Name: ______________________________ (please print)

Signature: ___________________________

ECE 3455
Exam 2
April 25, 2009

Exam duration: 90 minutes

- You may have one 8 ½ x 11 in. "crib" sheet, written on both sides, during the quiz. You may have any calculator you choose, but no computers. No other notes or materials will be allowed.
- Show all work necessary to complete the problem on these pages. A solution without the work shown will receive no credit.
- Show units in intermediate and final results, and in figures.
- If your work is sloppy or difficult to follow, points will be subtracted.

This exam has 9 pages, including the cover sheet. Raise your hand if you are missing a page.

1 ________________/40

2 ________________/30

3 ________________/30

Total ________________/100

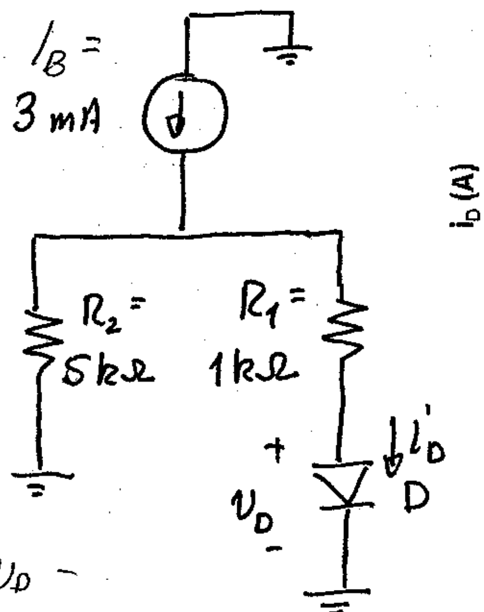
1. (**40 points**) The circuit below uses a resonant tunneling diode D, which has the unusual linear piece-wise i_D-v_D characteristics shown in the figure. Biasing regions I – IV are labeled on the plot.

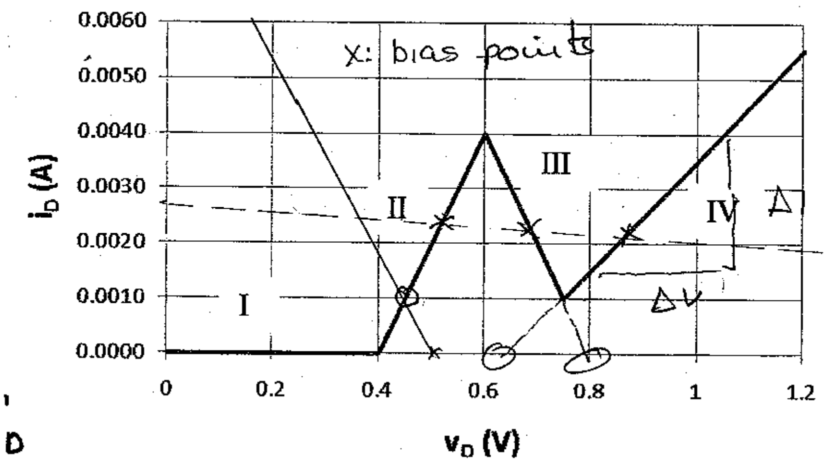
If we were to use the "guess and test" method to determine which biasing region the diode is in, we may find, depending on the current source and resistor values, that the diode has a stable bias point in more than one region.

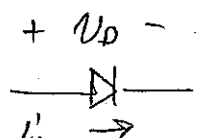
i) Using the given i_D-v_D characteristics, find circuit models valid for each region (I – IV) of diode operation.

ii) For the given circuit, find all possible bias points; in other words, calculate the diode current i_D and diode voltage v_D for each point at which the diode could be biased.

iii) Find current source and resistor values that will bias the diode in region II at the point i_D = 1 mA, v_D = 0.45 V. *Hint: Think "load line", and about what that means in terms of the biasing.*







i) Region I : no current ⇒

+ v_D −

$0 \le v_D \le 0.4$ V

Region II : 0.4 V, r_D = 50 Ω

$$r_D = \frac{\Delta V}{\Delta I} = \frac{0.2}{0.004} = 50\,\Omega$$

Region III : $r_D = \frac{\Delta V}{\Delta I} = \frac{0.15}{-0.003} = -50\,\Omega$

0.8 V, r_D = −50 Ω

Region IV : 0.65 V, r_D = 100 Ω

$$r_D = \frac{1.05-0.8}{0.0025} = 100\,\Omega$$

(see note on page 2)

Room for Extra Work

In finding the voltage source for the models in Regions III and IV, we do not need to rely on extrapolation back to the voltage axis. For example, in III,

$$v_D = V_{D0} + i_D' r_D \qquad \text{with } r_D = -50\,\Omega.$$

Then we can see that $v_D = 0.6V \Rightarrow i_D' = 0.004\,A$, so

$$0.6 = V_{D0} - 0.004 \cdot 50 \quad \Rightarrow V_{D0} = 0.8\ V.$$

Similarly in Region IV:

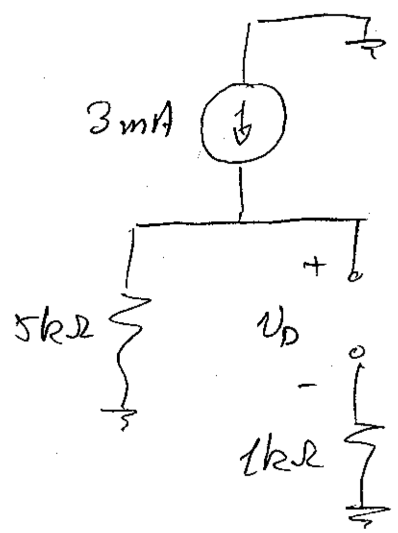
$$v_D = 1.2 = V_{D0} + 100\, i_D' \quad \text{and } i_D' = 0.0055\ A$$

$$\Rightarrow V_{D0} = 1.2 - 0.55 = 0.65\ V.$$

Room for extra work

ii) We need to substitute each of our models into the biasing circuit to see which i_D, v_D points are valid (i.e., are on the i_D'-v_D plot).

I

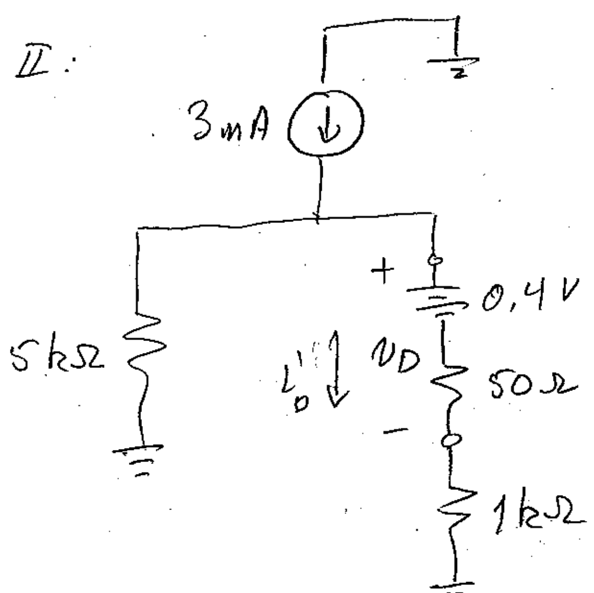


In region I, $i_D' = 0$ so the circuit predicts

$$v_D = 0.003(5000) = 15\text{ V}$$

which is clearly not on the graph: at 15V, i_D is large and positive, not 0.

II:



We could solve this circuit as is, but let's replace it with a Thevenin equivalent at the terminals of the diode.
We already have open circuit voltage from the calculation above

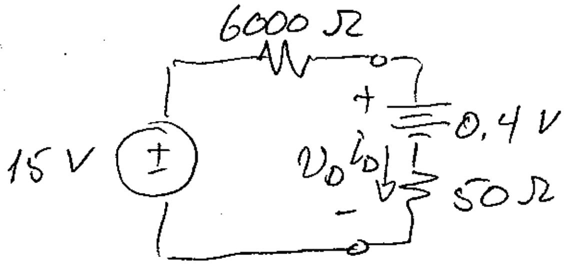
$$v_{oc} = V_{Th} = 15\text{ V}.$$

short circuit current:

$$i_{D,sc} = 3\text{mA} \cdot \frac{5k}{5k + 1k} = 2.5\text{ mA}$$

$$\therefore R_{Th} = \frac{V_{Th}}{i_{D,sc}} = 6000\ \Omega$$

(over)

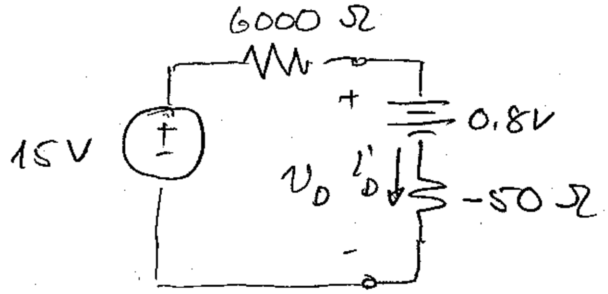


$$i_D' = \frac{15-0.4}{6050} = 2.413\ mA$$

$$v_D = 0.4 + 50 i_D' = 0.521\ V$$

This point is indeed on the graph in R I, so it is a valid bias point.

Region III:

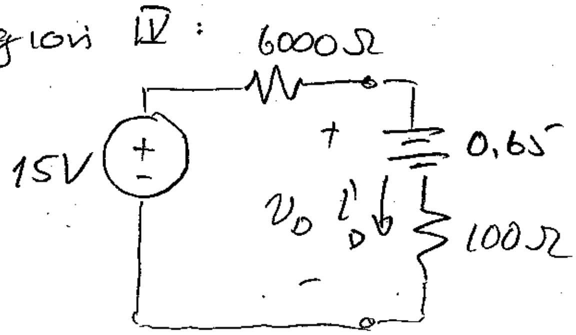


$$i_D' = \frac{15-0.8}{5950} = 2.387\ mA$$

$$v_D = 0.8 - 50 i_D' = 0.681\ V.$$

This point is again on the graph and is a valid bias point.

Region IV:



$$i_D' = \frac{15-0.65}{6100} = 2.352\ mA$$

$$v_D = 0.65 + 100 i_D' = 0.885\ V$$

Again we have a valid bias point in R IV.

We could have anticipated all of these points if we had plotted a load line, which is defined by the Thevenin Equivalent: $i_D' = \frac{15 - v_D}{6000} = 2.5mA - \frac{v_D}{6000}$

This line is shown as - - - - on the plot.

iii). The propose bias point is marked on the $i_D'-V_D$ plot as O. There are many load lines that go through that point; one is shown as a solid line. That line is

$$i_D' = I_D - \frac{V_{TH}}{R_{Th}}$$

where V_{Th} & R_{Th} are the Thevenin equivalent of the biasing circuit. We have:

$$V_{Th} = 0.5\ V \quad \text{and} \quad R_{Th} = \frac{\Delta V}{\Delta I} \approx \frac{0.05}{0.001} = 50\ \Omega$$

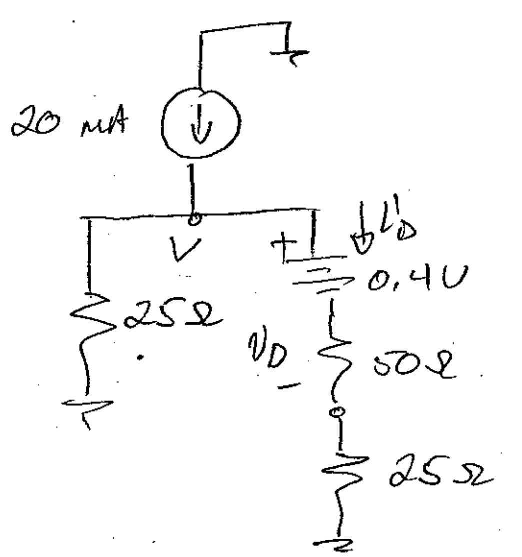
by inspection from the graph. Now we have to relate I_B, R_1, and R_2 to our Th. Eq:

$$V_{Th} = V_{OC} = I_B R_1 \qquad R_{Th} = R_1 + R_2$$

So $I_B R_1 = 0.5$ and $R_1 + R_2 = 50\ \Omega$.

if $\underline{R_1 = R_2 = 25\ \Omega}$, then $\underline{I_B = \frac{0.5}{25} = 20\ mA}$.

Let's see if this works:



$$\frac{V}{25} + \frac{V-0.4}{75} = 0.02$$

$$\therefore V = 0.475V$$

$$\Rightarrow i_D' = \frac{V-0.4}{75} = 1mA$$

$$V_D = 0.4 + 50 i_D' = 0.45V$$

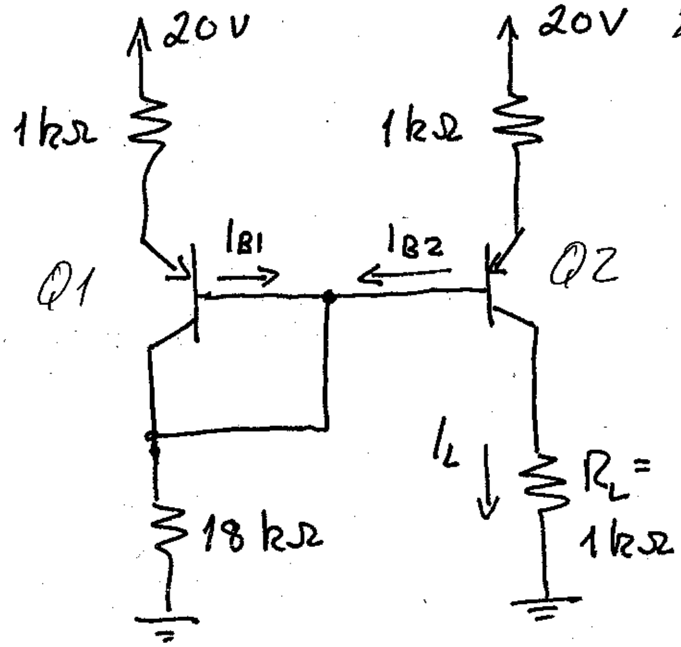
So it check!

2. (**30 points**) The circuit below is a variation of what is known as a current mirror. It is intended to provide a fixed current to the load R_L. The BJTs are assumed to be identical. They have $\beta = 100$ and $V_{CE,SAT} = 0.3$ V.

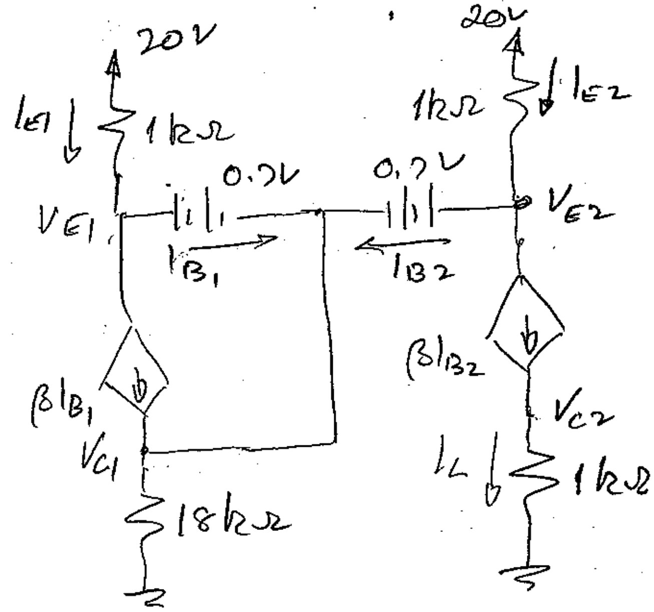
i) Find the <u>load current</u> I_L, and <u>base currents</u> I_{B1} and I_{B2}.

ii) If Q2 is required to stay in the linear region, what range of values can R_L take?

iii) Suppose that transistors with $\beta = 40$ are accidentally chosen when constructing the circuit. What then will the load current I_L be?



i) Assuming linear region for both Q1 and Q2:



<u>KVL</u>: $-20 + I_{E1} \cdot 1000 + 0.7 - 0.7 - I_{E2} \cdot 1000 + 20 = 0$

$$1000\, I_{E1} - 1000\, I_{E2} = 0 \quad \Rightarrow \quad I_{E1} = I_{E2}$$

But $I_{E1} = \frac{\beta+1}{\beta} I_{C1} = (\beta+1) I_{B1} \quad \Rightarrow \quad I_{B1} = I_{B2}, \quad I_{C1} = I_{C2}$.

That simplifies things...

<u>KVL</u>: $-20 + 1000\, I_E + 0.7 + 18000(\beta I_B + I_B + I_B) = 0$

$$I_E = (\beta+1) I_B$$

Room for extra work

$$-20 + (\beta+1) I_B \cdot 1000 + 0.7 + (\beta+2) I_B \cdot 18000 = 0$$

$$\therefore \quad I_B = \frac{20 - 0.7}{(\beta+1)\cdot 1000 + (\beta+2)\cdot 18000}$$

$\beta = 100 \quad \Rightarrow \quad I_B = 9.964\ \mu A$

$\beta = 40 \quad \Rightarrow \quad I_B' = 24.22\ \mu A$

(That last step was in anticipation of part iii)

$$\left\{ \begin{array}{l} \text{So } I_{B1} = I_{B2} = 9.964\ \mu A \text{ and} \\ I_L = \beta I_B = 0.996\ mA . \end{array} \right.$$

check linear region : $Q1: V_{C1} - V_{E1} = -0.7\ V$ ✓

$Q2: V_{C2} - V_{E2} = (\beta I_B \cdot 1000) - (20 - (\beta+1) I_B \cdot 1000)$

$\cong (0.1) - (19) = -18.9\ V$ ✓

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ii) with $V_{E2} \simeq 19\ V$, we can allow V_{C2} to go to $\sim 18.7\ V$ to keep $V_{C2} - V_{E2} \leq -0.3\ V$.

So $\beta I_B \cdot R_{L\,max} = 18.7\ V \Rightarrow$

$$\underline{R_{L\,max} = 18.7\ k\Omega}$$

(iii). From above, we now have

$$I_B' = 24.22 \text{ } \mu A.$$

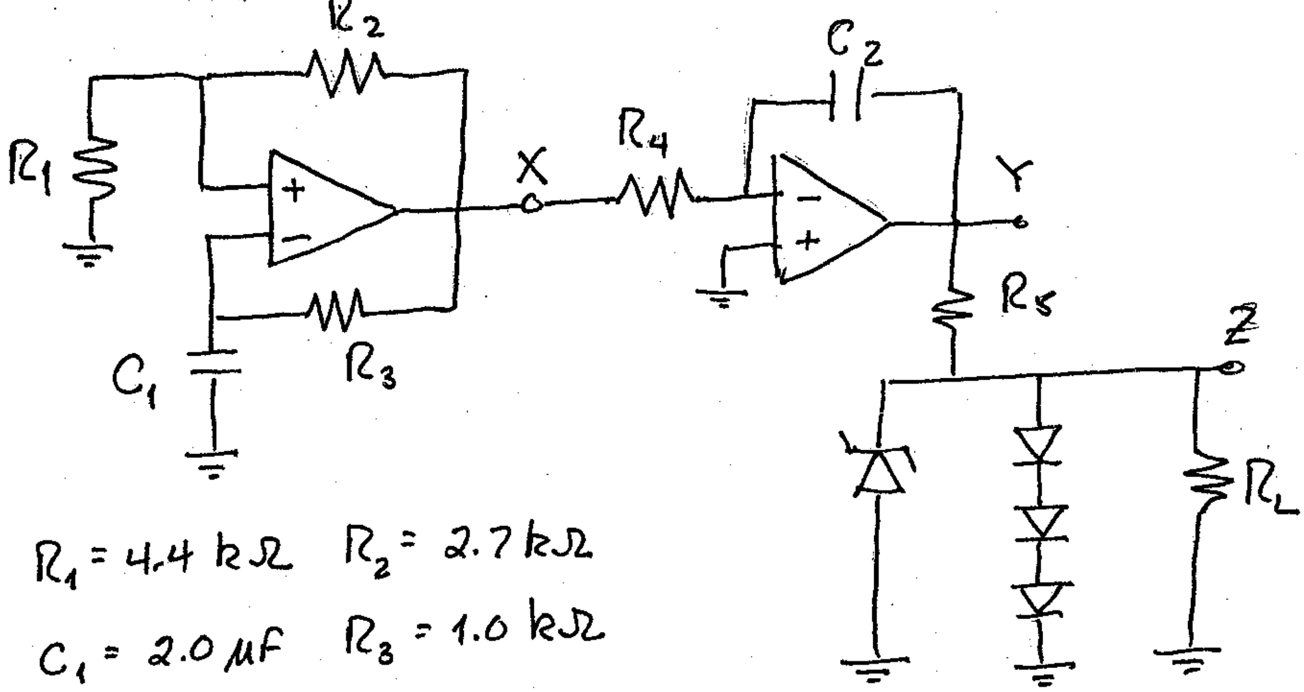
$$\therefore \underline{I_L' = \beta I_B' = 0.969 \text{ mA}}$$

whereas previously it was 0.996 mA! So with a change in β of a factor of 2½, the current in the load is unchanged to within $\sim$ 2.7%.

3. (**30 points**) The circuit below is a simple function generator. It has outputs at X, Y, and Z. Use the graphs on the next page to make a plot of the waveforms (voltage vs. time) at these three points. Plot two cycles of the waveforms; plot the total instantaneous values (ac + dc). Assume the following.

- The op amps are ideal and are powered by + 12 V and – 12 V supplies.
- The diodes (including the Zener) can be modeled using a constant voltage drop in forward bias of 0.6 V and $I_S = 0$. The Zener diode goes into reverse bias conduction at - 3V.
- The capacitors have 0 volts across them when the circuit is turned on.

To earn full credit, your graphs must be neat and easily readable. They must have clearly labeled tic marks, and they must include information concerning the period and amplitude of all waveforms. I will deduct significant credit if the graphs are sloppy or ambiguous.



$R_1 = 4.4\ k\Omega$ $R_2 = 2.7\ k\Omega$

$C_1 = 2.0\ \mu F$ $R_3 = 1.0\ k\Omega$

$C_2 = 98.\ \mu F$ $R_4 = 100\ \Omega$

$R_S = 500\ \Omega$

$R_L = 2.2\ k\Omega$

Room for extra work

X is the output of an astable multivibrator. Since the power supplies are symmetric at +/- 12 V, we have

$$T = 2\tau \ln \frac{1+\beta}{1-\beta} \qquad \tau = R_3 C_1 = 2\,ms$$

$$\beta = \frac{R_1}{R_1 + R_2} = \frac{4.4}{4.4+2.7} = 0.62 \Rightarrow T \simeq 5.8\ ms.$$

and amplitude 12 V.

Y is the result of integrating X. If C_2 starts at 0 V and if $t=0$ corresponds to the positive peak of X, then

$$v_Y(t) = -\frac{1}{R_4 C_2}(12)t \qquad 0 \le t \le T/2$$

so that at $T/2$, $v_y = -3.55$ V, which is the peak amplitude of the waveform. If the output at X started at the negative peak, the sign would be flipped. In general, since we don't know what $v_X(t=0)$ is, we cannot say what $v_Y(t=0)$ is, and so we cannot say what the dc offset is. I have made my plot assuming $v_X(t=0) = 12$ V.

Z: Given my output at Y, and

$$v_Z = \frac{R_L}{R_L + R_5} v_Y = 0.815\, v_Y ,$$

so max/min values of v_Z are $0/-2.9$ V, BUT this will forward-bias the Zener, and the other diodes will stay off. So v_Z looks like v_Y but is reduced in slope, and clips at 0.7 V.

If I had chosen v_Y to go positive, then the diodes would turn on at $v_Z = 2.1$ V, but the Zener would stay off.

Plot your waveforms (ac and dc components) here.