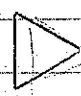
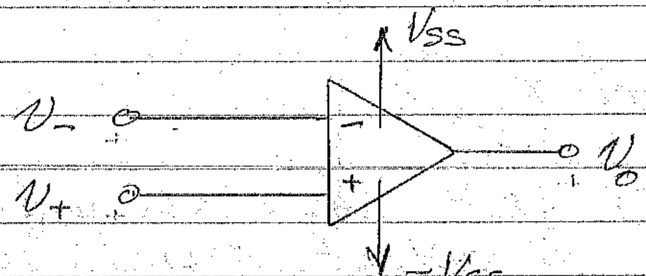
OPERATIONAL AMPLIFIERS

The operational amplifier, or Op Amp, is an Integrated Circuit (IC) that is inexpensive and extremely versatile.

With the addition of a few resistors and capacitors, it can be used for a very wide variety of electronic gadgets.

CIRCUIT SYMBOL

We often use  as a symbol for a generic amplifier:

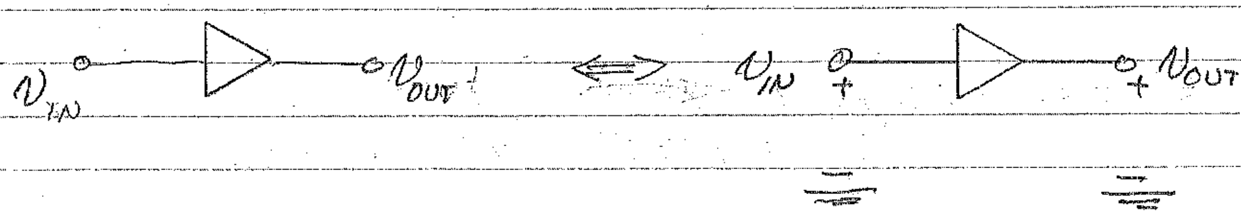


v_- : Inverting input $\quad\quad$ v_o : output

v_+ : Non-inverting input

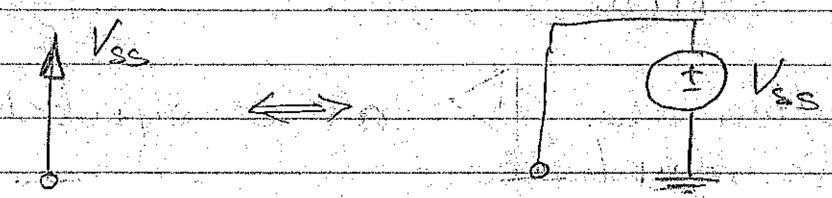
NOTATION

We introduce the following notation:



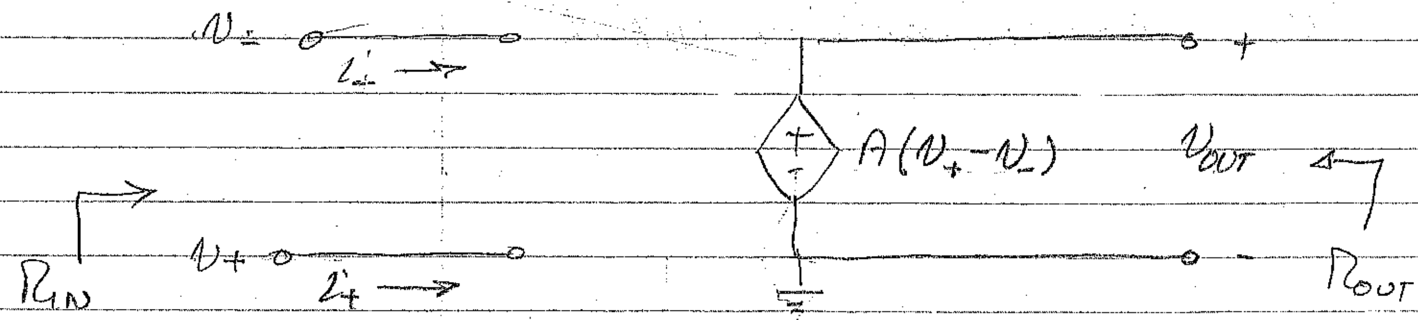
Whenever we write v at a circuit node, we will always mean that v is the voltage at that node with respect to ground. The ground terminal may or may not be shown but it is implied, if not.

$\pm V_{SS}$ are the power supplies. Very often they will not be shown. The arrow means that the other terminal is connected to ground:



EQUIVALENT CIRCUIT

The IDEAL OP AMP has an infinite input resistance, 0 output resistance, and infinite voltage gain:



$A \equiv$ OPEN LOOP GAIN
or DIFFERENTIAL GAIN

Note that the input currents $i_+ = i_- = 0$.

These properties are for the IDEAL device, but real op amps approximate to these properties quite well.

Ideal Op Amp:

$$A = \infty \qquad R_{IN} = \infty \qquad R_{OUT} = 0$$

A closer look at the output:

The op amp output is constrained to be within the power supply limits:

$$-V_{SS} < v_{OUT} < +V_{SS}$$

Now for $v_+ = v_-$, $v_{OUT} = 0$. But for any non-zero $(v_+ - v_-)$, the output will be infinite (it will go to the power supply value):

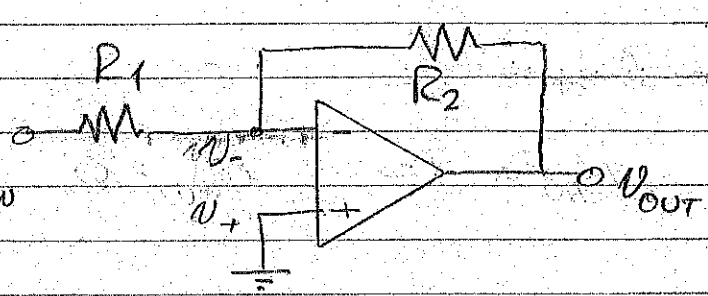
$$v_+ > v_- \Rightarrow v_{OUT} = +V_{SS}.$$

$$v_+ < v_- \Rightarrow v_{OUT} = -V_{SS}$$

As it turns out, this is quite useful. But we can do much more if we have...

NEGATIVE FEEDBACK

The circuit below adds two resistors to our op amp, and connects the non-inverting terminal to ground.



Resistor R_2 connects the output back to the inverting input. We make the following argument:

The larger is v_{OUT}, the more output signal is "fed back" to the inverting input. As a result, the signal getting into the op amp $(v_+ - v_-)$ is reduced. This action tends to stabilize v_{OUT}, so that instead of becoming infinite, it stays finite.

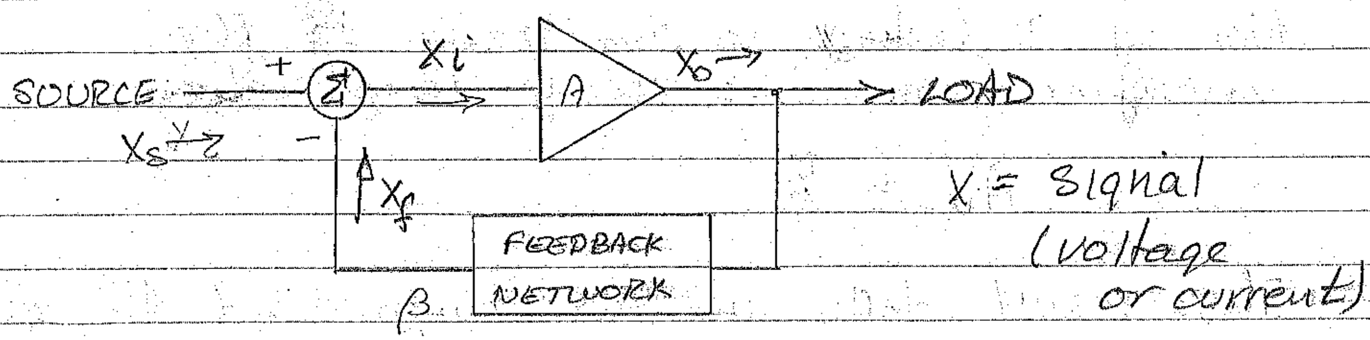
e.g. assume v_{IN} positive and $v_+ = 0$.

No feedback (R_2 removed) $\Rightarrow v_+ - v_- = -v_{IN}$

$$\therefore \; v_{OUT} = A(v_+ - v_-) \rightarrow -\infty \quad (-V_{SS}).$$

With feedback, some of v_{OUT} goes back to v_- and reduces the signal (makes it less positive). This reduces the magnitude of v_{OUT} until it stabilizes.

This argument does not prove that V_{out} will be less than infinity, but we can be a little more specific:



Interpretation: We have attached a feedback network to our amplifier. The feedback network is characterized by the parameter β, so that the feedback signal is

$$X_f = \beta X_o$$

The feedback is negative because it enters the summing node Σ at the − sign, meaning it is subtracted from the source X_s. Thus what gets in to the amplifier is

$$X_i = X_s - \beta X_o$$

And of course $X_o = A X_i$. So we have

$$X_o = A X_i = A X_s - A\beta X_o$$

$$X_o (1 + A\beta) = A X_s$$

$$A_V \equiv \frac{X_o}{X_s} = \frac{A}{1+A\beta}$$

where we have defined a new gain A_V which we will call the CLOSED LOOP GAIN.

Now if A (which is the gain of the amplifier alone) is very large, then

$$A_V \simeq 1/\beta$$

This result tells us two important things.

- If we have feedback, i.e., $\beta \neq 0$, then the output will be finite (i.e., less than $\pm V_{ss}$).
- If the open loop gain A is large, the closed loop gain will be independent of the amplifier properties!!

In other words, by providing negative feedback, we have stabilized the output AND arranged things so that the gain depends only on things we connect to the amp (i.e., the feedback network).

Bottom line: We will be able to construct op amp circuits with a wide variety of functions simply by connecting R's, C's, and L's to them in the appropriate way.

The configuration above (with R_1 and R_2) is called the INVERTING CONFIGURATION. We will find its gain A_V shortly. But first...

... we re-visit our op amp:

$$v_{OUT} = A(v_+ - v_-)$$

If we say that v_{OUT} is finite (less than the power supply voltages) and A is very large, then

$$v_+ - v_- = \frac{v_{OUT}}{A} \simeq 0$$

So if we have negative feedback so that v_{OUT} is finite, we have that

$$v_+ \simeq v_-$$

This will be crucial to our analysis of op amps. In particular:

IF we have NEGATIVE FEEDBACK,

THEN $v_+ = v_-$

So: how do we know if we have negative feedback?

For our purposes we will say that we have negative feedback when there is any connection from the output back to the inverting input.

This statement is ALMOST true. There are circumstances where connection exists between output and inverting input and we do not have negative feedback, but we will leave that complication for later.

In any case it is true that if we do not have a connection from output to inverting input, then we do not have negative feedback.

Finally, if $v_+ = v_-$ because of negative feedback, we call this a VIRTUAL SHORT. It is a SHORT in the sense that the voltages at two different nodes are equal, but VIRTUAL in the sense that there is not actually a wire (short circuit) connecting these things.

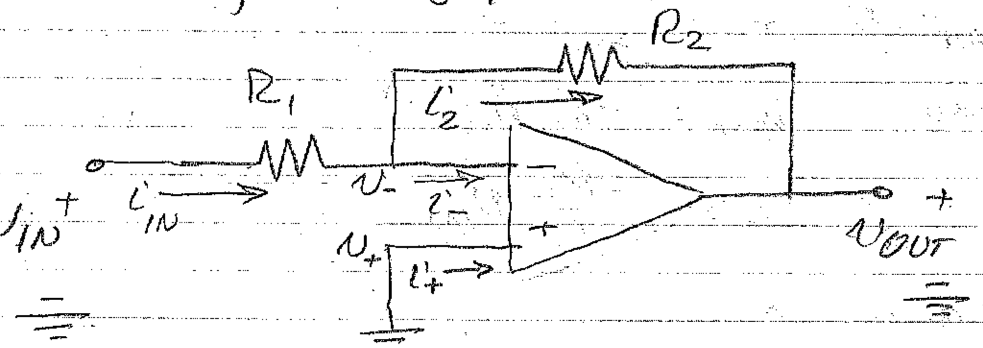
ANALYSIS OF THE INVERTING OP AMP

We will assume IDEAL OP AMPS which means that

- A (open-loop gain) = ∞
- Input currents to inverting and non-inverting terminals are 0.

The analysis shown here will follow steps that will be taken for solving any op amp circuit with negative feedback. Only the details will be different.

Inverting Configuration



Analysis:

- Do we have negative feedback?

  With very few exceptions, we will have negative feedback if there is something connected between the output and the inverting terminal.

If YES, Then $v_+ = v_-$ (virtual short)

If NO, Then we will need a different sort of analysis, which will come later.

So for our inverting configuration, $v_+ = v_-$

$i_- = i_+ = 0$ (infinite impedance looking into op amp terminals)

So...

$$v_+ = 0 \text{ (grounded)} \Rightarrow v_- = 0$$

$$\therefore \; i_{IN} = v_{IN}/R_1$$

$$i_- = 0 \Rightarrow i_2 = i_1$$

Now we do a KVL that includes v_o:

$$v_{OUT} - v_+ + i_2 R_2 = 0$$

$$v_+ = 0 \Rightarrow v_{OUT} = -i_2 R_2$$

$$v_{OUT} = -\frac{R_2}{R_1} v_{IN}$$

So the gain is

$$\frac{v_{OUT}}{v_{IN}} = \frac{-R_2}{R_1}$$

The ratio $-R_2/R_1$ is the CLOSED LOOP GAIN.

The closed loop gain is determined only by the resistances the designer adds to the circuit. It is not the same as the OPEN LOOP GAIN, which is infinite in the ideal op amp.

Note: $v_+ \simeq v_-$ because (i) we have negative feedback AND (ii) $A_{\text{open loop}} = \infty$.

FINITE OPEN-LOOP GAIN

What if A is not infinite (as it will not be in a real op amp)?

In this case we have

$$v_+ - v_- = \frac{v_{OUT}}{A} \qquad (A < \infty)$$

If $v_+ = 0$, $\quad v_- = {-v_{OUT}}/A$

Then $\quad i'_{IN} = \dfrac{v_{IN} - \left(-\frac{v_{OUT}}{A}\right)}{R_1} = i'_2$

(we still assume $i'_- = i'_+ = 0 \Rightarrow i'_{IN} = i'_2$).

KVL: $\quad v_{OUT} + \dfrac{v_{OUT}}{A} + i'_2 R_2 = 0$

$$\Rightarrow \quad v_{OUT}\left(1 + \frac{1}{A}\right) + \frac{R_2}{R_1}\left(v_{IN} + \frac{v_{OUT}}{A}\right) = 0$$

$$v_{OUT}\left(1 + \frac{1}{A} + \frac{R_2}{R_1 A}\right) = -\frac{R_2}{R_1} v_{IN}$$

$$A_V = \frac{v_{OUT}}{v_{IN}} = \frac{-R_2/R_1}{1 + \left(1 + \frac{R_2}{R_1}\right)/A}$$

Some numbers?

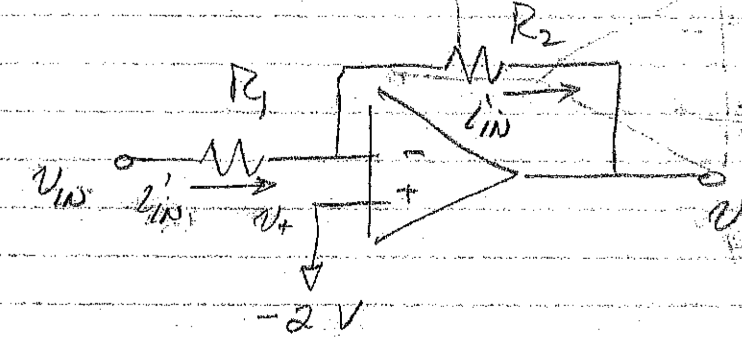
$$R_2 = 10\,k\Omega \qquad R_1 = 5\,k\Omega$$

$$A = 10^5 \quad \text{(a typical value)}$$

With these numbers, $A_V = -1.99994$

as opposed to the value for $A = \infty$, $-R_2/R_1 = -2$.

INVERTING CONFIGURATION with a source at the non-inverting input.



We need to be careful here since $v_+ = -2V$, we have

$$i_{IN}' = \frac{v_{IN} - (-2)}{R_1}$$

Now KVL is

$$v_O - (-2) + i_{IN}' R_2 = 0$$

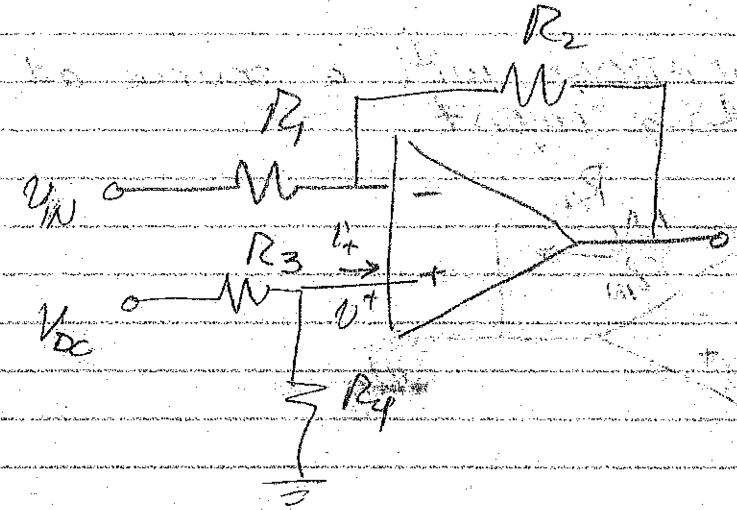
$$v_O = -i_{IN}' R_2 - 2$$

$$= -\frac{R_2}{R_1} v_{IN} - \frac{R_2}{R_1}(2) - 2$$

$$= -\frac{R_2}{R_1} v_{IN} - 2\left(1 + \frac{R_2}{R_1}\right)$$

so we have a "dc offset" at the output.

Note that the following circuit would have the same effect:

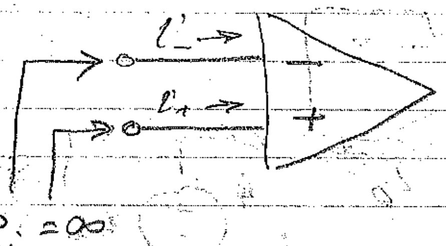


Now $v_+ = V_{DC} \dfrac{R_4}{R_3 + R_4}$

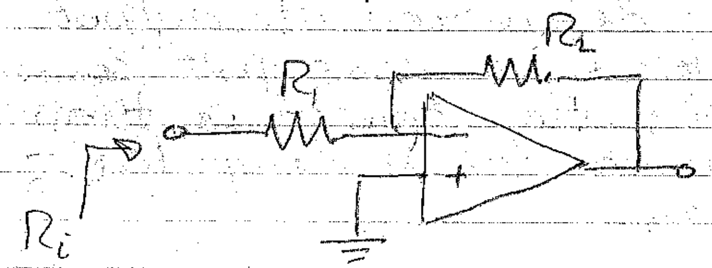
because i_+ is 0 so we have a simple VDR at the non-inverting input.

INPUT RESISTANCE

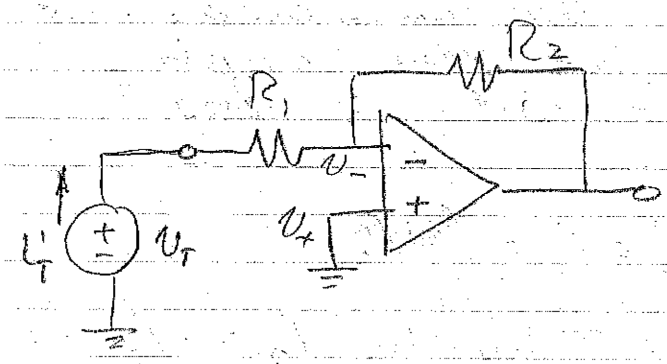
We said that $i_+ = i_- = 0$, so the impedance looking into the op amp terminals is ∞.



But what about the impedance looking into the input to the inverting configuration?



We can apply a test source:



$v_- = v_+ = 0$

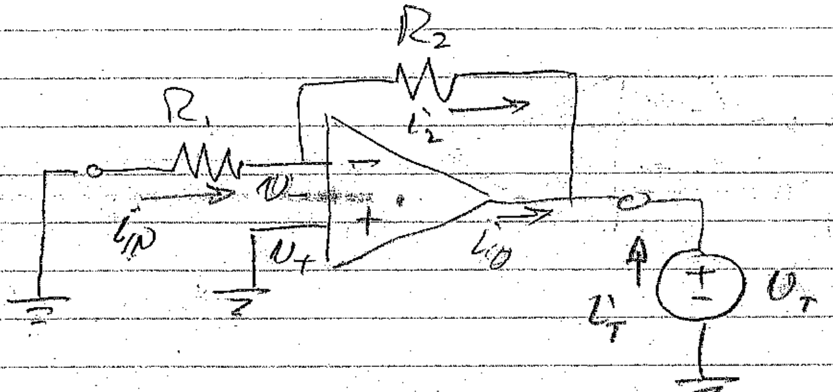
$\therefore \; i_T = v_T/R_1$

$\Rightarrow \; R_i = v_T/i_T = R_1$

Of course, for a different op amp configuration, the input impedance may be very different.

OUTPUT RESISTANCE

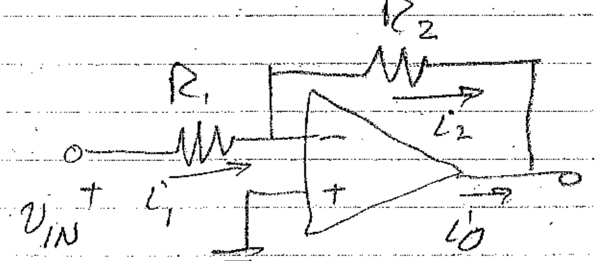
What about the output resistance? Apply a test source there:



As required by our definition of output resistance from Chapter 1, we have put a source at the input; the source is then de-activated, as required for finding R_{Th} using a test source.

Now... $v_- = v_+ = 0 \Rightarrow i_{IN}' = i_2' = 0$.

But what about i_0'? We cannot assume it is 0 - indeed, look at the simple inverting configuration with no load:



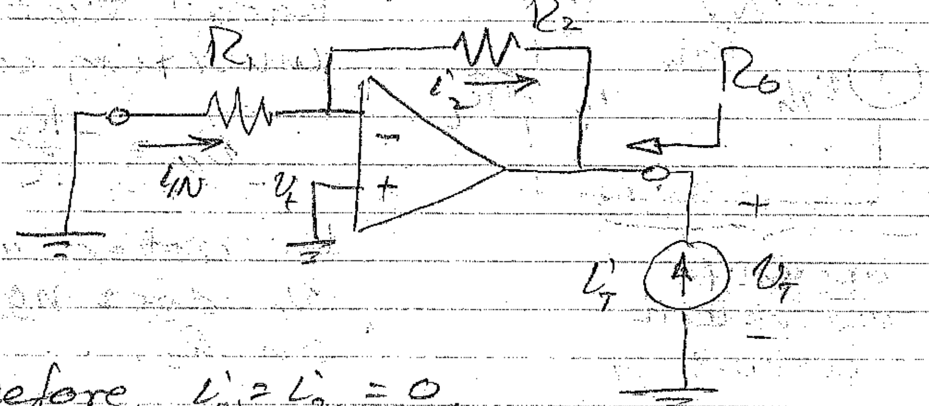
$i_1' = i_2' = v_{IN}/R_1$

and clearly $i_0' = -i_2'$

So we cannot assume $i_0' = 0$, in general.

But there does not seem to be any way to find i_0; which means we can't find i_T.

Let's try something else:



As before, $i_{IN} = i_2 = 0$.

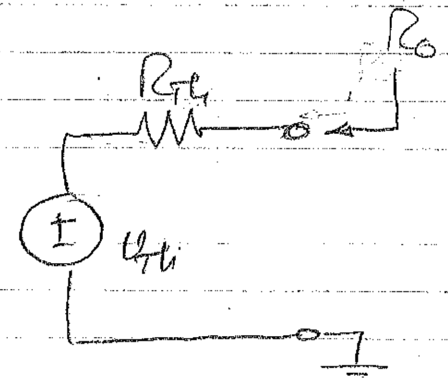
Now, however, we can find v_T by KVL.

$$v_T - v_+ + i_2' R_2 = 0$$

$$v_+ = 0, \quad i_2' = 0 \Rightarrow v_T = 0 \,!$$

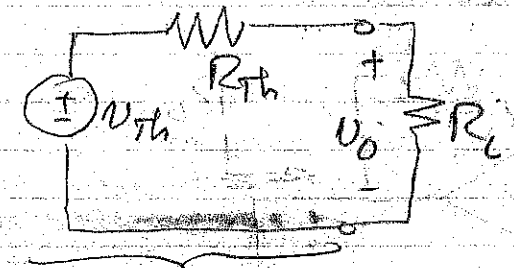
So $R_O = 0$.

Now we see why the test voltage source doesn't work: Since we're saying the op amp can be modeled as a Thevenin equivalent looking into the output, we have:



Now if $R_{Th} = 0$, putting a test voltage source at the output violates KVL!

Drawing the Thevenin equivalent circuit allows us to think about the output resistance a different way:



op-amp equivalent circuit

Recall that for the inverting config.,

$$v_0 = -\frac{R_2}{R_1} v_{IN}$$

In other words, v_0 does not depend on R_L.

Now look at the Thevenin equivalent above. The only way we can get v_0 not to depend on R_{Th} is if R_{Th} is 0:

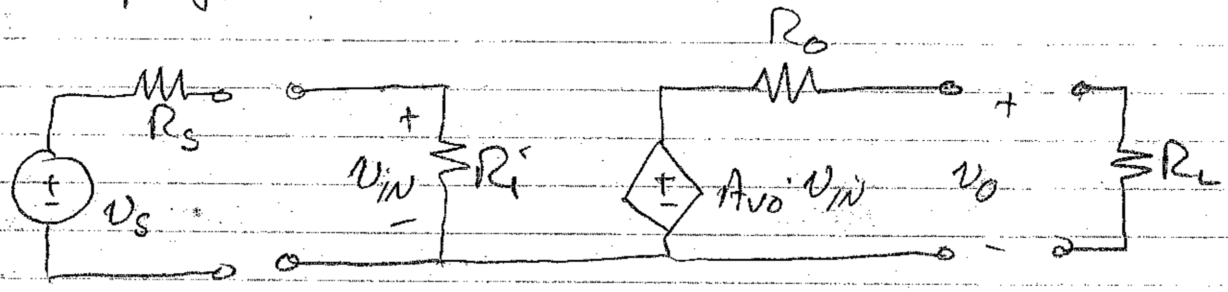
$$v_0 = v_{Th} \frac{R_L}{R_L + R_{Th}}$$

If v_0 is not a function of R_L, we must have $R_{Th} = 0$.

So if the output of the inverting configuration doesn't depend on R_L, it must be that $R_0 = R_{Th} = 0$.

CONNECTION WITH AMPLIFIER MODELS

Now let's go back to our voltage amplifier model:



We said that in the ideal case,

$$R_i \gg R_s \quad \& \quad R_o \ll R_L$$

What about the inverting configuration? That has

$$R_i = R_1 \qquad\qquad R_o = 0$$

So it looks like we have the ideal output resistance. What about R_i?

Whether or not $R_1 \gg R_S$ depends on R_S and our choice for R_1, which in turn depends on the gain we want:

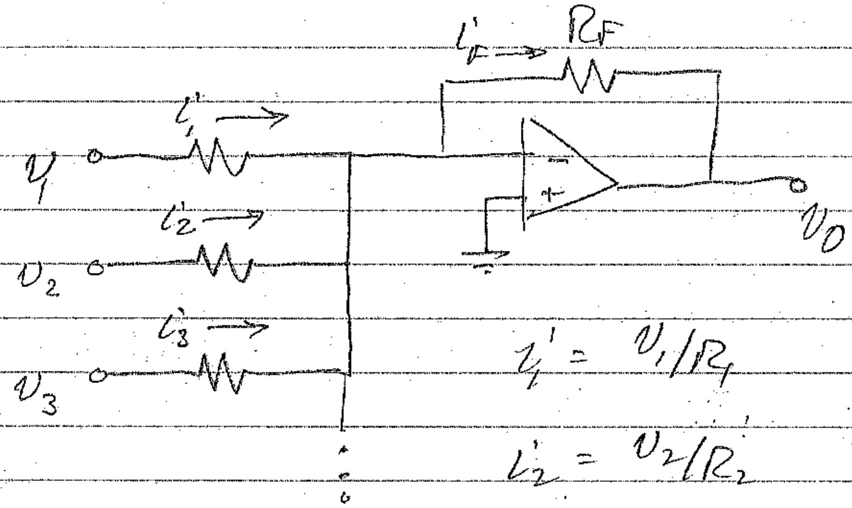
$$\frac{v_O}{v_{IN}} = -\frac{R_2}{R_1}$$

So it looks like we may have trouble getting high gain <u>and</u> high input resistance.

There are other configurations where we can get both, however...

SUMMING CONFIGURATION

We can add sources ...



$i_1' = v_1/R_1$

$i_2' = v_2/R_2$

$i_3' = v_3/R_3$

$i_F' = i_1' + i_2' + i_3'$

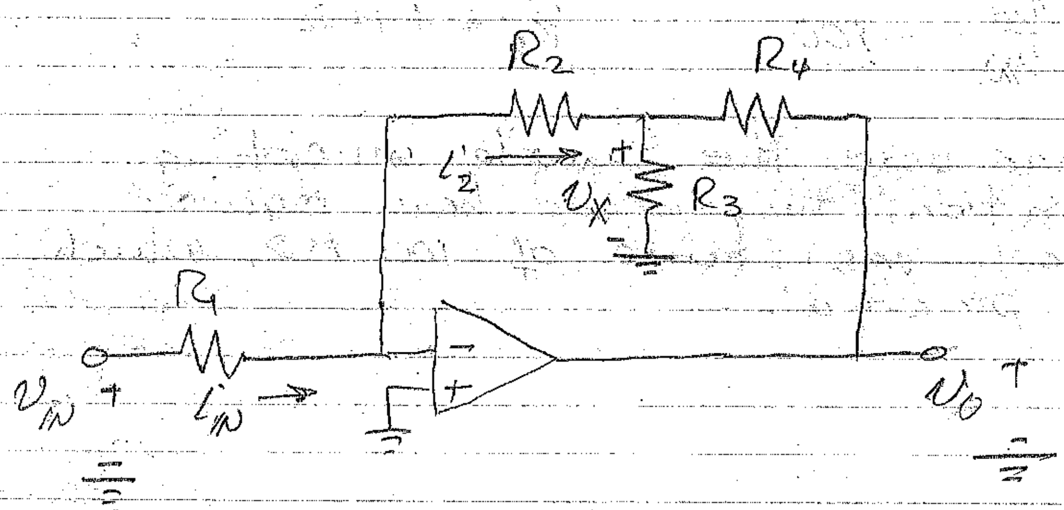
$\therefore \quad v_O = -i_F' R_F = -v_1 \frac{R_F}{R_1} - v_2 \frac{R_F}{R_2} - v_3 \frac{R_F}{R_3}$

So we can have different gain and input resistance for each source, which is handy.

We call the output a WEIGHTED SUM of the inputs. The factors R_F/R_i are PARTIAL GAINS.

High R_i, High Gain

Here is the high R_i, high A_v op amp we mentioned earlier.



Analysis: $i_{IN}' = v_{IN}/R_1 = i_2'$

KVL: $v_X + i_2' R_2 = 0$

$$\Rightarrow \quad v_X = -i_2' R_2 = -\frac{R_2}{R_1} v_{IN}$$

Node Voltage Method:

$$\frac{v_X}{R_2} + \frac{v_X}{R_3} + \frac{v_X - v_O}{R_4} = 0$$

$$v_X \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \right) = \frac{v_O}{R_4}$$

$$\frac{v_O}{v_{IN}} = -\frac{R_2}{R_1} R_4 \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \right)$$

For high R_i, let's choose $R_1 = 1\,M\Omega$.

Then with $R_2 = 1\,M\Omega$, $R_4 = 1\,M\Omega$, $R_3 = 10.2\,k\Omega$,

$$\frac{v_O}{v_{IN}} = -100 \qquad R_i = 1\,M\Omega$$

If we had used the simple inverting configuration, this would have required a feedback resistance of $100\,M\Omega$, which is not practical.