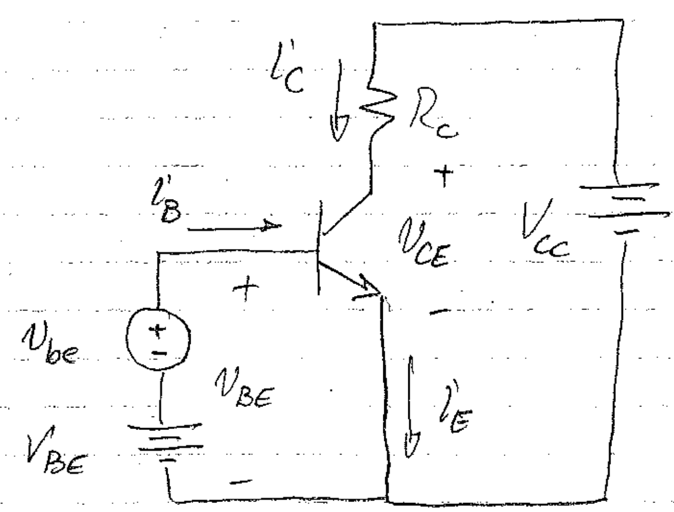
THE BJT AS AN AMPLIFIER

IDEA: If we bias the BJT into the linear (forward-active) region, we can superimpose an ac signal at the input and obtain an amplified version at the output.

NOTE: For most amplifier applications, we want the output to be simply a multiple of the input, that is, we want linear response. If the BJT is not properly biased or the signal is too large, we will get non-linear behavior; that is, the output will be a distorted copy of the input.

CONCEPTUAL ANALYSIS:



DC CONDITIONS:

$$I_C = I_S e^{V_{BE}/V_T} \quad (V_{BE} \gg V_T)$$

$$I_E = I_C/\alpha$$

$$I_B = I_C/\beta$$

$$V_C = V_{CC} - I_C R_C$$

Now consider current components when a signal is present:

$$v_{BE} = v_{be} + V_{BE}$$

COLLECTOR CURRENT

$$i_C = I_S e^{V_{BE}/V_T} e^{v_{be}/V_T}$$
$$= I_C e^{v_{be}/V_T}$$

For $v_{be} \ll V_T$ (say $\sim 10\,mV$),

$$i_C \simeq I_C (1 + v_{be}/V_T)$$
$$= I_C + \frac{I_C}{V_T} v_{be}$$

So the ac component is

$$i_c = \frac{I_C}{V_T} v_{be} \equiv g_m v_{be}$$

The TRANSCONDUCTANCE is $g_m \equiv \frac{I_C}{V_T}$

INPUT RESISTANCE AT THE BASE

We also have

$$i_B = \frac{i_C}{\beta} = \frac{I_C}{\beta} + \frac{I_C}{\beta V_T} v_{be}$$

and the ac component is

$$i_b = \frac{g_m}{\beta} v_{be}$$

We define $\quad r_\pi \equiv \dfrac{v_{be}}{i_b} = \beta / g_m$

which is the small-signal input resistance, looking into the base.

We can also write $\quad r_\pi = V_T / I_B \quad$ since

$$\frac{\beta}{g_m} = \frac{I_C/I_B}{I_C/V_T} = V_T/I_B$$

INPUT RESISTANCE AT THE EMITTER

$$i_E = \frac{i_C}{\alpha} = \frac{I_C}{\alpha} + \frac{i_c}{\alpha}$$

The ac component is

$$i_e = \frac{i_c}{\alpha} = \frac{I_C}{\alpha V_T} v_{be}$$

$$= \frac{I_E}{V_T} v_{be}$$

We define the small signal resistance between base and emitter, looking into the emitter, as

$$r_e \equiv \frac{V_T}{I_E} = \frac{\alpha}{g_m} \simeq \frac{1}{g_m}$$

We can also show that

$$r_\pi = (\beta + 1) r_e$$

VOLTAGE GAIN

If we take as output the voltage v_{CE}, then

$$v_{CE} = V_{CC} - i_C R_C$$

ac component is

$$v_{ce} = -i_c R_C = -g_m v_{be} R_C$$

So

$$A_V \equiv \frac{v_{ce}}{v_{be}} = -g_m R_C$$

SMALL SIGNAL EQUIVALENT CIRCUIT MODELS

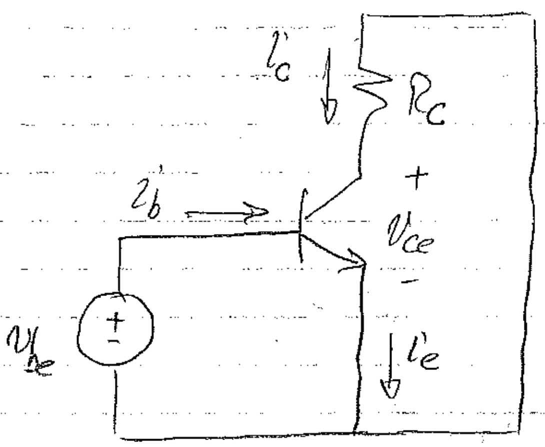
We can use these calculations to construct equivalent circuits that describe the BJT's response to ac circuits.

AC-CIRCUIT: The "ac-portion" of the circuit shown above is

$i_c = g_m v_{be}$

$i_b = v_{be}/r_\pi$

$i_e = v_{be}/r_e$



NOTE: DC sources were removed (de-activated). In other words,

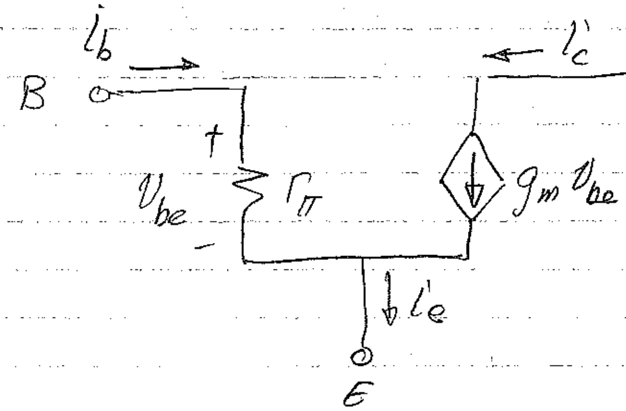
$V_{DC} \longrightarrow 0$ (short circuit)

$I_{DC} \longrightarrow \infty$ (open circuit)

We can draw an equivalent circuit to replace the BJT as follows.

(The equivalent circuit applies to the BJT only. It does not include external components like v_{be}, R_C, ...)

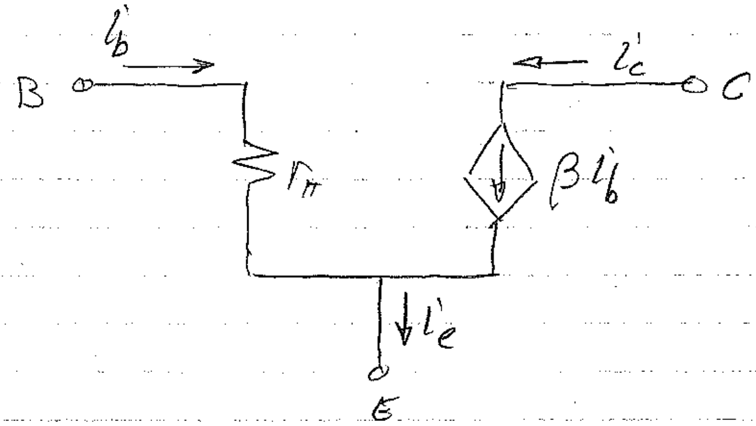
HYBRID - PI MODEL



$g_m = I_C / V_T$

$r_\pi = \beta / g_m$

Alternatively...



NOTES:

- Although r_e is not included, the model correctly predicts i_e:

$$i_e = \frac{v_{be}}{r_\pi} + g_m v_{be} = \frac{v_{be}}{r_\pi}(1 + g_m r_\pi)$$

$$= \frac{v_{be}}{r_\pi}(1+\beta) = v_{be} \Big/ \left(\frac{r_\pi}{1+\beta}\right)$$

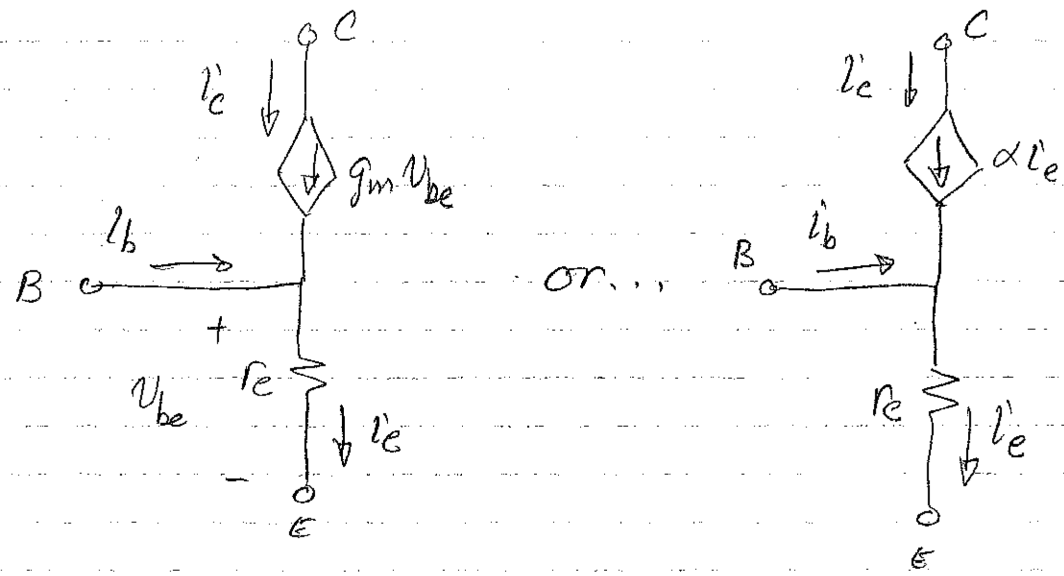
$$= v_{be}/r_e$$

- The parameters g_m, r_π depend on the biasing conditions through I_C. So if biasing conditions change, they have to be re-calculated.

- The models apply to npn and pnp BJTs without any modifications.

T MODEL

Sometimes it is convenient to use a model that contains r_e explicitly.



Although r_π is not included, the model correctly predicts i_b. We can show that

$$i_b = v_{be}/r_e - g_m v_{be}$$

$$= v_{be}/r_\pi$$

SMALL SIGNAL MODELING

We now have the following prescription:

Given a BJT amplifier circuit...

1. Find the dc operating point and I_C in particular.

   (We assume the BJT is in the linear mode.)

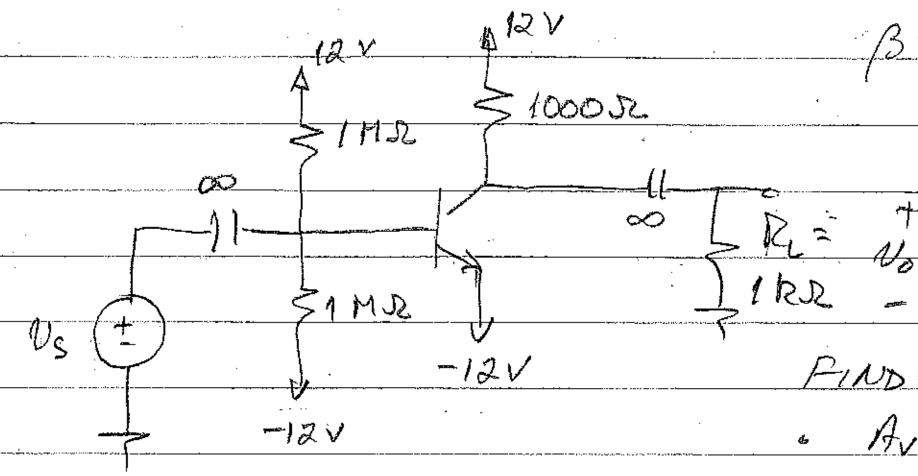
2. Calculate the small signal parameters r_π, g_m, r_e.

3. De-activate dc sources.

4. Replace the BJT with the Hybrid-π or T-Model.

5. Perform the ac analysis: voltage gain, input impedance, ...

EXAMPLE (From Shattuck's notes)



$\beta = 200$

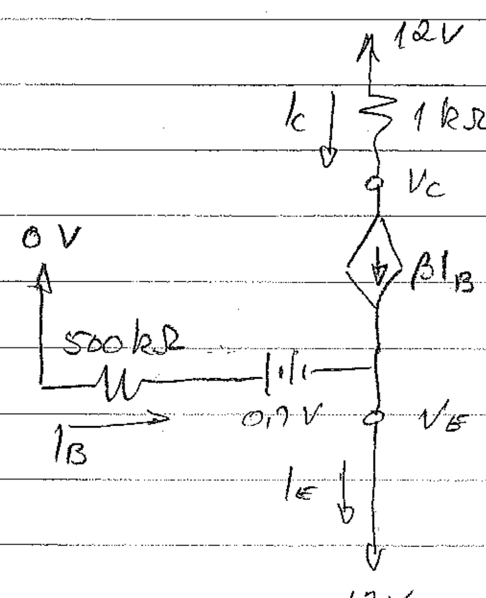
FIND:

- $A_V = v_o / v_s$
- Input/Output Impedances

DC Analysis

- Thevenin equivalent at base
- C → open circuit
- ac source v_s goes away (not relevant for dc)

Assume active region



$I_B = \frac{12 - 0.7}{500\ k} = 22.6\ \mu A$

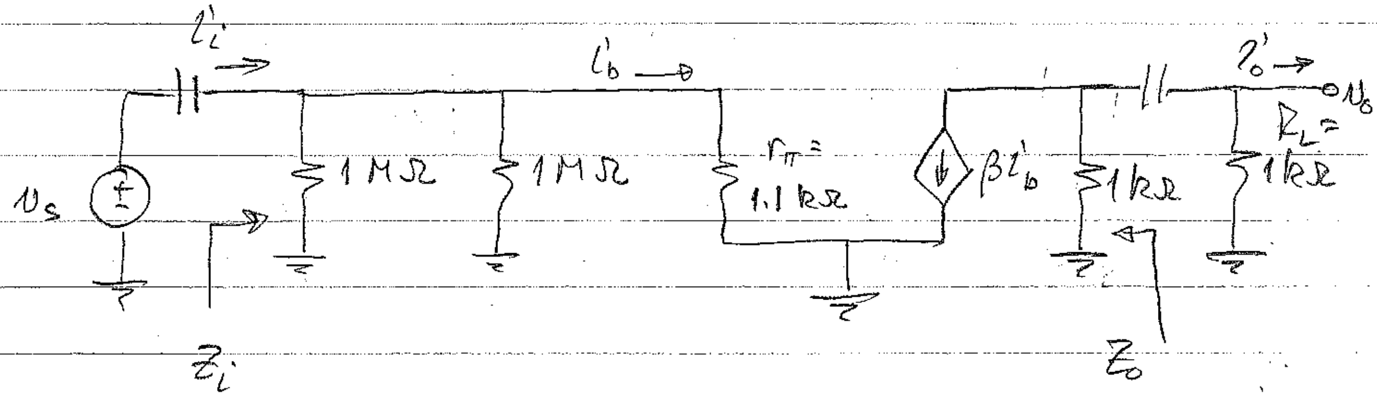
$I_C = \beta I_B = 4.52\ mA$

$V_C = 12 - 1000\, I_C = 7.48\ V$

So ACTIVE region is correct.

Ac Analysis

- $C \rightarrow$ short circuit $\left(\frac{1}{j\omega C} \rightarrow 0 \text{ since } C \rightarrow \infty \right)$
- dc sources $\rightarrow$ short



$$r_\pi = V_T / I_B = \frac{25\,mV}{22.6\,\mu A} = 1.1\ k\Omega$$

Analysis : $i_b' = v_s / r_\pi$

$$v_o = -\beta i_b' \cdot (1k\Omega \,//\, 1k\Omega)$$

$$= -\frac{200\, v_s}{r_\pi} 500$$

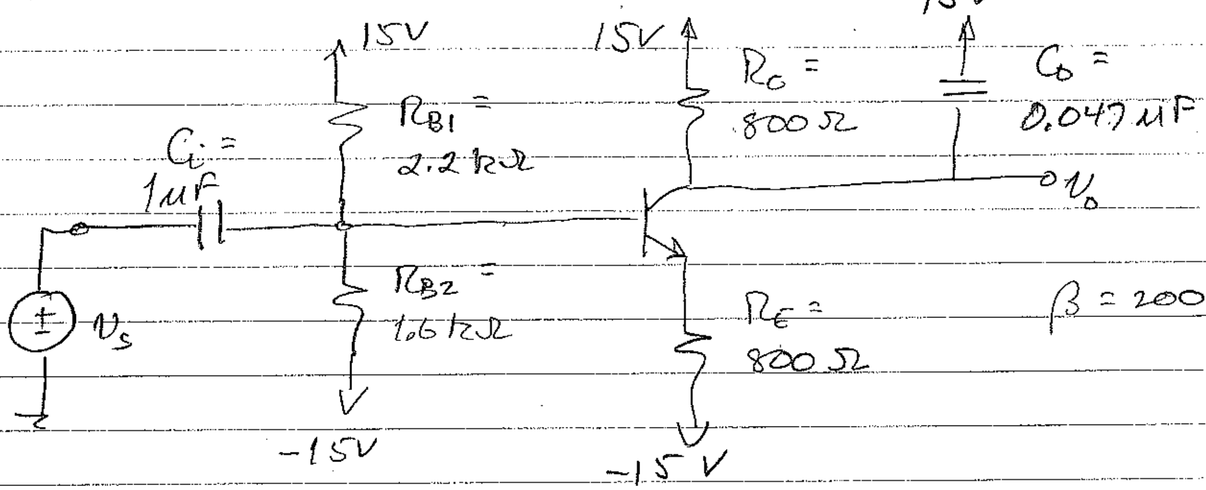
$$\therefore \quad A_V = \frac{v_o}{v_s} = -200(500)/r_\pi = -91$$

$$Z_i = \left(\frac{1}{1M} + \frac{1}{1M} + \frac{1}{r_\pi} \right)^{-1} \simeq r_\pi = 1.1\ k\Omega$$

$$Z_o = 1\ k\Omega$$

EXAMPLE: Common Emitter Amplifier

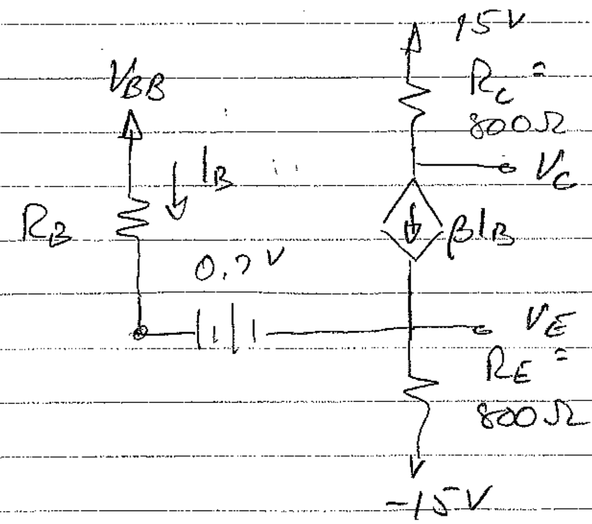
Find $T(\omega) = \bar{V}_o/\bar{V}_s$. Find the gain in the pass band
Find the 3dB band width.



This is a small-signal analysis problem, but we need a dc analysis to find the necessary ac model parameters. We should also check to be sure we are biased in the linear region.

DC: Capacitors $\rightarrow$ open.

Thevenizing the base, we have:



$$V_{BB} = 30 \cdot \frac{R_{B2}}{R_{B1}+R_{B2}} - 15$$

$$= -2.4\ V$$

$$R_B = R_{B1} \,//\, R_{B2} = 926\,\Omega$$

KVL: $-V_{BB}'' + R_B I_B + 0.7 + (\beta+1) R_E - 15 = 0$

$\Rightarrow \quad +2.4 + 926 I_B + 0.7 + 201(800) = 15$

$I_B = 73.6\ \mu A$ ✓

$V_{CE} = 15 - \beta I_B \cdot R_C - ((\beta+1) I_B R_E - 15)$

$= 15 - 200(73.6\times10^{-6})(800)$

$- [201(73.6\times10^{-6})(800) - 15]$

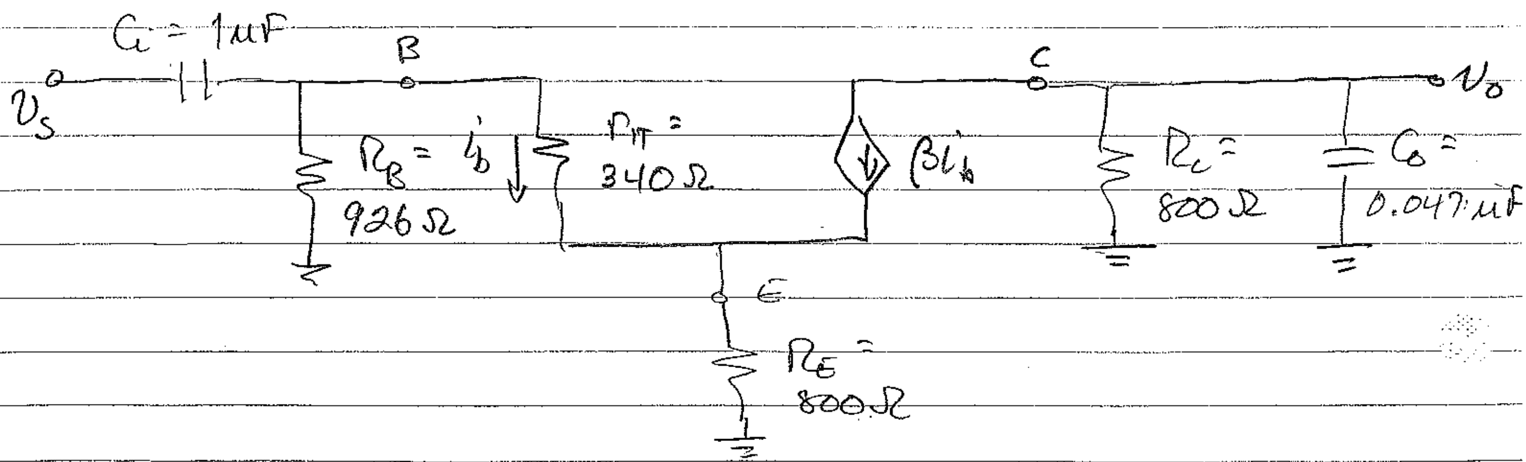
$= 3.23 - (-3.17) = 6.40\ V$ ✓

So we are in fact in the linear region. We can now find

$$r_\pi = \frac{V_T}{I_B} = \frac{25\times10^{-3}}{73.6\times10^{-6}} = 340\ \Omega$$

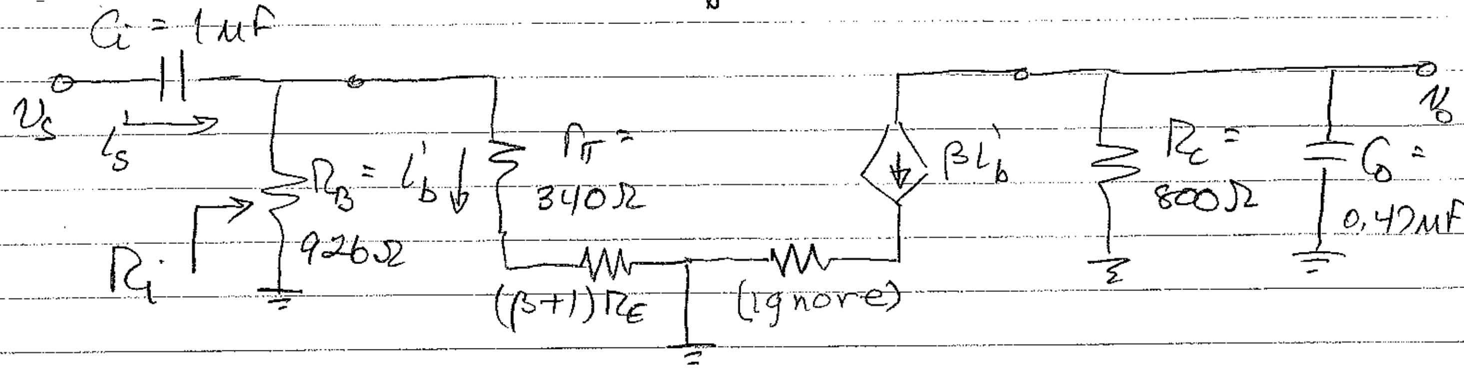
AC MODEL

DC voltage sources $\rightarrow$ short



Miller's Dual is an option here. Let's do that:

Current ratio $J \equiv \frac{\beta i_b'}{i_b'} = \beta$



Let's simplify the input a bit. The resistance R_i is

$$R_i = R_B \,//\, (r_\pi + (\beta+1)R_E)$$

But $r_\pi \lll (\beta+1)R_E$ and $R_B \ll (\beta+1)R_E$ so

$$R_i \simeq R_B \,//\, (\beta+1)R_E \simeq R_B.$$

Actual numbers:

$$R_i = 926 \,//\, (340 + 201 \times 800) = 920.7\,\Omega$$

so we are not far off if we say $R_i \simeq R_B$.

This simplification will help us find i_b ...

Analysis...

$$\bar{V}_o = -\beta \bar{I}_b \left(R_C \,/\!/\, 1/j\omega C_o \right) = -\beta \bar{I}_b \frac{R_C}{1+j\omega C_o R_C}$$

$$\bar{I}_s = \frac{\bar{V}_s}{1/j\omega C_i + R_i} = \frac{j\omega C_i \bar{V}_s}{1+j\omega C_i R_i}$$

But from above we have $R_i \simeq R_B$ so

$$\bar{I}_s = \frac{j\omega C_i \bar{V}_s}{1+j\omega C_i R_B}$$

$$\bar{I}_b = \bar{I}_s \cdot \frac{R_B}{R_B + r_\pi + (\beta+1)R_E} \simeq \bar{I}_s \cdot \frac{R_B}{R_B + (\beta+1)R_E}$$

So $$T(\omega) = \frac{\bar{V}_o}{\bar{V}_s} = -\beta \frac{R_C}{1+j\omega C_o R_C} \cdot \frac{j\omega C_i}{1+j\omega C_i R_B} \cdot \frac{R_B}{R_B+(\beta+1)R_E}$$

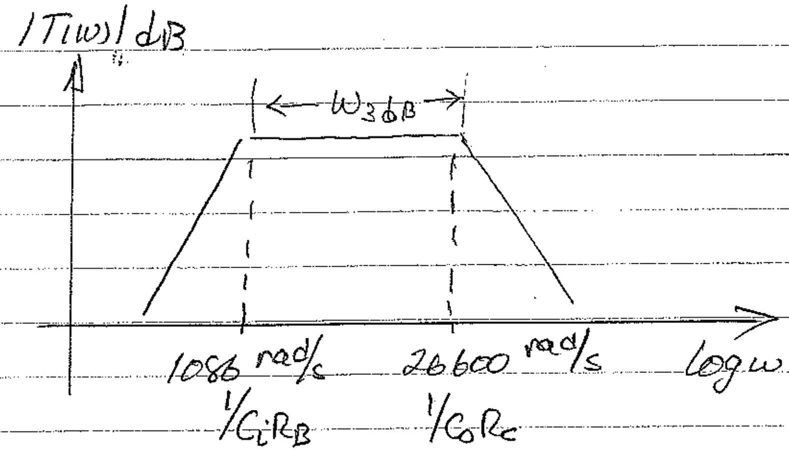
FREQUENCY ANALYSIS.

We have a zero at 0 and poles at

$$p_1 = 1/C_o R_C = 26\,600 \text{ rad/s}$$

$$p_2 = 1/C_i R_B = 1086 \text{ rad/s} \quad (\text{I used } R_B = 921\ \Omega)$$

So we have a pass-band response:



We can define the pass band as the roughly flat region between poles, i.e.

$$\omega_{3dB} = 26,600 - 1086 \simeq 25,500 \text{ rad/s}$$

This is the 3 dB BANDWIDTH.

We can also find the GAIN IN THE PASSBAND. This is the gain in the region where $|T(\omega)|$ is approximately constant.

The brute-force way to do this is to substitute a value of ω in the pass band region into T(ω) and calculate the gain. We choose $\omega_0 = 10,000$ rad/s and find

$$T(\omega = \omega_0) = -200 \, \frac{800}{1 + j(10^4)(4.7\times10^{-8})(800)} \cdot \frac{j10^4(10^{-6})}{1 + j10^4(10^{-6})(921)}$$

$$\times \frac{921}{921 \times (201)(800)}$$

$$= 0.92 \angle 165^\circ \text{ V}$$

$$|T(\omega_0)|_{dB} \cong 0 \text{ dB.}$$

There is a simpler way: Note that in the pass band, $T(\omega)$ is not a function of ω (otherwise it wouldn't be flat!). That means terms containing $j\omega$ are not having an effect. Why?

Look at the terms with $j\omega$ and evaluate at ω_o:

$$\frac{R_C}{1+j\omega C_o R_C} = \frac{800}{1+j\,10^4(4.7\times10^{-8})(800)}$$

$$= \frac{800}{1+j\,(0.38)} \approx \frac{800}{1+j\,0}$$

$$\frac{j\omega_o C_i}{1+j\omega_o C_i R_B} = \frac{j\,(10^4)(10^{-6})}{1+j\,10^4(10^{-6})(921)} = \frac{j\,(0.01)}{1+j\,9.21} \approx \frac{j\,(0.01)}{j\,9.21}$$

More generally

$$\omega_o \ll \frac{1}{C_o R_C} \quad \text{and} \quad \omega_o \gg \frac{1}{C_i R_B}$$

– this is evident from the Bode plot. So...

$$C_o \ll \frac{1}{\omega_o R_C} \quad \text{and} \quad C_i \gg \frac{1}{\omega_o R_B}$$

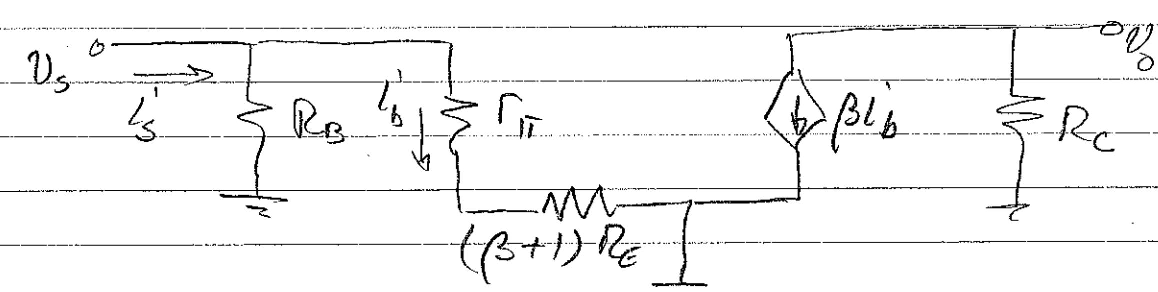
For C_o small, Z_{C_o} large, i.e., C_o is an open ckt.

For C_i large, Z_{C_i} small, i.e. C_i is a short ckt.

We can generalize this: In the passband, capacitors are either short or open (approximately), which is why $T(\omega)$ is not a function of frequency.

But which is which? In our example, C_i must be a short because if it were open, there would be no source. C_o must be open because if it were a short, v_o would be 0.

We now re-do our ac analysis with
$C_i \rightarrow$ short $\quad C_o \rightarrow$ open



$$v_o = -\beta i_b' R_C$$

$$i_s' \approx \frac{v_s}{R_B} \qquad i_b' = i_s' \cdot \frac{R_B}{R_B + (\beta+1)R_E}$$

$$\Rightarrow \quad \frac{v_o}{v_s} = -\beta \frac{R_C}{R_B} \cdot \frac{R_B}{R_B + (\beta+1)R_E} = -\beta \frac{R_C}{R_B + (\beta+1)R_E}$$

$$\frac{v_o}{v_s} = -200 \frac{800}{800 + 201(800)}$$

$$= -0.99 \text{ V/V}$$

This is close to the value we got from plugging ω_0 into $T(\omega)$.

We could also have done this from the transfer function:

$$T(\omega) = \frac{-\beta R_C}{1+j\omega C_o R_C} \cdot \frac{j\omega C_i}{1+j\omega C_i R_B} \cdot \frac{R_B}{R_B+(\beta+1)R_E}$$

But $\omega_0 \ll 1/C_o R_C \Rightarrow 1+j\omega C_o R_C \approx 1$

$\omega_0 \gg 1/C_i R_B \Rightarrow 1+j\omega C_i R_B \approx j\omega C_i R_B$

Then

$$T(\omega) \approx \frac{-\beta R_C}{1} \cdot \frac{j\omega C_i}{j\omega C_i R_B} \cdot \frac{R_B}{R_B+(\beta+1)R_E}$$

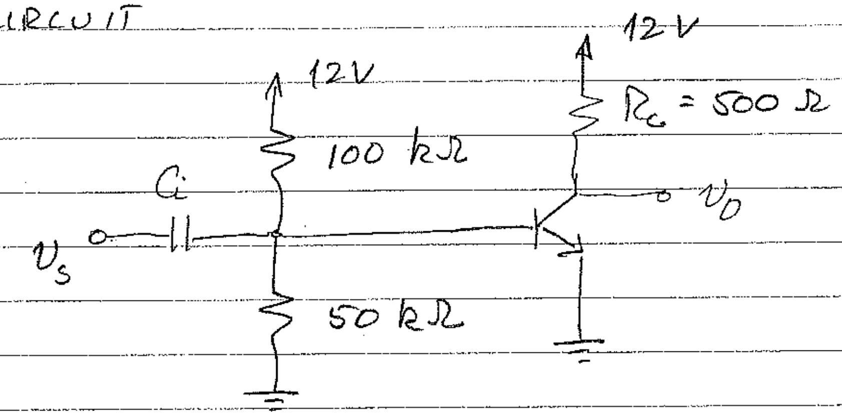
$$= \frac{-\beta R_C}{R_B+(\beta+1)R_E}$$

... as we got before!

LOAD LINE ANALYSIS

We will do a load-line analysis on a simple BJT to see how this all fits together. We will use a plot of i_C vs. v_{CE} from Sedra & Smith 5 ed. to record the results.

THE CIRCUIT

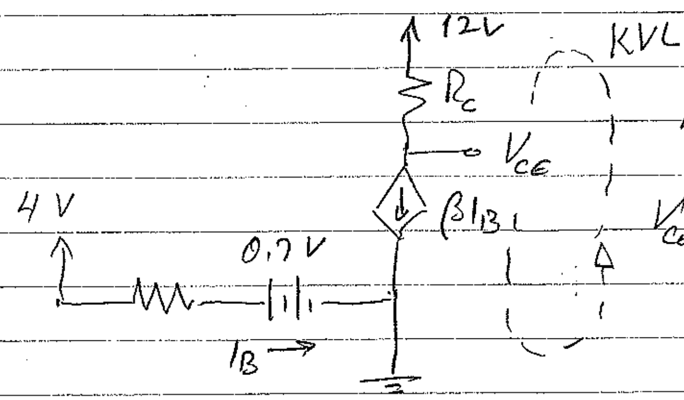


$\beta = 60$

$V_{CE,sat} = 0.3V$

DC ANALYSIS

Thevenizing the base, we have ($C \to$ open)



$$I_B = \frac{4-0.7}{33.33k} = 99\mu A$$

$$V_{CE} = 12 - 500 \cdot \beta I_B = 9.0 V$$

We can now go to the i_C - v_{CE} plot and locate our Q-point. It is at $I_B = 99 \mu A$ and $V_{CE} = 9.0 V$.

Also, $I_C = \beta I_B = 5.94 mA$. We have also indicated this on the plot.

LOAD LINE

The load line is based on the KVL at the output - that is, an equation relating I_C to V_{CE}.

KVL (indicated as dashed line on dc circuit):

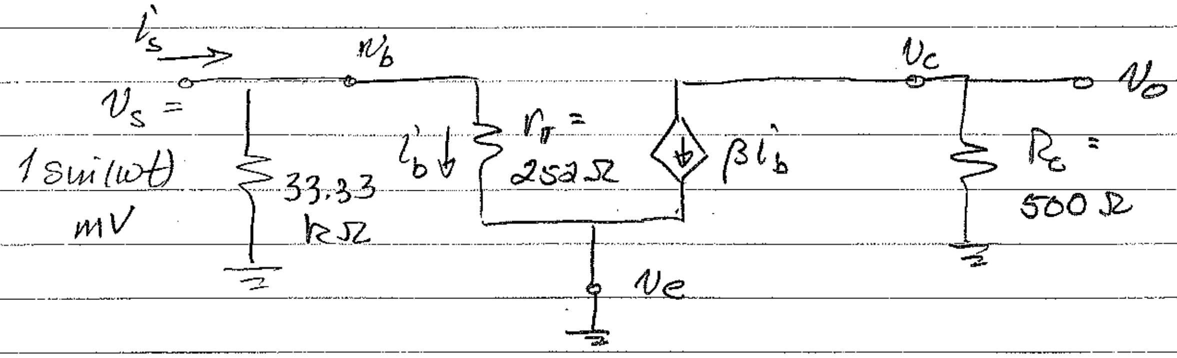
$$-12 + I_C \cdot R_C + V_{CE} = 0$$

$$\Rightarrow \quad I_C = \frac{12 - V_{CE}}{R_C}$$

This is a straight line of slope $-1/R_C$, y-intercept $12/R_C$, and x-intercept 12V. These intercepts are indicated on the plot

AC ANALYSIS

We have $r_\pi = \frac{V_T}{I_B} = \frac{25\times10^{-3}}{99\times10^{-6}} = 252\,\Omega$



We have assume large C_i so that for any ω we will consider, $C_i \rightarrow$ short.

We will take $v_s = 1 \sin(\omega t)$ mV

Now $\quad i_b' = \underbrace{\dfrac{v_s}{R_B \parallel r_\pi}}_{i_s'} \cdot \dfrac{R_B}{R_B + r_\pi} \simeq \dfrac{v_s}{r_\pi}$

The approximation is based on the fact $R_B \gg r_\pi$.

At max/min $v_s = \pm 1\,mV$, we have

$$i_b' = \pm \frac{0.001}{r_\pi} = \pm 3.97\,\mu A \simeq \pm 4\,\mu A$$

So the total signal i_B' is

$$i_B' = i_b' + I_B = \pm 4\,\mu A + 99\,\mu A$$

$$= 103\,\mu A,\ 95\,\mu A$$

Also $\quad i_c' = \beta i_b' = 240\,\mu A = 0.240\,mA$

$$\Rightarrow \quad i_C' = \pm 0.240 + 5.94\ mA$$

$$= 6.18\,mA,\ 5.70\,mA$$

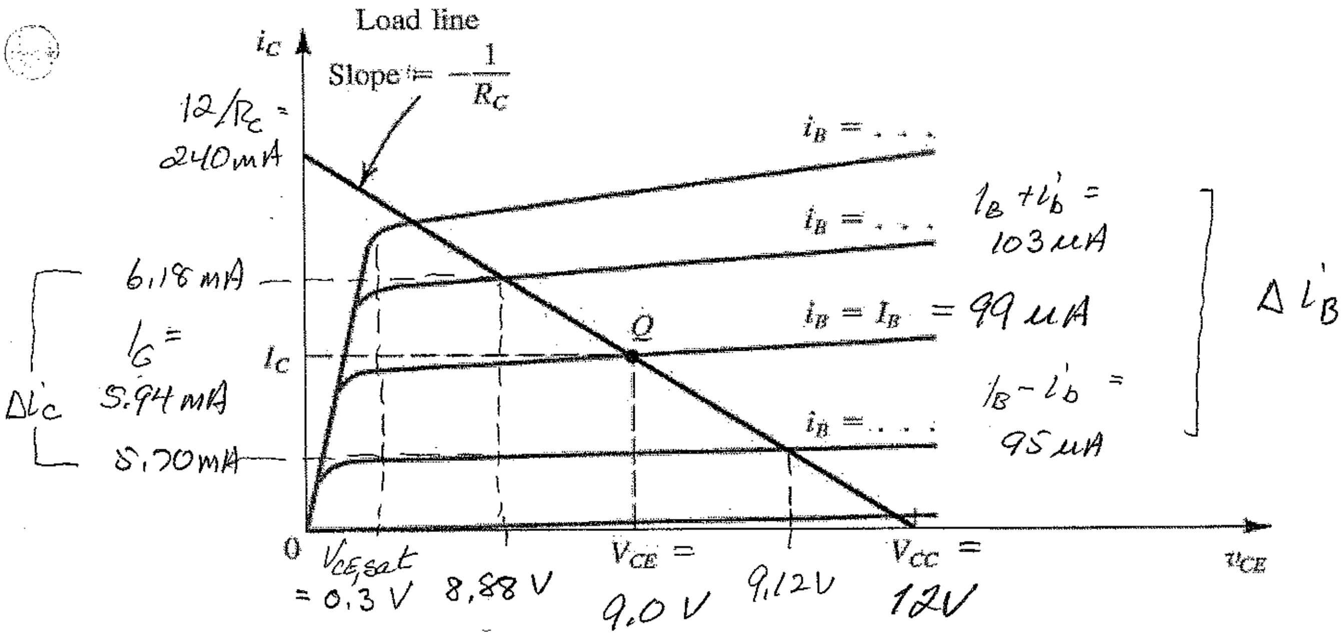
$$v_{ce} = \beta i_b' \cdot 500 = 0.12\,V$$

$$\Rightarrow \quad v_{CE} = \pm 0.12 + 9\ V$$

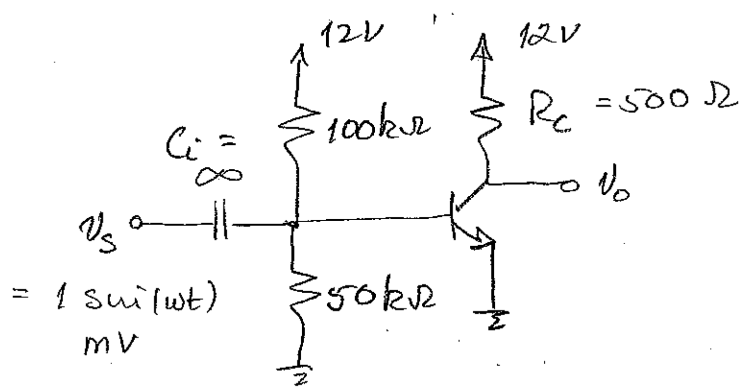
$$= 9.12\,V,\ 8.88\,V$$

These points are also indicated on the plot.

This analysis is valid for any BJT operating in linear mode as an amplifier. We also note that our signal causes v_{CE} to move closer to 12V (which would be cut-off) and to 0.3V (which would be saturation). So if v_s is too large, or β is too large, the signal will drive the BJT out of the linear region, in which case we will get distortion.

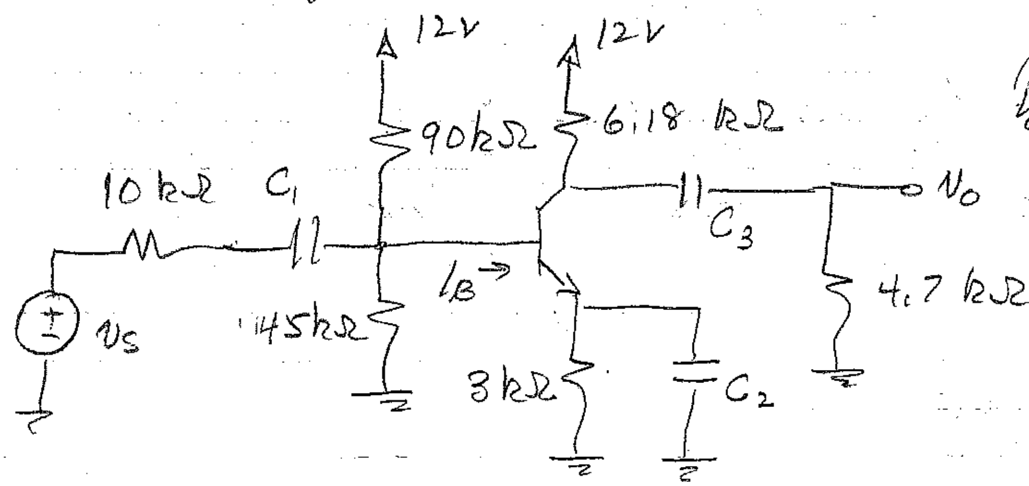


$$\frac{\Delta i'_C}{\Delta i'_B} = \frac{0.240}{0.004} = 60$$



EXAMPLE

Find the gain in the passband.



$\beta = 100$

$V_{CESAT} = 0.3\ V$

DC ANALYSIS

C → OPEN CKT

$$V_{BB} = 12 \cdot \frac{45}{45+90} = 4\ V$$

$$R_B = 90k \parallel 45k = 30\ k\Omega$$

$$\therefore \quad I_B = \frac{4-0.7}{30k\Omega + (101)\,3k\Omega} = 0.01\ mA$$

$$\Rightarrow \quad I_C = 1\ mA$$

$$g_m = I_C/V_T = 40\ m\mho$$

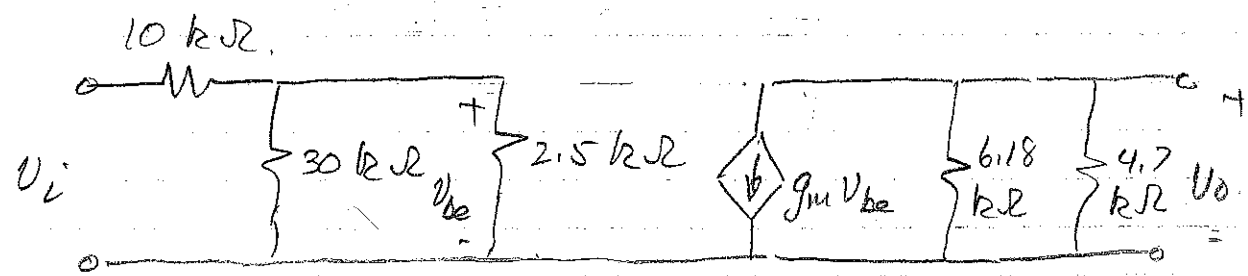
$$r_\pi = \beta V_T / I_C = 2.5\ k\Omega$$

AC ANALYSIS

In the passband, $C_1 \to$ SHORT (otherwise the input is cut off), $C_2 \to$ SHORT, $C_3 \to$ SHORT (otherwise the output is cut off).

It's not terribly obvious that C_2 should be SHORT, but this is usually the case in the passband. Typically C_2 is large, so $1/\omega C \to 0$.

MODEL



$$\frac{v_{be}}{v_i} = \frac{2.5k \| 30k}{2.5k\|30k + 10k} = 0.187$$

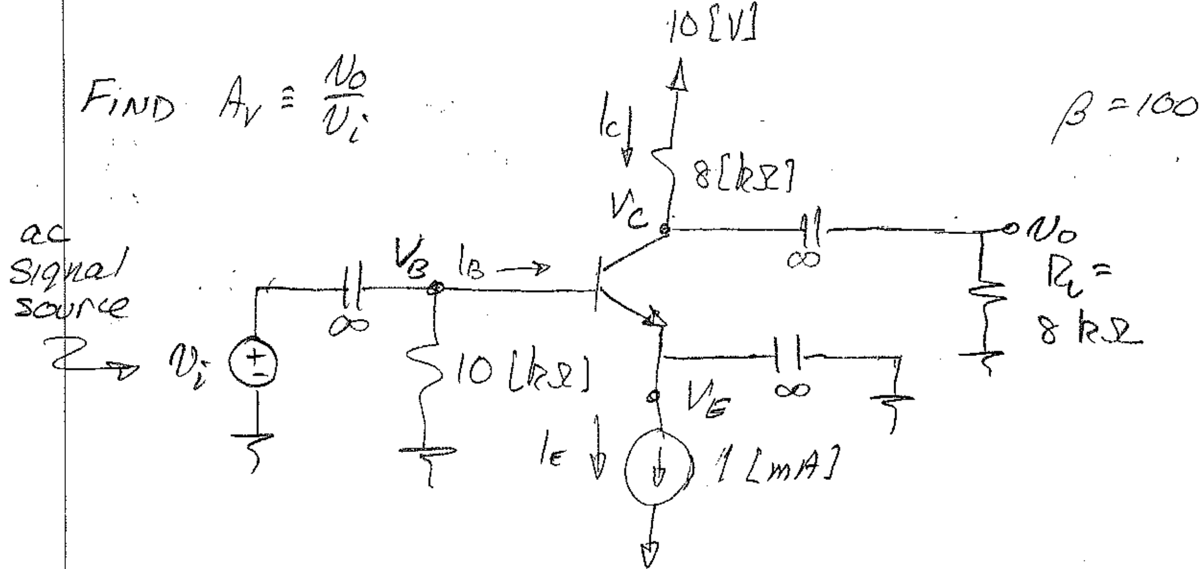
$$\frac{v_o}{v_{be}} = -g_m \; 4.7k \| 6.18k$$

$$\therefore \quad \frac{v_o}{v_i} = -0.187 \cdot 40\times10^{-3} \cdot 4.7k\|6.18k = -20 \text{ V/V}$$

EXAMPLE

FIND $A_V \equiv \frac{v_o}{v_i}$

$\beta = 100$



DC ANALYSIS: ASSUME LINEAR

a) $I_E = 1 \text{ [mA]}$; $I_C = \frac{\beta}{1+\beta} I_E = 0.99 \text{ [mA]}$

$I_B = I_E - I_C = 0.01 \text{ [mA]}$

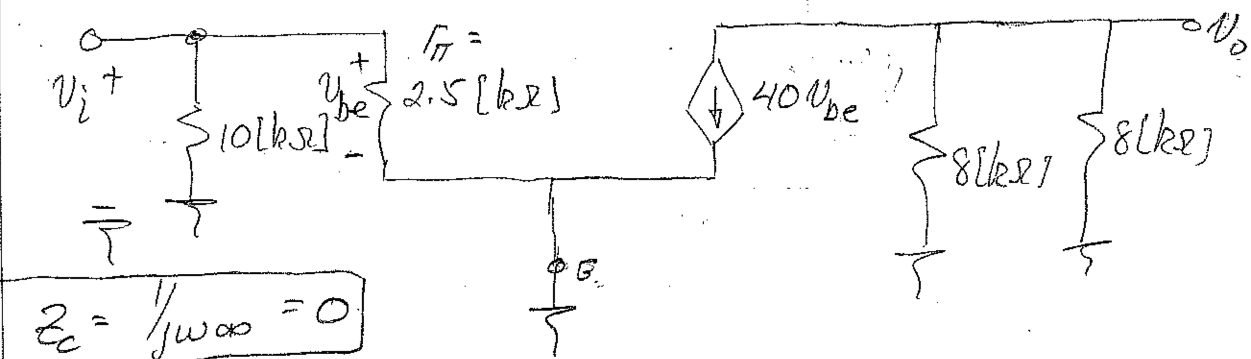
$\therefore$

$$\boxed{\begin{aligned} V_B &= -I_B(10k) = -0.1 \text{ [V]} \\ V_E &= V_B - 0.7 \text{ [V]} = -0.8 \text{ [V]} \\ V_C &= 10 - I_C(8k) = 2.1 \text{ [V]} \end{aligned}}$$

b) $g_m = \frac{I_C}{V_T} = \frac{0.99}{25} = 39.6 \left[\frac{mA}{V}\right]$

$r_\pi = \beta / g_m = 2.5 \text{ [kΩ]}$

c)



$\boxed{Z_c = 1/j\omega\infty = 0}$

<u>NOTE</u>: To an ac signal, +10 V looks like ground; 1 [mA] looks like $R = \infty$.

We say +10 V = SIGNAL GROUND
1 mA = SIGNAL OPEN CKT

Then $v_o = -40 [\frac{mA}{V}] \times 4 [k\Omega]\ v_{be}$

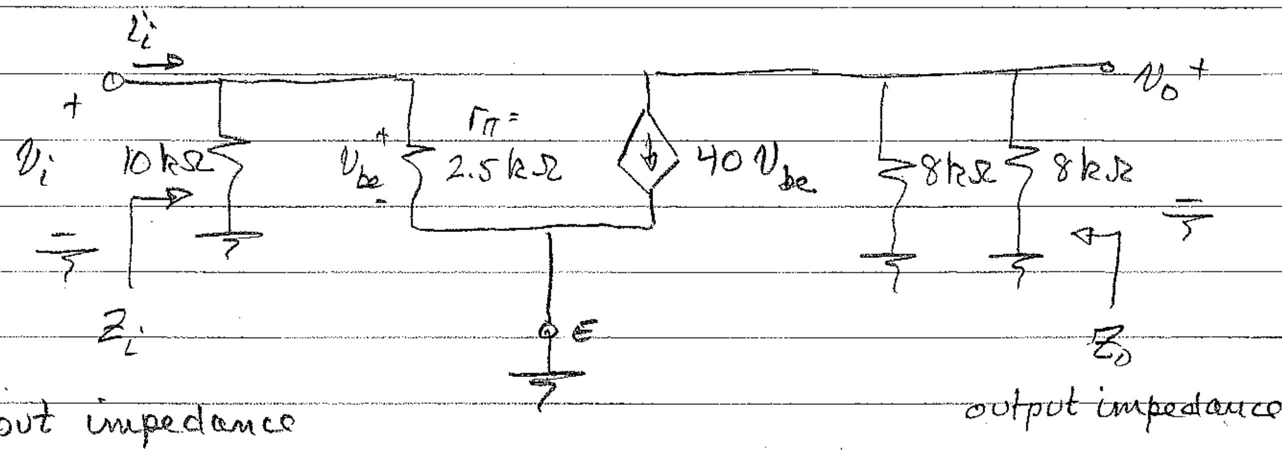
$= -160 [V/V]\ v_{be}$

$v_{be} = v_i$ so

$$\boxed{\frac{v_o}{v_i} = A_v = -160 [V/V]}$$

We can also find input and output impedances. In general, there may be capacitance in the circuit, in which case this is a phasor analysis problem. For the example above, $C \to \infty \Rightarrow Z_c \to 0$, so this remains a resistive circuit only.

Redraw, defining impedances:



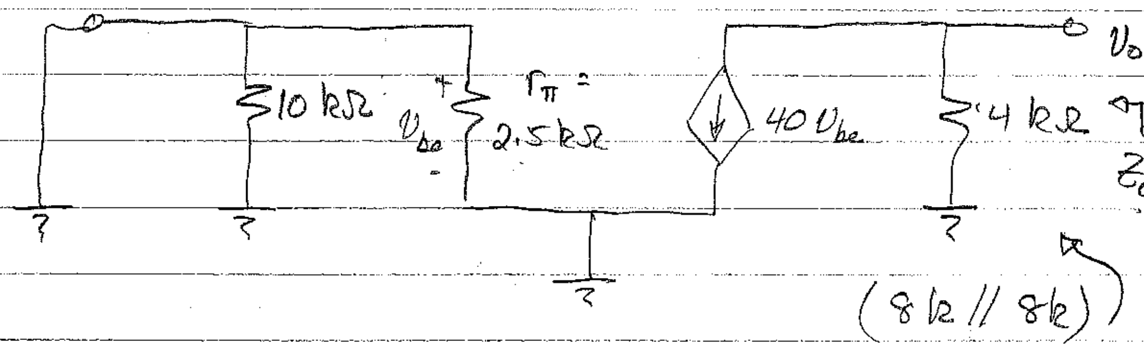
Q. How do we find Z_i, Z_o?
A. Test-source method.

$$Z_i = \frac{V_i}{I_i} \longrightarrow \frac{v_i}{i_i} \quad \text{(resistors-only)}$$

$$i_i = \frac{v_i}{10k} + \frac{v_{be}}{2.5k}, \quad v_{be} = v_i$$

$$\Rightarrow \quad z_i = \frac{v_i}{i_i} = \left(\frac{1}{10k} + \frac{1}{2.5k}\right)^{-1} = 2\ k\Omega$$

Output Impedance is usually taken with the source connected. In using a test source, however, we short-circuit the independent sources. So...

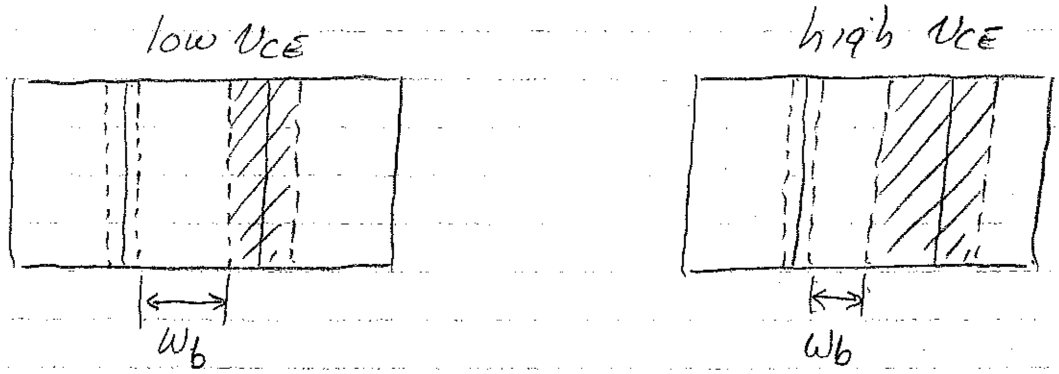


Now $v_{be} = 0 \Rightarrow 40\,v_{be} = 0 \Rightarrow \underline{\underline{Z_o = 4\,k\Omega}}$

EARLY EFFECT

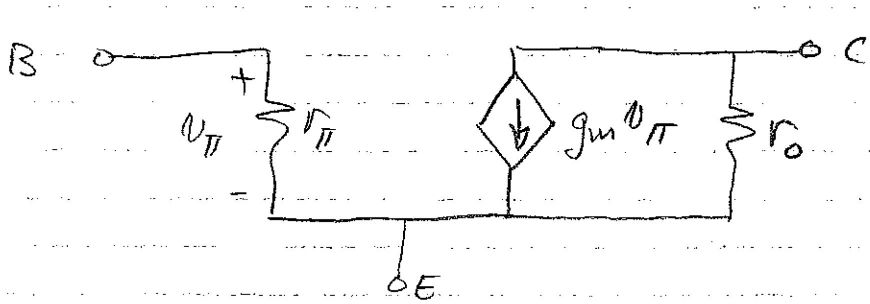
OBSERVATION: In the linear region, $i_C - v_{CE}$ curves are not flat, but tilt upwards slightly. In other words, i_C is not completely independent of v_{CE} but increases slightly.

REASON: As CB junction becomes increasingly reverse-biased, the "depletion region" in the CB junction increases and the base width is reduced. Smaller base $\Rightarrow$ higher β, so i_C increases.



$W_b \equiv$ based width

MODELING We can account for this in our small-signal model by adding an output resistance r_o:



$$r_o = \frac{|V_A|}{I_C}$$