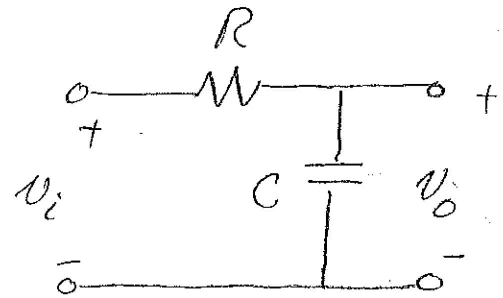
FREQUENCY RESPONSE

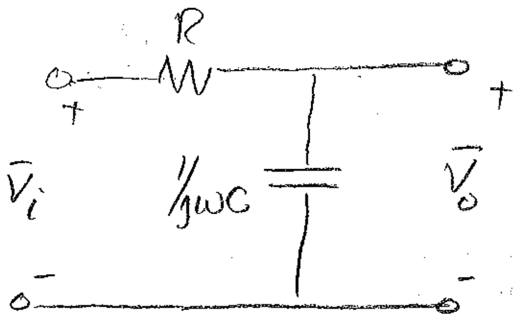
The output of a circuit containing capacitance or inductance will be a function of the input frequency.

example:

time domain:



phasor domain:



Analysis: $$\bar{V}_o = \bar{V}_i \frac{1/j\omega C}{1/j\omega C + R}$$

$$\bar{V}_o = \bar{V}_i \frac{1}{1 + j\omega CR}$$

If we are given R, C and $v_i = V_i \cos(\omega t + \varphi_i)$

we can do the phasor transform: $\bar{V}_i = V_i \angle \varphi_i$ and find $\bar{V}_o$. Then

$$\bar{V}_o \longrightarrow v_o = V_o \cos(\omega t + \varphi_o)$$

The time dependence of v_o is the same as that of v_i, but the magnitude and phase have changed.

Note also that the magnitude and phase of V_o depend on ω:

$$\frac{\bar{V}_o}{\bar{V}_i} = \frac{1}{1+j\omega CR}$$

magnitude:

$$\left|\frac{\bar{V}_o}{\bar{V}_i}\right| = \frac{1}{\sqrt{1+\omega^2C^2R^2}}$$

phase:

$$\angle \bar{V}_o/\bar{V}_i = -\tan^{-1}(\omega CR)$$

If our signal contains a single frequency ω_0, this is a simple problem. But what if our signal is more complicated, and has many frequency components?

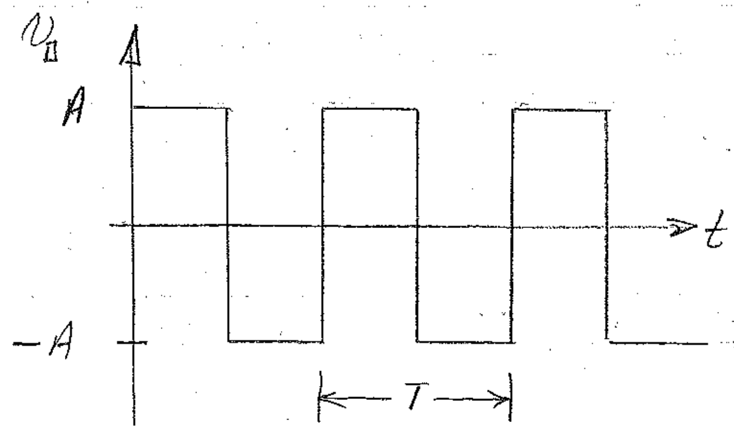
FOURIER ANALYSIS

Any periodic waveform can be expressed mathematically as a sum (usually with an infinite number of terms) of sinusoids with appropriate frequencies and amplitudes.

example: A square wave $v_\square(t)$:

$$v_\square(t) = \frac{4A}{\pi}\left[\sin(\omega_0 t) + \frac{1}{3}\sin(3\omega_0 t) + \frac{1}{5}\sin(5\omega_0 t) + \cdots\right]$$

where $f_0 = 1/T$ and $\omega_0 = 2\pi f_0$.



So what if our input is $v_\square(t)$?

If our circuit is LINEAR and if we know its response to a sinusoid (a signal with a single frequency component) of arbitrary frequency, then we can use SUPERPOSITION and FOURIER ANALYSIS to determine the response of the system to any periodic waveform (such as $v_\square(t)$).

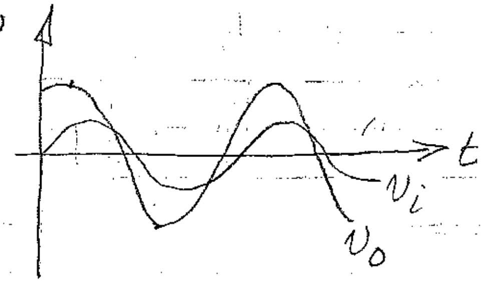
This is an extremely powerful idea with far-reaching implications for electronics.

LINEAR SYSTEM (LINEAR CIRCUIT)

"A linear system is a system to which superposition applies."

— Hambley, 2 ed. p. 41

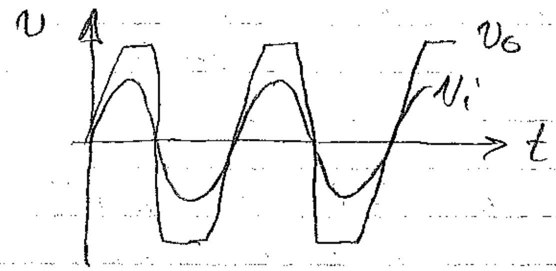
Another viewpoint: The output of a linear system contains the same frequency components as the input (no more, no fewer). Note that amplitudes and phases may be different, however.



v_o has different φ and V_m, but same period and shape.

$\Rightarrow$ LINEAR

If an amplifier is driven to saturation, "clipping" will occur. The output (the clipped signal) is not sinusoidal even if the input is sinusoidal, so it contains more frequency components.



v_o has more ω components than v_i

$\Rightarrow$ NON-LINEAR

DEFINITIONS

1) TRANSFER FUNCTION

The transfer function is

$$T(\omega) \equiv \bar{V}_o / \bar{V}_i$$

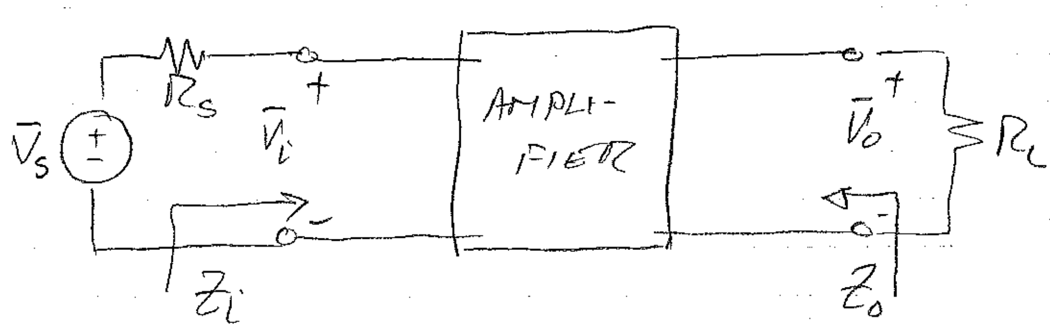
The magnitude, $|\bar{V}_o/\bar{V}_i|$, is just the gain A_v; in general, it is frequency-dependent.

2) INPUT & OUTPUT IMPEDANCE

The INPUT IMPEDANCE of a circuit or amplifier is the Thevenin equivalent impedance seen at the input terminals with a load connected.

The OUTPUT IMPEDANCE of a circuit or amplifier is the Thevenin equivalent impedance seen at the output terminals with a source connected at the input.

Input Impedance Z_i Output Impedance Z_o



NOTES ON NOTATION

We use two different but related representations in Electronics: time domain and frequency (PHASOR) domain.

TIME DOMAIN: circuit variables are a function of time t, e.g.

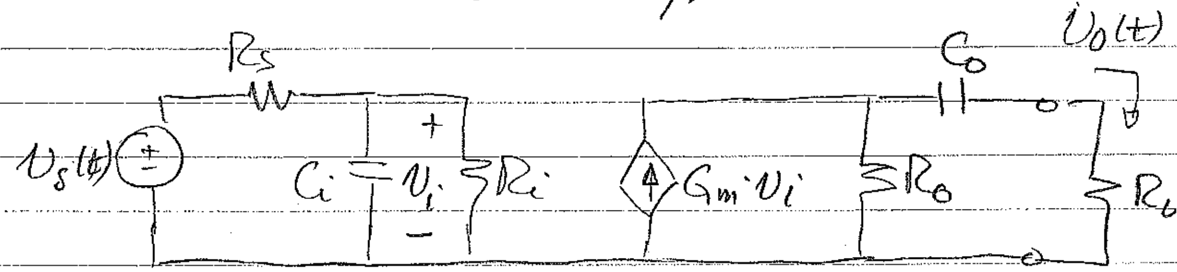
$$i_s(t) = 3.5 \sin(\omega t + \varphi) \ [mA]$$

$$v(t) = 6.0(1 - e^{-t/\tau}) \ [V]$$

$$V_{CC} = 5 \ [V]$$

This last quantity, though constant, can still be considered a function of time.

Time Domain circuit diagram:

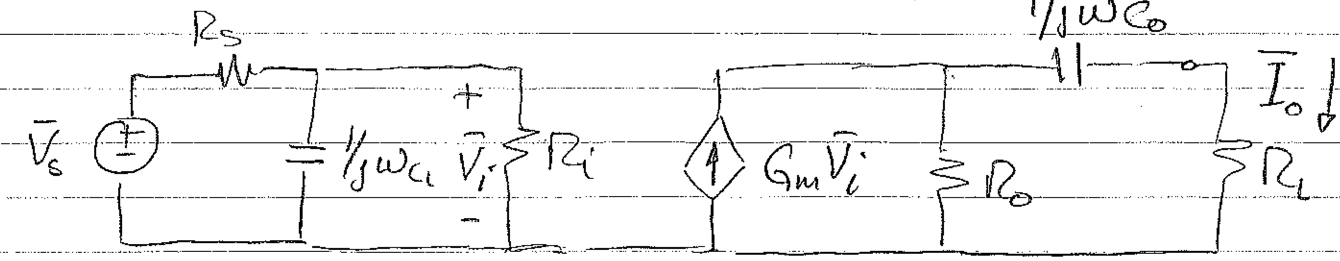


PHASOR DOMAIN: circuit variables are specifically NOT a function of time. They are represented as quantities with amplitude and phase only, eg.,

$$\bar{V}_x = 5.0\angle 30^\circ \ [V]$$

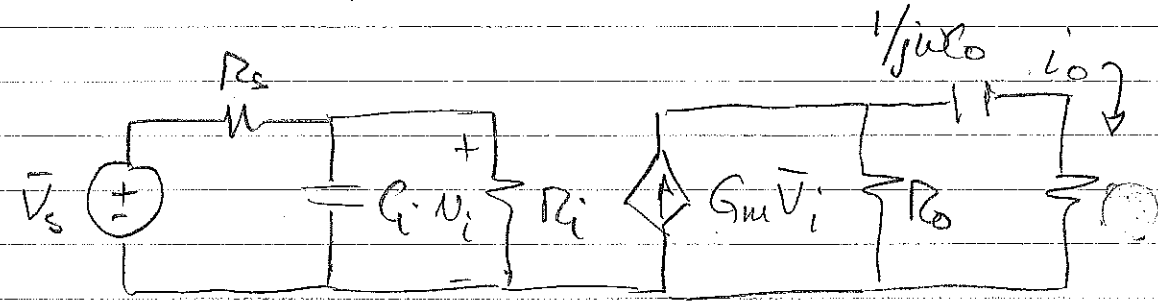
$$\bar{I}_s = (620 + j\,455) \ [mA]$$

Phasor Domain circuit diagram:



We cannot mix the notation between these representations. A circuit diagram with mixed notation, for example, does not mean anything.

<u>BAD</u>:



(This will cause an immediate reduction in quiz and exam grades.)

Same thing for equations:

<u>BAD</u>:

$$v_s = 5 \angle 30^\circ \ |V|$$

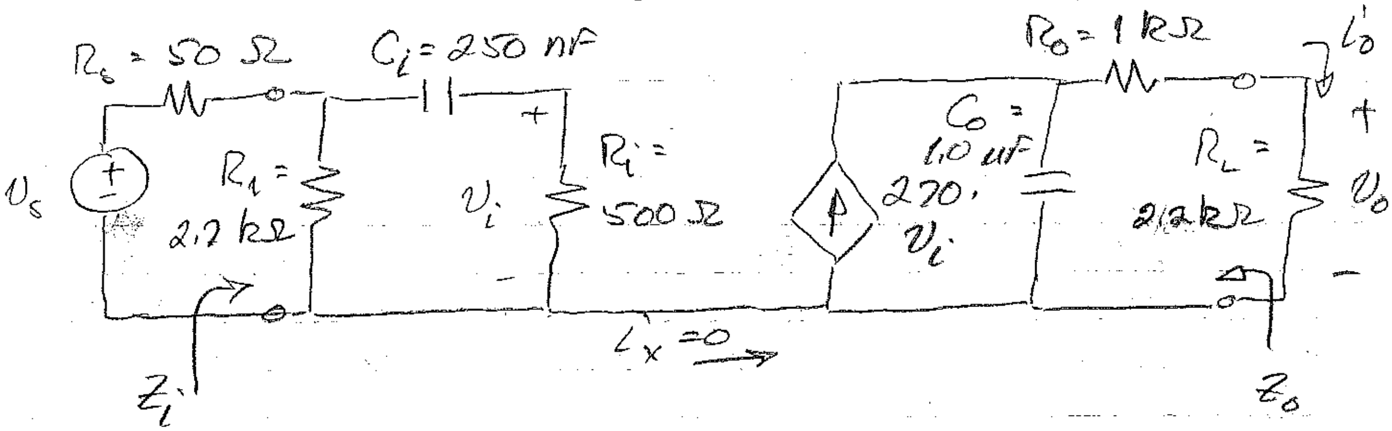
$$\bar{I} = 450 \ |mA|$$

This will similarly result in grade reduction.)

EXAMPLE: IMPEDANCE & TRANSFER FUNCTION

FIND the input and output impedances Z_i, Z_o for the circuit below.

FIND the transfer function $T(\omega) = V_o/V_s$



Z_i: Since $i_x = 0$, the right-hand side of the circuit has no effect on Z_i. So...

$$Z_i = R_1 \parallel (1/j\omega C_i + R_i)$$

$$= \frac{R_1 \cdot (1/j\omega C_i + R_i)}{R_1 + (1/j\omega C_i + R_i)} = \frac{R_1 (1 + j\omega C_i R_i)}{1 + j\omega C_i (R_1 + R_i)}$$

NOTES: What we have calculated is the impedance seen by the source. The arrow associated with Z_i indicates this, and in fact this is a common thing to want to find.

We can't go any farther than this without knowing ω...

Suppose $\omega = 1.5 \times 10^5$ rad/s. Then

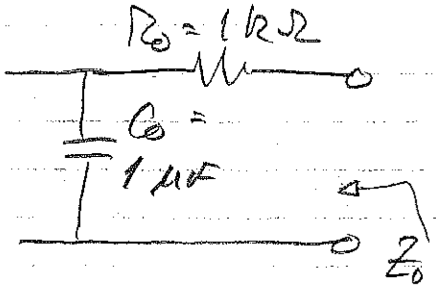
$$Z_i = \frac{2700\,[1 + j(1.5\times10^5)(250\times10^{-9})(500)]}{1 + j(1.5\times10^5)(2700+500)(250\times10^{-9})}$$

$$= (422 - j\,19.0)\ \Omega$$

Z_o is the impedance seen by the load. We have stated that this calculation requires a source, which of course we already have (it is v_s). But we now apply a test source at the output so

$$v_s \to 0 \;\Rightarrow\; v_i \to 0 \;\Rightarrow\; 250\,v_i = 0$$

So the dependent current source is an open circuit and we have...



$$Z_o = R_o + 1/j\omega C_o$$

Again if $\omega = 1.5\times10^5$ rad/s, $Z_o = (1000 - j\,6.67)\ \Omega$

TRANSFER FUNCTION

Going back to the original circuit, we have

$$\bar{V}_o = \bar{I}_o R_L$$

$$= R_L \cdot 270\, \bar{V}_i \cdot \frac{1/j\omega C_o}{1/j\omega C_o + R_o + R_L}$$

$$= 270\, \bar{V}_i \frac{R_L}{1 + j\omega C_o (R_o + R_L)}$$

But we want $\bar{V}_o / \bar{V}_s$ so we need $\bar{V}_s$ in terms of $\bar{V}_i$. The input is messy, but we will make an approximation to simplify: we will ignore R_s since it is small compared to the other resistances. Then...

$$\bar{V}_i \simeq \bar{V}_s \cdot \frac{R_i}{R_i + 1/j\omega C_i}$$

$$= \bar{V}_s \frac{j\omega C_i R_i}{1 + j\omega C_i R_i}$$

Finally $\quad T(\omega) = \bar{V}_o / \bar{V}_s$

$$= \frac{270 \cdot R_L \cdot j\omega C_i R_i}{[1 + j\omega C_o (R_o + R_L)][1 + j\omega C_i R_i]}$$

Reducing, we get

$$T(\omega) = \frac{j\omega (3.375)}{(1 + j\omega (3.2 \times 10^{-3}))(1 + j\omega (1.25 \times 10^{-4}))}$$

An aside ...

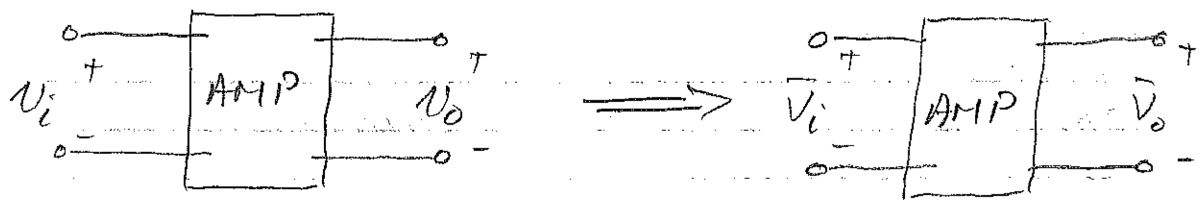
Q: where does C and L come from in the first place?

A: There are several possibilities:

- Caps or inductors placed in the circuit intentionally for design reasons
- Device capacitance: as we will see, solid state devices behave like capacitors in some situations.
- Parasitic elements: unavoidable effects from wires/cables or other geometric effects.

BODE PLOTS

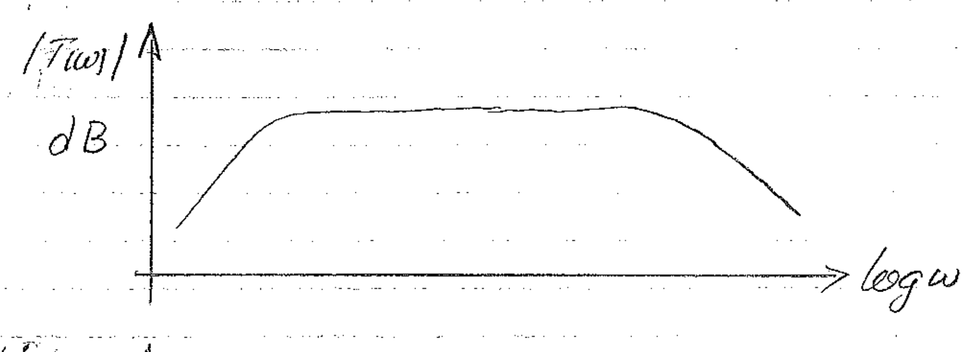
When discussing the effect of a linear circuit (an amplifier, say) on the phase and amplitude of a signal, we will find it convenient to express the circuit behavior in the form of BODE PLOTS.

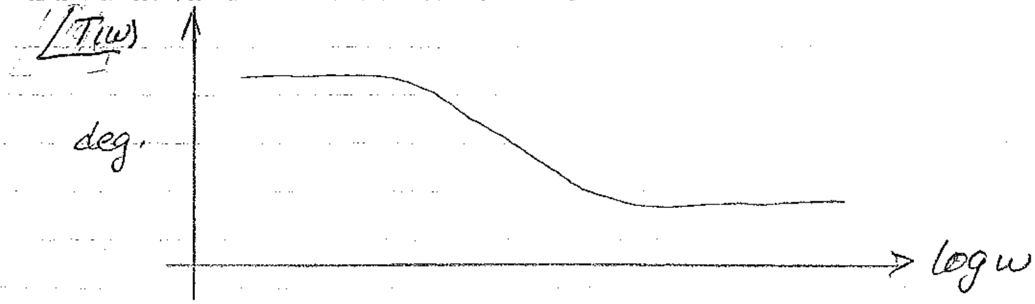


$$T(\omega) = \frac{\bar{V}_o}{\bar{V}_i} = \left|\frac{\bar{V}_o}{\bar{V}_i}\right| \underline{/\bar{V}_o/\bar{V}_i}$$

magnitude phase

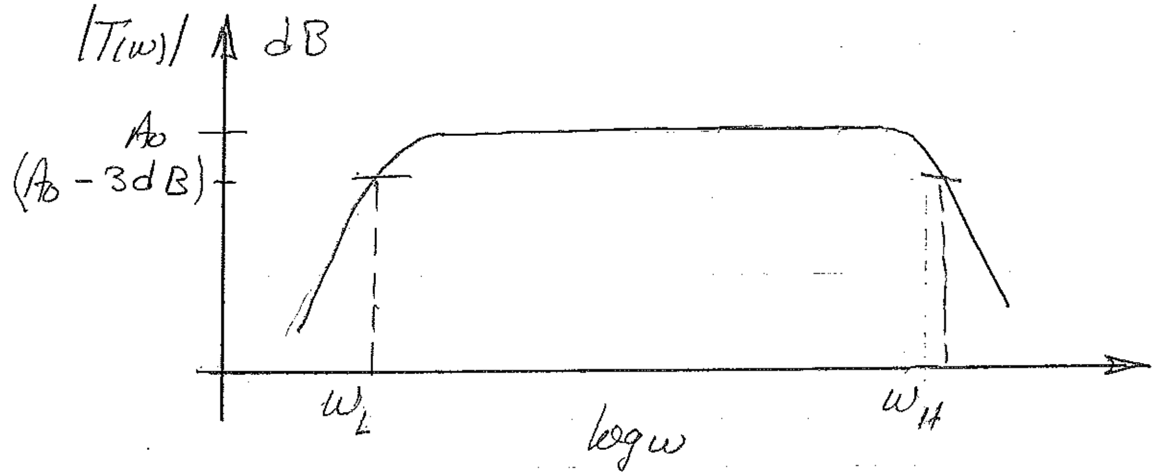
Magnitude and phase are functions of frequency. This information can be plotted:





AMPLIFIER FREQUENCY RESPONSE

Typical amplifier frequency response is the following.



The frequency range over which the gain is approximately constant is called the PASSBAND.

We can be more specific: if the gain in the passband is A_0, we can find the frequencies at which the gain is reduced by 3dB - call these ω_H and ω_L.

Then the passband covers the range ω_L to ω_H and the 3dB BANDWITH is

$$BW_{3dB} = \omega_H - \omega_L$$

FILTERS

We can also think in terms of filters: the Bode plot above indicates a BANDPASS filter, which attenuates signals with frequencies above ω_H or below ω_L.

LOW-PASS FILTER attenuates signals with frequencies that are high relative to some reference.

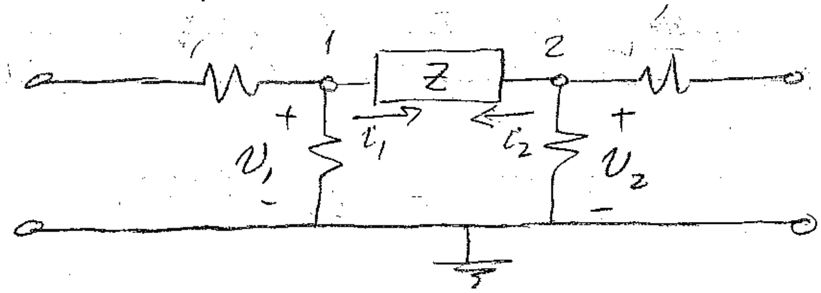
HIGH-PASS FILTER attenuates signals with frequencies lower than some reference.

SEDRA FIGS 1.23, 1.24

We look at a low-pass filter as a specific example. This is not an amplifier, but a simple RC circuit.

MILLER'S THEOREM

Miller's Theorem is a circuit simplification tool that comes in handy sometimes, when analyzing amplifier circuits.

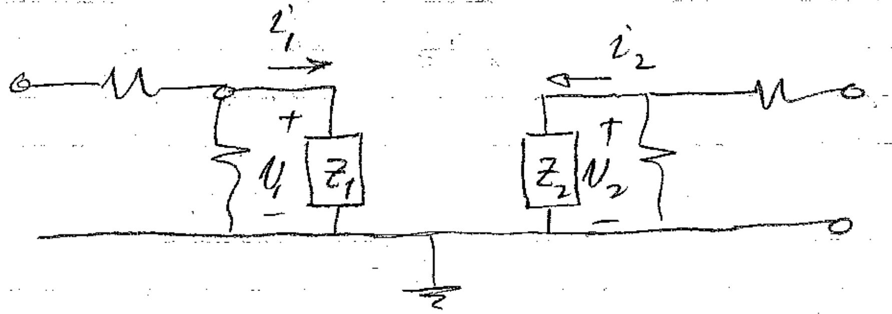


Suppose a circuit has a segment such as that above, where an arbitrary impedance Z is connected between nodes 1 and 2.

Suppose also that we know the ratio of v_2 to v_1, i.e.

$$v_2/v_1 \equiv K$$

Then we can transform the circuit to the one below:



where $\boxed{Z_1 = \frac{Z}{1-K}}$ $\boxed{Z_2 = \frac{ZK}{K-1}}$

Analysis: We need to keep i_1 and i_2 the same in the original circuit and its equivalent. Then...

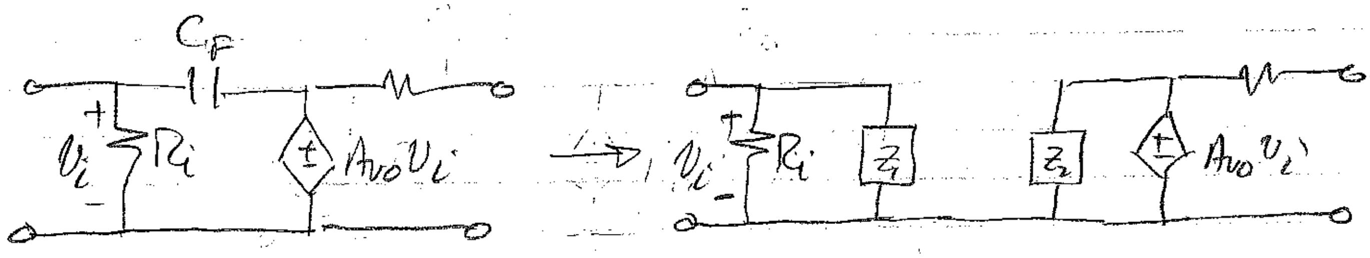
$$i_1' = \frac{v_1 - v_2}{Z} = v_1\left(\frac{1-K}{Z}\right)$$

$$i_2' = \frac{v_2 - v_1}{Z} = v_2\left(\frac{1-1/K}{Z}\right)$$

But then $Z_1 \equiv \frac{v_1}{i_1'} = \frac{Z}{1-K}$

$$Z_2 \equiv \frac{v_2}{i_2'} = \frac{Z}{1-1/K} = \frac{ZK}{K-1}$$

This theorem is useful in voltage or transresistance amplifier circuits containing a "feedback" impedance, such as C_F below.



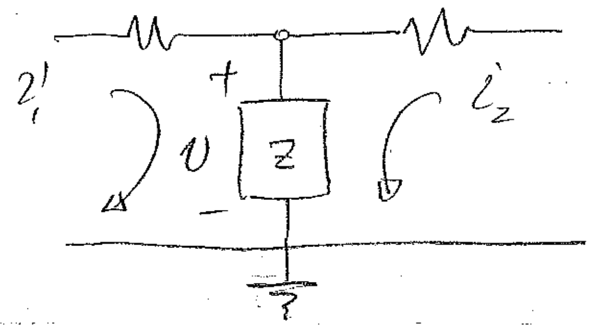
Here, $Z \equiv 1/j\omega C_F$ and $K = \frac{A_{vo} \cdot v_i}{v_i} = A_{vo}$

$$Z_1 = \frac{1/j\omega C_F}{1-A_{vo}} \qquad Z_2 = \frac{1/j\omega C_F \cdot A_{vo}}{A_{vo}-1}$$

MILLER'S DUAL

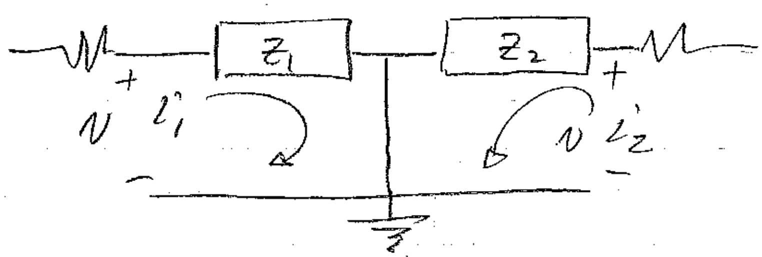
A similar technique is useful for amplifier circuits containing current sources.

If an impedance Z is shared between two meshes (loops), and if we know the ratio of the loop currents, we can make the following transformation.



GIVEN $i_2 / i_1 \equiv J$

Transform to:

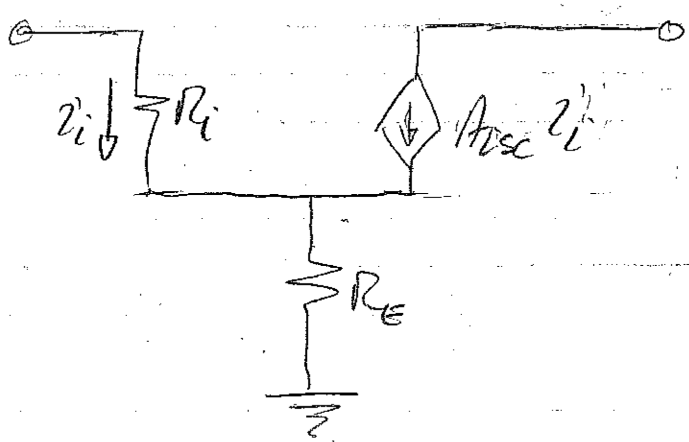


Analysis: $v = (i_1 + i_2) Z = i_1 (1 + J) Z$

$= i_2 (1/J + 1) Z$

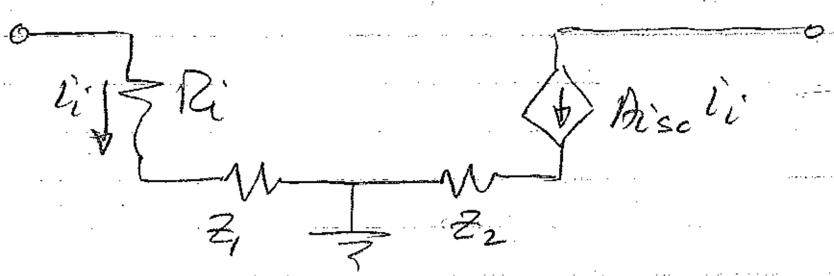
Then $\boxed{Z_1 = \frac{v}{i_1} = (1 + J) Z}$ $\boxed{Z_2 = \frac{v}{i_2} = \left(\frac{J+1}{J}\right) Z}$

This is useful in amplifier circuits with an impedance to ground, as sometimes occurs in BJT circuits (which we will study later.



Here, $Z \equiv R_E$ and $J = \dfrac{A_{isc}\, i_i}{i_i} = A_{isc}$

Transform is



$Z_1 = (A_{isc}+1)R_E \qquad Z_2 = \dfrac{A_{isc}+1}{A_{isc}} R_E$