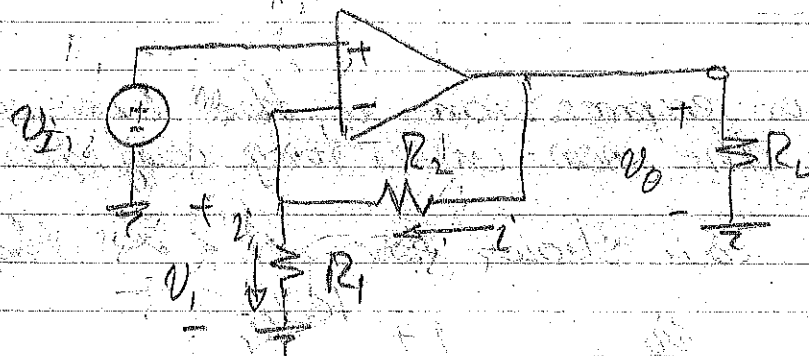


## NON-INVERTING CONFIGURATION

We can also apply a signal to the non-inverting input. We still want negative feedback, however. Let's look at the following circuit...



ANALYSIS

$$\text{Virtual Short} \Rightarrow V_1 = V_2$$

$$\text{Hence } i = \frac{V_1}{R_1} = \frac{V_I}{R_1}$$

$$\text{KVL: } V_O - V_2 - i R_2 = 0$$

$$V_O = V_I + \frac{V_I}{R_1} R_2$$

$$\boxed{\frac{V_O}{V_I} = 1 + \frac{R_2}{R_1}}$$

NOTE: gain is POSITIVE, hence NON-INVERTING

### Alternative Derivation:

By voltage divider rule,

$$V_{in} = V_1 = V_0 \cdot \frac{R_1}{R_1 + R_2}$$

$$A_v = \frac{V_0}{V_{in}} = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1}$$

This expression is the closed loop gain for the non-inverting configuration.

We can show for  $A < \infty$  that

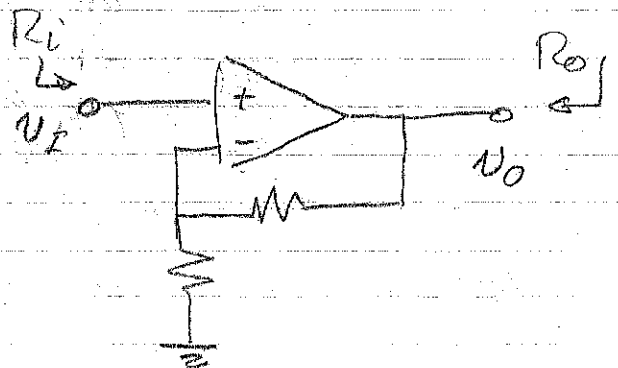
$$\frac{V_0}{V_2} = \frac{1 + \frac{R_2}{R_1}}{1 + \frac{1 + \frac{R_2}{R_1}}{A}}$$

for the non-inverting configuration.

### NON-INVERTING CONFIGURATION:

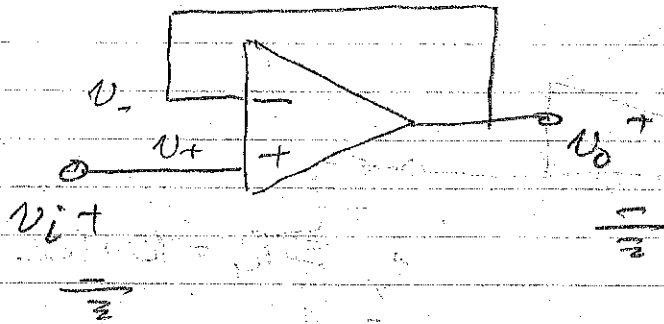
$$R_i = \infty$$

$$R_o = 0$$



## BUFFER AMPLIFIER

What about this?

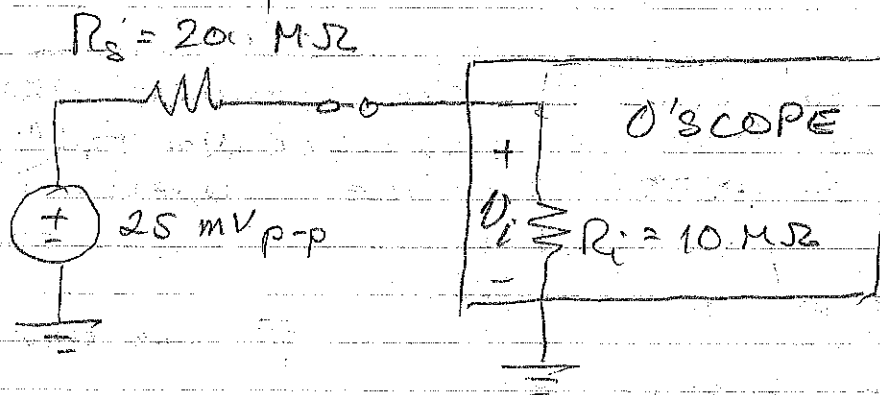


$$V_+ = V_- = V_i$$

$$\underline{\underline{V_o = V_i}}$$

Is this useful? Yes! The important property of this configuration is that  $R_i = \infty$  (and  $R_o = 0$ ).

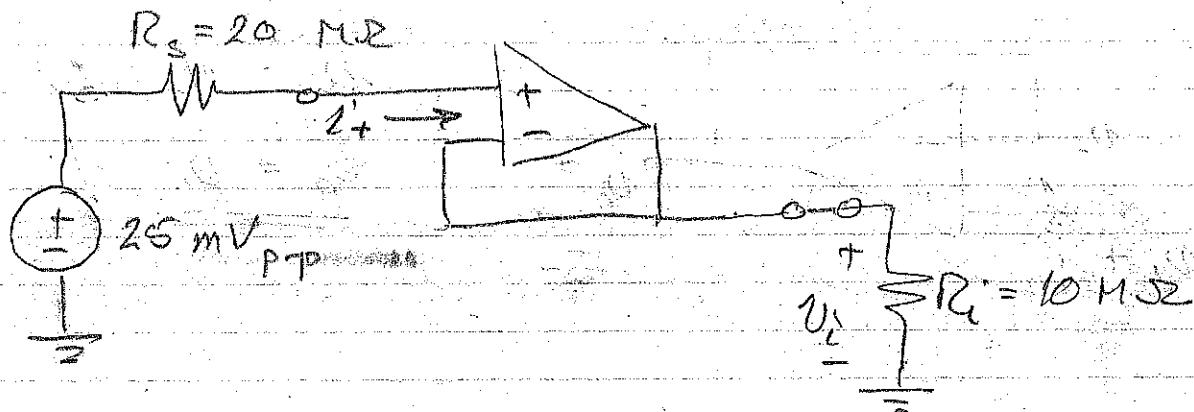
Suppose we had a source with the following Thevenin equivalent:



We have connected a source having large  $R_s$  to an oscilloscope with a 10 MΩ input. The signal to the scope is

$$v_i = 0.025 \frac{10M}{10M + 200M} = 0.0083 V_{p-p}$$

In this case the scope isn't go to show us much. But if we insert a buffer amp:



Now  $V_L = 25 \text{ mV p-p}$ . If that still isn't enough, we can amplify before going to the scope, and our amplifier will not need a large  $R_i$ .

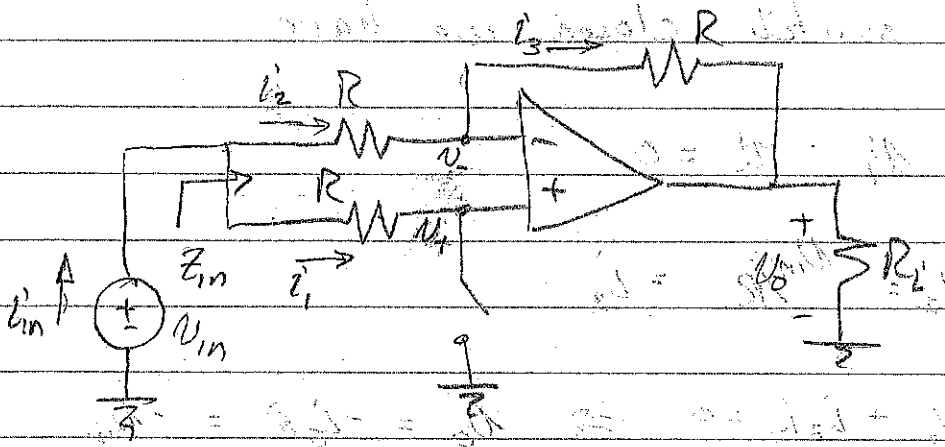
Because  $Z_i \approx 0$ , the power drawn from the load is negligible. But the power delivered to the scope is

$$P_{\text{del to } R_L} = \frac{V_L^2}{R_L}$$

$$P_{\text{del by source}} \approx 0$$

$\Rightarrow$  power gain  $\sim \infty!$

## HAMBLEY 2ed. EXERCISE 2.4 73



a) To start, note that we do have negative feedback, so we can say  $v_- = v_+$ .

With switch open and the op amp input resistance very large ( $\infty$ ), we have  $i_1 = 0$ .  
Therefore

$$v_- = v_+ = v_{in}$$

This is a curious result, since it means that  $i_2 = 0$  and  $i_3 = 0$ ! So

$$v_o = v_{in} \Rightarrow |A_v| = 1$$

The impedance  $Z_{in}$  can be found by considering  $v_{in}$  to be a test source. Then

$$Z_{in} = \frac{v_{in}}{i_{in}} = \frac{v_{in}}{0} = \infty !$$

b) With the switch closed, we have

$$V_1 = V_2 = 0$$

$$\therefore I_1 = I_2 = \frac{V_{in}}{R} = I_3$$

KVL:  $V_0 + I_3 R = 0 \Rightarrow V_0 = -I_3 R = -V_{in}$

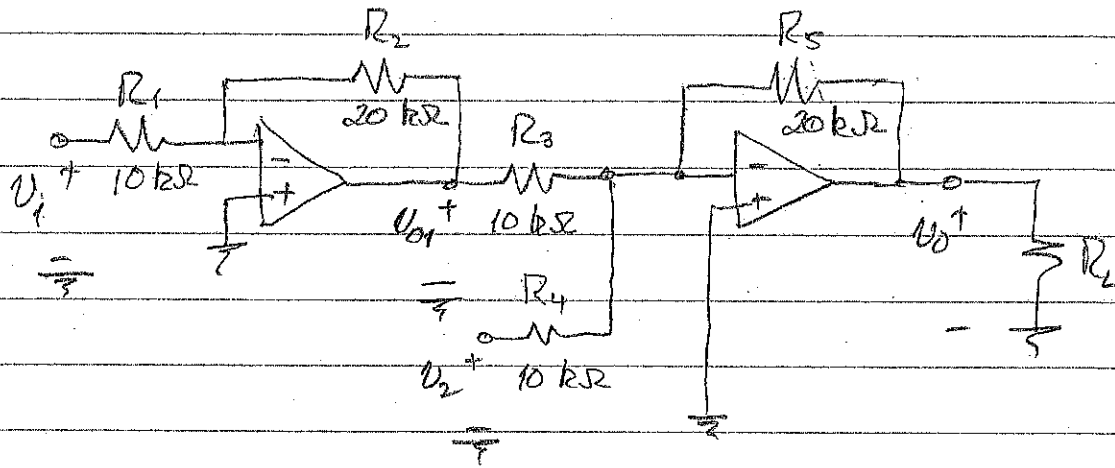
So  $A_v = -1$

We have  $Z_{in} = Z_1 + Z_2$

$$= \frac{V_{in}}{R} + \frac{V_{in}}{R} = V_{in} \frac{2}{R}$$

$\therefore \frac{V_{in}}{Z_{in}} = \boxed{I_{in} = \frac{R}{2}}$

HARTBLEY 2 ed. Ex. 2.3 p. 69



As is often the case in op-amp circuits, this problem is much easier than it looks. Note that the second "stage" is a summing configuration, and the first is a simple inverter. So...

$$V_0 = -\frac{R_5}{R_4} V_2 - \frac{R_5}{R_3} V_{01}$$

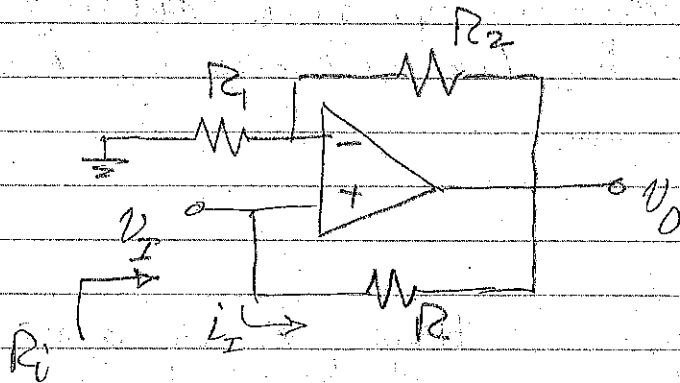
$$V_{01} = -\frac{R_2}{R_1} V_1$$

$$\therefore V_0 = -\frac{R_5}{R_4} V_2 + \frac{R_5 R_2}{R_3 R_1} V_1$$

Plugging in numbers...

$$V_0 = -2V_2 + 4V_1$$

## NEGATIVE IMPEDANCE CONVERTER



The analysis of  $V_O/V_I$  is the same as for the simple non-inverting configuration - the resistor  $R$  has no effect on that. So,

$$V_O/V_I = 1 + R_2/R_1$$

But the input resistance  $R_i$  is interesting...

$$R_i = V_I/I_I$$

$$I_I = \frac{V_I - V_O}{R} = \frac{V_I - (1 + R_2/R_1)V_I}{R}$$

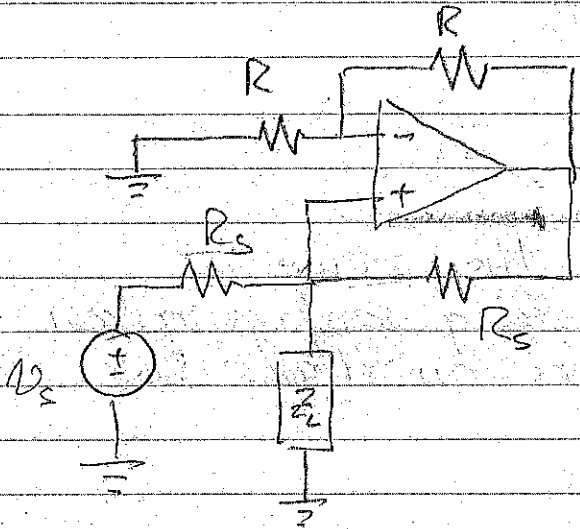
$$= -\frac{1}{R} \frac{R_2}{R_1} V_I$$

So  $R_i = -\frac{R R_1}{R_2}$  which is negative!

What can we do with this?  $\rightarrow$

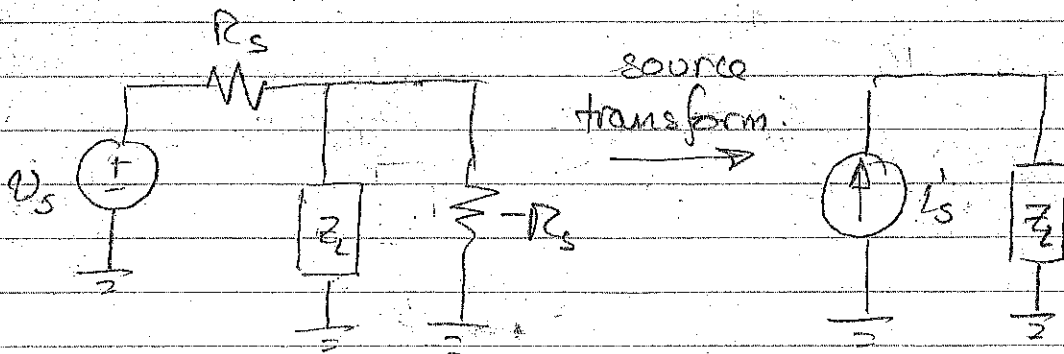


If we take the entire op amp circuit as a resistance  $-R_s$  (by setting  $R_2 = R_1$ ), we have



Here,  $Z_L$  is an arbitrary load, and we have attached a source with series resistance  $R_s$ .

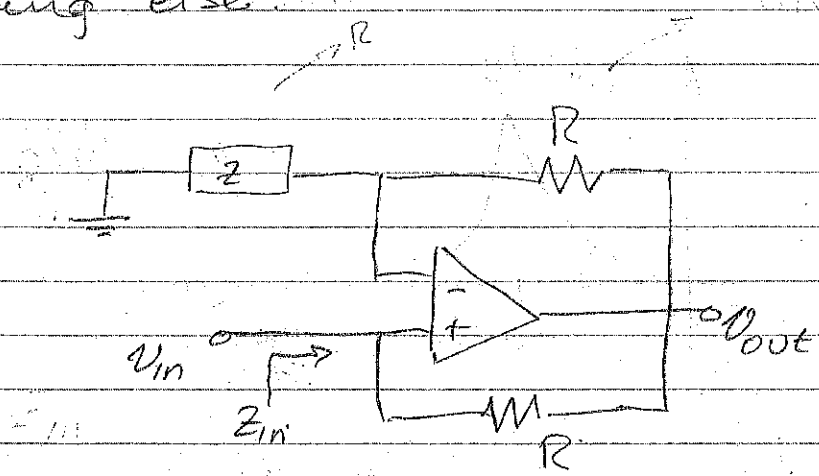
We can re-draw as



In the last step we have combined  $R_s$  and  $-R_s$  in parallel ( $R_s \parallel -R_s \rightarrow \infty$ ).

So we have constructed an IDEAL CURRENT SOURCE, i.e., one with  $R_s = \infty$ .

Something else:



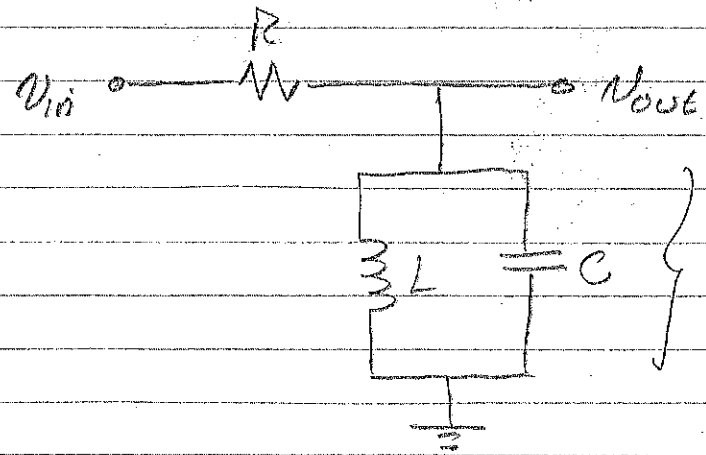
In this circumstance,  $Z_{in} = -Z$ . So if

$$Z = 1/j\omega c \quad (\text{a capacitor})$$

Then  $Z_{in} = 1/j\omega c$

which looks like an inductor. An inductor cannot be produced on an integrated circuit, but this will simulate one.

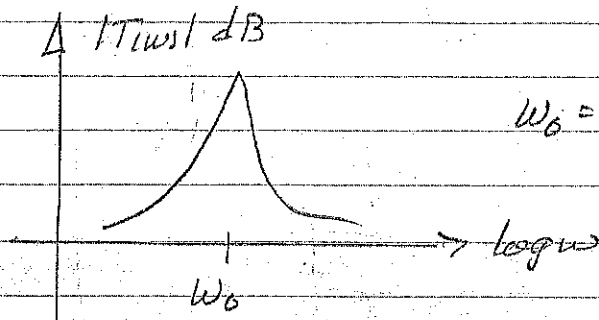
With both capacitors and inductors available, we can make a special class of filters.



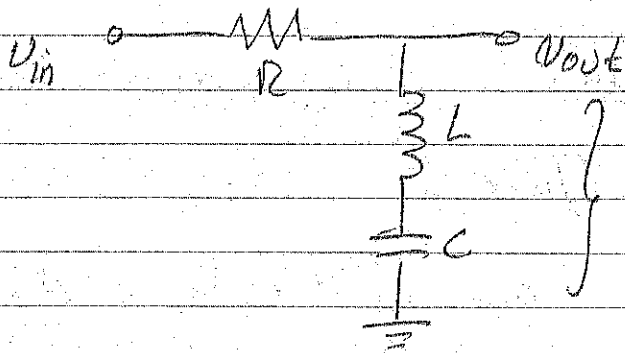
$$Z = j\omega L \parallel 1/j\omega c = \frac{4c}{j(\omega L - 1/\omega c)}$$

At  $\omega = \frac{1}{\sqrt{LC}}$ ,  $Z \rightarrow \infty$  and  $V_{out} = V_{in}$ .

Bode:



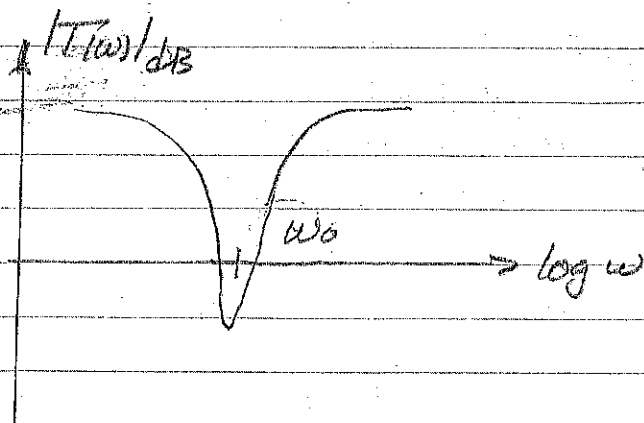
Alternatively, we can do this:



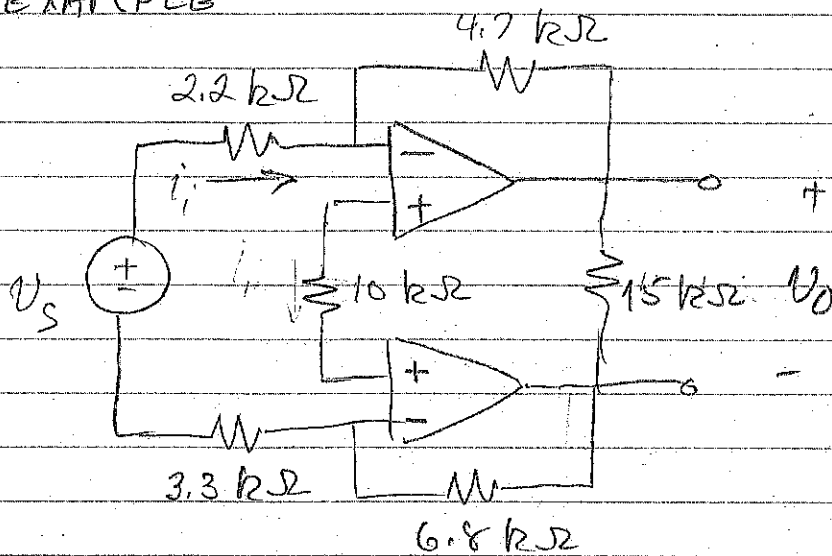
$$Z = \frac{1}{j\omega C} + j\omega L$$

$\rightarrow 0$  at  $\omega_0$

BODE:



## EXAMPLES



FIND

$$V_o/V_s$$

$$R_i$$

By KVL,

$$-V_s + 2200i + 3300i = 0$$

because there is no current in the  $10\text{ k}\Omega$  resistor.

$$\text{So } i = \frac{V_s}{5500}$$

KVL around outside gives

$$-V_s + i(2200 + 4700) + V_o + i(6800 + 3300) = 0$$

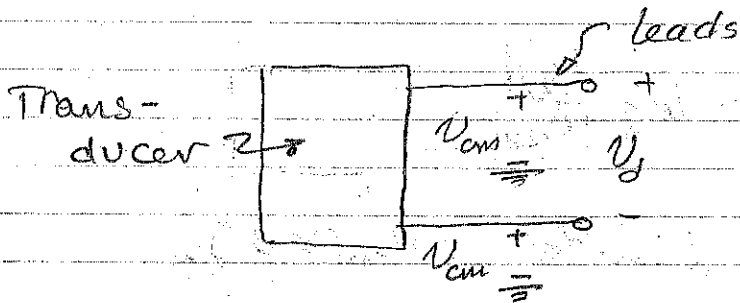
$$V_s \left( -1 + \frac{6900}{5500} + \frac{10100}{5500} \right) = -V_o$$

$$\frac{V_o}{V_s} = -2.09$$

$$\text{Also } R_i = \frac{V_i}{i} = 5500 \Omega.$$

## DIFFERENCE AMPLIFIER

We sometimes have the following situation.



Transducer: a measuring device, e.g. for temperature or sound intensity...

$V_d$  is the difference in potential between the two leads; we want to amplify this.

$V_{cm}$  is a voltage (the "common mode" voltage) between each lead and ground, which may be large compared to  $V_d$  and which we do not want to amplify.

We need a DIFFERENCE AMPLIFIER here.

In general,

$$V_o = A_d V_{Id} + A_{cm} V_{Icm}$$

where  $V_{Id}$  and  $V_{Icm}$  are the differential

and common mode input signals, and  $A_d$  and  $A_{cm}$  are the differential and common mode gains.

We would like the difference amplifier to reject the common mode signal; i.e.,

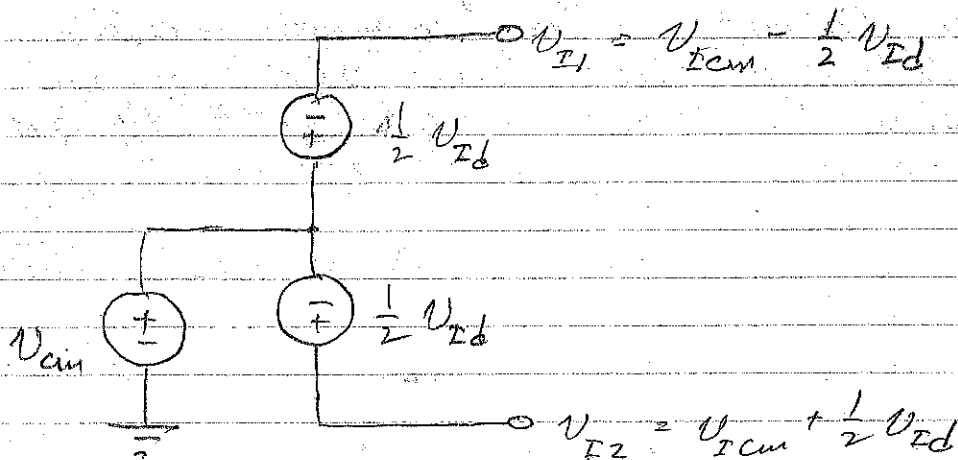
We would like  $A_{cm} \rightarrow 0$ . A measure of an amplifier's ability to do this is the "common mode rejection ratio" CMRR:

$$CMRR \equiv 20 \log \frac{|A_d|}{|A_{cm}|}$$

Ideally this is infinite ( $A_{cm} = 0$ ).

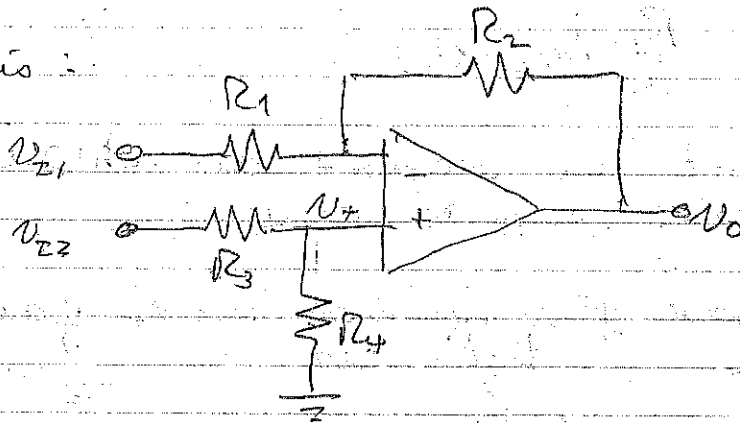
ANALYSIS of a difference amplifier:

We can represent the total signal (difference and common mode) as



Let's look at a configuration that will amplify  $V_{Id}$  and reject  $V_{cm}$  ...

Try this:



So  $V_{Id} = V_{I2} - V_{I1}$ . What is the output?

Superposition:

$$V_{I2} \rightarrow 0 \Rightarrow V_{O1} = -V_{I1} \frac{R_2}{R_1}$$

$$V_{I1} \rightarrow 0 \Rightarrow V_{O2} = \left(1 + \frac{R_2}{R_1}\right) V_{I2} \frac{R_4}{R_3 + R_4}$$

$$\text{So } V_O = V_{O1} + V_{O2} = V_{I1} \left(-\frac{R_2}{R_1}\right) + V_{I2} \left(1 + \frac{R_2}{R_1}\right) \frac{R_4}{R_3 + R_4}$$

This is complicated, and for arbitrary values of  $R_1, R_2, R_3, R_4$ , it is not what we want.

However, if we choose  $\frac{R_4}{R_3} = \frac{R_2}{R_1}$ , we get

$$\left(1 + \frac{R_2}{R_1}\right) \left(\frac{R_4}{R_3 + R_4}\right) = \left(1 + \frac{R_2}{R_1}\right) \left(\frac{1}{\frac{R_3}{R_4} + 1}\right)$$

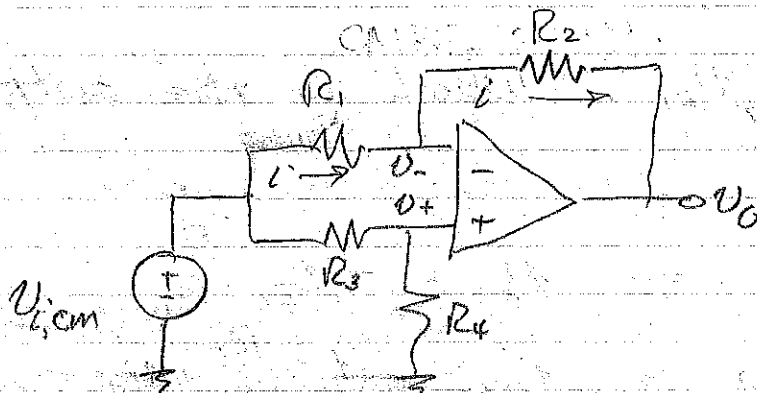
$$= \left(1 + \frac{R_2}{R_1}\right) \left(\frac{1}{R_1/R_2 + 1}\right)$$

$$= \left(\frac{R_1 + R_2}{R_1}\right) \left(\frac{R_2}{R_1 + R_2}\right) = \frac{R_2}{R_1}$$

$$\text{So } V_0 = \frac{R_2}{R_1} (V_2 - V_1)$$

This is what we want: an amplification of the difference signal only.

But what about the common mode gain of this circuit? Let's see.



$$V_0 = V_+ = V_{i,cm} \frac{R_4}{R_3 + R_4}$$

$$V_- = \frac{V_{i,cm} \left(1 - \frac{R_4}{R_3 + R_4}\right)}{R_1}$$

$$V_0 = V_{i,cm} \frac{R_4}{R_3 + R_4} - i_2 R_2$$

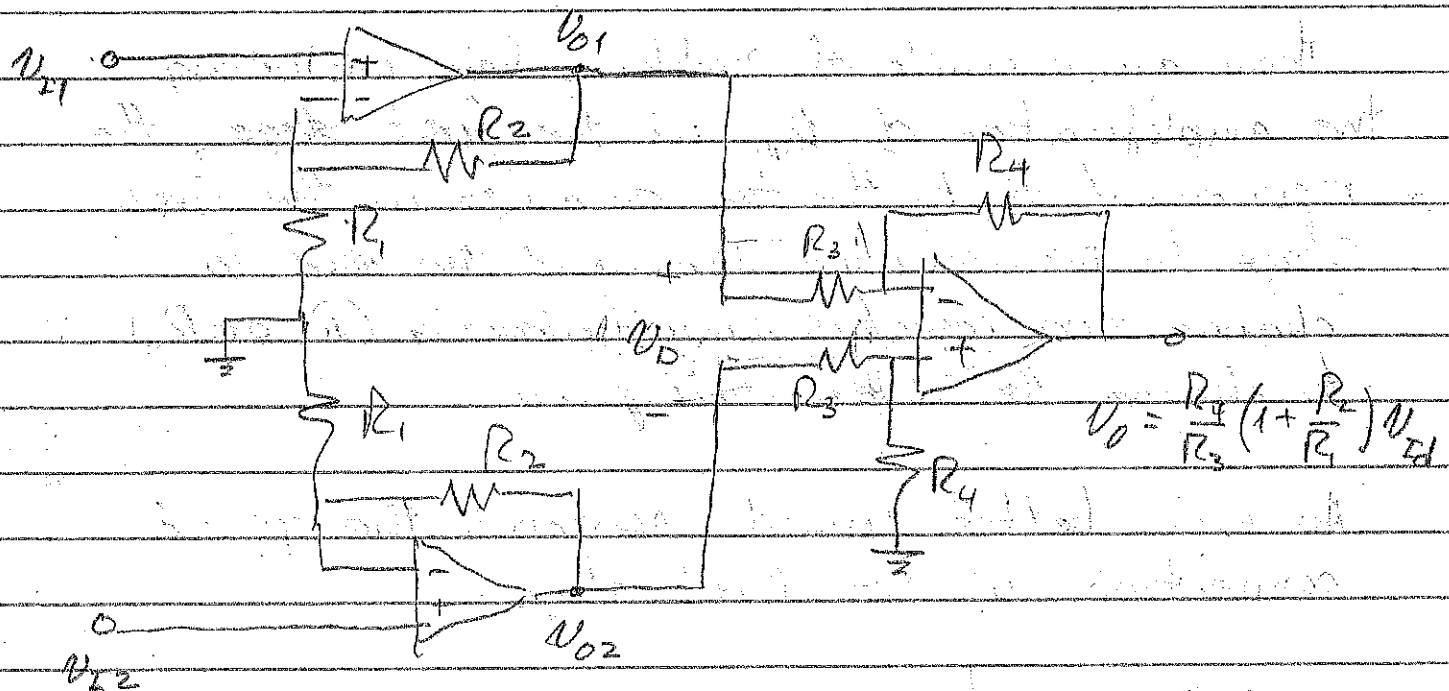
$$\text{and } A_{cm} = \frac{V_0}{V_{i,cm}} = \frac{R_4}{R_4 + R_3} \left(1 - \frac{R_2}{R_1} \frac{R_3}{R_4}\right)$$

If we choose  $\frac{R_4}{R_3} = \frac{R_2}{R_1}$  as before, then  $A_{cm} = 0$ , as desired.



## INSTRUMENTATION AMPLIFIER

A potential problem with the difference amplifier we just considered is the finite input resistance. We can improve this with the instrumentation amplifier:



The non-inverting op amps at the input provide high input resistance and gains of

$$V_{o1} = \left(1 + \frac{R_2}{R_1}\right) V_{i1} \quad V_{o2} = \left(1 + \frac{R_2}{R_1}\right) V_{i2}$$

Then  $V_o = V_{id} = \left(1 + \frac{R_2}{R_1}\right) (V_{i2} - V_{i1})$  is

amplified by the configuration we looked at previously.

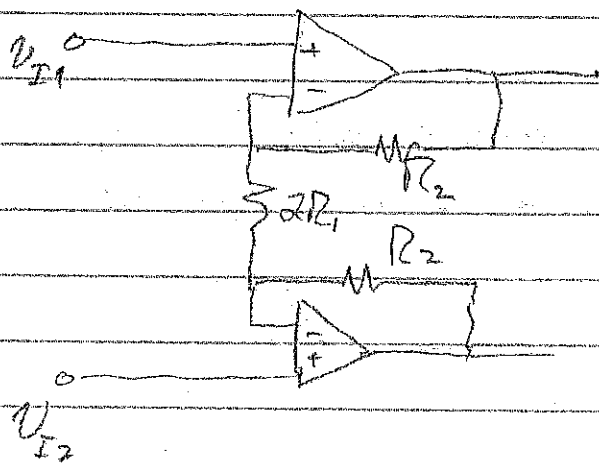
This second stage gives

$$V_o = \frac{R_4}{R_3} \left(1 + \frac{R_2}{R_1}\right) V_{Id}$$

So we have high  $R_{in}$ , and high CMRR.

There are a couple of problems here, including the amplification of  $V_{cm}$  in the first stage, the requirement that the two op amps in the first stage be precisely matched, and the need to change two resistors simultaneously ( $R_2$  or  $R_1$ ) to change the differential gain.

An even better circuit removes the ground connection in the first stage:



The resulting  $V_o$  is the same.