Name: SOLUTIONS (please print)

Signature: ____________________________

ECE 3355
Final Exam
July 26, 2017

Exam duration: 170 minutes

- You may have one 8 ½ x 11 in. "crib" sheet, written on both sides, during the exam. You may have any calculator you choose, but no computers or communication devices. No other notes or materials are allowed.
- Show all work necessary to complete the problem on these pages. A solution without the work shown will receive no credit.
- Show units in intermediate and final results, and in figures.
- If your work is sloppy or difficult to follow, points will be subtracted.

This exam has 11 pages, including the cover sheet. Raise your hand if you are missing a page.

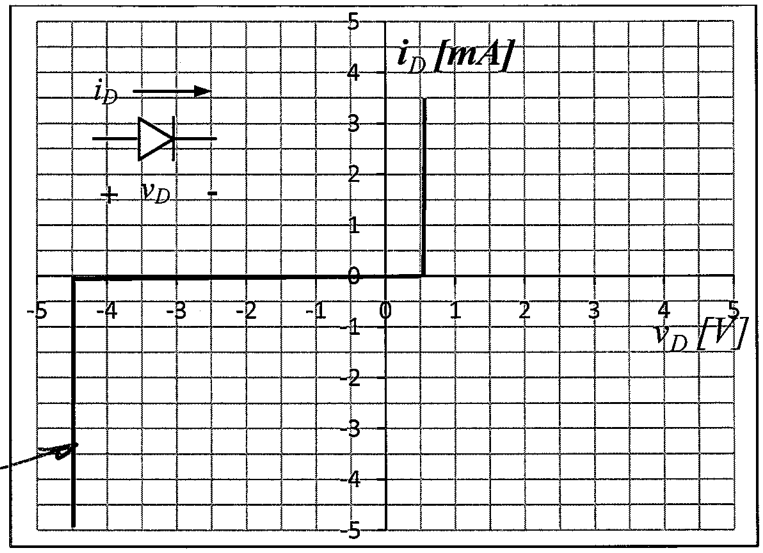
1	______/40
2	______/40
3	______/40
4	______/30
5	______/50
Total	______/200

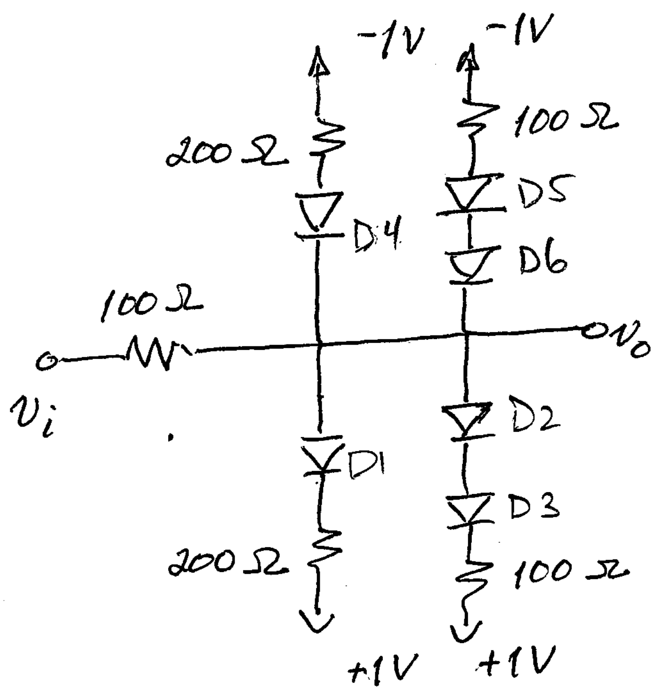
1. (**40 points**) In the circuit below, the diodes are all modeled as shown in the $i_D - v_D$ plot. The input v_i is a triangle wave with a period of 20 ms.

i) What is the largest amplitude the input triangle wave can take if we want to avoid driving any of the diodes into the reverse-bias breakdown region?

ii) Assuming an input amplitude up to but not including the value you found in part i), make a sketch of the transfer characteristics v_o as a function of v_i.

reverse-bias breakdown

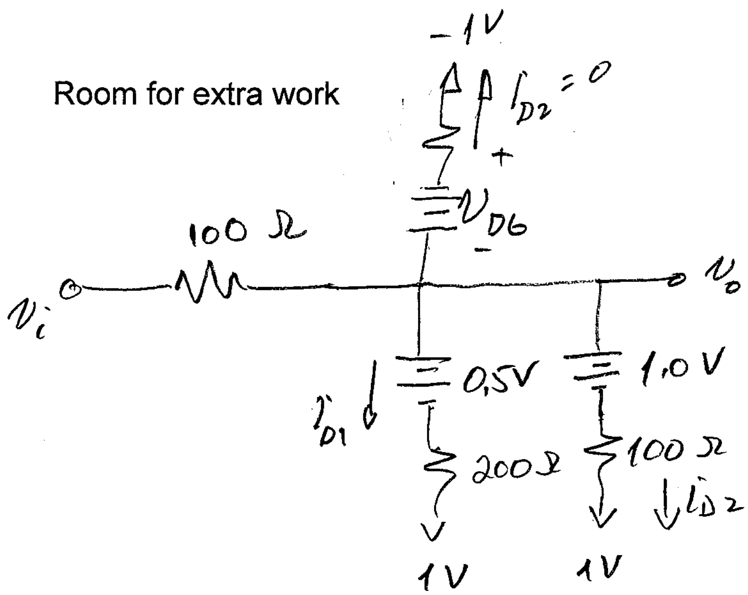




Note that the circuit is symmetric: what happens for $v_i > 0$ will happen for $v_i < 0$ but with opposite signs.

For $v_i > 0$, the first thing to happen is that D1 will turn ON. Eventually, D4 will go into breakdown. We will assume that's the case, and that in addition D2 and D3 are also ON:

Room for extra work



i)

$$-V_o - V_{D6} - 1 = 0$$

$$V_{D6} = -4.5V \Rightarrow$$

$$\boxed{V_o = 3.5V}$$

$$\frac{V_o - 0.5 - 1}{200} + \frac{V_o - 1 - 1}{100} + \frac{V_o - V_i}{100} = 0$$

$$\Rightarrow V_o = 1.1 + 0.4 V_i$$

$\boxed{V_o = 3.5V \Rightarrow V_i = 6V.}$

$i_{D1} = \frac{3.5 - 0.5}{200} > 0$ $\quad i_{D2} = \frac{3.5 - 1}{100} > 0$ so D2, D3 are in fact ON, and $\boxed{V_i = 6V}$ is the largest voltage for V_i $\quad +15$

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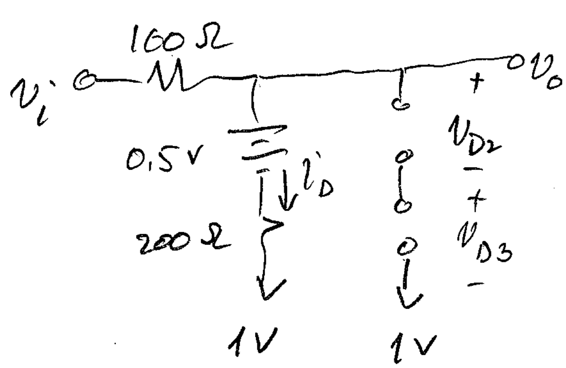
ii) For $6V > V_i > 0$, D1 will go ON, and then D2 and D3 will go ON. D4, D5, D6 will be OFF. Set D1 ON first:



$+10$

$$\frac{V_o - 0.5 - 1}{200} + \frac{V_o - V_i}{100} = 0$$

$$\Rightarrow V_o = 0.5 + \tfrac{2}{3} V_i\ [V]$$

This will happen at $V_o = 1.5V$, at which point i_D is just barely 0.

pg-3a.

$V_o = 1.5V \Rightarrow V_o = V_i$

Before that, $V_o = V_i$.

Room for extra work

Put D2, D3 ON: This is the circuit and equation above, so

$v_o = 1.1 + 0.4 v_i \quad [V]$

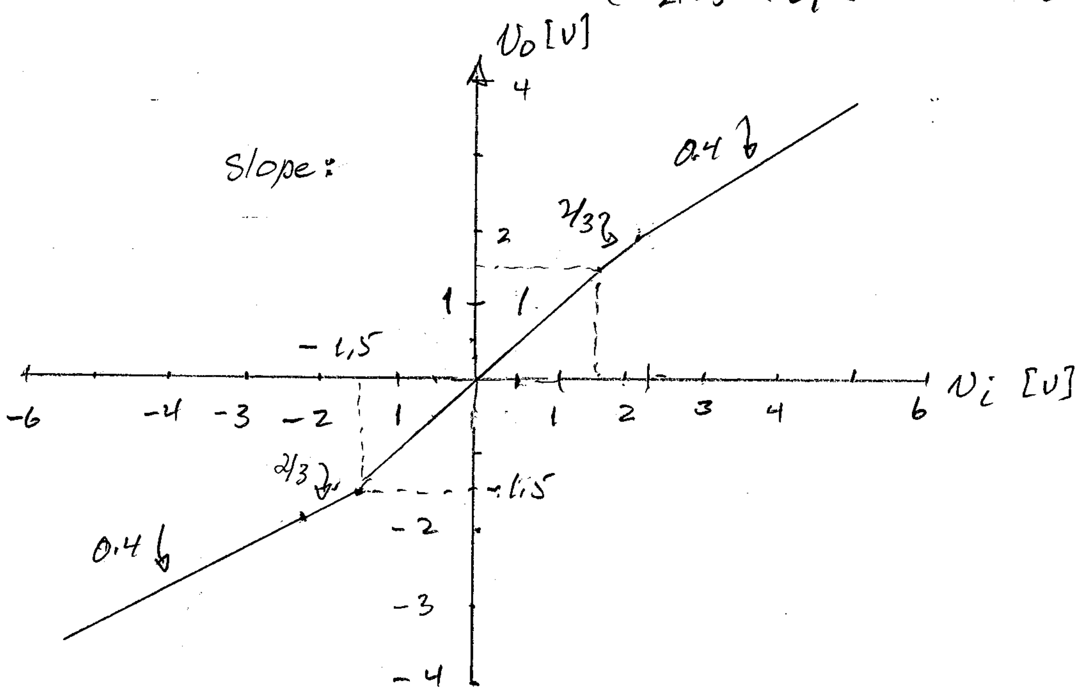
+10

Where does this happen? If D2 and D3 are ON but just barely, there will be no current in that branch.

So $v_o = 2V$ at that point, so $v_i = 2.25V$,

Everything will happen symmetrically for $v_i < 0$.

summary
$$\begin{cases} 0 < v_i < 0.5 \ [V] & v_o = v_i \\ 0.5 < v_i < 2.25 \ [V] & v_o = 0.5 + \frac{2}{3} v_i \\ 2.25 < v_i < 6 \ [V] & v_o = 1.1 + 0.4 v_i \end{cases}$$



+5

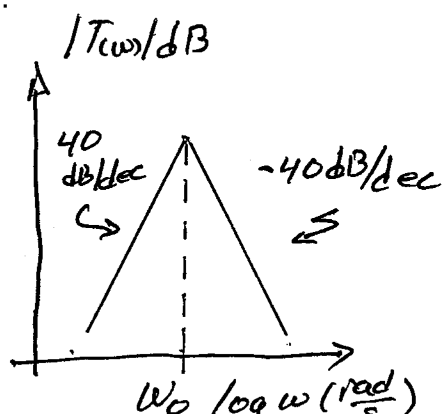
It's hard to draw accurately w/o graph paper!

2. (**40 points**) We wish to create a band-pass filter. The straight-line approximation to the Bode plot we wish to achieve is given in the figure below (axes are purposely left unlabeled). The specifications for the filter are as follows.

- The maximum amplitude is at ω_o = 500 rad/s.
- The roll-off of the amplitude is 40 dB/dec on either side of ω_o.
- The magnitude of the transfer function at ω_o must be at least 12 dB.

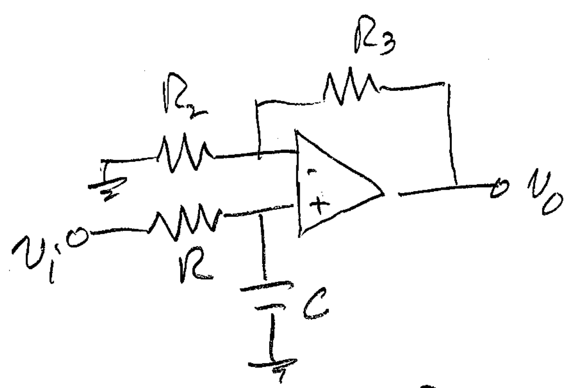
i) Using any number of ideal op amps, and any number and value of resistors and capacitors, design a filter that satisfies these specifications.

ii) Find the transfer function T(ω), and evaluate it at ω_o to prove that the amplitude is at least 12 dB.



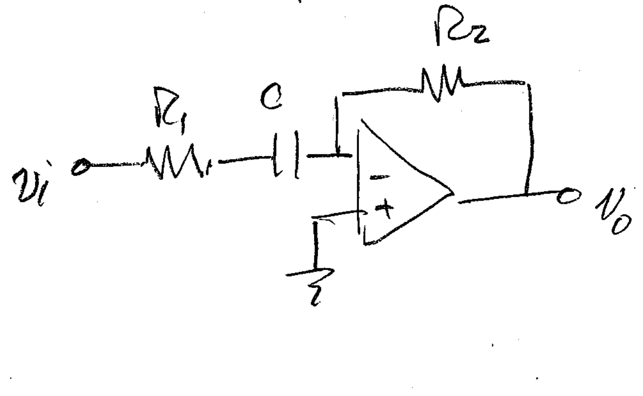
We will need 2 zeroes at 0 and four poles at ω_o.

Here are a couple of op-amp configurations that could be useful: There are others.



$$\frac{V_o}{V_i} = \left(1 + \frac{R_3}{R_2}\right)\frac{1}{1+j\omega CR}$$

We'll take 2 of these ...

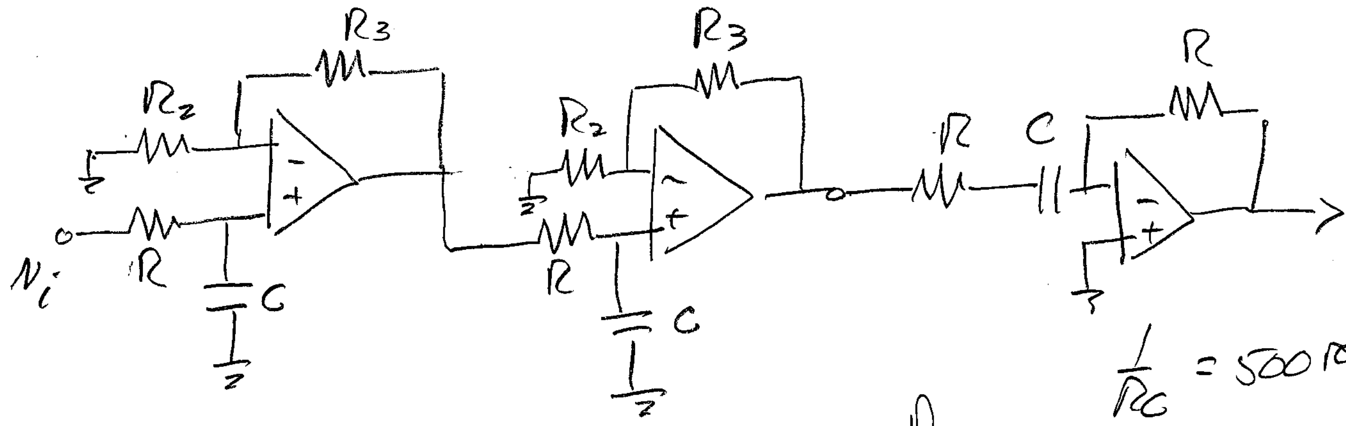


$$\frac{V_o}{V_i} = \frac{-j\omega CR_2}{1+j\omega CR_1}$$

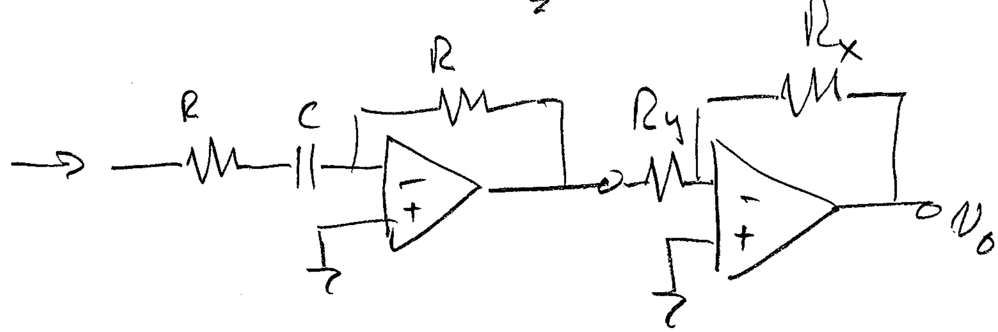
... and 2 of these!

Room for Extra Work

i)



$\frac{1}{RC} = 500$ rad/s



+30

ii)

$$T(\omega) = \left(\frac{j\omega CR}{1+j\omega CR}\right)^2 \left(1+\frac{R_3}{R_2}\right)^2 \left(\frac{1}{1+j\omega CR}\right)^2$$

We could choose $R_3 = R_2 \Rightarrow |T(\omega=\omega_0)| = 1. = 0$ dB

Then we need further amplification (last op amp)

and $R_x/R_y = 4. = 12$ dB

+10

Alternatively $\left(1+\frac{R_3}{R_2}\right)^2 = 16 \Rightarrow R_3/R_2 = 3$, Any choice will do, like $R_3 = 10\ k\Omega$, $R_2 = 3.3\ k\Omega$.

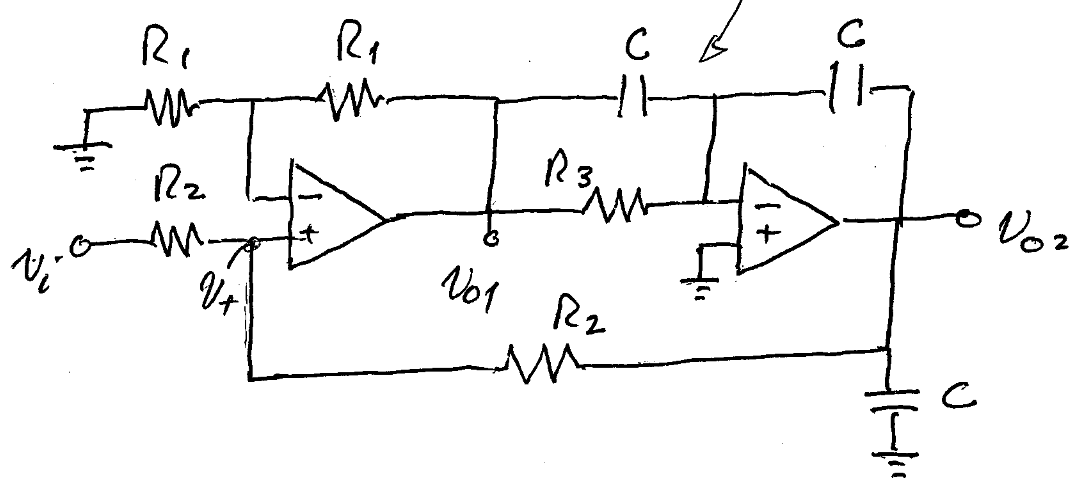
In the second case, $|T(\omega=\omega_0)| = \left|\left(\frac{j}{1+j}\right)^2 (16) \left(\frac{1}{1+j}\right)^2\right| = 4$

$\frac{1}{RC} = 500$ rad/s : $C = 0.1\ \mu F \Rightarrow R = 20\ k\Omega$

3. (**40 points**) The circuit below has two outputs and thus can provide two filtering functions. The op amps are ideal.

i) Find the transfer function $T_1(\omega) = V_{o1}/V_i$.
ii) Find the transfer function $T_2(\omega) = V_{o2}/V_i$.

$$R_3 /\!/ C = \frac{R_3}{1+j\omega C R_3}$$



start on the right, and note that the capacitor in parallel with V_{o2} has no effect.

$$\frac{V_{o2}}{V_a} = \frac{-1/j\omega C}{\frac{R_3}{1+j\omega C R_3}} = -\frac{1+j\omega C R_3}{j\omega C R_3}$$

+12

$$V_{o1} = 2V_+ \qquad \frac{V_+ - V_{in}}{R_2} + \frac{V_+ - V_{o2}}{R_2} = 0 \Rightarrow V_+ = \frac{1}{2}V_i + \frac{1}{2}V_{o2}$$

$$\text{So } V_+ = \frac{1}{2}V_{o1} = \frac{1}{2}V_i + \frac{1}{2}V_{o2}$$

+24

$$V_{o1} = V_i - \frac{1+j\omega C R_3}{j\omega C R_3} V_{o1} \qquad V_{o1}\left(1 + \frac{1+j\omega C R_3}{j\omega C R_3}\right) = V_i$$

+4

$$\boxed{\frac{V_{o1}}{V_i} = \frac{j\omega C R_3}{1+2j\omega C R_3}}$$

$$\frac{V_{o2}}{V_i} = \frac{V_{o2}}{V_{o1}}\frac{V_{o1}}{V_i} = -\left(\frac{1+j\omega C R_3}{j\omega C R_3}\right)\left(\frac{j\omega C R_3}{1+2j\omega C R_3}\right)$$

$$\boxed{\frac{V_{o2}}{V_i} = -\left(\frac{1+j\omega C R_3}{1+2j\omega C R_3}\right)}$$

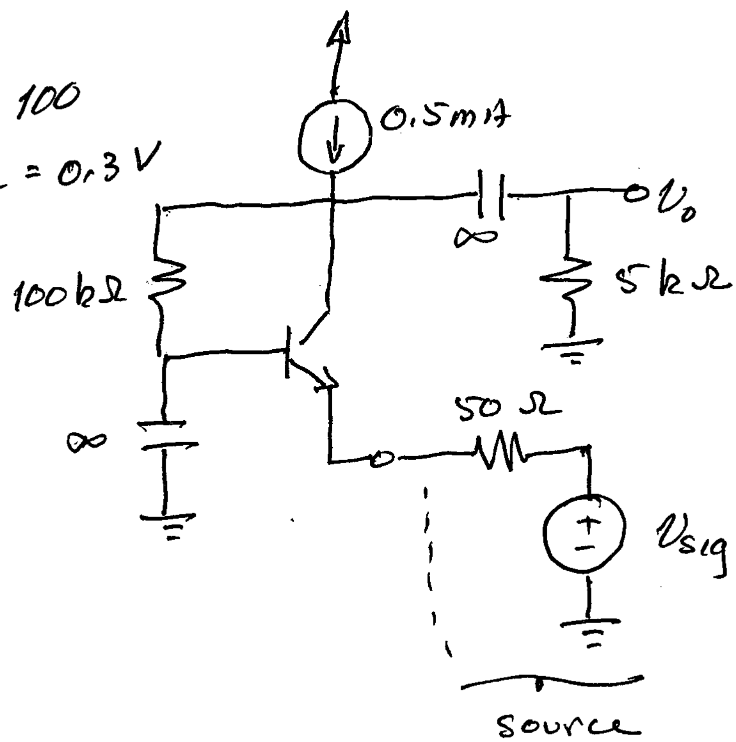
Room for extra work

4. (**30 points**) For the BJT circuit below, assume operation in the active region. The source v_{sig} is shown along with its source resistance of 50 Ω.

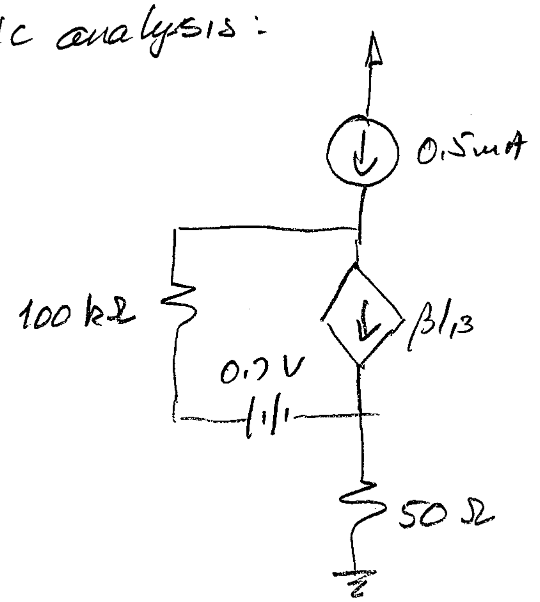
i) Find the gain v_o/v_{sig} in the passband.
ii) Find the input resistance R_i seen by the source, in the passband.

$\beta = 100$

$V_{CE,SAT} = 0.3\,V$



To construct the small-signal model we will need a dc analysis:



$$-0.5\,mA + (\beta+1)I_B = 0$$

$$I_B = \frac{0.5\,mA}{101} = 4.95\,\mu A$$

$$r_\pi = \frac{V_T}{I_B} = 5.05\,k\Omega$$

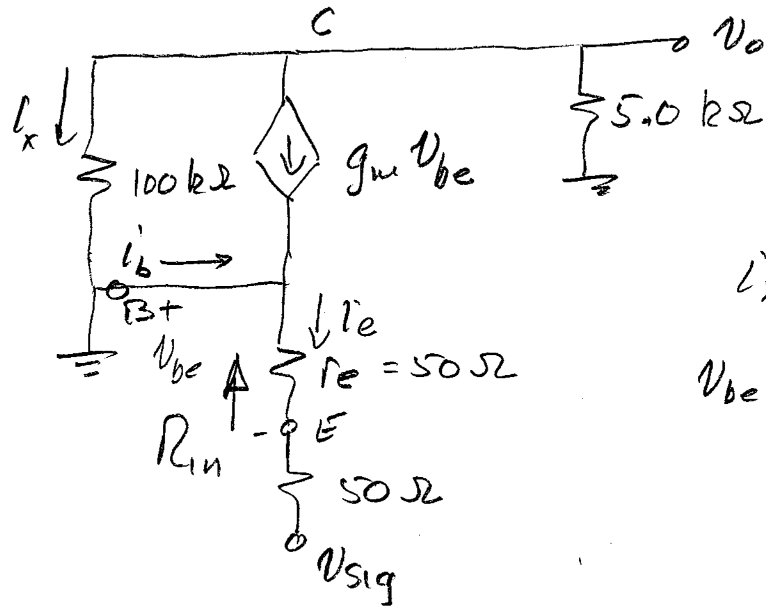
+6

$$g_m = \frac{I_C}{V_T} = \frac{\beta I_B}{V_T} = 19.8\,\frac{mA}{V}$$

$$r_e = 1/g_m = 50\,\Omega$$

Room for extra work

i)



$i_e = -v_{sig}/100$

$i_x = v_o/100000$ +12

$v_{be} = i_e r_e = -v_{sig}/2$

$$v_o = -5000(i_x + g_m v_{be}) = -5000\left(\frac{v_o}{100000} - \frac{19.8\times10^{-3}}{2} v_{sig}\right)$$ +8

$$\Rightarrow \boxed{\frac{v_o}{v_{sig}} = 49.5\ \frac{V}{V}}$$

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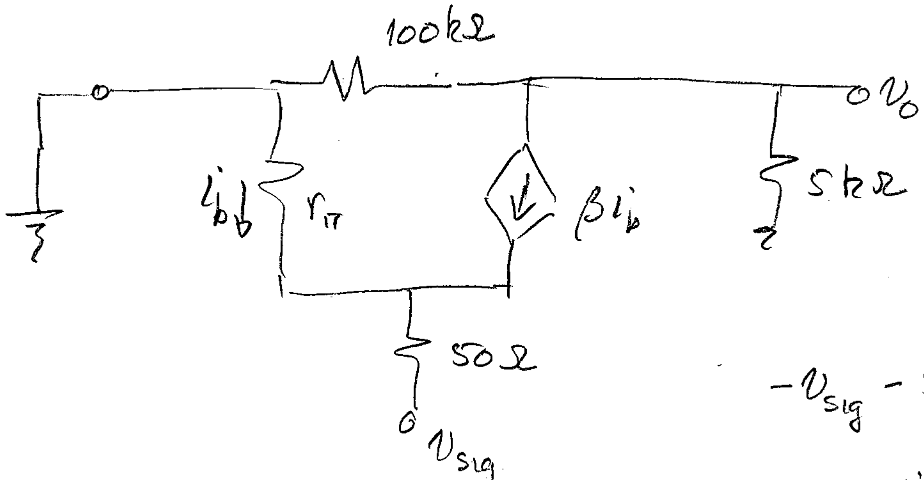
ii) $R_{in} = -\frac{v_{sig}}{i_e} - 50$ (because the source test current is $i_T = -i_e$, and we don't want to include the 50 Ω source resistance)

$$i_e = \frac{-v_{sig}}{100} \Rightarrow \boxed{R_{in} = r_e = 50\ \Omega\ !}$$ +4

Room for extra work

Alternative model:



+12

$$-v_{sig} - 50(\beta+1)i_b - r_\pi i_b = 0$$

$$i_b = \frac{-v_{sig}}{50(\beta+1) + r_\pi}$$

$$= -v_{sig}/10100$$

$$v_o = -\left(\beta i_b + \frac{v_o}{100000}\right)5000$$

$$v_o\left(1 + \frac{5000}{100\,000}\right) = -\beta(5000)\left(\frac{-v_{sig}}{10100}\right)$$

$$\frac{v_o}{v_{sig}} = 47.2 \ \frac{V}{V}$$

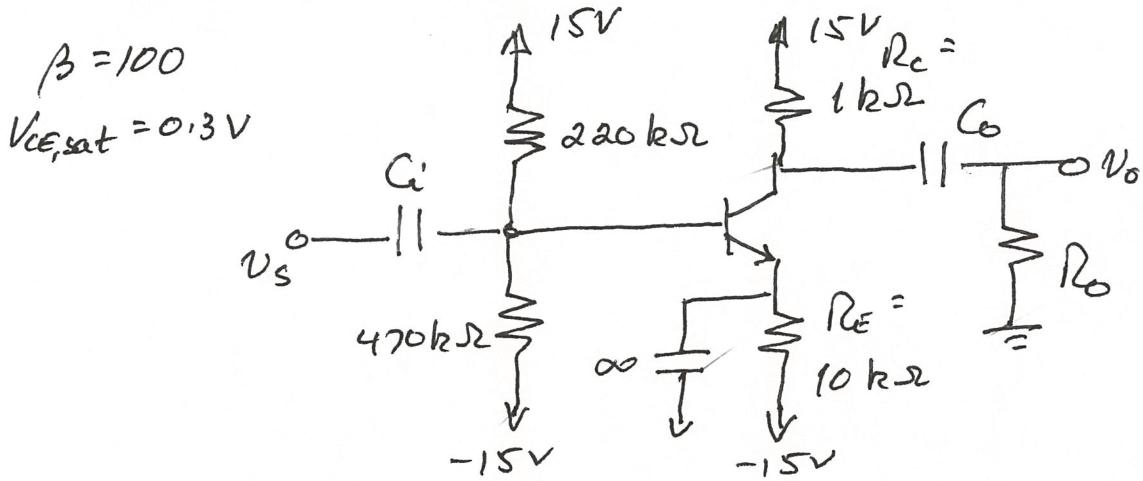
+8

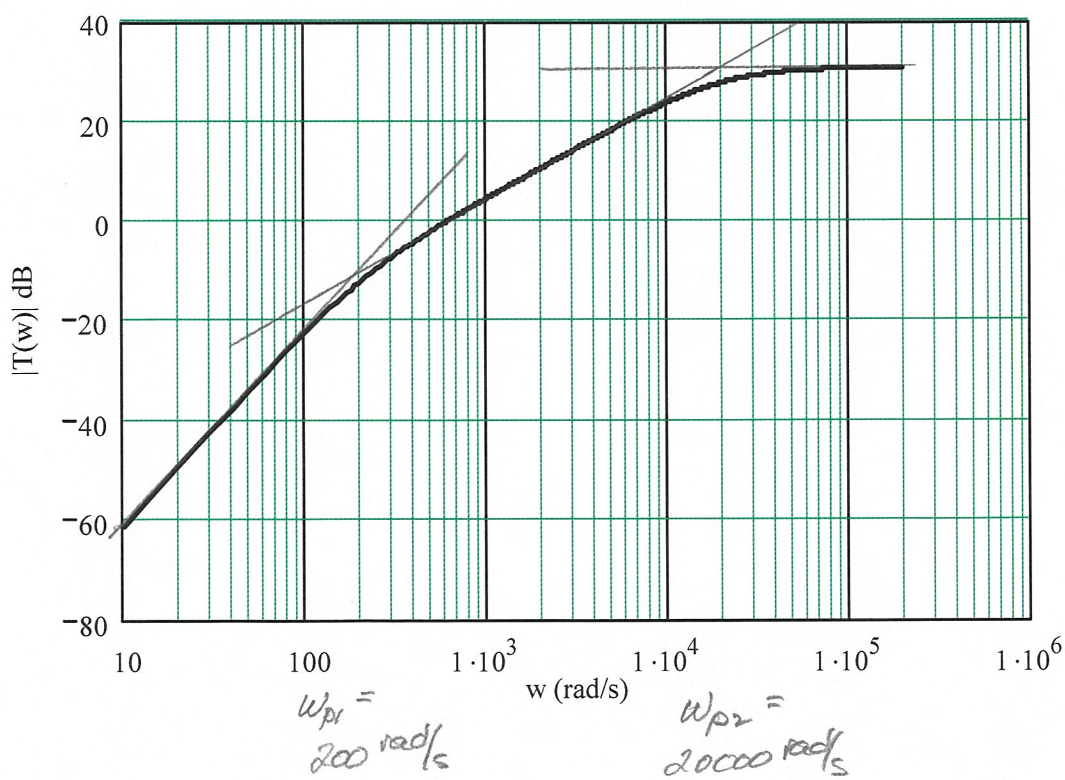
5. (**50 points**) The circuit below shows a common emitter amplifier that is to be designed to provide the frequency response shown in the Bode plot. The BJT is operating in the linear region.

i) Find the currents I_C and I_B, and the collector-emitter voltage V_{CE}, at the Q-point.

ii) Find the transfer function $T(\omega)$.

iii) Choose values for the capacitances and for the resistance R_o so that the frequency response and high-frequency gain of the BJT corresponds to the Bode plot.

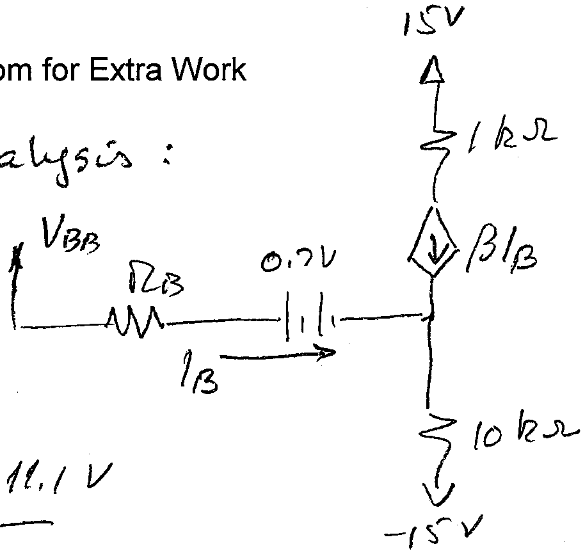




Room for Extra Work

DC analysis :

i)



$\underline{V_{CE} = 11.1\ V}$

$V_{BB} = 30 \cdot \frac{470}{470+220} - 15 = 5.435V$

$R_B = 220 \| 470 = 150 k\Omega$

$\underline{I_B} = 17.0\ \mu A$

$\underline{I_C} = \beta I_B = 1.70\ mA$

$V_C = 15 - \beta I_B (1000) = 13.3V$

$V_E = (\beta+1) I_B (10000) - 15 = 2.18V$

$-5.435 + 150000 I_B + 0.7 + 101 I_B (10000) - 15 = 0$

$I_B = 17.0 \mu A \Rightarrow r_\pi = \frac{V_T}{I_B} = 1.470 k\Omega$

I_B + 3

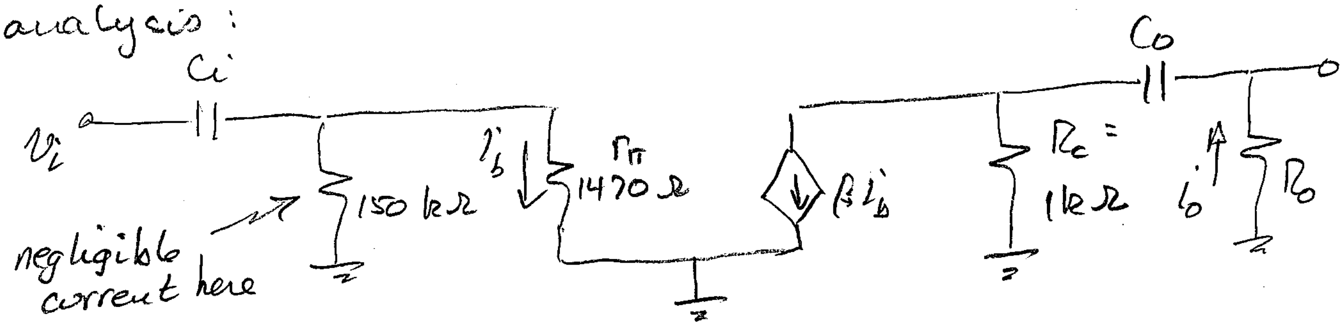
I_C + 2

V_{CE} + 4

9

AC analysis :

ii)



+12

$I_b = \frac{V_s}{r_\pi + 1/j\omega C_i} = \frac{j\omega C_i V_s}{1 + j\omega C_i r_\pi}$

$I_o = \beta I_b \frac{R_c}{R_c + R_o + 1/j\omega C_o}$

$= \beta I_b \frac{j\omega C_o R_c}{j\omega C_o (R_c + R_o)}$

$V_o = -I_o R_o$

+ 12

$T(\omega) = \frac{V_o}{V_s} = -\beta R_o \frac{j\omega C_o R_c}{1 + j\omega C_o (R_o + R_c)} \frac{j\omega C_i}{1 + j\omega C_i r_\pi}$

to 11a

24

Room for Extra Work

iii)

The Bode plot indicates two zeroes at 0, which is indicated in $T(\omega)$. A straight-line approximation to $T(\omega)$ also suggests poles at 200 rad/s and 20000 rad/s. So...

$$\omega_{p1} = 1/C_i r_\pi = 200 \text{ rad/s} \Rightarrow \boxed{C_i = \frac{1}{200\, r_\pi} = 3.4\ \mu F}$$ + 5

R_o will determine ω_{p2} as well as the gain at $\omega \to \infty$.

$$\omega \to \infty \Rightarrow |T(\omega)| \to \beta R_o \frac{R_c}{R_o + R_c} \cdot \frac{1}{r_\pi} = 32\text{ dB} = 39.8\ \frac{V}{V}$$

$$\beta R_o \frac{1000}{R_o + 1000} \frac{1}{1470} = 39.8 \Rightarrow R_o = 1411\ \Omega$$ + 7

So $$\omega_{p2} = 1/C_o(R_o + R_c) = 20000 \tfrac{\text{rad}}{\text{s}} \Rightarrow \boxed{C_o = 21\ nF}$$

+ 5

17