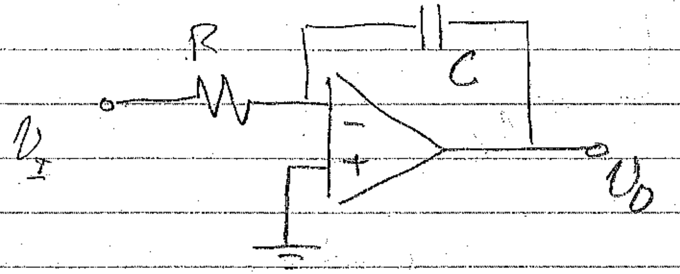
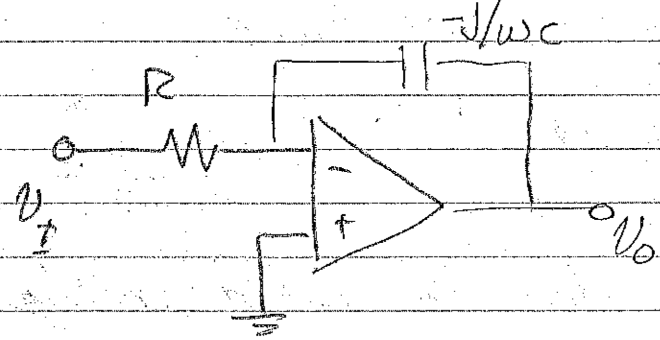
MILLER INTEGRATOR

Consider the following:



We can look at this circuit in either the time domain or the frequency (phasor) domain.

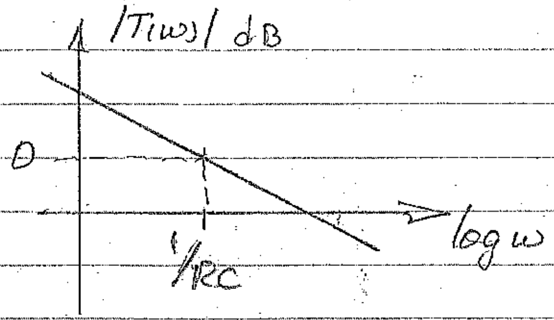
PHASOR DOMAIN



$$T(\omega) = \frac{+j/\omega C}{R}$$

$$= -\frac{1}{j\omega CR}$$

A Bode plot looks like this:

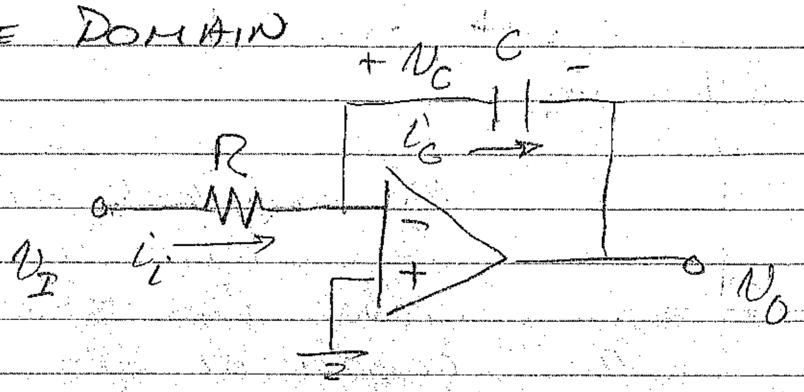


pole at $\omega = 0$

Also $\angle T(\omega) = 90°$

So this is a low-pass filter, in that frequencies of $\omega \ll 1/CR$ are "passed" whereas those above are attenuated.

TIME DOMAIN



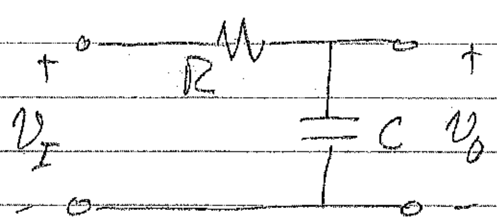
$$i_i = i_C = \frac{v_I}{R} \qquad v_O = -v_C = -\frac{1}{C}\int_{t_0}^{t} \frac{v_I}{R}\,dt + v_O(t_0)$$

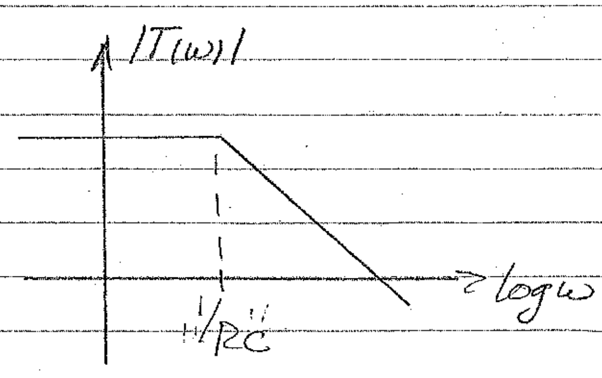
If we take $t_0 = 0$, $v_O(0) = 0$, we have

$$v_O = -\frac{1}{RC}\int_0^t v_I\,dt$$

so this circuit integrates the input voltage! This is a MILLER INTEGRATOR.

A simple RC circuit has some connection with this op amp:



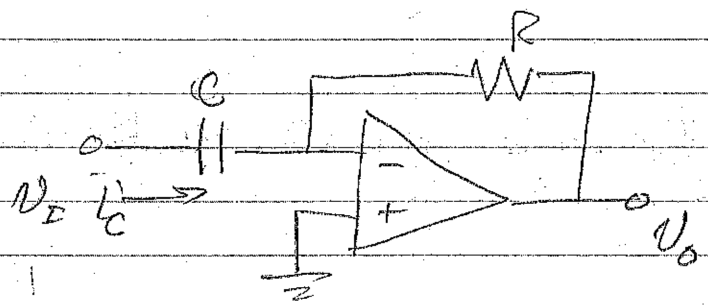


$$T(\omega) = \frac{1}{1 + j\omega RC}$$

So the MILLER INTEGRATOR is a low-pass filter with a break point at $\omega = 0$.
Looking from the RC circuit side, we can say that for $\omega \gg 1/RC$, the RC circuit is an integrator (based on the Bode plot).

MILLER DIFFERENTIATOR

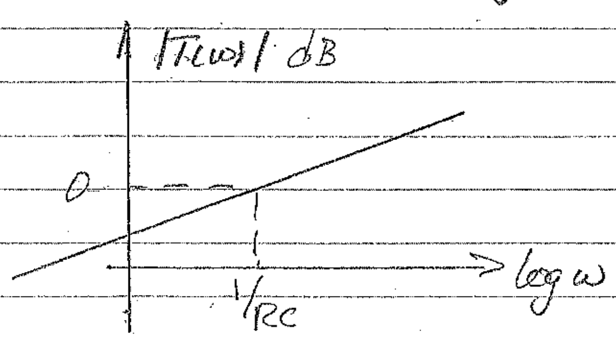
By interchanging R and C, we get a differentiator:



$$i_C = C\frac{dv_I}{dt} \qquad v_O = -i_C R = -RC\frac{dv_I}{dt}$$

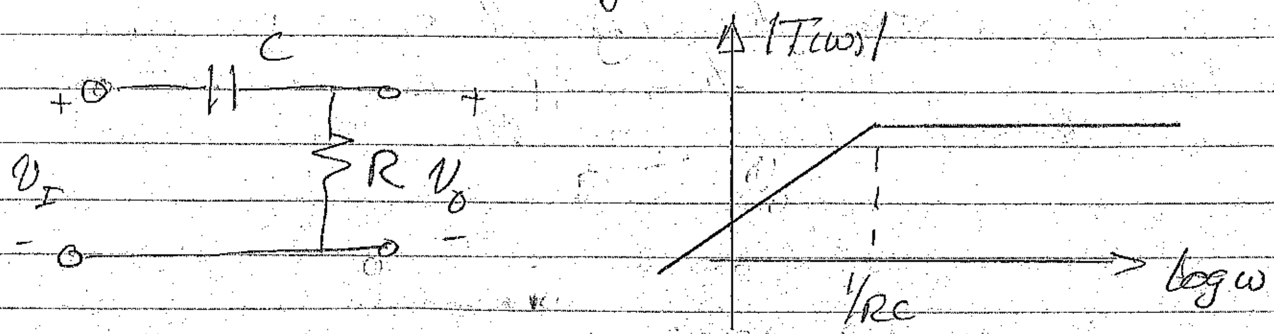
The phasor domain solution is

$$T(\omega) = -j\omega CR$$



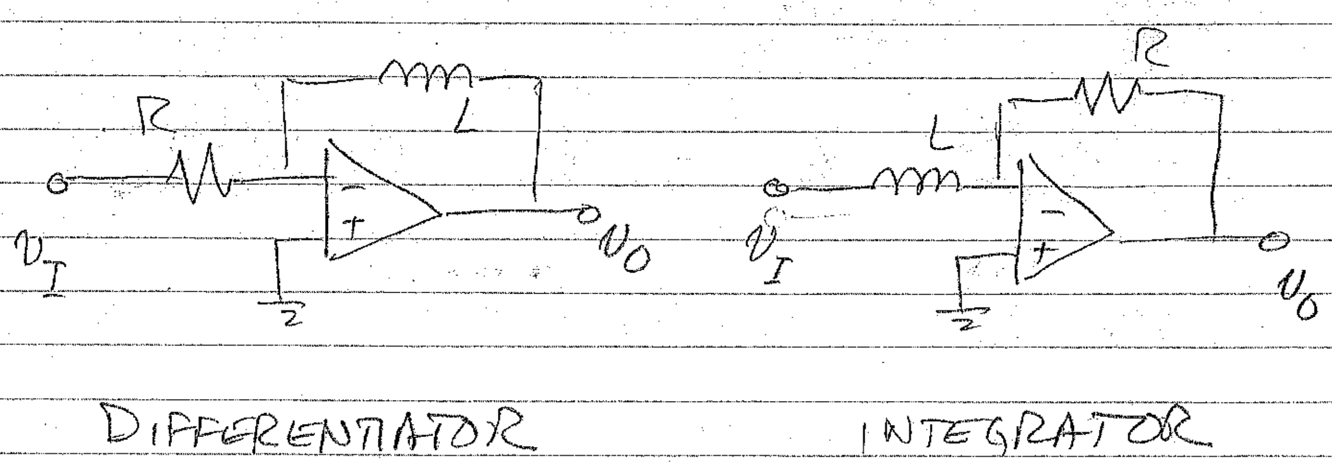
and $\angle T(\omega) = -90^\circ$

This is a high-pass filter. The RC circuit analog is



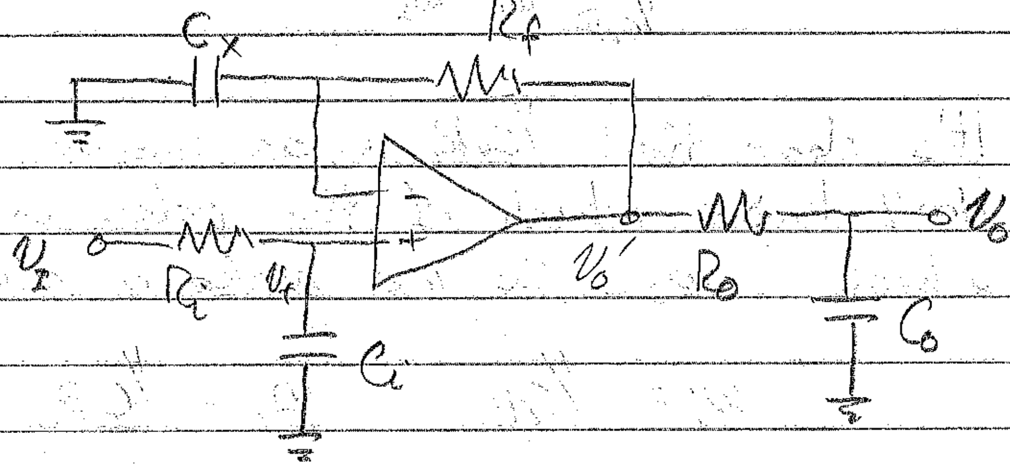
This has the same frequency response as the differentiator if $\omega << 1/RC$.

We can integrate and differentiate with inductors:



The circuit below is intended to be a band-pass filter. Choose resistor and capacitor values so that

- the 3 dB bandwidth is 20 krad/s.
- the upper breakpoint is 80 krad/s



Non-inverting configuration:

$$\bar{V}_o' = \bar{V}_+ (1 + j\omega C_x R_f)$$

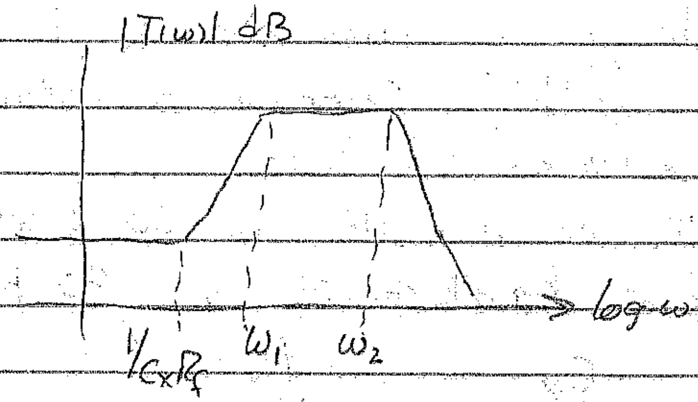
$$\bar{V}_+ = \bar{V}_I \frac{1}{1 + j\omega C_i R_i}$$

$$\bar{V}_o = \bar{V}_o' \frac{1/j\omega C_o}{1/j\omega C_o + R_o} = \bar{V}_o' \frac{1}{1 + j\omega C_o R_o}$$

From this we have

$$T(\omega) = \frac{\bar{V}_o}{\bar{V}_I} = \frac{(1 + j\omega C_x R_f)}{(1 + j\omega C_o R_o)(1 + j\omega C_i R_i)}$$

We have a zero and two poles, so...



It's clear that $1/C_xR_f$ is as shown, but we don't know which of the poles is ω_1 and which is ω_2. So we make a choice:

$$\omega_1 = 1/C_iR_i \qquad \omega_2 = 1/C_oR_o$$

We also have $\omega_2 = 80 \frac{krad}{s}$

and $\omega_2 - \omega_1 = 20 \frac{krad}{s} \Rightarrow \omega_1 = 60 \text{ krad/s}$

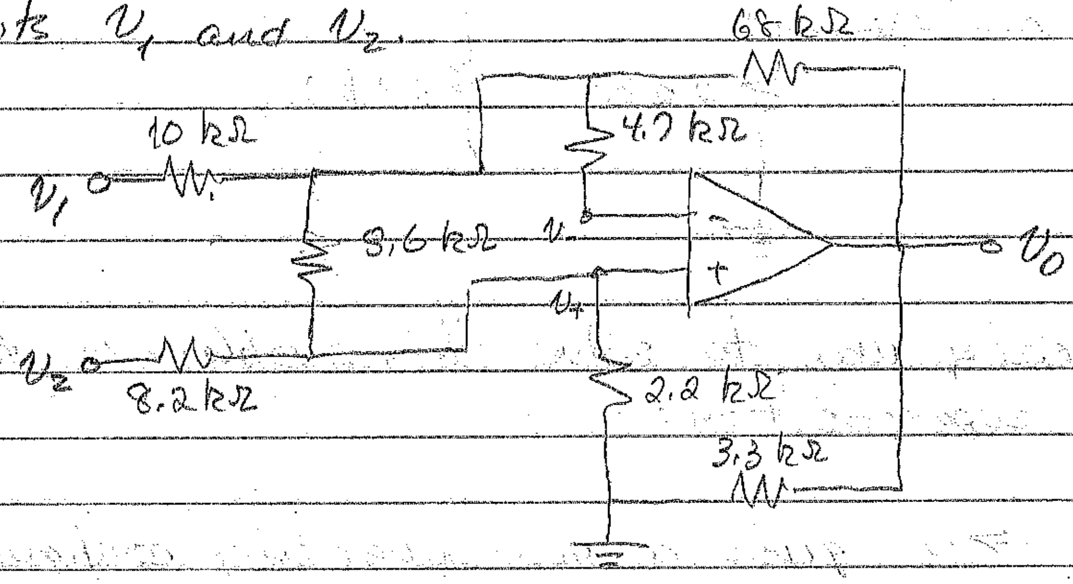
We now need to choose R's and C's, but we have no specifications so we pick something for the resistors:

$$R_i = 1\,k\Omega \Rightarrow C_i = \frac{1}{\omega_1 R_i} = 16.7\,nF$$

$$R_2 = 1\,k\Omega \Rightarrow C_o = \frac{1}{\omega_2 R_o} = 12.5\,nF$$

Quiz 3 Fall 2000

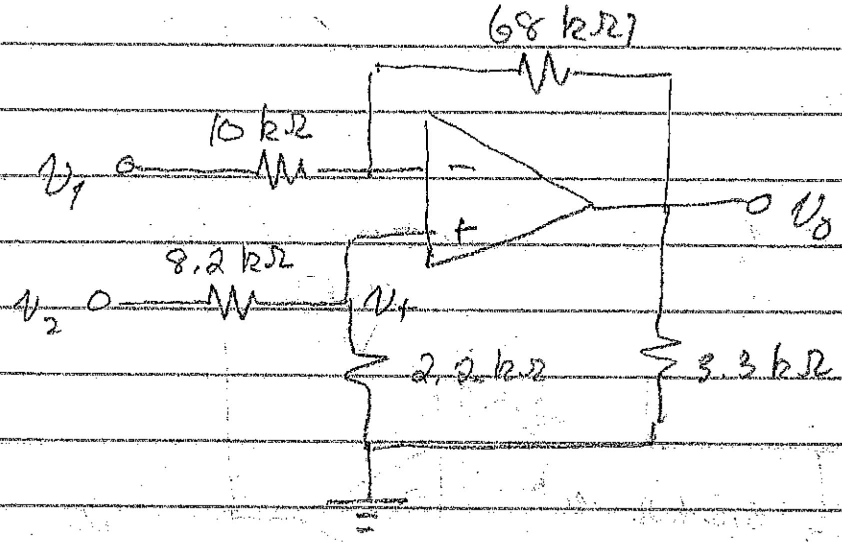
Find the output v_o as a function of the inputs v_1 and v_2.



We can simplify a bit here... Since we have negative feedback, $v_+ = v_-$. Also, there is no current in the 4.7 kΩ resistor, so that can be replaced by a short, in which case there's no voltage across 5.6 kΩ, and we can open circuit that!

We have re-drawn the circuit, incorporating these changes, on the next page.

We have left the 3.3 kΩ resistor in place, but it should be clear that v_o is not affected by it (because the op amp output resistance is zero).



An easy way to solve this problem is to use superposition:

$v_1 \to 0$ gives a non-inverting configuration, with

$$v_+ = v_2 \cdot \frac{2.2}{2.2+8.2} = 0.21\, v_2$$

Then $v_O^{NI} = v_+\left(1 + \frac{68}{10}\right) = 7.8 v_+ = 1.64\, v_2$.

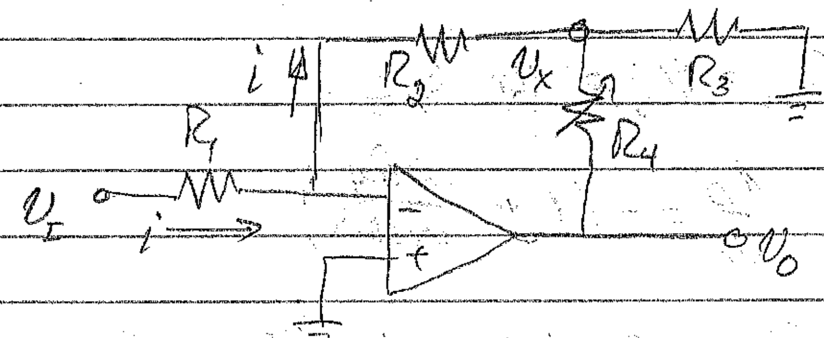
With $v_2 \to 0$ we have an inverting configuration.

$$v_O^{I} = -\frac{68}{10} v_1 = -6.8\, v_1$$

So $\boxed{v_O = v_O^{I} + v_O^{NI} = -6.8\, v_1 + 1.64\, v_2}$

Choose values for R_1, R_2, R_3 and a range for R_4 so that

- the input resistance seen by the source is 100 kΩ
- the gain v_O/v_I varies between −2 and −5.



Let's find v_O/v_I in terms of R's ...

$$\frac{v_X - v_O}{R_4} + \frac{v_X}{R_2} + \frac{v_X}{R_3} = 0$$

This gives v_O in terms of v_X:

$$v_O = v_X R_4 \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \right)$$

So we need v_X in terms of v_I:

$$v_X = -iR_2 = -\frac{v_I}{R_1} R_2$$

Substituting that above gives

$$\frac{v_O}{v_I} = -\frac{R_2}{R_1}\left(1 + \frac{R_4}{R_3} + \frac{R_4}{R_2}\right)$$

To get the required R_{in} we need $R_1 = 100\,k\Omega$. There are a lot of choices for the other resistors, so let's choose something:

$$R_2 = R_3 = 100\,k\Omega$$

$$\Rightarrow \quad \frac{v_O}{v_I} = -\left(1 + \frac{2R_4}{100\,k\Omega}\right)$$

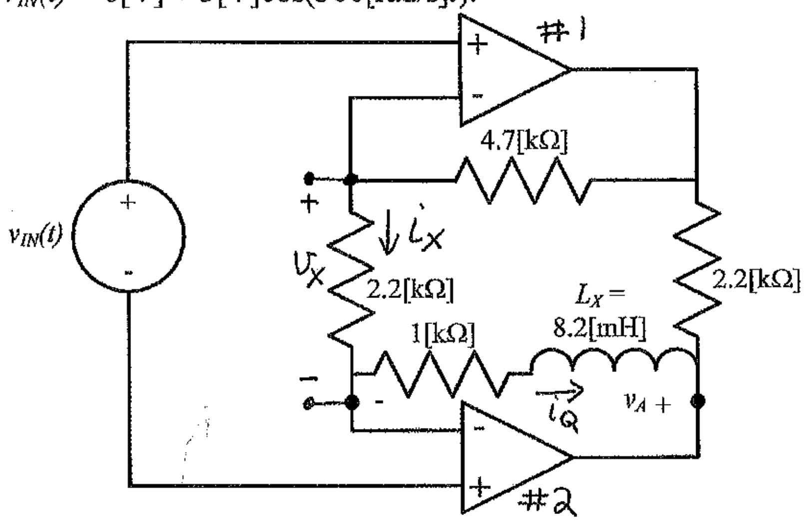
Now $R_4 = 50\,k\Omega \Rightarrow \frac{v_O}{v_I} = -2$

$R_4 = 200\,k\Omega \Rightarrow \frac{v_O}{v_I} = -5$

So the range for R_4 is

$$50\,k\Omega \leq R_4 \leq 200\,k\Omega$$

2. {45 Points} Use the circuit below to solve this problem. Assume an ideal op amp.
a) Find the voltage gain V_A / V_{IN}.
b) Find the voltage gain V_a / V_{in} as a function of angular frequency.
c) Find the voltage gain v_a / v_{in} as a function of time, if
$v_{IN}(t) = 6[V] + 3[V]\cos(300[rad/s]t)$.



op amp #1 has neg. fdbk, through 4.7{kΩ}.
Op amp #2 has neg. fdbk, through L_X and 1[kΩ] resistor.

So, by KVL, $v_X = v_{IN}$.

$$i_X = \frac{v_X}{2.2\{k\Omega\}} = \frac{v_{IN}}{2.2\{k\Omega\}} = i_Q$$

a) $\frac{V_A}{V_{IN}} = ?$ Now @ dc, L_X acts as a short circuit, so

$$V_A = -I_Q \, 1\{k\Omega\} = -\frac{V_{IN}}{2.2\{k\Omega\}} 1\{k\Omega\}$$

$$\frac{V_A}{V_{IN}} = -\frac{1}{2.2} = \boxed{-0.455}$$

see next page

b) $\dfrac{V_a}{V_{in}} = ?$ In the phasor domain,

$$V_a = -I_q\left(1\{k\Omega\} + j\omega\, 8.2\{mH\}\right)$$

$$V_a = -\frac{V_{in}}{2.2\{k\Omega\}}\left(1\{k\Omega\} + j\omega\, 8.2\{mH\}\right)$$

$$\frac{V_a}{V_{in}} = -0.455 - j\omega(3.727)\left[\frac{\mu H}{\Omega}\right]$$

Now, we remember that $1\{H\} = 1\dfrac{\{V\}\{s\}}{\{A\}}$,
so $1\left\{\dfrac{H}{\Omega}\right\} = 1\{s\}$.

$$\boxed{\frac{V_a}{V_{in}} = -0.455 - j\omega\, 3.727\{\mu s\}}$$

c) $\dfrac{v_a}{v_{in}} = ?$ Since these are signal quantities,
$$v_{in} = 3\{V\}\cos\left(300\left\{\tfrac{rad}{s}\right\}t\right)$$

$$v_a(t) = -i_Q\, 1\{k\Omega\} - 8.2\{mH\}\frac{d\, i_Q}{dt}$$

Since $i_Q = \dfrac{3\{V\}\cos\left(300\left\{\frac{rad}{s}\right\}t\right)}{2.2\{k\Omega\}} = 1.364\{mA\}\cos 300\left\{\frac{rad}{s}\right\}t$

$$v_a(t) = -1.364\{V\}\cos\left(300\left\{\tfrac{rad}{s}\right\}t\right) +$$
$$-11.18\left[\mu\frac{H}{A}\right]300\left[\tfrac{rad}{s}\right]\left(-\sin\left(300\left\{\tfrac{rad}{s}\right\}t\right)\right)$$

see next page

So, $$\frac{v_a}{v_{in}} = -\left(\frac{1.364}{3}\right) + 1.118\ \frac{\sin\left(300\left\{\frac{\text{rad}}{\text{s}}\right\}t\right)}{\cos\left(300\left\{\frac{\text{rad}}{\text{s}}\right\}t\right)} \times 10^{-3}$$

$$\boxed{\frac{v_a}{v_{in}} = -0.455 + 0.001118\ \tan\left(300\left\{\frac{\text{rad}}{\text{s}}\right\}t\right)}$$

3. {45 Points} Assume an ideal op amp in the schematic in Figure 1. Find the values of R_2, C_X, and L_X that will produce the magnitude Bode plot that has the straight-line approximation given in the plot in Figure 2.

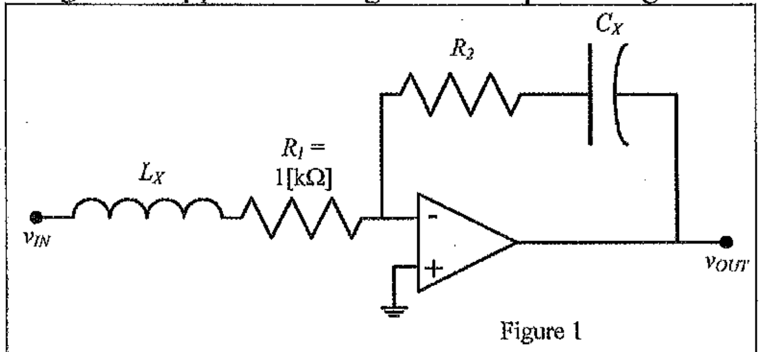


Figure 1

← This is the inverting configuration of the op amp, and so we know the gain. By going to the phasor domain, we can get the transfer function.

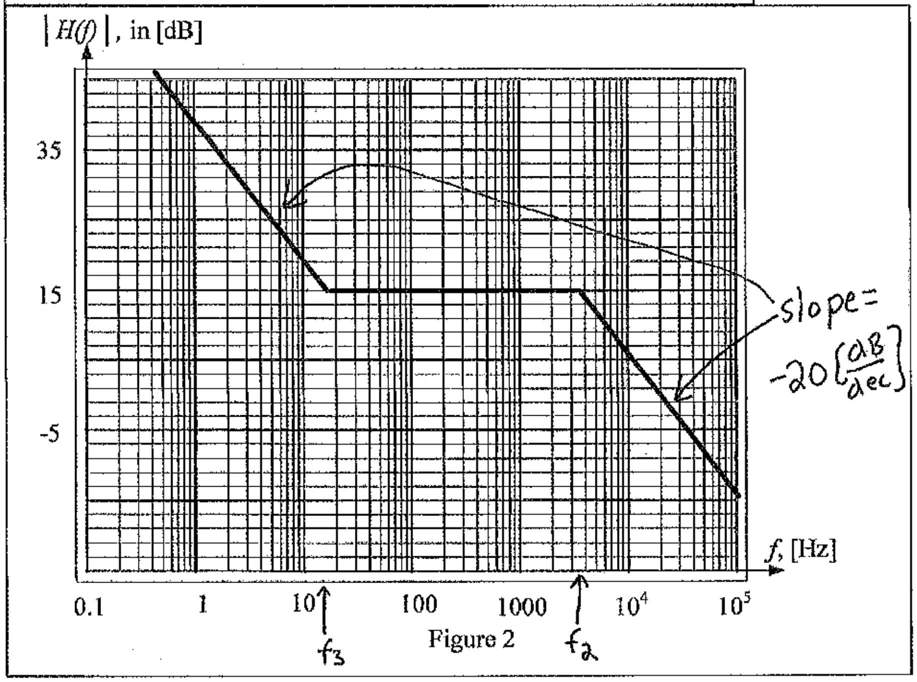


Figure 2

The transfer function, $\frac{V_{out}}{V_{in}} = \frac{-\left(R_2 + \frac{1}{j\omega C_X}\right)}{R_1 + j\omega L_X} =$

$$H(\omega) = \frac{-(j\omega C_X R_2 + 1)}{j\omega C_X\,(R_1 + j\omega L_X)}$$

See next page

This has 2 poles, and 1 zero.

One pole is at $\omega_1 = 0$

and one at $\omega_2 = \frac{R_1}{L_X}$

The zero is at $\omega_3 = \frac{1}{R_2 C_X}$

The pole is at $f_2 = 3{,}500[Hz]$, which corresponds to $\omega_2 = 2\pi f_2 = 22.0\left[\frac{krad}{s}\right]$

$$22.0\left[\frac{krad}{s}\right] = \frac{1[k\Omega]}{L_X} \text{ , so}$$

$$\boxed{L_X = 45[mH]}$$

In the frequency range below f_2 (and ω_2),

$|R_1| > |j\omega L_X|$, so we can neglect L_X.

In a similar way, above f_3 (and ω_3),

$|R_2| > \left|\frac{1}{j\omega C_X}\right|$, so we can neglect C_X.

So, between f_3 and f_2, the gain is $-\frac{R_2}{R_1}$,

and $$20 \log_{10}\left|\frac{R_2}{-R_1}\right| = 15[dB].$$

$$10^{(15/20)} = \frac{R_2}{R_1} = 5.62 \text{, So } \boxed{R_2 = 5.62[k\Omega]}$$

see next page

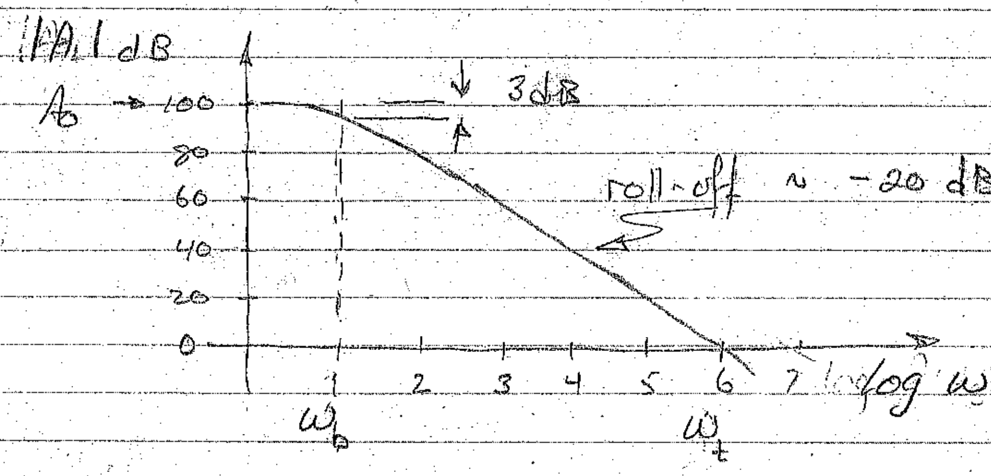
The zero is at $f_3 = 15\{Hz\}$, so

$$w_3 = 2\pi\, 15 = 94.2\left\{\frac{rad}{s}\right\} = \frac{1}{R_2 C_X}$$

$$C_X = \frac{1}{(94.2)(5620)} = \boxed{1.89\{\mu F\}}$$

NONIDEAL PERFORMANCE OF OP AMPS

We have been assuming that the open-loop gain A is large (∞) and constant with frequency. In fact its behavior in typical op amps is as follows:



3 dB frequency $\equiv \omega_b$

Generally we have

$$A(\omega) = \frac{A_0}{1 + j\omega/\omega_b}$$

which for high frequencies is

$$A(\omega) \simeq \frac{A_0 \omega_b}{j\omega} \qquad (\omega \to \infty)$$

Now $|A(\omega)| \longrightarrow 1 \Rightarrow \omega \longrightarrow \omega_t = A_0 \omega_b$

where ω_t is the <u>UNITY GAIN BANDWITH</u>.

So we have $A(\omega) \simeq \frac{\omega_t}{j\omega}$

and $|A(\omega)| \simeq \omega_t/\omega$

Op amp data sheets generally specify ω_t so we can easily estimate the open loop gain as a fn of frequency.

INVERTING CONFIGURATION REVISITED

For noninfinite open loop gain we had

$$\frac{v_o}{v_i} = \frac{-R_2/R_1}{1 + (1 + R_2/R_1)/A}$$

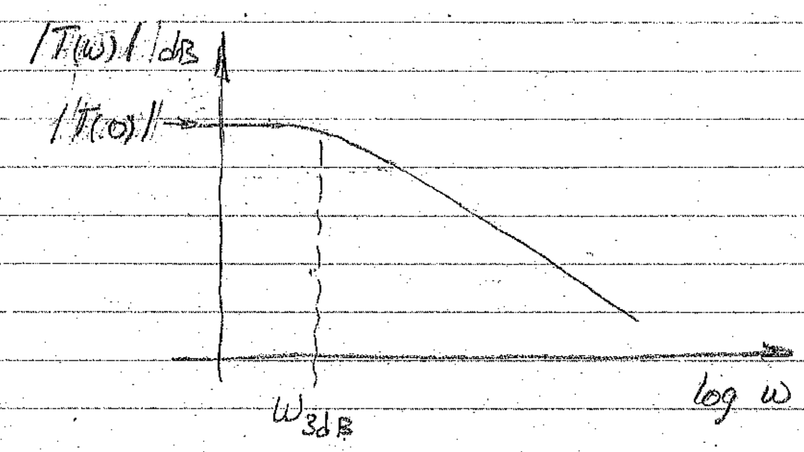
Substituting $A(\omega) = \frac{A_0}{1 + j\omega/\omega_b}$ gives

$$\frac{\bar{V}_o(\omega)}{\bar{V}_i(\omega)} = \frac{-R_2/R_1}{1 + (1 + R_2/R_1)/A_0 + \frac{j\omega}{\omega_t/(1 + R_2/R_1)}}$$

and if $A_0 \gg 1 + R_2/R_1$ we can simplify to

$$T(\omega) = \frac{\bar{V}_o(\omega)}{\bar{V}_i(\omega)} = \frac{-R_2/R_1}{1 + \frac{j\omega}{\omega_t/(1 + R_2/R_1)}}$$

Thus the inverting configuration has a magnitude bode plot that looks like...



where $\omega_{3dB} = \dfrac{\omega_t}{1 + R_2/R_1}$

and $\omega \rightarrow 0 \Rightarrow T(0) \rightarrow -R_2/R_1$

NONINVERTING CONFIGURATION

Analysis of the noninverting configuration shows that

$$T(\omega) \equiv \frac{\bar{V}_o(\omega)}{\bar{V}_i(\omega)} = \frac{1 + R_2/R_1}{1 + \dfrac{j\omega}{\omega_t/(1 + R_2/R_1)}}$$

So $\omega_{3dB} = \dfrac{\omega_t}{1 + R_2/R_1}$ and $|T(0)| = 1 + R_2/R_1$

So the GAIN - BANDWIDTH PRODUCT,

$|T(0)| \cdot \omega_{3dB} = \omega_t$ is CONSTANT.

This is also approximately true for the inverting configuration.

Thus we have a fundamental trade-off between gain and bandwidth (i.e. the frequency range over which the gain is constant).

SLEW RATE

In addition to ω-dependent gain, real op amps can provide output voltages that change at a finite maximum rate. The max rate is the SLEW RATE SR:

$$SR = \left.\frac{dV_o}{dt}\right|_{max}$$

SEDRA & SMITH, FIG. 2.43

Note that a finite slew rate is not the same thing as limited frequency response. For ω-dependent gain, the output amplitude will be reduced, but V_o is still a good representation of the input signal. In other words, the amp response is still linear.

However, a finite slew rate can change the shape of the output, so the amp is no longer linear. For example, a sinusoid has a varying slope, which maximizes at the zero-crossings, and will be distorted by finite slew rate.

SEDRA & SMITH FIG 2.44

Specification: FULL-POWER BANDWIDTH f_M:

$$\omega_M V_{omax} = SR$$

$$\Rightarrow \quad f_M = \frac{SR}{2\pi V_{omax}}$$

V_{omax} is the rated max output voltage, and f_m is the frequency at which a sinusoid of amplitude V_{omax} will begin to distort.

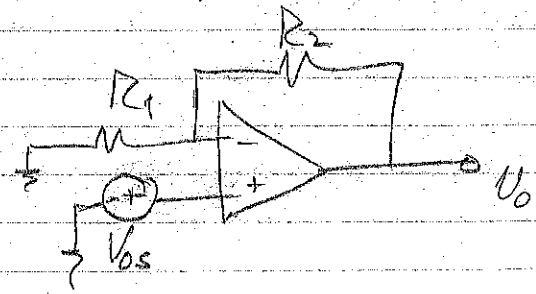
Smaller amplitude sinusoids can operate at higher frequencies before distortion sets in. For amplitude V_o we have distortion free output until a frequency ω such that

$$V_o = V_{omax} \frac{\omega_H}{\omega}$$

INPUT OFFSET VOLTAGE (V_{OS})

For an ideal op amp, $v_1 = v_2 = 0 \Rightarrow v_0 = 0$.
For $V_{OS} \neq 0$, $v_1 = v_2 = 0$ gives a non-zero v_0.
Assuming $V_{OS} > 0$, we can analyze the effect of V_{OS} on both inverting and noninverting configurations by shorting the input signal:

$v_{IN} \rightarrow 0 \Rightarrow$

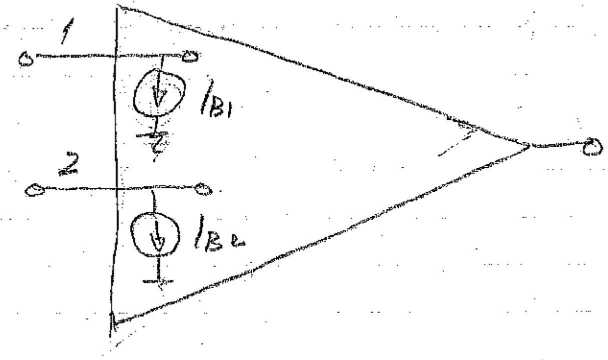


Here $v_0 = V_{OS}(1 + R_2/R_1)$ which will be superimposed on the output due to v_{IN}.

Possible Solution: Some amps are provided with "trim" pins to which a simple circuit can be connected to reduce v_0 to 0 for $v_1 = v_2 = 0$.

Another dc-related problem is INPUT BIAS CURRENT.

Internal op amp structure results in the following equivalent circuit:

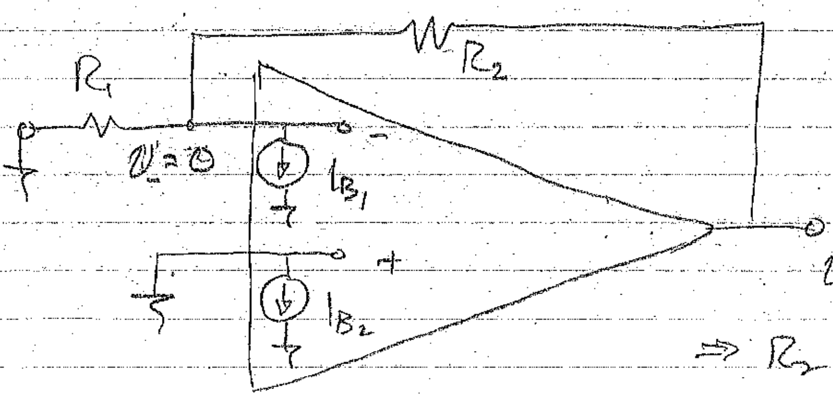


Define INPUT BIAS CURRENT $I_B = \frac{1}{2}(I_{B_1} + I_{B_2})$

and INPUT OFFSET CURRENT $I_{OS} = |I_{B_1} - I_{B_2}|$

Now for the INVERTING CONFIGURATION:

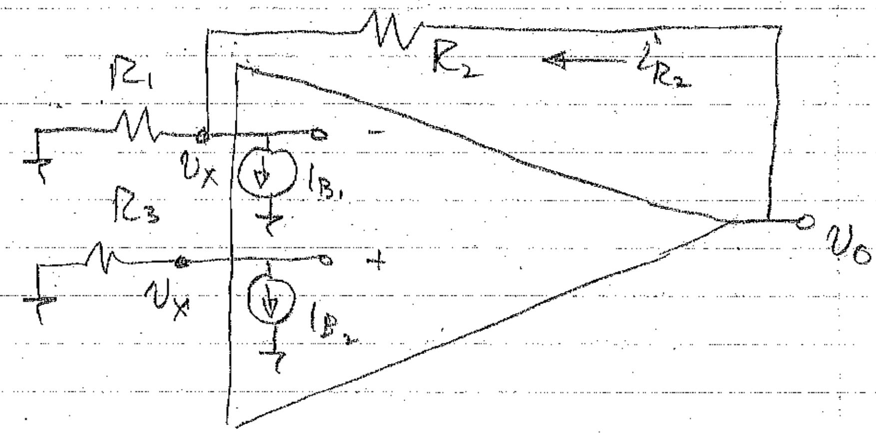
Sedra/Smith Fig 2.32, 2.33



$v_o = I_{B_1} R_2$

$\Rightarrow R_2$ can't be large.

Compensation: add series R_3 to + input:



$$v_x = -I_{B_2} R_3$$

$$i_{R_2} = \frac{v_x}{R_1} + I_{B_1} = \frac{-I_{B_2} R_3}{R_1} + I_{B_1}$$

$$v_o - v_x - i_{R_2} R_2 = 0$$

$$\therefore \quad v_o = v_x + i_{R_2} R_2 = -I_{B_2} R_3 + I_{B_1} R_2 - \frac{I_{B_2} R_2 R_3}{R_1}$$

If $I_{os} = 0$, i.e. $I_{B_1} = I_{B_2} = I_B$, we have

$$v_0 = I_B [R_2 - R_3(1 + R_2/R_1)]$$

and $v_0 = 0$ if $R_3 = R_1 R_2/(R_1 + R_2)$

If R_3 is chosen as such, and $I_{os} \neq 0$, say $I_{B_1} = I_B + I_{os}/2$, $I_{B_2} = I_B - I_{os}/2$, we have

$$v_0 = I_{os} R_2$$

which is much smaller than w/o R_3.

General result: To minimize the effect of I_B, place a resistance at the noninverting input equal to the dc resistance seen by the inverting terminal.