Name: Solutions! (please print)

Signature: ____________________

ECE 3355
Mid-Term Exam
July 5, 2017

- You may have one 8 ½ x 11 in. "crib" sheet, written on both sides, during the quiz. You may have any calculator you choose, but no computers.
- Show all work necessary to complete the problem on these pages. A solution without the work shown will receive no credit.
- Show units in intermediate and final results, and in figures.
- If your work is sloppy or difficult to follow, points will be subtracted.
- You have 90 minutes to work on the exam.

This exam has 10 pages, including the cover sheet. Raise your hand if you are missing a page.

1 ____________/25

2 ____________/25

3 ____________/30

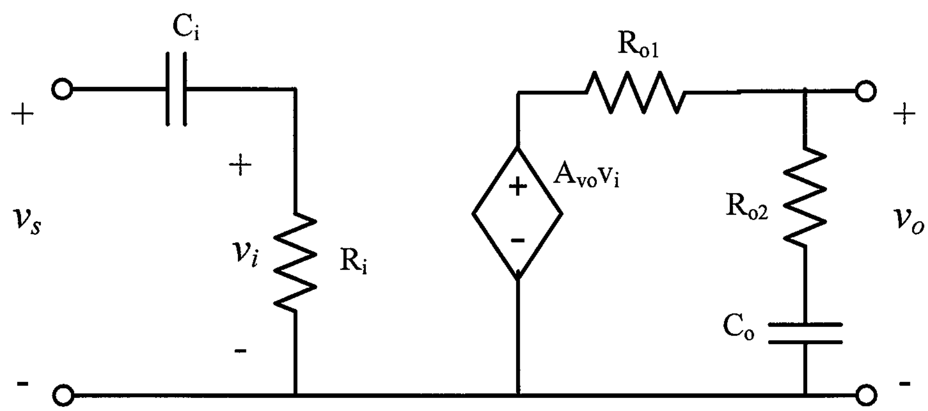
4 ____________/20

Total ____________/100

1. (**25 points**) The amplifier circuit model below has a magnitude Bode plot whose straight-line approximation is shown on the next page. The Bode plot is for the transfer function $T(\omega) = V_o/V_s$.

a) Find values for the circuit components that will produce the Bode plot shown.

b) Find the transfer function for the circuit, and evaluate it at one frequency (your choice) to show that it agrees with the Bode plot.



$$V_o = A_{vo} V_i \frac{R_{o2} + 1/j\omega C_o}{R_{o1} + R_{o2} + 1/j\omega C_o} \qquad V_i = V_s \frac{j\omega C_i R_i}{1 + j\omega C_i R_i}$$

$$T(\omega) = \frac{V_o}{V_s} = A_{vo} \frac{j\omega C_i R_i}{1 + j\omega C_i R_i} \frac{1 + j\omega C_o R_{o2}}{1 + j\omega C_o (R_{o1} + R_{o2})}$$

Bode plot has 20 dB/dec slopes with...

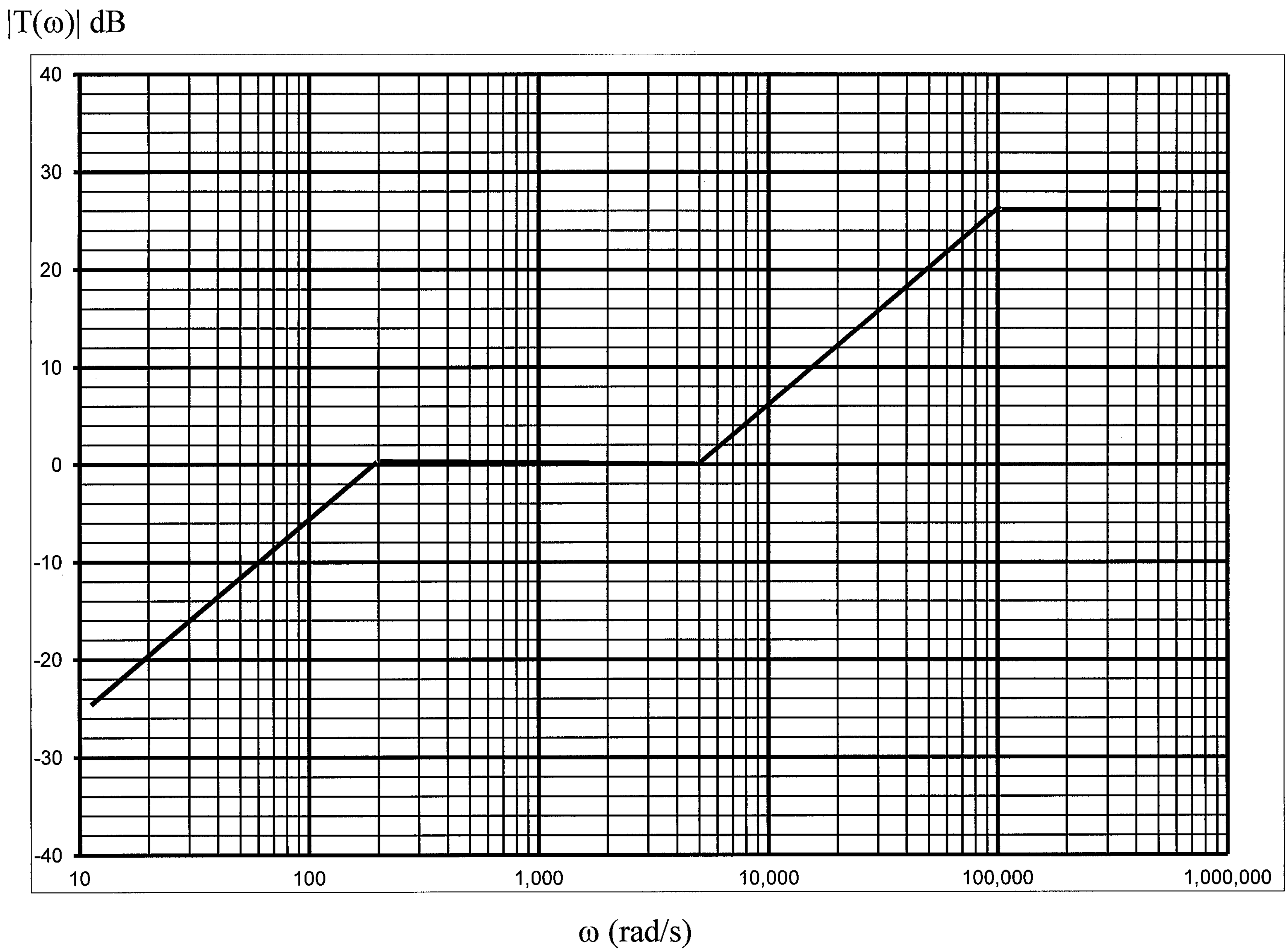
zeroes @ 0, 5000 rad/s

poles @ 200, 100 000 rad/s

We set $1/C_o R_{o2} = 5000$ rad/s $\qquad 1/C_i R_i = 200$ rad/s

$1/C_o(R_{o1} + R_{o2}) = 100\,000$ rad/s

Uh-oh... we have a problem here... ↗ p. 5



|T(ω)| dB
40
30
20
10
0
-10
-20
-30
-40
10
100
1,000
10,000
100,000
1,000,000
ω (rad/s)

Room for Extra Work

The zero at 5000 rad/s along with the pole at 100000 rad/s means that $R_{o1} + R_{o2} < R_{o2} \Rightarrow R_{o1}$ is negative. While this is technically possible, it's not in the spirit of this type of problem. The resolution is to change the Bode plot to show a pole at 5000 rad/s and a zero at 100000 rad/s. We will not do that here, but instead go ahead with it as is.

Choose $C_o = 0.1\,\mu F \Rightarrow R_{o2} = \frac{1}{10^{-7}(5000)} = 2\,k\Omega$

a) Then $1/C_o(R_{o1}+R_{o2}) = 100\,000$ rad/s $\Rightarrow R_{o1} + R_{o2} = 100\,\Omega$

$\Rightarrow R_{o1} = -1900\,\Omega$

Choose $C_i = 0.1\,\mu F \Rightarrow R_i = 50\,k\Omega$

P 5'

b) We note that as $\omega \to \infty$, $T(\omega) \to A_{vo} \frac{R_{o2}}{R_{o1}+R_{o2}}$

$T(\omega) \to A_{vo} \frac{2000}{100} = 26\,dB = 19.95$ V/V

$\therefore A_{vo} = 19.95 \frac{100}{2000} \cong 1.0$

$T(\omega = 1000$ rad/s$) \sim 0\,dB$ ✓

Room for Extra Work

There is another possibility here, which is to reassign the poles:

$$1/C_o(R_{o1}+R_{o2}) = 200\ \tfrac{rad}{s} \Rightarrow R_{o1}+R_{o2} = 50\ k\Omega$$

$$\underline{R_{o2} = 2\ k\Omega} \Rightarrow \underline{R_{o1} = 48\ k\Omega}$$

$$1/C_i R_i = 500\ \tfrac{rad}{s} \Rightarrow \underline{R_i = 20\ k\Omega}$$

We can keep $C_i = C_o = 0.1\ \mu F$.

Now $$T(\omega \to \infty) = A_{vo}\frac{R_{o2}}{R_{o1}+R_{o2}} = A_{vo}\frac{2000}{50000} = 19.95\ V/V$$

$$\Rightarrow \underline{A_{vo} = 498.8}$$

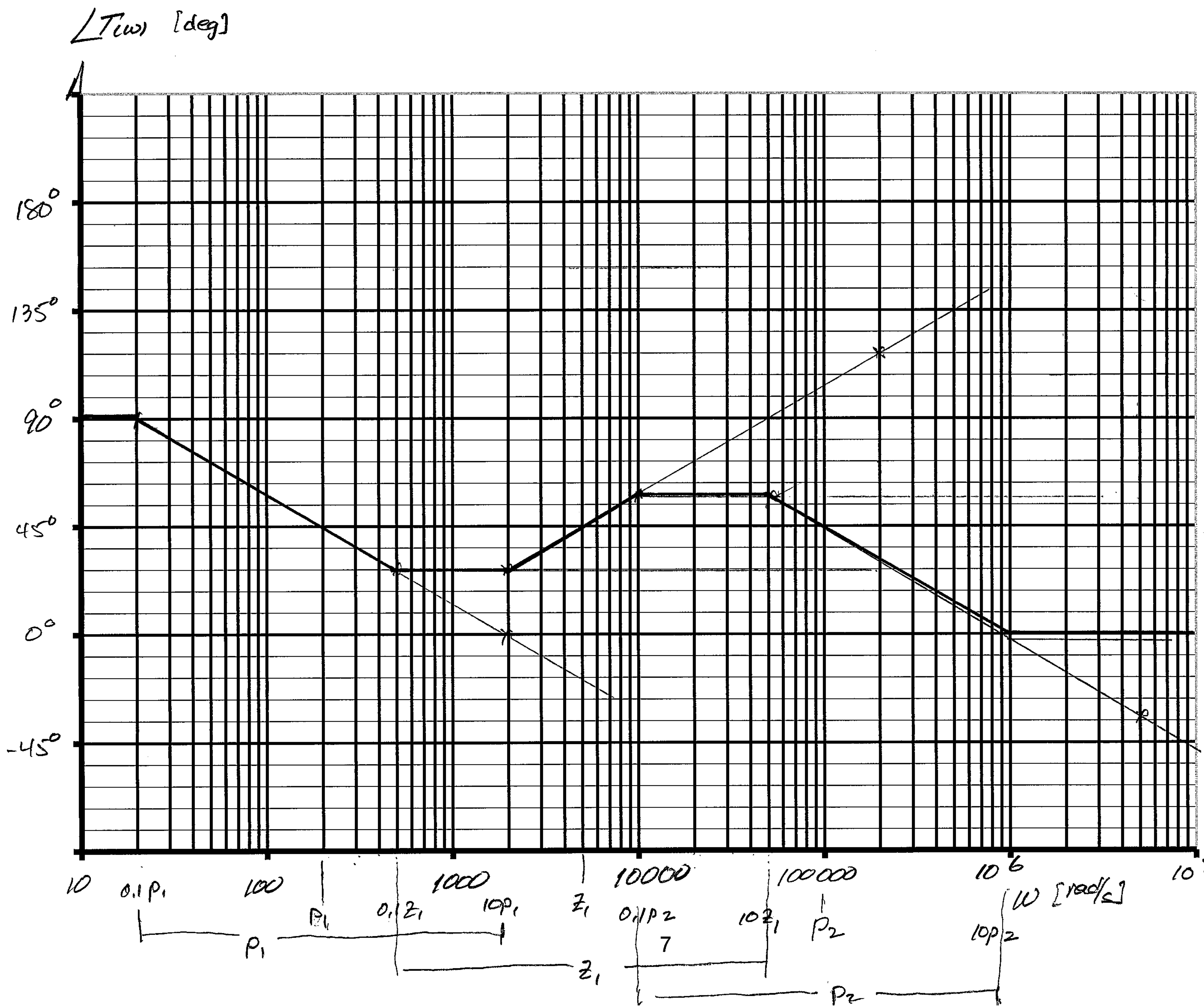
2. (**25 points**) On the graph paper given on the next page, plot the phase Bode plot for the circuit given in Problem 1

For $\omega \to 0$, $T(\omega) \to j\omega C_i R_i \Rightarrow \angle T(\omega) \to 90°$.

We have indicated poles and zeroes, and 10x and 0.1x these values on the graph.

$z_1 = 5000$ rad/s $p_1 = 200$ rad/s $p_2 = 100\,000$ rad/s

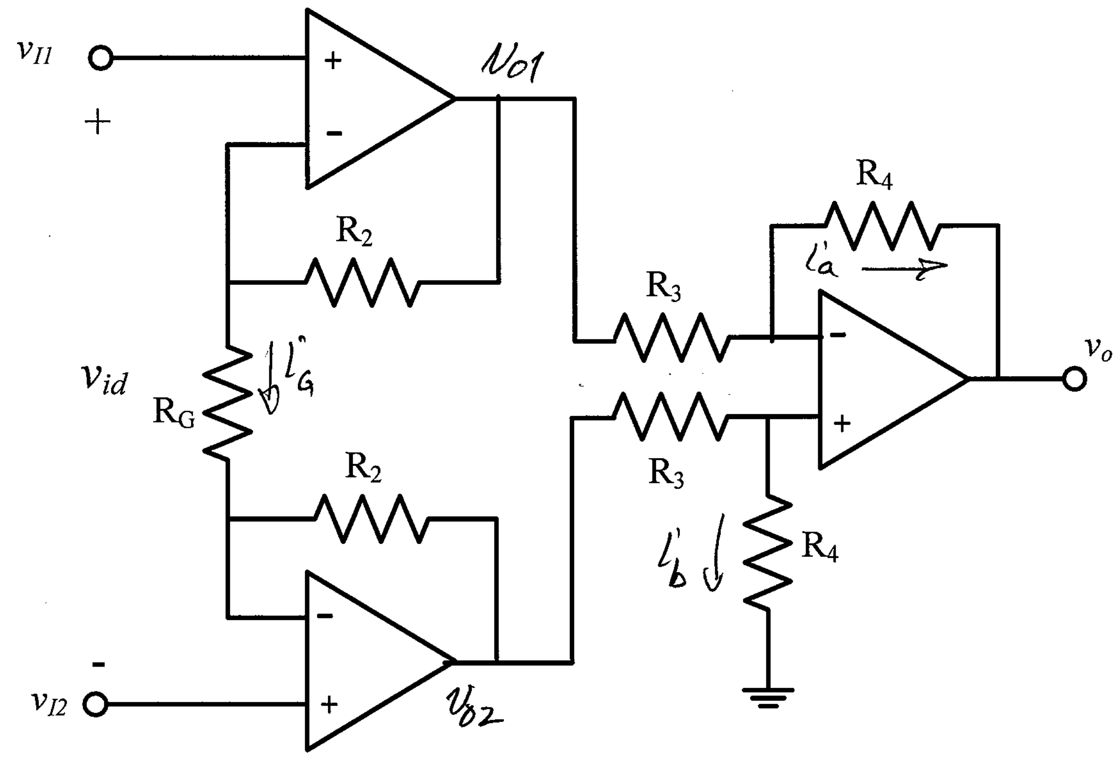
For $\omega \to \infty$ $\angle T(\omega) \to 0°$ so this looks ok!


∠T(ω) [deg]
180°
135°
90°
45°
0°
-45°
10
0.1p₁
100
p₁
0.1z₁
1000
10p₁
z₁
0.1p₂
10000
10z₁
100000
p₂
10⁶
10p₂
10⁷
ω [rad/s]
p₁
z₁
7
p₂

3. (**30 points**) The circuit below is a differential amplifier.

a) Find the output voltage v_o in terms of the resistances and the differential voltage $v_{id} = v_{I1} - v_{I2}$.

b) By replacing one of the resistors in this circuit with a 10 kΩ potentiometer in series with a fixed resistor, we can make the gain v_o/v_{id} variable over the range 2 to 100. Make this replacement and choose resistance values to realize gain over the indicated range.



a)

All op amps have negative feedback, so...

$$v_{o1} - v_{I1} - i_G R_2 = 0 \quad \Rightarrow \quad v_{o1} = v_{I1} + i_G R_2$$

$$v_{o2} - v_{I2} + i_G R_2 = 0 \quad \Rightarrow \quad v_{o2} = v_{I2} - i_G R_2$$

$$i_G = \frac{v_{I1} - v_{I2}}{R_G} = \frac{v_{id}}{R_G}$$

↗ p.9

$$v_{o1} - v_{o2} = v_{I1} - v_{I2} + 2 i_G R_2$$
$$= v_{id} + 2 \frac{v_{id}}{R_G} R_2$$

Room for Extra Work

$$i_a' = \frac{v_{O1} - v_O}{R_3 + R_4} \qquad i_b' = \frac{v_{O2}}{R_3 + R_4}$$

KVL $\quad v_O - i_b' R_4 + i_a' R_4 = 0$

$$v_O = v_{O2} \frac{R_4}{R_3 + R_4} - (v_{O1} - v_O) \frac{R_4}{R_3 + R_4}$$

$$v_O \left(1 - \frac{R_4}{R_3 + R_4}\right) = (v_{O2} - v_{O1}) \frac{R_4}{R_3 + R_4}$$

$$v_O \underbrace{\left(\frac{R_4}{R_3 + R_4} - 1\right)}_{= R_3/(R_3+R_4)} = v_{id} \left(1 + 2\frac{R_2}{R_G}\right) \frac{R_4}{R_3 + R_4}$$

$$\Rightarrow \boxed{\frac{v_O}{v_{id}} = \left(1 + 2\frac{R_2}{R_G}\right) \frac{R_4}{R_3}}$$

b) We want to replace R_G with 10 kΩ variable + R_{fixed},
Set $R_4 = R_3$ and R_G to it's lowest value: R_{fixed}

$$\frac{v_O}{v_{id}} = 100 = \left(1 + 2\frac{R_2}{R_{fixed}}\right) \cdot 1$$

$$\therefore \frac{R_2}{R_{fixed}} = \frac{99}{2}$$

P 9'

Room for Extra Work

Now set R_G to it's highest value: $10k\Omega + R_{fixed}$.

$$\frac{v_o}{v_{id}} = 2 = \left(1 + 2\frac{R_2}{10k + R_{fixed}}\right) \cdot 1$$

$$\therefore \frac{R_2}{10000 + R_{fixed}} = \frac{1}{2}$$

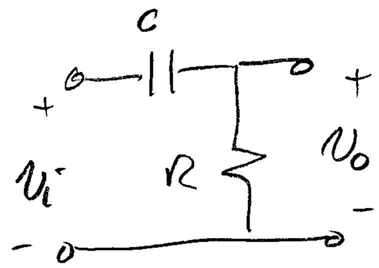
Solving gives $R_{fixed} = 1.02\ k\Omega$

$R_2 = 50.5\ k\Omega$

4. (**20 points**) Using any components you choose, design a high-pass filter with the following specifications.

i) breakpoint at ω_0 = 200 rad/s
ii) gain at high frequency ($\omega >> \omega_0$) of 20 dB.

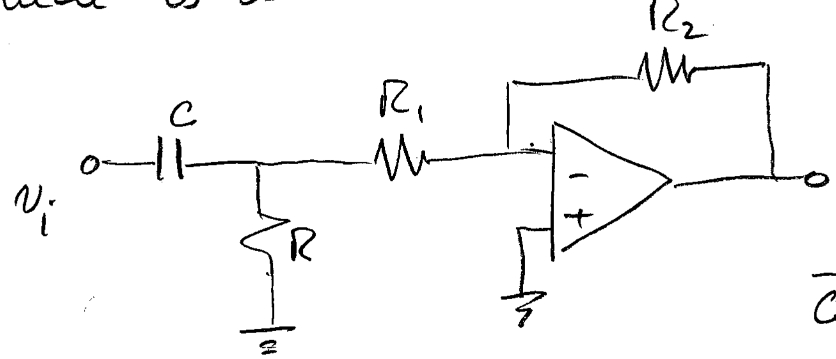
There is more than one possibility...



$$T(\omega) = \frac{j\omega CR}{1+j\omega CR}$$

This has the high-pass configuration with a breakpoint at 1/CR.

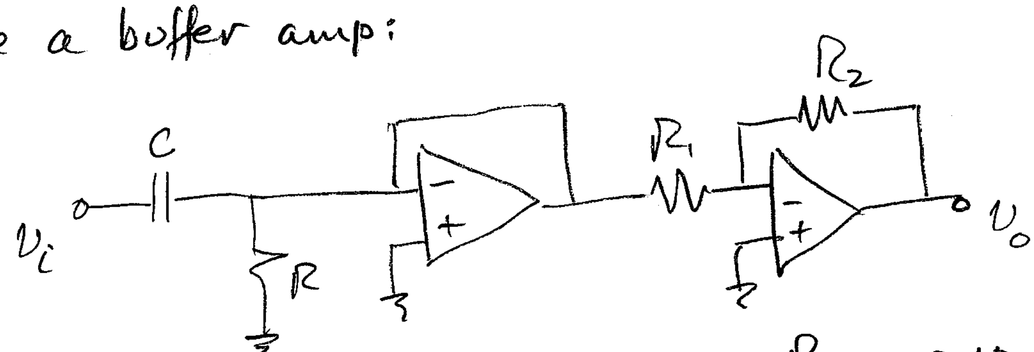
If we use this, we will need to amplify it by 10x, which is 20 dB:



$$R_2/R_1 = 10$$

$$\frac{1}{C(R_1 \| R)} = 200 \text{ rad/s}$$

If we want to isolate the filter first, we can use a buffer amp:



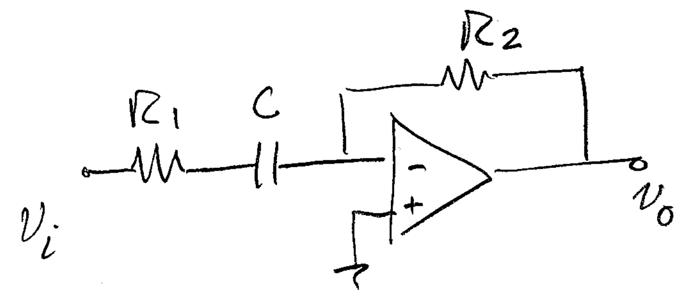
$$R_2/R_1 = 10$$

$$1/RC = 200 \text{ rad/s}$$

p 2.

Room for Extra Work

We could also go this way:



$$T(\omega) = -\frac{R_2}{R_1 + 1/j\omega C}$$

$$= -\frac{j\omega C R_2}{1 + j\omega C R_1}$$

For $\omega \to \infty$, $T(\omega) \to -\frac{R_2}{R_1}$,

so we choose $R_2/R_1 = 10$ and $1/CR_1 = 200$ rad/s.