Name: SOLUTIONS (please print)

Signature: ____________________

ECE 3455
Mid-Term Exam
July 13, 2010

Exam duration: 100 minutes

- You may have one 8 ½ x 11 in. "crib" sheet, written on both sides, during the quiz. You may have any calculator you choose, but no computers. No other notes or materials will be allowed.
- Show all work necessary to complete the problem on these pages. A solution without the work shown will receive no credit.
- Show units in intermediate and final results, and in figures.
- If your work is sloppy or difficult to follow, points will be subtracted.

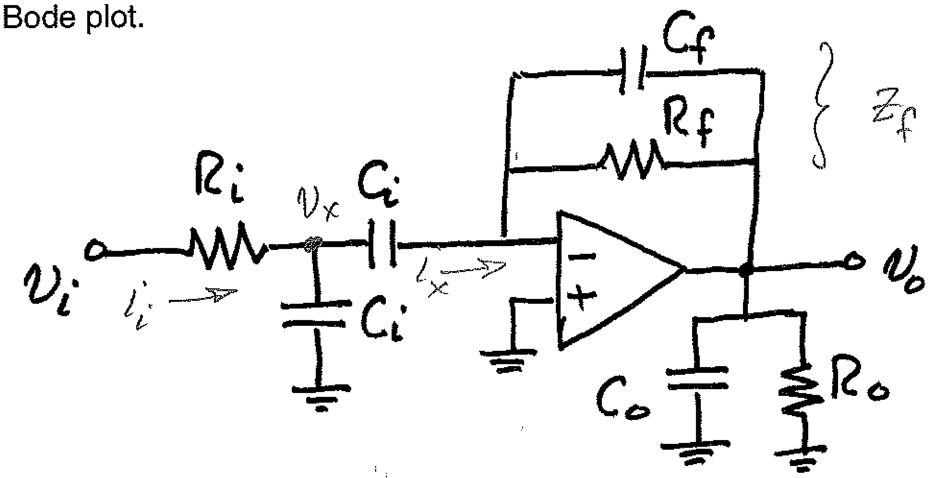
This exam has 10 pages, including the cover sheet. Raise your hand if you are missing a page.

1	____/20
2	____/25
3	____/25
4	____/30

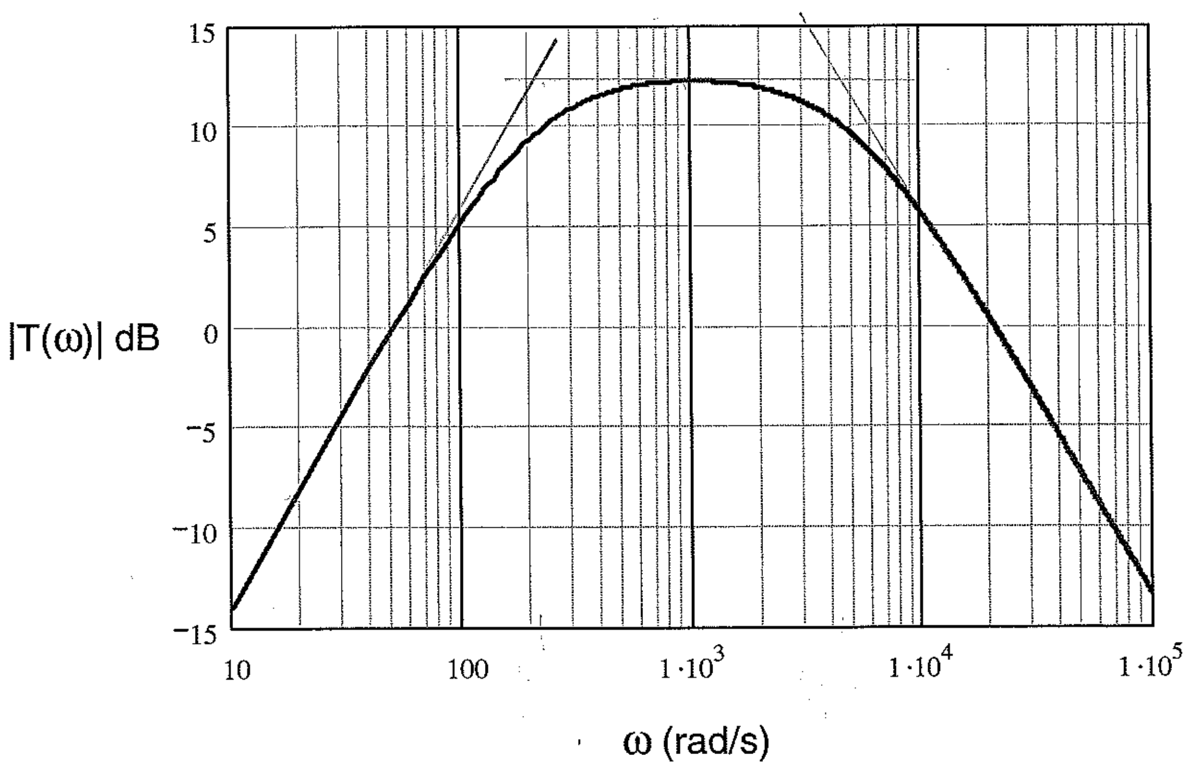
Total ____/100

1. (**20 points**) For the circuit shown below, do the following.

   i. Find the transfer function T(ω) = **V**$_o$/**V**$_i$.
   ii. The graph below is a magnitude Bode plot for the circuit; you can think of it as providing specifications for the circuit. Choose circuit component values to meet these specifications. Be sure your choices reproduce the important features of the Bode plot.

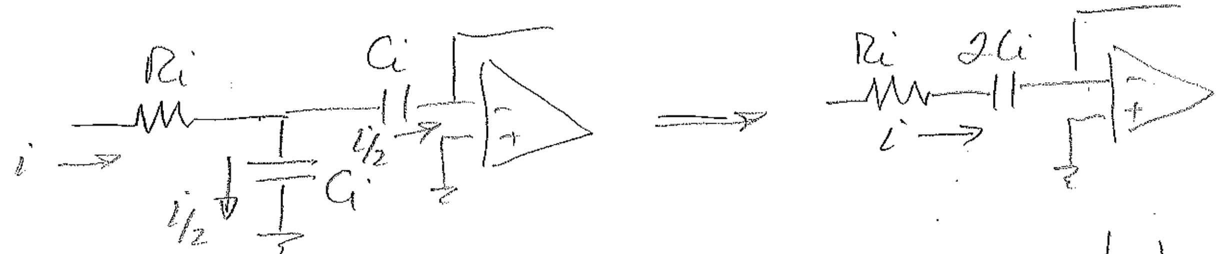


$$Z_f = \frac{R_f}{1+j\omega C_f R_f}$$



i) Room for Extra Work

There is something interesting going on in the input of this circuit. It would appear that the capacitors C_i are in parallel so that all we need do is:



This correctly predicts the input impedance but it does not give the correct output. For that we need a more complete analysis. Looking at the original circuit:

$$\frac{\bar V_x}{1/j\omega C_i} + \frac{\bar V_x}{1/j\omega C_i} + \frac{\bar V_x - \bar V_i}{R_i} = 0 \Rightarrow \bar V_x = \frac{\bar V_i}{1 + j\omega 2 C_i R_i}$$

Thus $\bar I_x = j\omega C_i \bar V_x = \frac{j\omega C_i \bar V_x}{1 + j\omega 2 C_i R_i} = \bar I_f$

So $\bar V_o = -\bar I_x Z_f = -\frac{j\omega C_i V_x}{(1+j\omega 2C_i R_i)} \frac{R_f}{(1+j\omega C_f R_f)}$

+9

$$\boxed{T(\omega) = \frac{\bar V_o}{\bar V_i} = \frac{-j\omega C_i R_f}{(1+j\omega 2 C_i R_i)(1+j\omega C_f R_f)}}$$

This is a factor of 2 <u>less</u> than if we had done a simple inverting configuration with the input impedance $R_i + 1/j\omega 2C_i$. The reason is that the current in the feedback resistor has been reduced by half because of the capacitor to ground.

Room for extra work

In any case, C_o and R_o have no effect.

ii) From the graph we have two poles at

$\omega_{z1} \simeq 210$ rad/s $\quad \omega_{z2} \simeq 4300$ rad/s.

Our transfer function shows poles at

+6 $\quad 1/C_f R_f = 4300$ rad/s, $\quad 1/2C_i R_i = 210$ rad/s

We have arbitrarily set the poles of $T(\omega)$ equal to the poles indicated on the plot. The opposite assignment is also valid. So we have

4 $\quad C_f R_f = 2.33 \times 10^{-4} \qquad R_i C_i = 2.38 \times 10^{-3}$

But before we assign values, we note that

$$|T(\omega = 10^3 \text{ rad/s})| = 12.5\,dB \rightarrow 4.22\ V/V$$

From our transfer function, we have

$$|T(\omega = 10^3 \text{ rad/s})| = \left| \frac{j(1000)(2.38 \times 10^{-3}/R_i) \cdot R_f}{[1 + j(1000)(2)(2.38 \times 10^{-3})][1 + j(1000)(2.33 \times 10^{-4})]} \right|$$

+3 $$\simeq \left| \frac{j\, R_f/R_i \cdot 2.38}{(1 + j\,4.76)(1 + j0)} \right| \cong \frac{R_f}{R_i} \frac{4.76}{9.52} = \frac{1}{2} \frac{R_f}{R_i}$$

$\Rightarrow R_f = 2(4.22) R_i$

+2

So $R_i = 10\ k\Omega \qquad C_i = 0.476\ \mu F$

$\Rightarrow R_f = 84.4\ k\Omega \qquad C_f = 2.76\ nF$

2. (**25 points**) On the next page you will find the phase Bode plot for a certain circuit. There are no breakpoints outside the frequency range shown. The magnitude Bode plot (not shown) increases with frequency at 20 dB/dec at very low frequencies, and has a constant value of 35 dB at $\omega = 2{,}000$ rad/s. Find the transfer function for this circuit.

+6
- Drop of $90°$/dec at $\omega = 2$ rad/s $\Rightarrow$ double pole at $\omega = 20$ rad/s.

+3
- Drop reduced to $45°$/dec at 40 rad/s $\Rightarrow$ zero at $\omega = 400$ rad/s.

  $\omega = 200$ rad/s : effect of double pole ends.

+3
- Increase of $45°$/dec at $\omega = 10^3$ rad/s $\Rightarrow$ zero at $\omega = 10^4$ rad/s.

  $\omega = 4000$ rad/s : zero ends

  $\omega = 10^5$ rad/s : zero ends.

+5
$$T(\omega) = -K \cdot \frac{j\omega(j\omega + 400)(j\omega + 10^4)}{(j\omega + 20)^2}$$

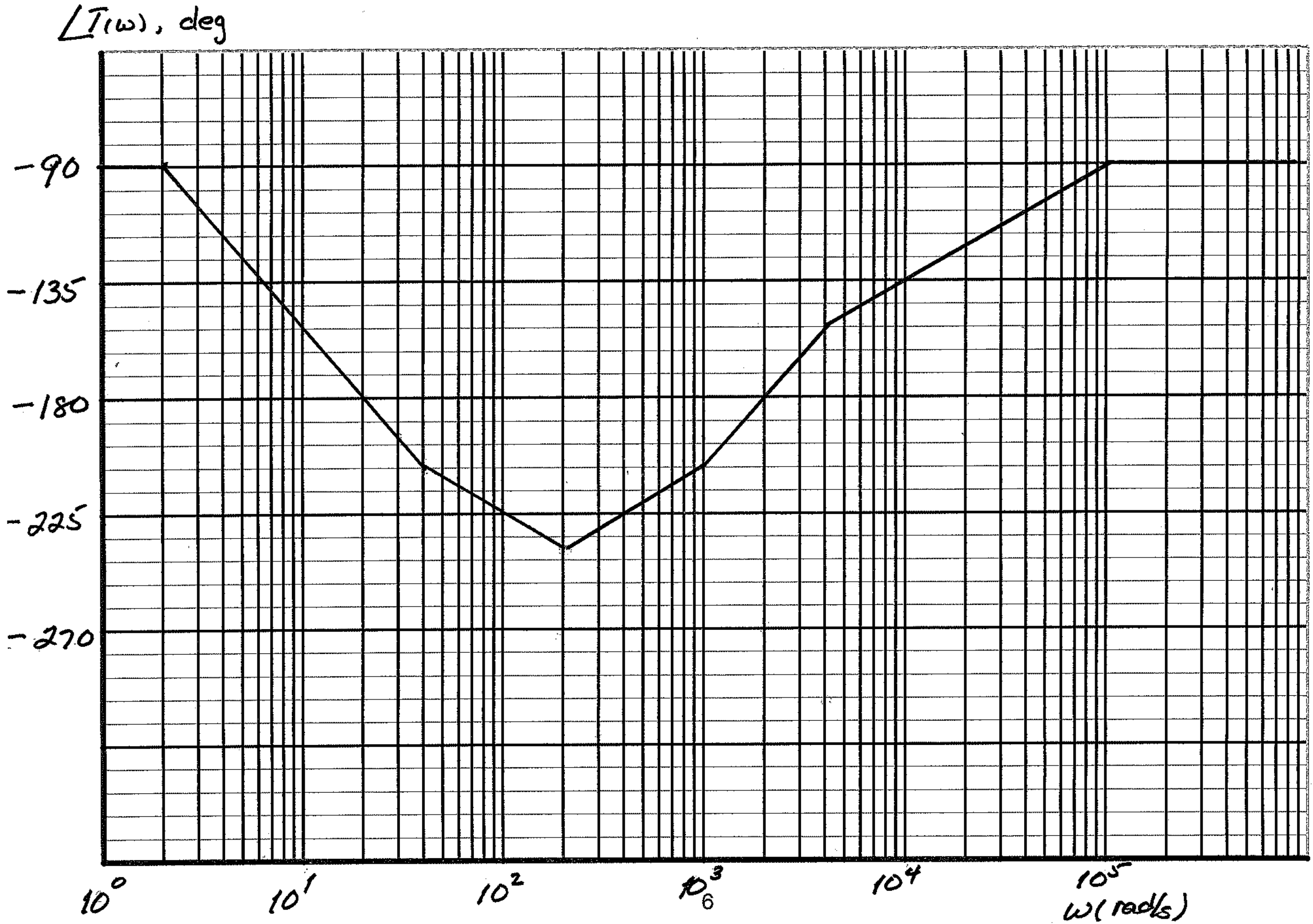
+3
- The $j\omega$ in the numerator accounts for the 20 dB/dec slope indicated in the magnitude Bode plot.

Also, $|T(\omega)|_{\omega = 2000 \text{ rad/s}} = 35 \text{ dB} \Rightarrow |T(\omega)|_{\omega = 2 \text{ krad/s}} = 56.2$

+3 But $|T(\omega)|_{\omega = 2000 \text{ rad/s}} = 1.04 \times 10^4 K = 56.2$

$\Rightarrow \underline{K = 5.4 \times 10^{-3}}$

+2 Finally, we note that $\angle T(\omega \to 0) \to 90°$, but the plot indicates $\angle T(\omega \to 0) \to -90°$, so there must also be a '−' sign in $T(\omega)$.


∠T(ω), deg
-90
-135
-180
-225
-270
10^0
10^1
10^2
10^3
10^4
10^5
ω (rad/s)

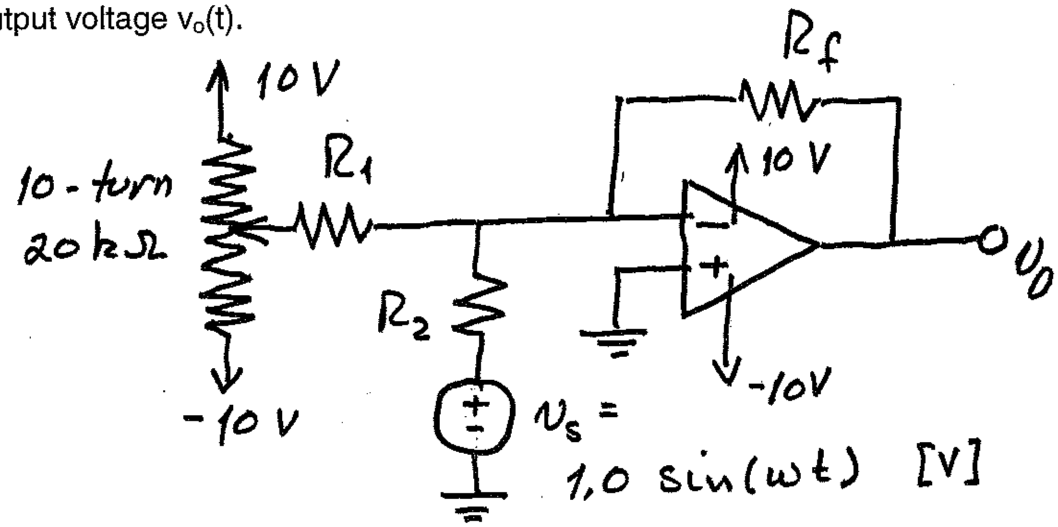
3. (**25 points**) For the circuit below, do the following.

i. Choose component values to produce the output

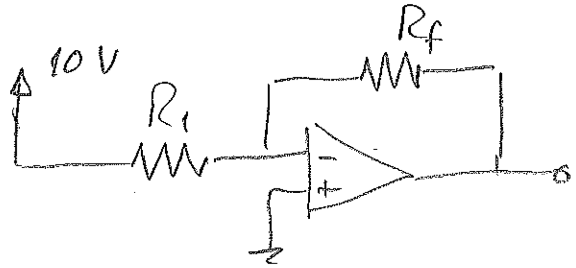
$$v_O(t) = -2.5\sin(\omega t) + V_{offset} \quad [\mathrm{V}]$$

where V_{offset} is a dc offset voltage that ranges from +4 V to –4 V, and the potentiometer is a 10-turn, 20 kΩ potentiometer.

ii. The ~~10~~ 20 kΩ potentiometer is set to 4 turns from its end value, with the smaller resistance above the wiper (as it is oriented in the figure). Write an expression for the output voltage $v_o(t)$.



i) If we turn the pot all the way to the top, we will have (ignoring v_s input):



$$v_o = -\frac{R_f}{R_1} \times 10$$

In this case, we need $v_o = -4\ \mathrm{V}$, $\Rightarrow \frac{R_f}{R_1} = 0.4$

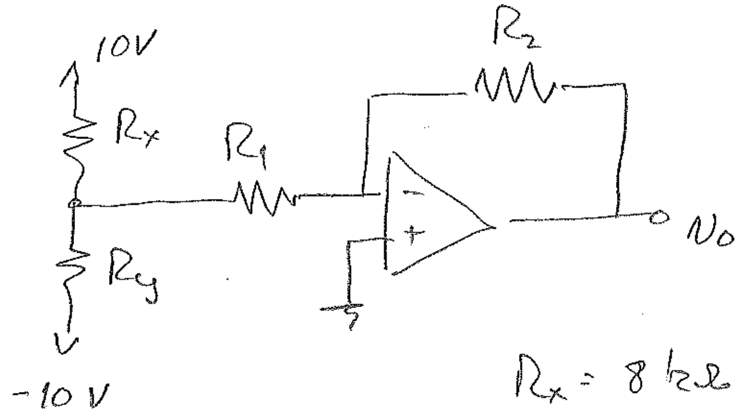
Room for extra work

+6 choosing $R_f = 10\ k\Omega$ $\Rightarrow$ $R_1 = 25\ k\Omega$

+4 To get an ac component of $-2.5\ V$ (amplitude), we will need

$$R_2 = 4\ k\Omega$$

ii At 4 turns, which is $0.4 \times 20\ k\Omega = 8\ k\Omega$, we have:

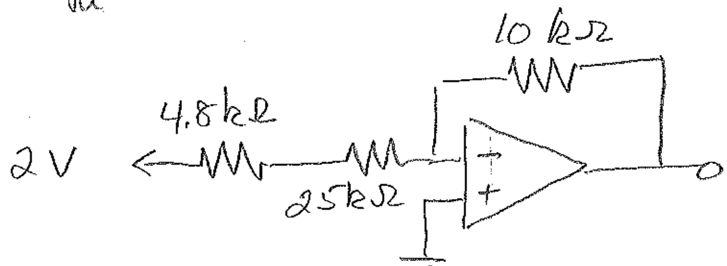


We again ignore v_s and focus on the dc component.

$R_x = 8\ k\Omega$ $R_y = 12\ k\Omega$

Taking a Thevenin equivalent, we have (dc component)

+10 $R_{Th} = 8k // 12k = 4.8\ k\Omega$ $V_{Th} = 20 \cdot \frac{12}{20} - 10 = 2\ V.$

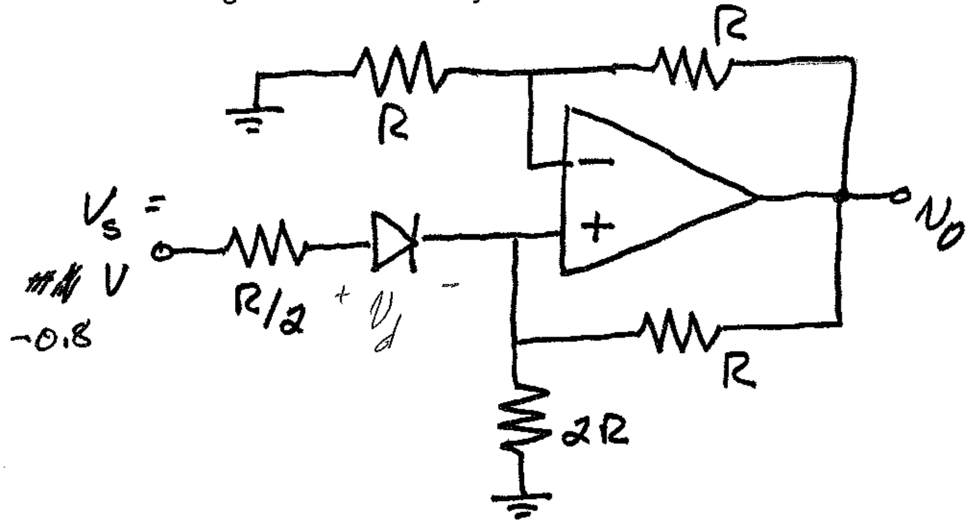
+5 

$v_0 = -\frac{10}{29.8} \cdot 2 = -0.67\ V.$

$$\boxed{\therefore\ v_0 = -0.67 - 2.5 \sin(\omega t)}$$

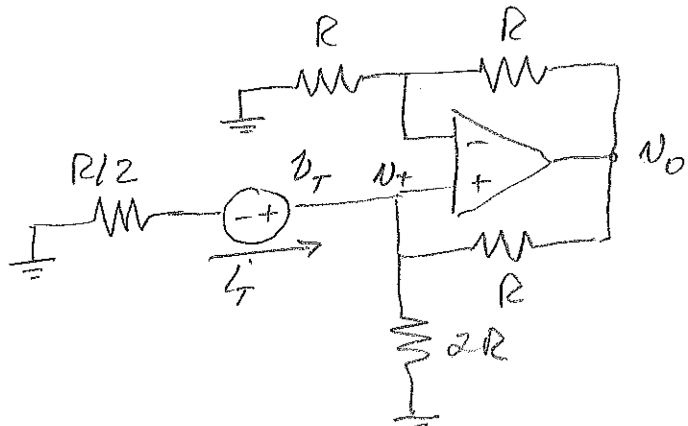
4. (**30 points**) For the circuit below, do the following.

i. Find the Thevenin equivalent circuit with respect to the terminals of the diode.
ii. The diode $i_d - v_d$ characteristics are shown in the graph on the next page. On this graph, plot the load line for the circuit.
iii. Based on your plot, estimate all possible operating points, i.e., the current and voltage values taken by the diode.



$R = 50\ k\Omega$

This looks like the negative impedance converter but it's not identical. Let's do the analysis:



$$i_T' = \frac{v_+}{2R} + \frac{v_+ - v_o}{R}$$

$$v_o = v_+\left(1 + \frac{R}{R}\right) = 2v_+$$

$$\Rightarrow \quad i_T' = \frac{-v_+}{2R}$$

KVL: $v_+ + \frac{R}{2} i_T' - v_T = 0$

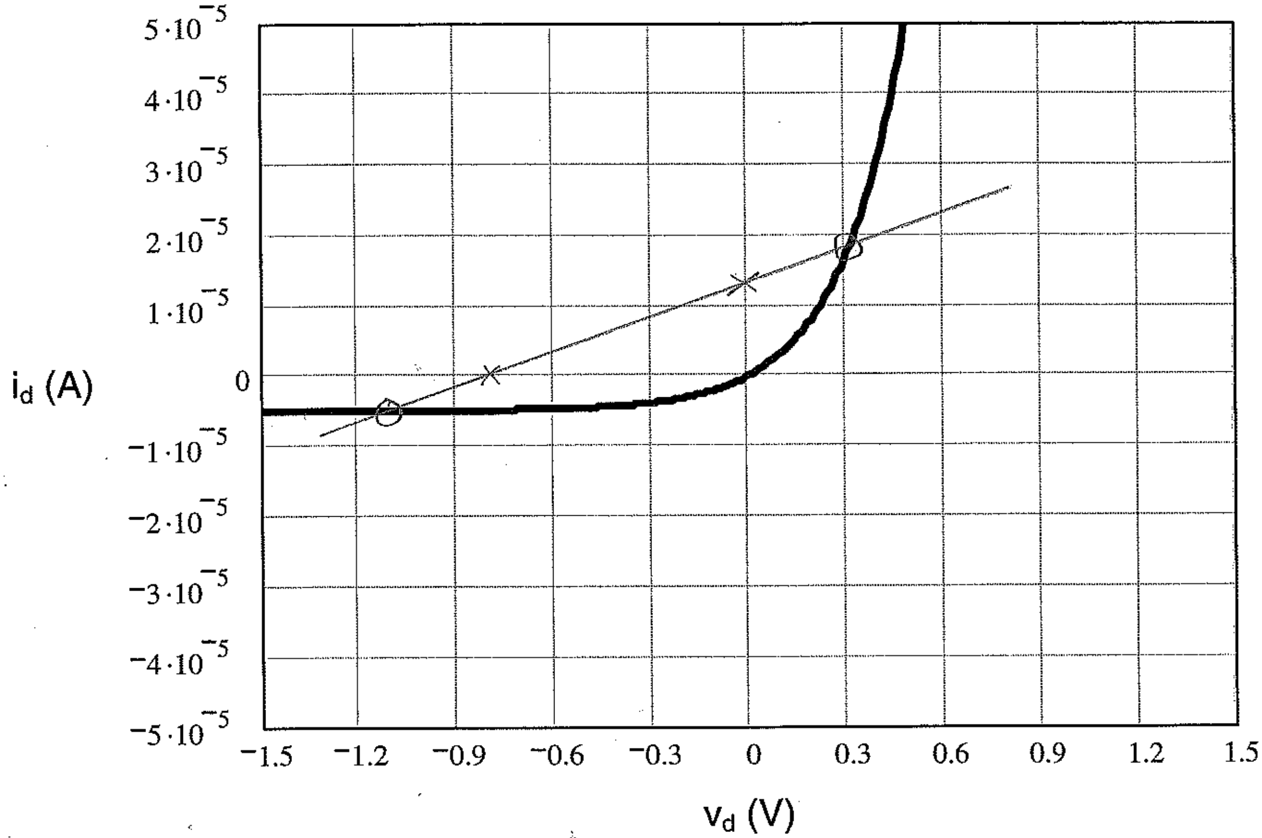
$$v_+ = v_T - \frac{R}{2} i_T'$$

$$\Rightarrow \quad i_T' = \frac{-v_T}{2R} + \frac{1}{4} i_T'$$

$$\frac{v_T}{i_T'} = -\frac{3}{2} R = R_{Th}.$$

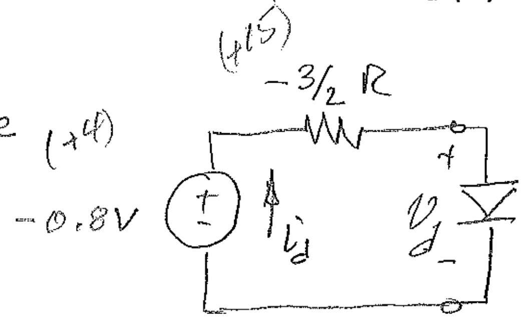
With diode removed,

$v_d = -0.8\ V. = v_{oc} = V_{Th}$



+19

So we have (+4)



$$0.8 - i_d \cdot \tfrac{3}{2}R + v_d = 0 \quad (+4)$$

$$i_d = \frac{v_d + 0.8}{3/2\,R} = \frac{2}{3R}v_d + \frac{1.6}{3R}$$

+8 So this is a straight line w/ slope $= +\frac{2}{3R}$ (+4)

For $i_d = 0$, $v_d = -0.8V$; For $v_d = 0$, $i_d = \frac{2}{3R} = 13.3\ \mu A$. These points are plotted above. The load line implies two possible operating points:

+3 $i_d \simeq -5\ \mu A$, $V_d \simeq -1.1\ V$; $i_d \simeq 19\ \mu A$ $V_d \simeq 0.3\ V$.