Name: ______________________________ (please print)

Signature: ___________________________

ECE 3455
Final Exam
May 6, 2009

Exam duration: 170 minutes

- You may have ***two*** 8 ½ x 11 in. "crib" sheets, written on both sides. You may have any calculator you choose, but no computers. No other notes or materials will be allowed.
- Show all work necessary to complete the problem on these pages. A solution without the work shown will receive no credit.
- Show units in intermediate and final results, and in figures.
- If your work is sloppy or difficult to follow, points will be subtracted.

This exam has 11 pages, including the cover sheet. Raise your hand if you are missing a page.

1 ________________/35

2 ________________/40

3 ________________/40

4 ________________/45

5 ________________/40

Total ________________/200

1. (**35 points**) A certain amplifier has a magnitude Bode plot with the following characteristics.

- A double zero at 0;
- a single pole at ω = 2,000 rad/sec;
- a single pole at ω = 8,000 rad/sec;
- a double pole at ω = 30,000 rad/sec;
- a gain of 20 dB at ω = 10,000 rad/sec.

i) Find the transfer function $T(\omega)$.
ii) On the graph provided on the next page, plot the **PHASE** Bode plot for this amplifier.

i)

$$T(\omega) = K \cdot \frac{(j\omega)^2}{(2000 + j\omega)(8000 + j\omega)(30{,}000 + j\omega)^2} \qquad +4$$

We need K:

$$K\,|T(\omega = 10\,\mathrm{krad/s})| = K \cdot 7.66 \times 10^{-10}$$

$$20\text{ dB} \Rightarrow \left|\frac{\bar{V}_o}{\bar{V}_i}\right| = 10 \Rightarrow \underline{K = 1.3 \times 10^{10}}. \qquad +2$$

ii) Plot next page.

$$\omega \to 0 \Rightarrow \angle T(\omega) \to 180^\circ$$

$$\omega \to \infty \Rightarrow \angle T(\omega) \to -180^\circ$$

Alternative:

$$T(\omega) = K \cdot \frac{(j\omega)^2}{(1 + j\omega/2000)(1 + j\omega/8000)(1 + j\omega/30000)^2}$$

$$K \cdot |T(\omega = 10\,\mathrm{krad/s})| = K \cdot 1.103 \times 10^7 = 10$$

$$\Rightarrow K = 9.07 \times 10^{-7}$$

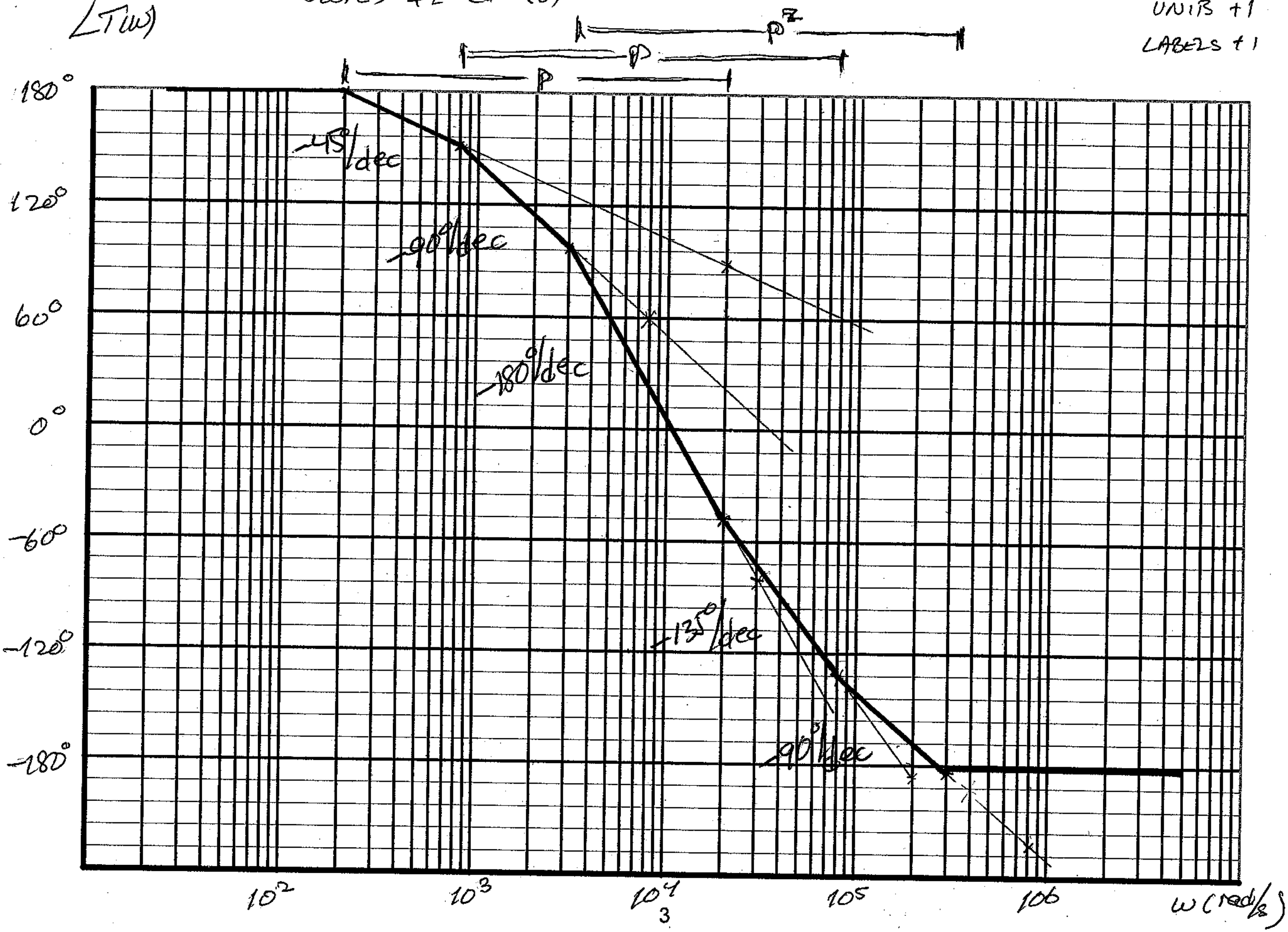
SCALE: +3

BP +2 ea (6)

SLOPES +2 ea (6)

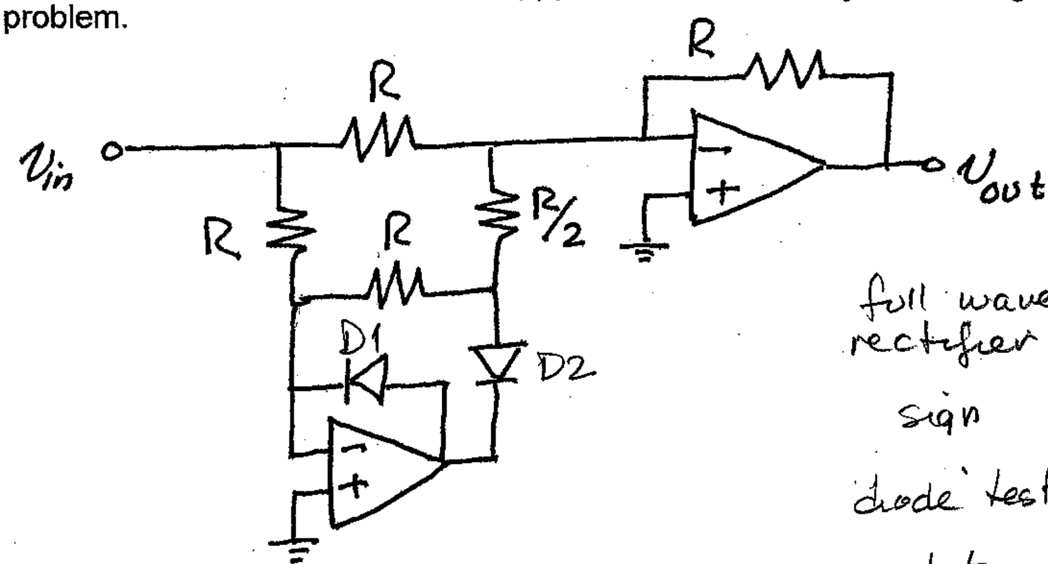
UNITS +1

LABELS +1



2. (**40 points**) The op amp in the circuit below may be considered ideal. The diodes are approximated by a constant voltage model with a threshold of 0.7 V. The resistance R is 2.2 kΩ.

Assuming the input (and period 1 ms) to this circuit is a triangle wave with 0 mean (no dc offset) and an amplitude of 2 V, make a **neat** sketch of the output v_{out} as a function of time. Show at least two cycles. If your sketch is sloppy or difficult to read, you will not get credit for the problem.



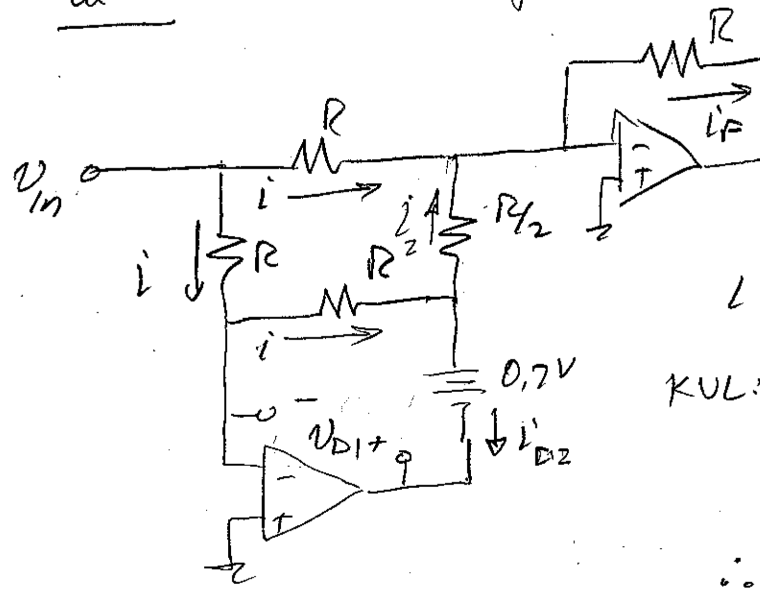
full wave rectifier +24

sign +2

diode tests +8

plot +6

For $v_{in} > 0$ we will guess D2 ON, D1 OFF, so ...



$i = v_i/R$

KVL: $iR - i_2\frac{R}{2} - iR - iR = 0$

$\Rightarrow \quad i_2 = -2i$

$\therefore \quad i_F = i + i_2 = -i$

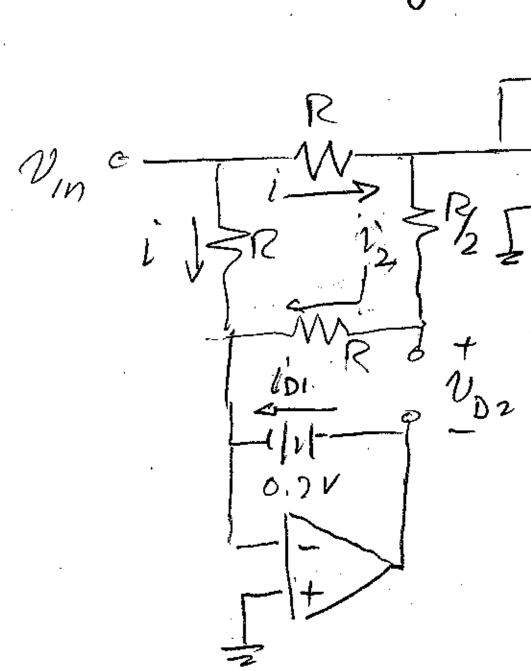
$\underline{v_{out}} = -i_F R = iR = \underline{\underline{v_{in}}}$

TEST:

- $i_{D2} = i - i_2 = 3i > 0$ ✓
- $v_{D1} = -iR - 0.7 \leq -0.7V$ for $i \geq 0$, so OK ✓

Room for extra work

$v_{in} < 0$ we will guess D_2 OFF, D1 ON ..



$i = \frac{v_{in}}{R}$ (negative)

KVL: $iR + i_2' \frac{3R}{2} - iR = 0$

$\Rightarrow i_2' = 0$

$\therefore i_F = i$

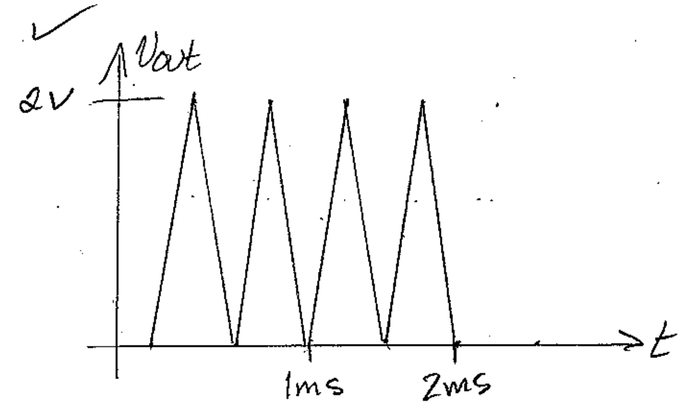
$\Rightarrow v_{out} = -iR = -v_{in}$

But v_{in} is negative, so v_{out} is the abolute value of v_{in} - it is a full-wave rectified version of v_i!

TEST: $i_{D_1} = i + i_2' = i$ (negative) ✓

$v_{D2} = i_2' R - 0.7 = -0.7$ ✓

There is no diode voltage drop (v_D never enters in to the calculation of v_{out}).



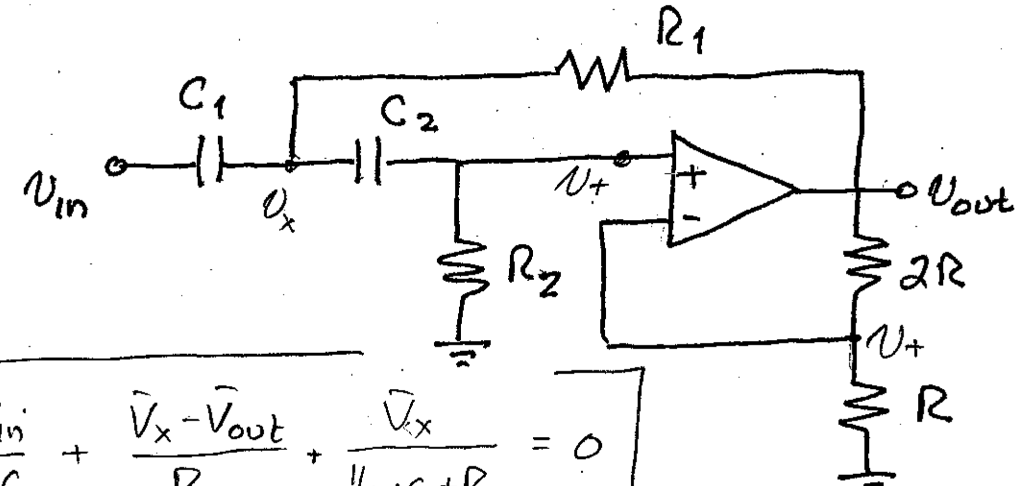
3. (**40 points**) The op amp in the circuit below is ideal. The circuit is a filter whose type (bandpass, notch, low pass…) we will try to determine.

+20 i) Write a set of equations that could be used to find the transfer function $T(\omega) = \mathbf{V}_{out}/\mathbf{V}_{in}$ for the circuit. In this step, do not try to solve or simplify the equations. Draw a box around the equations that are part of your solution; equations not in a box will not be counted.

+10 ii) Find an expression for $T(\omega)$; any valid algebraic expression involving only $\mathbf{V}_{out}$ and $\mathbf{V}_{in}$ (as well as the capacitances and resistances) will do. There is no need to simplify; do not spend time multiplying terms or reducing complex fractions to get your expression into a standard form.

+4 +4 iii) We can guess what is going on with this filter by letting $\omega \rightarrow 0$ and $\omega \rightarrow \infty$. Let ω approach these limits, and for <u>each</u> case (0 and ∞), find $T(\omega)$. At these values of ω, your expression for $T(\omega)$ will simplify dramatically.

+2 Iv) Based on iii), what type of filter do you think this is?



i)

$$\boxed{\frac{\bar V_x - \bar V_{in}}{1/j\omega C_1} + \frac{\bar V_x - \bar V_{out}}{R_1} + \frac{\bar V_x}{1/j\omega C_2 + R_2} = 0}$$

$$\boxed{\bar V_+ = \bar V_x \frac{R_2}{R_2 + 1/j\omega C_2}} \qquad \boxed{\bar V_+ = \bar V_{out} \cdot \frac{R}{R + 2R} = \frac{\bar V_{out}}{3}}$$

This all we need. The second two equations give us $\bar V_+$ in terms of $\bar V_{out}$ and hence $\bar V_x$ in terms of $\bar V_{out}$. This allows us to write $\bar V_o$ as a function of $\bar V_{in}$ using the first equation...

Room for extra work.

ii) $$\bar{V}_x = \bar{V}_+ \cdot \frac{R_2 + 1/j\omega C_2}{R_2} = \bar{V}_+ \cdot \frac{1+j\omega C_2 R_2}{j\omega C_2 R_2} = \frac{\bar{V}_{out}}{3} \frac{1+j\omega C_2 R_2}{j\omega C_2 R_2}$$

From the first equation:

$$\bar{V}_x \left(j\omega C_1 + \frac{1}{R_1} + \frac{j\omega C_2}{1+j\omega C_2 R_2} \right) - \frac{\bar{V}_{out}}{R_1} = j\omega C_1 \bar{V}_{in}$$

$$\bar{V}_{out} \left[\frac{1}{3} \frac{1+j\omega C_2 R_2}{j\omega C_2 R_2} \left(\frac{1+j\omega C_1 R_1}{R_1} + \frac{j\omega C_2}{1+j\omega C_2 R_2} \right) - \frac{1}{R_1} \right] = j\omega C_1 \bar{V}_{in}$$

$$* \quad \frac{\bar{V}_{out}}{\bar{V}_{in}} = \left[\frac{1}{3} \frac{1+j\omega C_2 R_2}{j\omega C_2 R_2} \left(\frac{1+j\omega C_1 R_1}{R_1} + \frac{j\omega C_2}{1+j\omega C_2 R_2} \right) - \frac{1}{R_1} \right]^{-1} \cdot j\omega C_1$$

This satisfies the requirements of ii).

iii) For $\omega \to \infty$, we have

$$\frac{\bar{V}_{out}}{\bar{V}_{in}} \approx \left[\frac{1}{3} \cdot 1 \cdot \left(j\omega C_1 + 1 \right) - \frac{1}{R_1} \right]^{-1} \cdot j\omega C_1$$

$$\approx \left[\frac{1}{3} \cdot j\omega C_1 \right]^{-1} \cdot j\omega C_1 = 3.$$

($\frac{1}{R_1}$ and 1 are small compared to $j\omega C_1$ at $\omega \to \infty$)

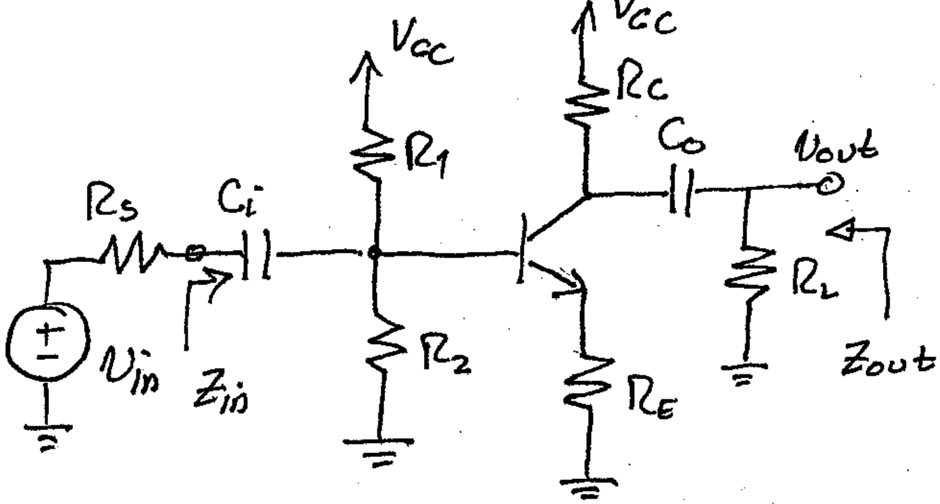
For $\omega \to 0$, we have

$$\frac{\bar{V}_{out}}{\bar{V}_{in}} \approx \left[\frac{1}{3} \cdot \frac{1}{j\omega C_2 R_2} \cdot \left(\frac{1}{R_1} + 0 \right) - \frac{1}{R_1} \right]^{-1} \cdot j\omega C_1 \longrightarrow 0$$

iv) HIGH PASS FILTER!

4. (**45 points**) The BJT in the circuit below has $\beta = 100$ and $V_{CE,SAT} = 0.2$ V. It is biased in the linear region; there is no need to prove this is the case.

i) Find the input impedance Z_{in}. Express the result symbolically; do not substitute component values.
ii) Find the output impedance Z_{out}. Express the result symbolically; do not substitute component values.
ii) Find the voltage gain v_{out}/v_{in} in the pass band. For the gain, provide a numerical value.

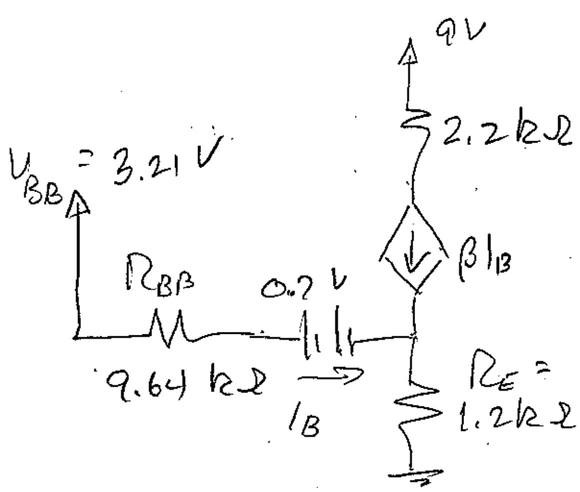


$V_{cc} = 9$ V
$R_1 = 27$ kΩ
$R_2 = 15$ kΩ
$R_C = 2.2$ kΩ
$R_E = 1.2$ kΩ
$R_S = 10$ kΩ
$R_L = 2$ kΩ
$C_i = C_o = 1\,\mu F$

We will look at the dc case because we need r_π or r_e:

$V_{BB} = V_{cc} \cdot \frac{R_2}{R_1 + R_2} = 3.21$ V , $R_{BB} = R_1 \| R_2 = 9.64$ kΩ

+4



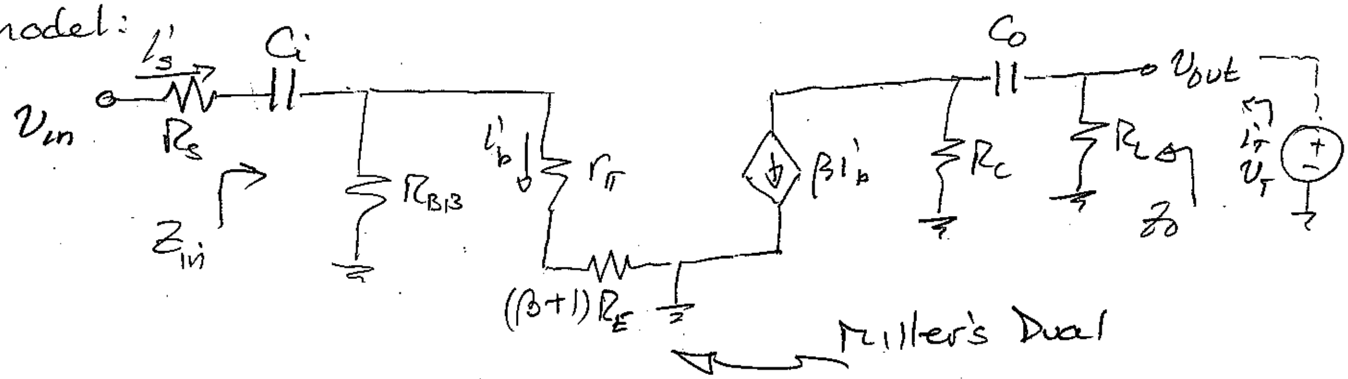
$I_B = \frac{V_{BB} - 0.7}{R_{BB} + (\beta+1)R_E}$

$= 19.2\,\mu A$

$\Rightarrow r_\pi = \frac{V_T}{I_B} = 1.3$ kΩ

Room for extra work.

ac model:



+15

Z_o: with source attached and de-activated (after applying a test source at the output), we have

+6 $$Z_o = \frac{V_T}{I_T} = R_L \,//\, (1/j\omega C_o + R_C) = \frac{j\omega C_o R_L R_C + R_L}{1 + j\omega C_o (R_L + R_C)}$$

Z_i: $$Z_i = 1/j\omega C_i + R_{BB} \,//\, (r_\pi + (\beta+1)R_E)$$

+6 $(r_\pi = 1.3\ k\Omega;\ (\beta+1)R_E = 121\ k\Omega \Rightarrow r_e + (\beta+1)R_E \simeq (\beta+1)R_E)$

In the passband, both C_i and $C_o \to$ short, so...

$$v_{out} = -\beta i_b' \cdot \overset{1.05k\Omega}{(R_L // R_C)} \qquad i_s' = \frac{v_{in}}{R_s + R_{BB} // (r_\pi + (\beta+1)R_E)}$$

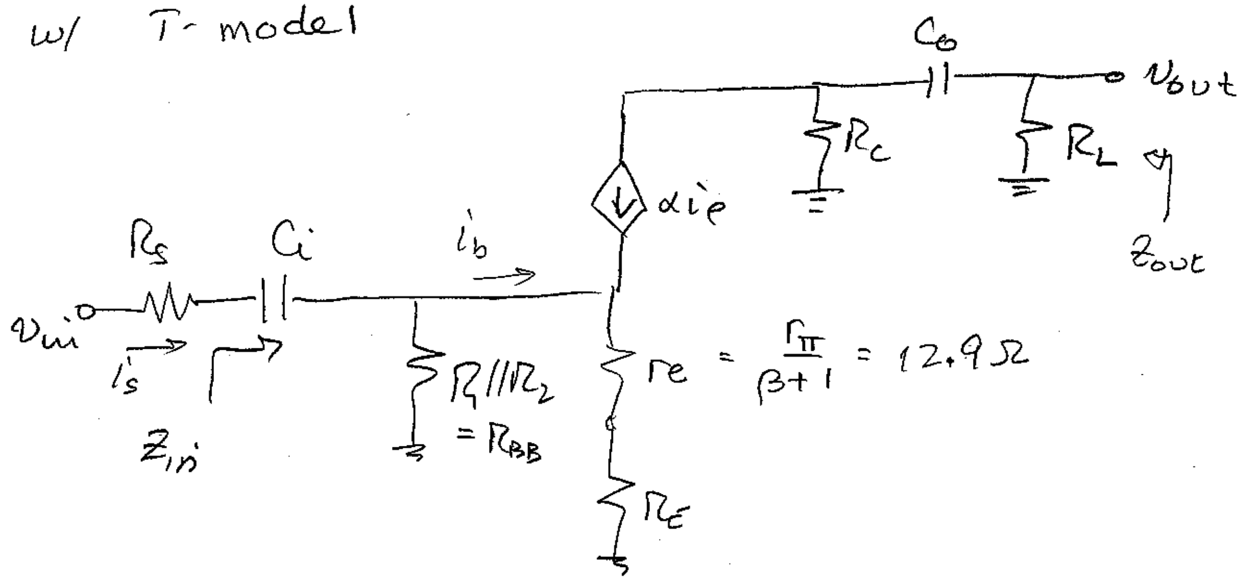
+14 $$i_b' = i_s' \cdot \frac{R_{BB}}{R_{BB} + (r_\pi + (\beta+1)R_E)}$$ Taking $r_\pi \ll (\beta+1)R_E$ we get

$$\frac{v_{out}}{v_{in}} = -\frac{\beta R_L // R_C}{R_{BB} + (\beta+1)R_E} \cdot \frac{R_{BB}}{R_s + R_{BB} // (\beta+1)R_E} = -\frac{1.058\times10^5}{1.308\times10^5} \cdot \frac{9.614\times10^3}{1.89\times10^4}$$

$R_{BB} // (\beta+1)R_E = 8.93k\Omega$

9 $\boxed{\frac{v_{out}}{v_{in}} = -0.414}$

#4 w/ T-model



$$z_{in} = 1/j\omega C_i + (R_{BB} // (\beta+1)(r_e + R_E))$$

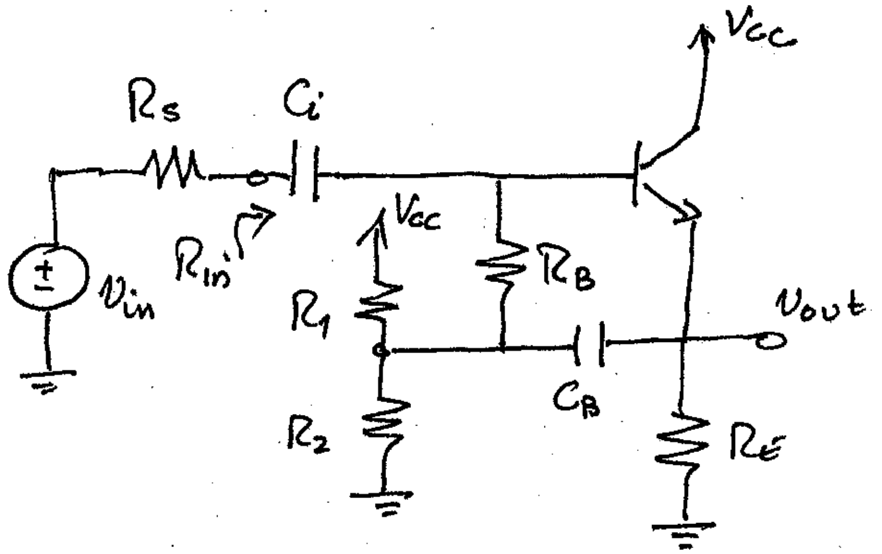
$$z_{out} = R_L // (R_C + 1/j\omega C_o)$$

$$v_{out} = -\alpha i_e \cdot R_C // R_L = -(\beta+1) i_b' (R_C // R_L)$$

$$i_s' = \frac{v_{in}}{R_s + R_{BB} // (\beta+1)(r_e + R_E)} \qquad i_b' = i_s' \cdot \frac{R_{BB}}{R_{BB} + (\beta+1)(r_e + R_E)}$$

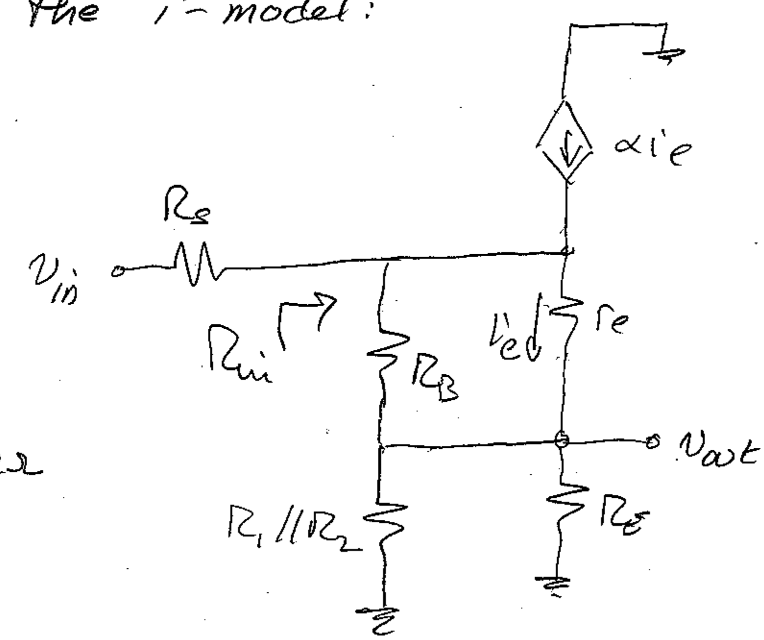
5. (**40 points**) The BJT in the circuit below has $\beta = 100$ and $V_{CE,SAT} = 0.2$ V. It is biased in the linear region; there is no need to prove this is the case. The configuration shown is called a bootstrap follower: the addition of the capacitor allows for a trade-off between input resistance and voltage gain, which we will explore.

i) With the capacitor C_B in place, find the input resistance R_{in} and the voltage gain v_{out}/v_{in}.
ii) With the capacitor removed, again find R_{in} and v_{out}/v_{in}.
iii) Explain what is gained or lost by adding the capacitor C_B.



$V_{cc} = 9$ V
$R_1 = R_2 = 20$ kΩ
$R_B = 10$ kΩ
$C_i = \infty$
$C_B = \infty$
$R_s = 10$ kΩ
$R_E = 2$ kΩ

Since the capacitors are large (∞!) they are shorts:
we will use the T-model:



$R_1 // R_2 = 10$ kΩ

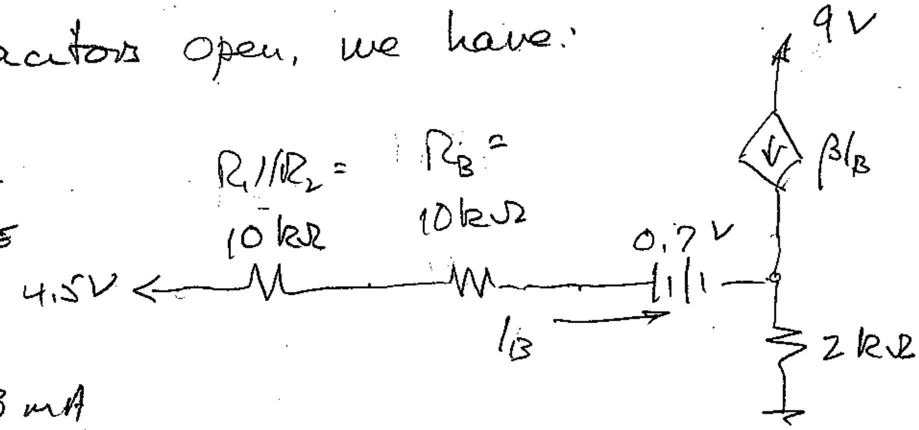
We need r_e:

Room for extra work. $V_{BB} = 9 \cdot \frac{R_2}{R_1 + R_2} = 4.5\ V$

$R_{BB} = R_1 // R_2 = 10\ k\Omega$

with both capacitors open, we have:

$$I_B = \frac{9 - 0.7}{10k + 10k + (\beta+1) R_E} = 17.1\ \mu A$$



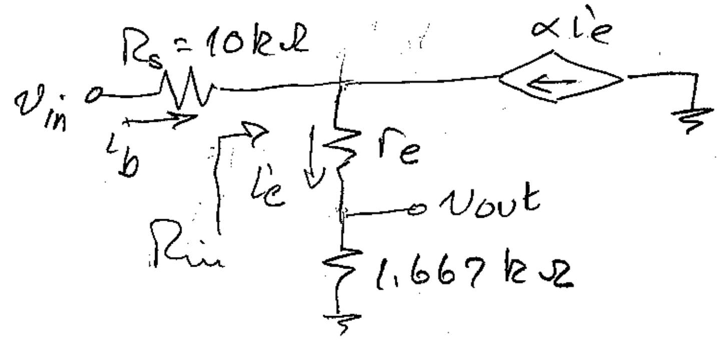
$I_E = (\beta+1) I_E = 1.73\ mA$

+2 $\Rightarrow r_e = \frac{V_T}{I_E} = 14.5\ \Omega$

Now we can simplify the ac circuit:

$R_B // r_e \cong r_e \quad (R_1 // R_2) // R_E = 1.667\ k\Omega$

$\rightarrow$

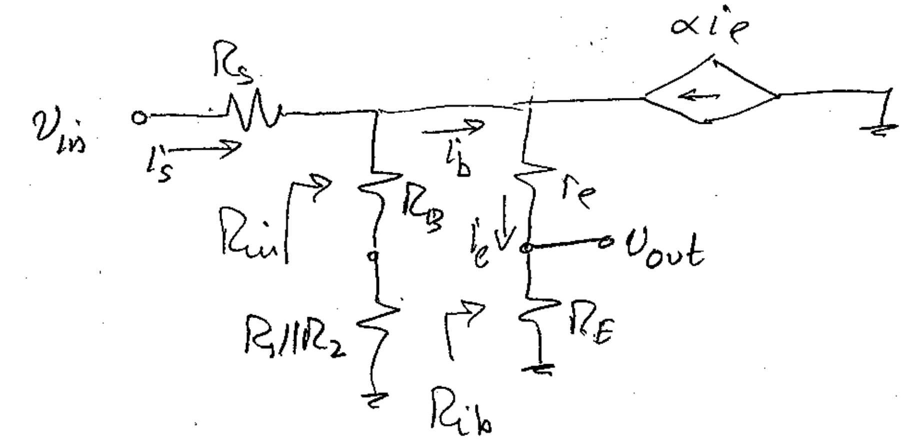


+8 $R_{in} = \frac{i_e (r_e + 1.667\ k\Omega)}{i_b} = (\beta+1)(r_e + 1.667\ k\Omega) \cong 169.8\ k\Omega$

$v_{out} = i_e (1.667\ k\Omega) \qquad v_{in} = i_b (R_s + (\beta+1)(r_e + 1.667\ k\Omega))$

+8 $\therefore \frac{v_{out}}{v_{in}} = \frac{(\beta+1) i_b \cdot (1.667\ k\Omega)}{i_b \cdot (R_s + (\beta+1) \cdot (r_e 11 + 1.667\ k\Omega)} = 0.936$

If we remove the capacitor:



$$R_{ib} = (\beta+1)(r_e + R_E) = 203.5\ k\Omega$$

$$R_{in} = (R_B + R_1 // R_2) // R_{ib} = 20\ k\Omega // 203.5\ k\Omega = 18.2\ k\Omega$$

$$i_s = \frac{v_{in}}{R_s + R_{in}} = \frac{v_{in}}{28.2\ k\Omega}$$

$$i_b = \frac{v_{in}}{28.2\ k\Omega} \cdot \frac{10k + 10k}{10k + 10k + R_{ib}} = v_{in} \times 3.173 \times 10^{-6}$$

$$v_{out} = (\beta+1) i_b \cdot (R_E + r_e) = 2.035 \times 10^5\ i_b = 0.641\ v_{in}$$

$$\Rightarrow \frac{v_{out}}{v_{in}} = 0.641$$

iii) The input resistance has increased from 18 kΩ to 170 kΩ when the capacitor is in place, but the voltage gain has dropped from 0.94 to 0.64.