

(30 Points) (1) In the circuits shown,  
 $R_s = 5 \text{ k}\Omega$ ,  $R_i = 1 \text{ k}\Omega$ , and  $G_m = 10 \text{ mA/V}$ ,  
 $R_o = 10 \text{ k}\Omega$  and  $R_L = 10 \text{ k}\Omega$ .

- a) In Fig.1, Find the gain  $A = V_o/V_s$ .
- b) In Fig.2, Find the gain  $A = V_o/V_s$ .

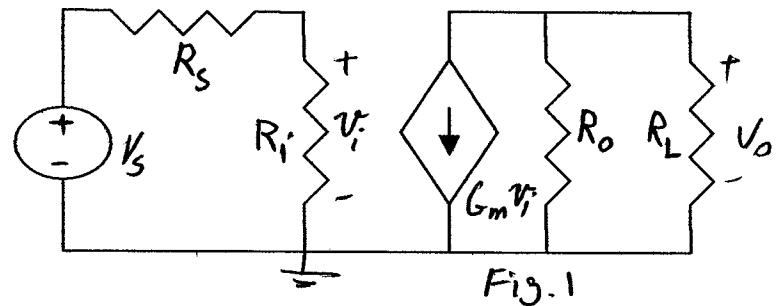


Fig. 1

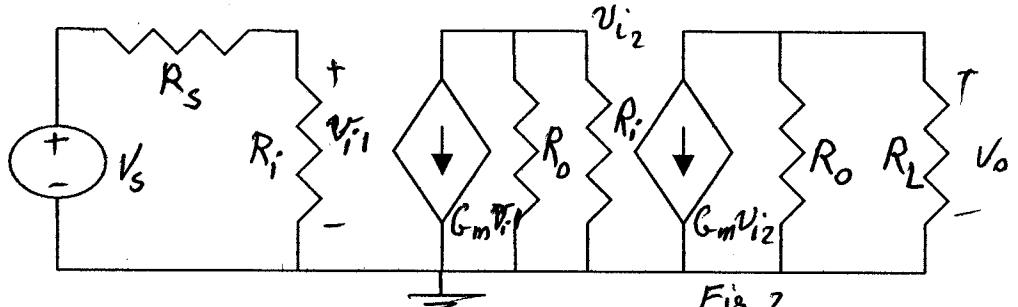


Fig. 2

Solution:

$$a) \frac{V_i}{V_s} = \frac{R_i}{R_i + R_s} = \frac{1}{1+5} = \frac{1}{6}$$

$$V_o = -G_m V_i \times (R_o || R_L) = -10 (10^3 || 10^3) V_i = -10 \times 5 V_i = -50 V_i$$

$$\frac{V_o}{V_i} = -50$$

$$\text{Gain} = A = \frac{V_o}{V_i} = \frac{V_o}{V_s} \cdot \frac{V_i}{V_s} = \frac{1}{6} (-50) = \boxed{-8.33}$$

$$b) \frac{V_{i1}}{V_s} = \frac{R_i}{R_i + R_s} = \frac{1}{6}$$

$$V_{i2} = -G_m V_{i1} (R_o || R_i) = -10 (10 || 1) V_{i1} = -9.1 V_{i1}$$

$$\frac{V_{i2}}{V_{i1}} = -9.1$$

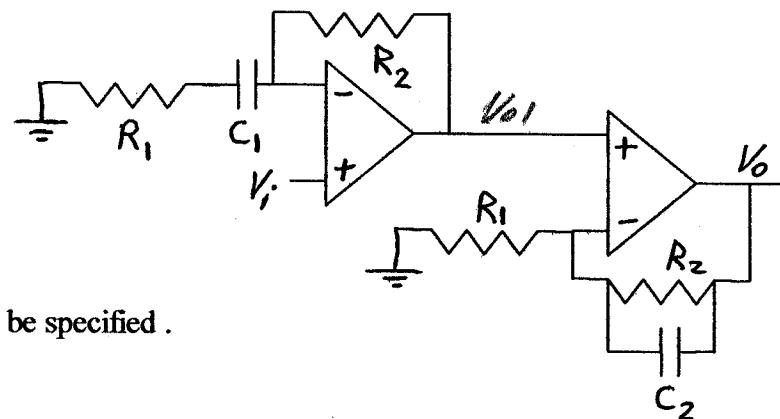
$$V_o = -G_m V_{i2} (R_o || R_L) = -50 V_{i2}$$

$$\frac{V_o}{V_{i2}} = -50$$

$$\frac{V_o}{V_s} = \frac{V_{i1}}{V_s} \cdot \frac{V_{i2}}{V_{i1}} \cdot \frac{V_o}{V_{i2}} = \frac{1}{6} \cdot (-9.1) (-50) = \boxed{75.8}$$

(40 Points) (2) In the given circuit,  $R_1=10\text{ K}$ ,  $R_2=50\text{ K}$ ,  $C_1=1\mu\text{F}$  and  $C_2=1\text{nF}$ . Find the transfer function  $T = V_o/V_i$ .  $T$  should be in the form

$$T = K(1+\tau_1 s) \dots / (1+\tau_2 s) \dots$$



And numerical values of  $K$ ,  $\tau_1$  and  $\tau_2$  should be specified.

Solution:

$$\frac{V_{o1}}{V_i} = 1 + \frac{R_2}{Z_1} = 1 + \frac{R_2}{R_1 + \frac{1}{sC_1}} = 1 + \frac{sR_2 C_1}{1 + sR_1 C_1} =$$

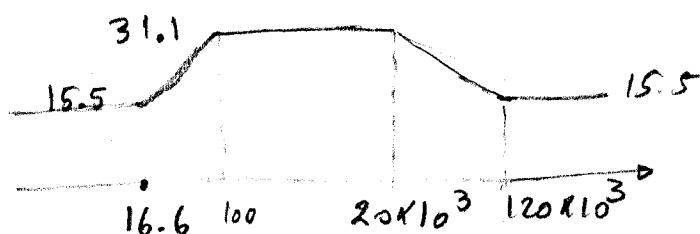
$$\frac{V_{o1}}{V_i} = \frac{1 + sC_1(R_1 + R_2)}{1 + sR_1 C_1}$$

$$\frac{V_o}{V_{o1}} = 1 + \frac{Z_2}{R_1} = 1 + \frac{\frac{1}{sC_2} \cdot R_2}{R_1} = 1 + \frac{R_2/R_1}{1 + sR_2 C_2}$$

$$\frac{V_o}{V_{o1}} = \frac{1 + R_2/R_1 + sR_2 C_2}{1 + sR_2 C_2} = \left(1 + \frac{R_2}{R_1}\right) \frac{1 + sC_2 \frac{R_2}{1 + R_2/R_1}}{1 + sR_2 C_2}$$

$$\frac{V_o}{V_i} = \frac{V_{o1}}{V_i} \cdot \frac{V_o}{V_{o1}} = \left(1 + \frac{R_2}{R_1}\right) \frac{(1 + sC_1(R_1 + R_2))(1 + sC_2 \frac{R_2}{1 + R_2/R_1})}{(1 + sR_1 C_1)(1 + sR_2 C_2)}$$

$$\frac{V_o}{V_i} = 6 \frac{(1 + 0.06s)(1 + 8.3 \times 10^{-6}s)}{(1 + 10^{-2}s)(1 + 5 \times 10^{-5}s)}$$



(30 Points) (3) For the given transfer function, sketch the magnitude and phase of its bode plot.

$$T = (1 + 0.06 S)(50 + 5 \cdot 10^{-5} S) / (1 + 0.01 S)(1 + 5 \cdot 10^{-5} S)$$

$$T = 50 \frac{(1 + 0.06 S)(1 + 10^{-6} S)}{(1 + 0.01 S)(1 + 5 \cdot 10^{-5} S)}$$

$$\begin{aligned}|T|_{dB} &= |1 + 0.06 S|_{dB} + |1 + 10^{-4} S|_{dB} + \left| \frac{1}{1 + 0.01 S} \right|_{dB} \\&\quad + \left| \frac{1}{1 + 5 \cdot 10^{-5} S} \right|_{dB}\end{aligned}$$

$$\angle T = \underbrace{\angle(1 + 0.06 S)}_{-27^\circ} + \underbrace{\angle(1 + 10^{-4} S)}_{+45^\circ} + \underbrace{\angle \frac{1}{1 + 0.01 S}}_{-90^\circ} + \underbrace{\angle \frac{1}{1 + 5 \cdot 10^{-5} S}}_{+90^\circ}$$

