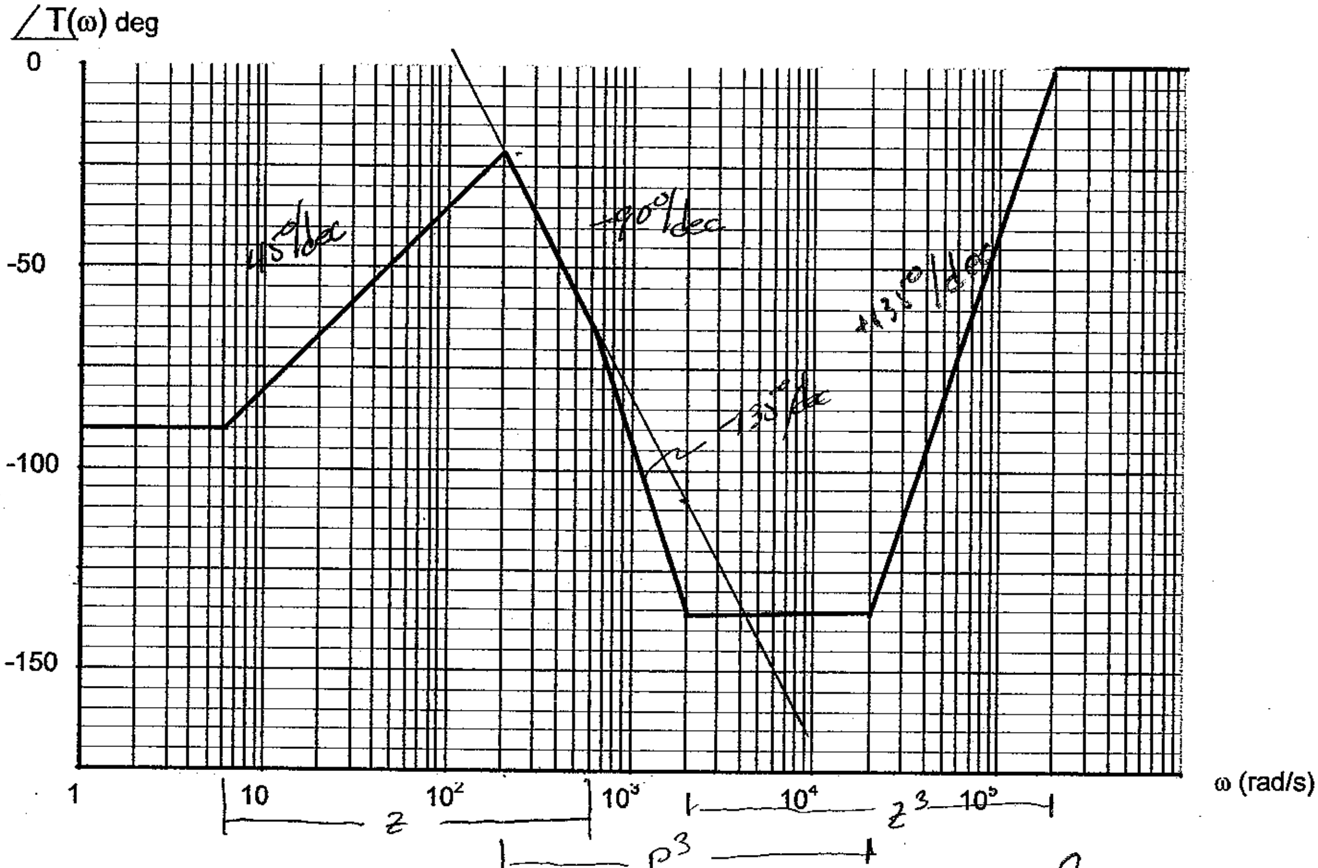
4 _________________/25

5 _________________/10

Total _________________/100

1. **(20 points)** A phase Bode plot corresponding to a certain transfer function is sketched below. The magnitude of the transfer function at very large ω is 15 dB, and as ω approaches 0, the transfer function magnitude approaches ∞.

i) Find the transfer function T(ω), including any constants necessary to specify the magnitude.
ii) Plot the magnitude Bode plot on the paper provided on the next page.



Analysis of slopes shows: +8

Zeros: 60, 3 at 20000 rad/s

poles: 3 at 2000 rad/s

$$\Rightarrow T(\omega) = K \cdot \frac{(j\omega+60)(j\omega+20000)^3}{(j\omega+2000)^3(j\omega)}$$

$\omega \to 0 \Rightarrow \angle T(\omega) \to -90°, \Rightarrow$ Pole at 0

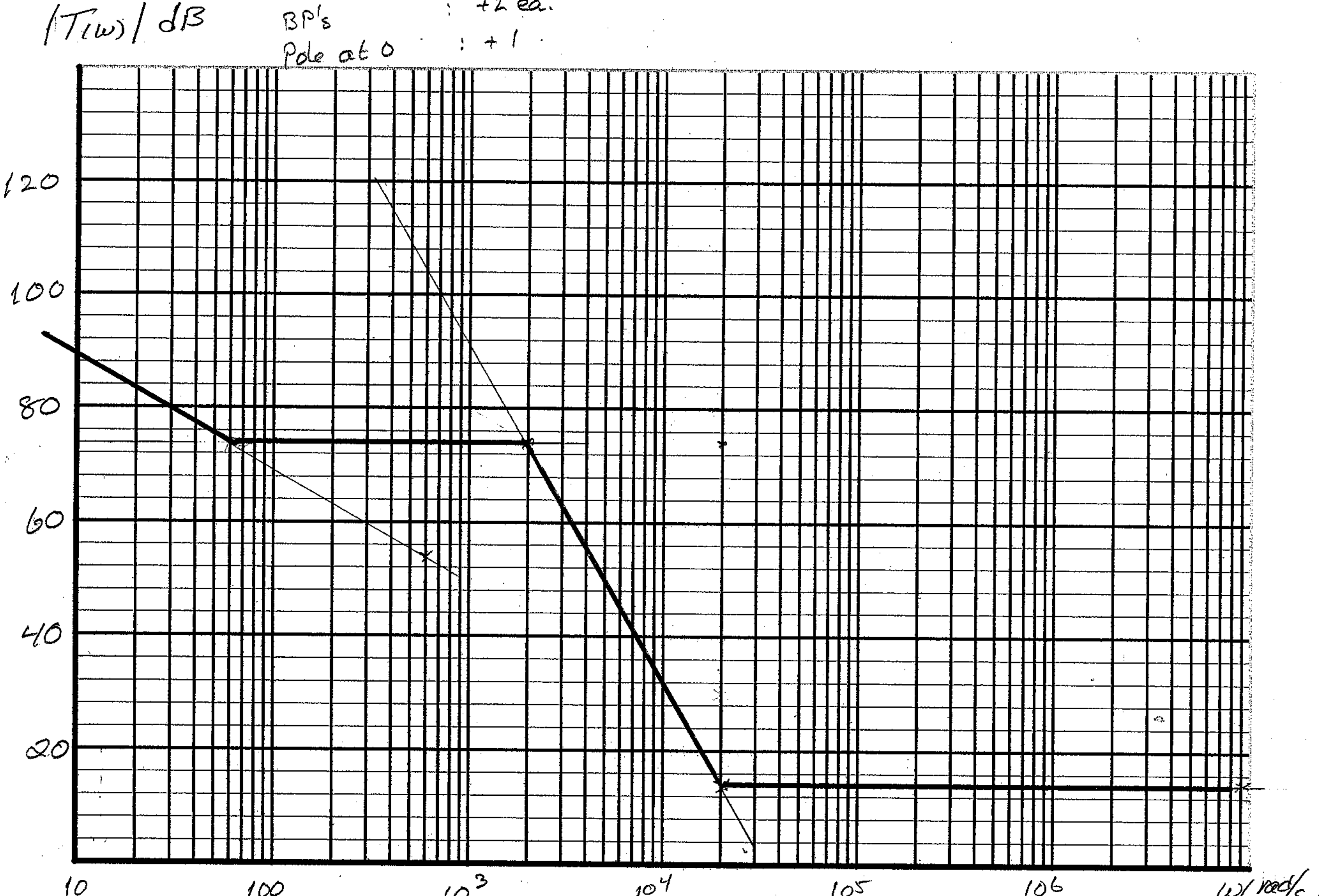
$\omega \to \infty \Rightarrow T(\omega) \to K \Rightarrow 20 \log K = 15$ dB +2

$\therefore K = 10^{15/20} = \underline{5.62}$.

Magnitude plot next page →

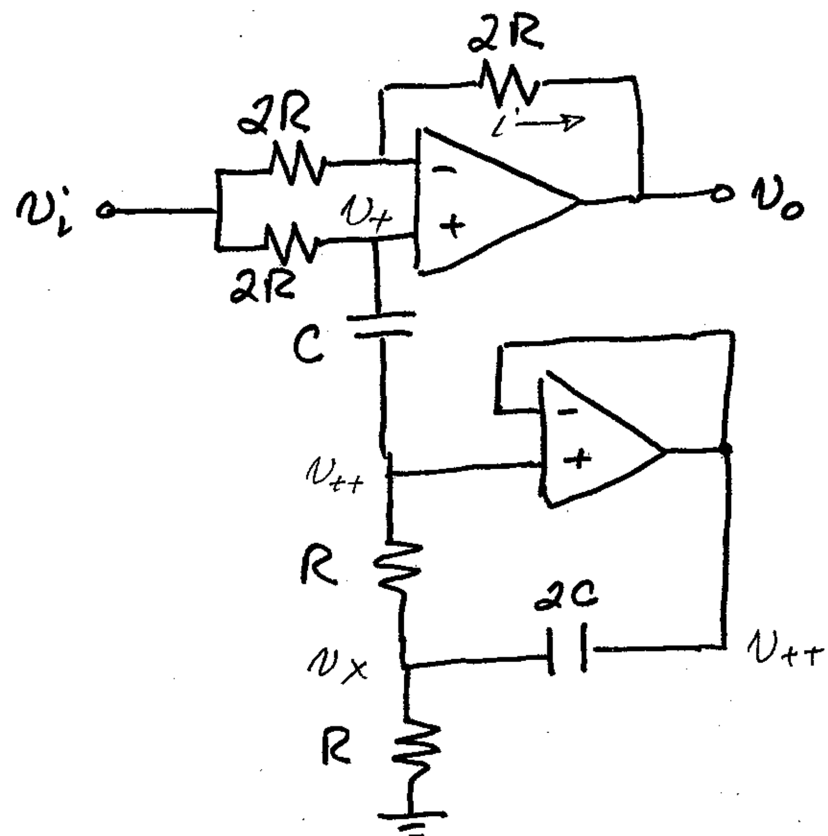
check: $|T(\omega)|_{300\ rad/s} \cong 75.$ 3 so plot is ok at this ω.

Labels, Units : +1
Magnitude : +2
BP's : +2 ea.
Pole at 0 : +1



2. (**20 points**) The op amps in the circuit below may be considered ideal.

Write a set of equations that could be used to find the transfer function $T(\omega) = \mathbf{V_o/V_i}$ for the circuit. Do not attempt to solve the equations. Draw a box around the equations that are part of your solution; equations not in a box will not be counted. Points will be deducted if you do not have a sufficient number of independent equations to find the transfer function, or if your equations are redundant or otherwise unnecessary.



$$\left.\begin{aligned} \bar{V}_o - \bar{V}_+ + 2R\cdot\bar{I} &= 0 \\ \bar{I} &= \frac{\bar{V}_i - \bar{V}_+}{2R} \end{aligned}\right\} \Rightarrow \quad \bar{V}_o = V_+ - \bar{V}_i + \bar{V}_+$$

$$\boxed{\bar{V}_o = 2\bar{V}_+ - \bar{V}_i} \qquad (1)$$

We need $\bar{V}_+$ in terms of $\bar{V}_i$:

$$\boxed{\frac{\bar{V}_+ - \bar{V}_i}{2R} + \frac{\bar{V}_+ - \bar{V}_{++}}{1/j\omega C} = 0} \qquad (2)$$

Now we need an equation for $\bar{V}_{++}$ and another for $\bar{V}_x$ →

Room for Extra work

$$\boxed{\frac{\bar{V}_{++} - \bar{V}_{+}}{1/j\omega C} + \frac{\bar{V}_{++} - \bar{V}_{x}}{R} = 0} \qquad (3)$$

$$\boxed{\frac{\bar{V}_{x} - \bar{V}_{++}}{R} + \frac{\bar{V}_{x} - \bar{V}_{++}}{1/2j\omega C} + \frac{\bar{V}_{x}}{R} = 0} \qquad (4)$$

Now (3) and (4) together will give us $\bar{V}_{++}$ in terms of $\bar{V}_{+}$ (by elliminating $\bar{V}_x$); this gets plugged into (2) to elliminate $\bar{V}_{++}$ and the result goes into (1).

The 4 equations above are all we were asked to do, but let's solve anyway...

From (4): $\bar{V}_x = \bar{V}_{++} \dfrac{1 + j\omega 2CR}{2 + j\omega 2CR}$

From (3); $\bar{V}_{++}\left(j\omega C + \frac{1}{R}\right) - \bar{V}_x\left(\frac{1}{R}\right) = \bar{V}_{+} j\omega C$

Substituting $\bar{V}_x$ and multiplying by R gives

$$\bar{V}_{++}\left(j\omega CR + 1 - \frac{1 + j\omega 2CR}{2 + j\omega 2CR}\right) = \bar{V}_{+} j\omega C$$

Solving for $\bar{V}_{++}$ and clearing fractions... $(\rightarrow 2)$

Room for Extra Work

$$\bar{V}_{++} = \bar{V}_{+} \frac{j\omega CR(2+j\omega 2CR)}{(1+j\omega CR)(2+j\omega 2CR)-(1+j\omega 2CR)}$$

From (2)

$$\bar{V}_{+}\left(\frac{1}{2R}+j\omega C\right) - \bar{V}_{++}\, j\omega C = \bar{V}_i \left(\frac{1}{2R}\right)$$

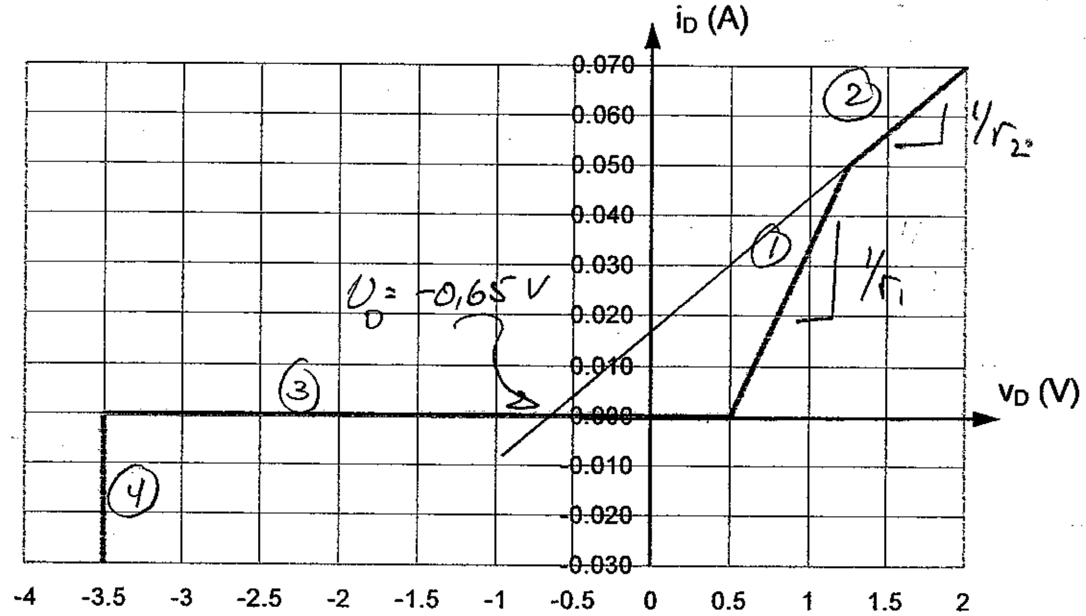
Multiplying by 2R and substituting V_{++} from above gives

$$\bar{V}_{+}\left[(1+j\omega 2CR) - \frac{j\omega CR(2+j\omega 2CR)(j\omega 2CR)}{(1+j\omega CR)(2+j\omega 2CR)-(1+j\omega 2CR)}\right] = \bar{V}_i$$

This now goes into (1) but we will leave that for the student ...

Although not obvious from the equations, this is a notch filter, For $C\cdot R = 10^{-3}$ s, the notch frequency is ~ 700 rad/s and the 3dB bandwidth is 1 decade.

3. **(25 points)** The current-voltage characteristics for a particular diode are shown below. Use the graph provided to plot the transfer characteristics (i.e., v_O/v_I) for the circuit shown. Be sure to account for an input voltage large enough to bias the diodes in all possible regions.

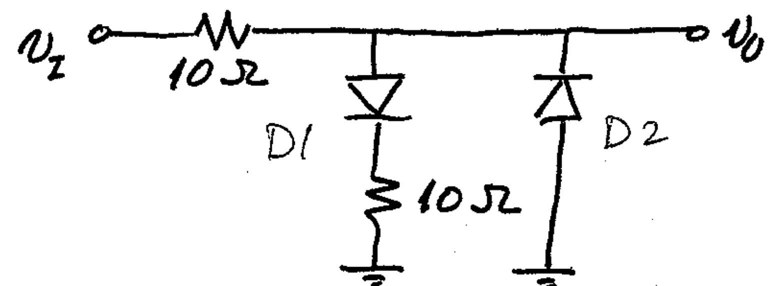


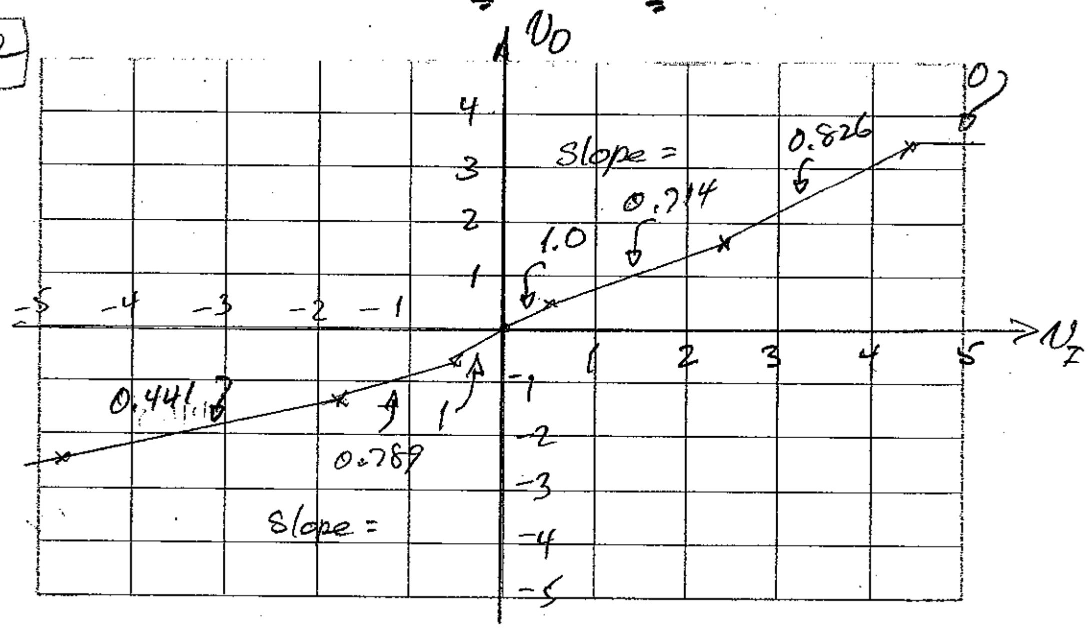
$r_1 = \frac{1.25-0.5}{0.05-0}$

$\boxed{r_1 = 15\,\Omega}$

$r_2 = \frac{2-1.25}{0.02}$

$\boxed{r_2 = 37.5\,\Omega}$

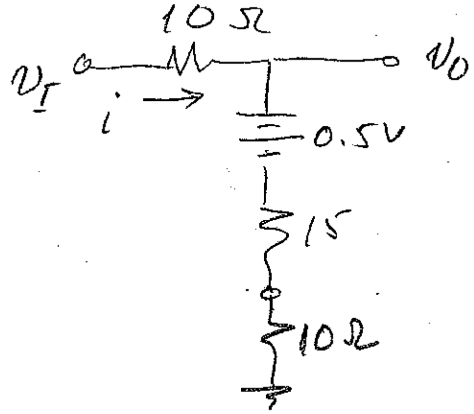




Room for extra work

$v_I > 0$: $0 \leq v_i \leq 0.5\,V \Rightarrow$ D1 OFF, D2 OFF $\Rightarrow v_O = v_I$

$0.5V \leq v_i \leq \, ?$ $\Rightarrow$ D1 ON, D2 OFF

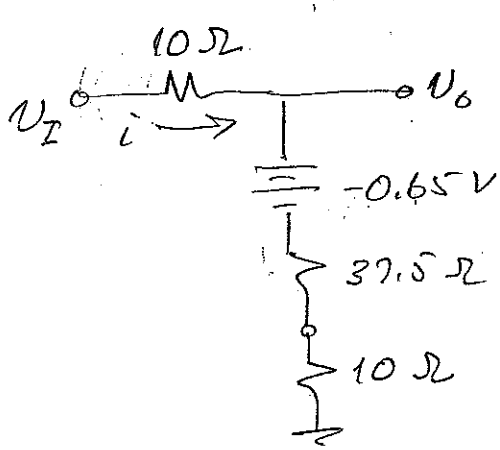


$$i = \frac{v_I - 0.5}{35}$$

$$v_O = 0.5 + \frac{25}{35} v_I - \frac{25}{35}(0.5)$$

$$v_O = 0.143 + 0.714\, v_I \quad [V]$$

Where does D1 go to region 2 ?



$$i = \frac{v_I + 0.65}{57.5}$$

$$v_O = -0.65 + \frac{47.5}{57.5}(v_I + 0.65)$$

$$= -0.113 + 0.826\, v_I$$

Set this result equal to that from previous region:

$$-0.113 + 0.826\, v_I = 0.143 + 0.714\, v_I \Rightarrow v_I = 2.29\ V$$

$$\approx 2.3\ V.$$

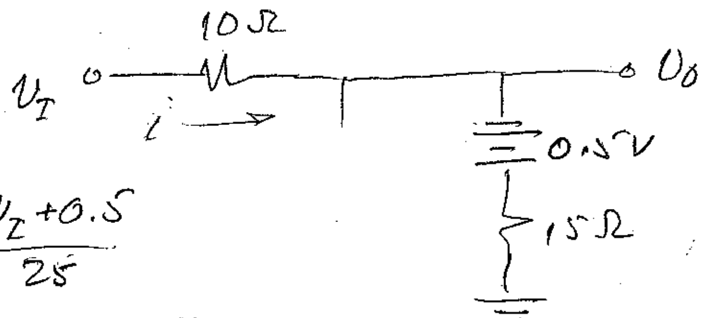
At this point, $v_O = 1.79$ V

Finally, for $v_O = 3.5\ V$, D2 goes ON (reverse-bias)
This happens at v_I such that

$$3.5 = -0.113 + 0.826\, v_I \Rightarrow v_I = 4.37\ V$$

After that, v_o is constant at 3.5 V

$v_I < 0$: $\quad v_I > -0.5\ V \Rightarrow$ D1 OFF, D2 OFF $\Rightarrow v_O = v_I$

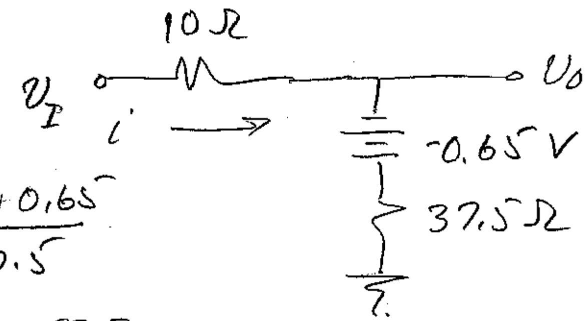


$$i = \frac{v_I + 0.5}{25}$$

$$v_O = -0.5 + \frac{15}{25}(v_I + 0.5)$$

$$v_O = -0.2 + 0.6\, v_I$$

To get D2 to region 2 we need:



$$i = \frac{v_I + 0.65}{47.5}$$

$$v_O = +0.65 + \frac{37.5}{47.5}(v_I - 0.65)$$

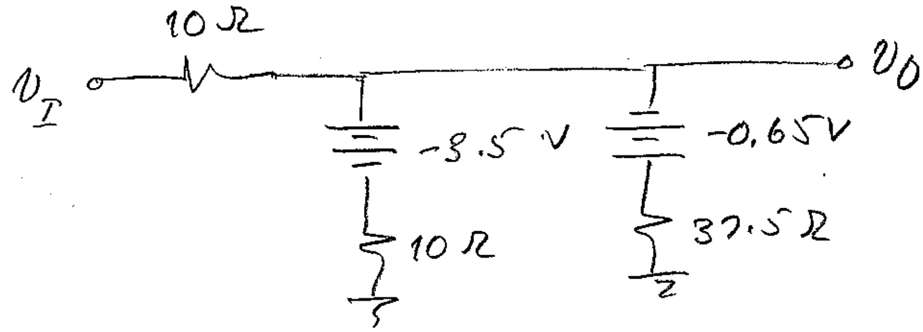
$$v_O = 0.137 + 0.789\, v_I$$

Again setting this equal to the previous result gives

$$+0.137 + 0.789\, v_I = -0.2 + 0.6\, v_I \Rightarrow v_I = -1.78 \text{ V.}$$

At this point, $v_O \cong -1.27$ V.

For D1 in reverse-breakdown we have



$$\frac{v_O + 3.5}{10} + \frac{v_O - 0.65}{37.5} + \frac{v_O - v_I}{10} = 0$$

$$v_O\left(\frac{1}{10} + \frac{1}{37.5} + \frac{1}{10}\right) = v_I\left(\frac{1}{10}\right) - \frac{3.5}{10} + \frac{0.65}{37.5}$$

$$v_O(0.2267) = 0.1\, v_I - 0.333$$

$$v_O = 0.441\, v_I - 1.47 \text{ V.}$$

To find where this happens, set

$$0.441\, v_I - 1.47 = 0.137 + 0.789\, v_I \Rightarrow v_I = -4.62 \text{ V}$$

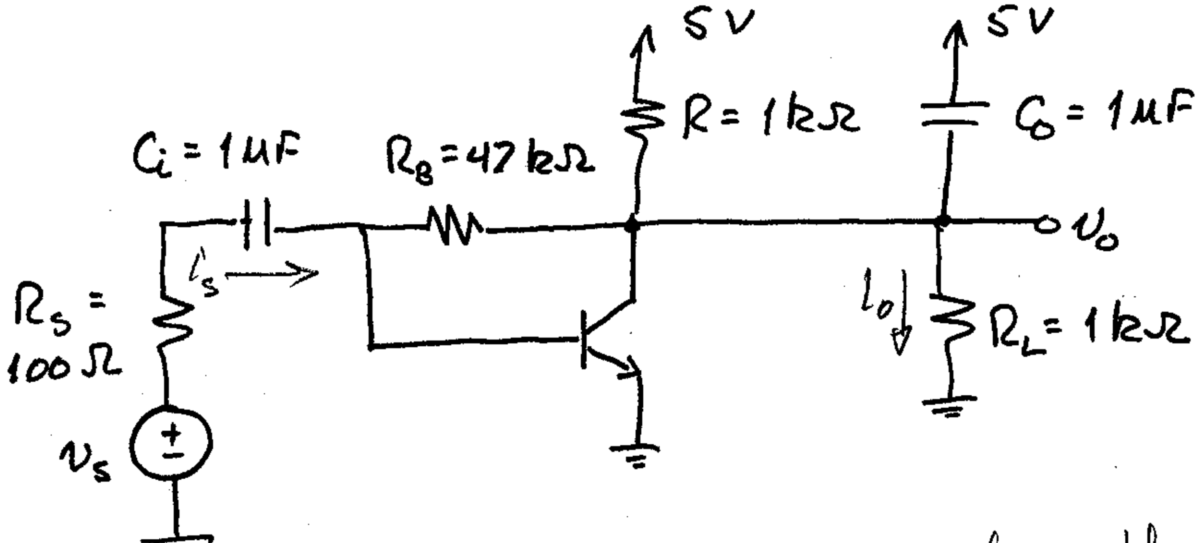
$$\Rightarrow v_O = -2.37 \text{ V.}$$

THE END

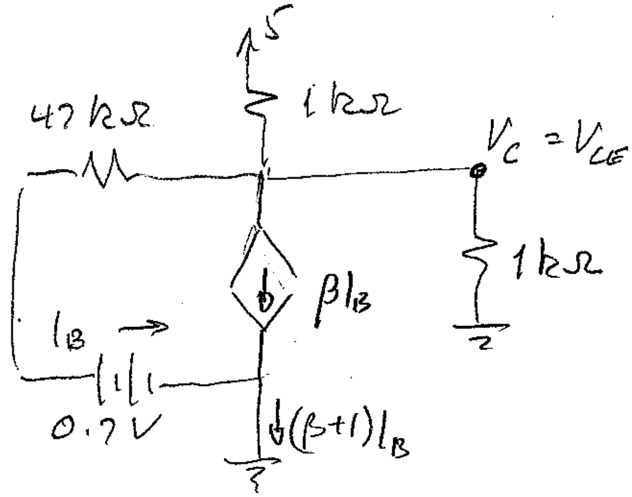
NOTE: This problem is a nice exercise but is TOO LONG !!!
(Sorry to SP 2008 class)

4. **(25 points)** The BJT in the circuit below has $\beta = 100$ and $V_{CE,SAT} = 0.2$ V. It is biased in the linear region; there is no need to prove this is the case.

i) Find the current gain i_o/i_s in the pass band.
ii) Find the voltage gain v_o/v_s in the pass band.



We need a dc analysis for the ac model parameters:



$$0 = \frac{V_c - 5}{1000} + \frac{V_c}{1000} + (\beta+1)\frac{V_c - 0.7}{47000}$$

$$V_c\left(\frac{1}{1000} + \frac{1}{1000} + \frac{101}{47000}\right) = \frac{5}{1000} + (\beta+1)\frac{0.7}{47000}$$

$$\Rightarrow V_c = 1.568 \text{ V}$$

$$I_B = \frac{V_c - 0.7}{47000} = 18.47 \mu A$$

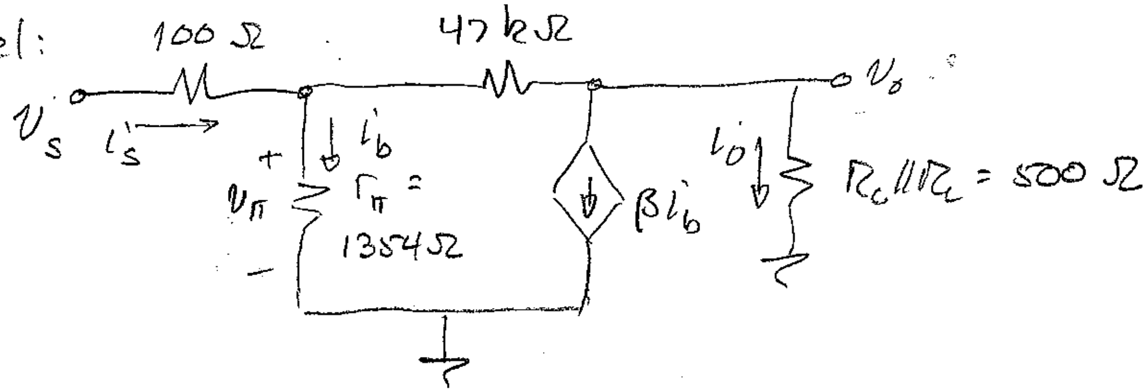
+3

$$\Rightarrow r_\pi = \frac{V_T}{I_B} = 1354 \Omega$$

$$g_m = \frac{I_C}{V_T} = 73.87 \text{ mS}$$

Room for extra work

ac model:



circuit: +12

Capacitors have been shorted since we are in the pass band. Analysis:

① $\dfrac{v_\pi - v_s}{100} + \dfrac{v_\pi - v_o}{47000} + \dfrac{v_\pi}{1354} = 0$

② $\dfrac{v_o - v_\pi}{47000} + 100\dfrac{v_\pi}{1354} + \dfrac{v_o}{500} = 0$

② ⇒ $v_\pi\left(\dfrac{100}{1354} - \cancel{\dfrac{1}{47000}}^{\approx 0}\right) = -v_o\left(\cancel{\dfrac{1}{47000}}^{\approx 0} + \dfrac{1}{500}\right)$

$$v_\pi \cong -0.0271\, v_o$$

① $v_\pi\left(\dfrac{1}{100} + \cancel{\dfrac{1}{47000}}^{\approx 0} + \dfrac{1}{1354}\right) - v_o\left(\dfrac{1}{47000}\right) = \dfrac{v_s}{100}$

$$v_\pi(0.0107) = -2.908\times10^{-4}\, v_o - \left(\frac{1}{47000}\right) v_o = \frac{v_s}{100}$$

$$v_o = -32.04\, v_s$$

+5

$$\boxed{\frac{v_o}{v_s} = -32.0}$$

+5

$$\boxed{\frac{i_o}{i_s} = -48.2}$$

over →

$$i_o' = \frac{v_o}{500} = \frac{-32v_s}{500}$$

$$i_s' = \frac{v_s - v_\pi}{100} = \frac{v_s - (-0.0271)v_o}{100}$$

$$= v_s\left(\frac{1}{100} + \frac{0.0271(-32)}{100}\right)$$

$$= 1.328 \times 10^{-3} v_s$$

$$\therefore \frac{i_o'}{i_s'} = \frac{-32/500}{1.328\times 10^{-3}} = -48.2$$

$$\boxed{\frac{i_o'}{i_s'} = -48.2}$$

$$\frac{i_b'}{i_s'} = \frac{v_\pi / r_\pi}{(v_s - v_\pi)/100} = \frac{100}{r_\pi}\frac{v_\pi}{v_s - v_\pi}$$

$$v_s - v_\pi = v_s + (0.0271)v_o$$

$$= v_s + (0.0271)(-32)v_s$$

$$= v_s(1 - 32 \times 0.0271)$$

$$= 0.1328\, v_s$$

$$v_\pi = -0.0271\, v_o$$

$$= 0.8672\, v_s$$

$$\therefore \frac{i_b'}{i_s'} = \frac{100}{r_\pi}\frac{0.8672}{0.1328}$$

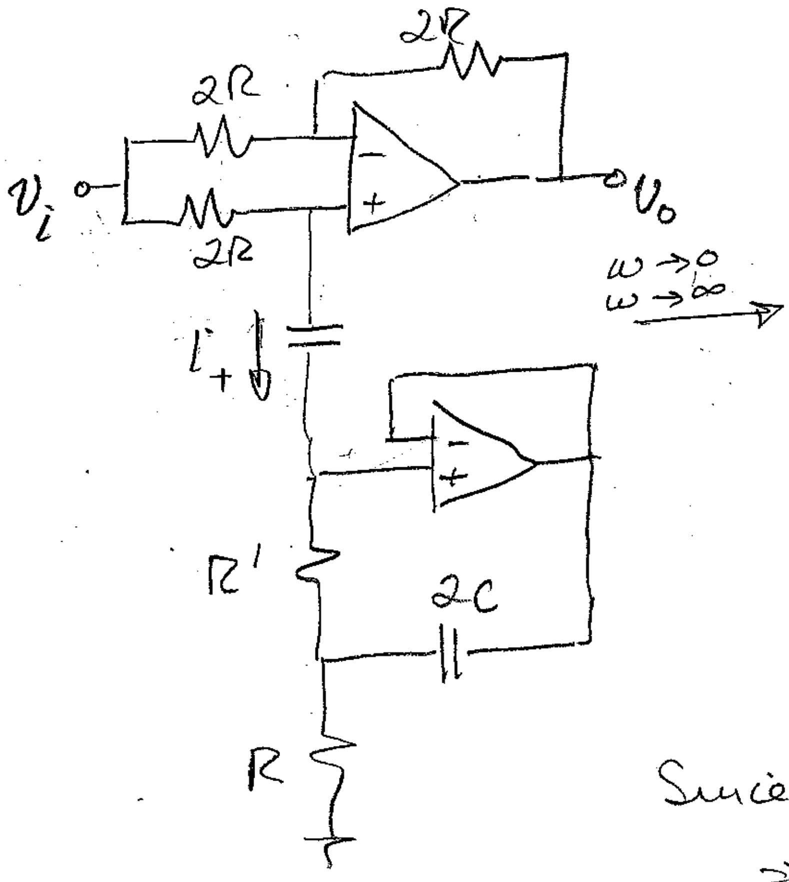
$$= 0.482$$

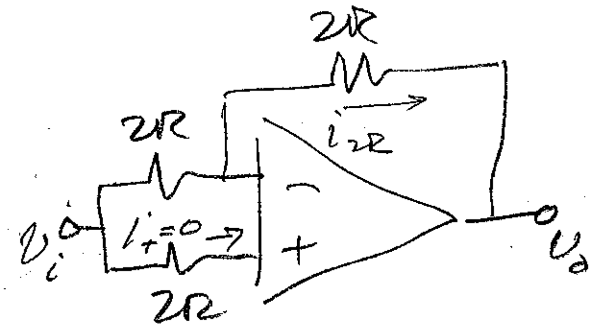
5. **(10 points)** The transfer function for the op amp in Problem 2 is greatly simplified in the case that the frequency is either very high or very low. For that op amp problem, do the following.

i) Find the transfer function for very low frequencies. Explain in a sentence or two how you arrived at your answer.
ii) Find the transfer function for very high frequencies. Explain in a sentence or two how you arrived at your answer.

For $\omega \to \infty$, capacitors short. That means the voltage across R' is 0 so i_+ is 0.

For $\omega \to 0$, capacitors open. In either case we are left with just the upper circuit:





$$\boxed{\frac{v_o}{v_i} = 1}$$

Since $i_+ = 0$, $v_+ = v_- = v_i$

$\Rightarrow i_{2R} = 0 \Rightarrow v_o = v_I$

correct circuit

+4 ea.