Creating Bode Plots using Straight-line Approximations

Step 1: Find the poles and zeros

Write the transfer function of the circuit in the form

$$H(j\omega) = A(\omega) \cdot \frac{\left(1 + j\frac{\omega}{z_1}\right)\left(1 + j\frac{\omega}{z_2}\right)\cdots}{\left(1 + j\frac{\omega}{p_1}\right)\left(1 + j\frac{\omega}{p_2}\right)\cdots}$$

and identify the location of the zeros and poles. Here, we refer to the z variables as *zeros* and the p variables as *poles*, and A may be a function of ω.

Example 1

$$H(j\omega) = 10 \cdot \frac{\left(1 + j\frac{\omega}{10}\right)}{\left(1 + j\frac{\omega}{100}\right)\left(1 + j\frac{\omega}{10000}\right)}$$

has one zero at 10 rad/s and two poles, one at 100 rad/s and another at 10,000 rad/s.

Example 2

$$H(j\omega) = 1000 \cdot \frac{j\frac{\omega}{1000}}{\left(1 + j\frac{\omega}{1000}\right)\left(1 + j\frac{\omega}{10000}\right)}$$

has no zeros and two poles, one at 1000 rad/s and another at 10,000 rad/s. Notice that $j\frac{\omega}{1000}$ in the numerator is not a zero, and it does make A a function of ω.

Step 2: Determine the starting values

Calculate the value of the transfer function for a frequency that is one tenth the value of the smallest pole or zero (the choice of one tenth is arbitrary: just choose a convenient value that is much less than the smallest pole or zero). Note that the imaginary part of all of the poles and zeros will be much smaller than the real part at these low frequencies and you can ignore the imaginary part for this approximation. If A is a function of ω, then the starting slope will be $-20\,\text{dB/dec}$ if ω is in the denominator and $20\,\text{dB/dec}$ if ω is in the numerator. If the ω has an exponent, the slope should be multiplied by the exponent.

Example 1

For

$$H(j\omega) = 10 \cdot \frac{\left(1 + j\frac{\omega}{10}\right)}{\left(1 + j\frac{\omega}{100}\right)\left(1 + j\frac{\omega}{10000}\right)}$$

choose $\omega = 1\,\text{rad/s}$, which is one tenth the value of the first pole or zero. At this frequency, the imaginary terms will be much smaller than 1, so we can simplify the expression to

$$H(1) = 10 \cdot \frac{\left(1 + j\frac{1}{10}\right)}{\left(1 + j\frac{1}{100}\right)\left(1 + j\frac{1}{10000}\right)} \simeq 10 \cdot \frac{(1)}{(1)\,(1)} = 10.$$

The magnitude is $20\log_{10}|10| = 20\,\text{dB}$ and the phase is $0°$. Because A is not a function of ω, the slope will be $0\,\text{dB/dec}$.

Example 2

For

$$H(j\omega) = 1000 \cdot \frac{j\frac{\omega}{1000}}{\left(1 + j\frac{\omega}{1000}\right)\left(1 + j\frac{\omega}{10000}\right)}$$

choose $\omega = 100\,\text{rad/s}$ so that

$$H(100) = 1000 \cdot \frac{j\frac{100}{1000}}{\left(1 + j\frac{100}{1000}\right)\left(1 + j\frac{100}{10000}\right)} \simeq \frac{j100}{(1)\,(1)} = j100.$$

The magnitude is $20\log_{10}|j100| = 40\,\text{dB}$ and the phase is $90°$. Because A is a function of ω and ω in the numerator, the slope will be $20\,\text{dB/dec}$.

Step 3: Sketch the graph

For the magnitude plot, mark the starting point on the graph and draw a straight line with the starting slope until you reach the frequency of a pole or zero. At this point, zeros change the slope by 20 dB/dec and poles change the slope by −20 dB/dec. For the phase plot, a pole will change the phase by −90° over two decades, starting one decade before the pole, and a zero will change the phase by 90° over two decades, starting one decade before the zero.

Example 1

For the magnitude plot, start the line at $x = 1$ rad/s and $y = 20$ dB and extend the line until you reach the first zero at $x = 10$ rad/s. The slope at this zero changes to 20 dB/dec until we reach the pole at $x = 100$ rad/s, where the slope returns to 0. At $x = 10{,}000$ rad/s the slope changes again to −20 dB/dec because of the second pole.

For the phase plot, we can sketch the ranges of influence for the poles and zeros and we label these with the appropriate slope. Then we plot our first point at $x = 1$ rad/s and $y = 0°$ and continue to sketch lines at slopes that are the sum of the overlapping slopes in our ranges of influence. The graph in Figure 1 shows it best. Note that I have included the actual plot of the function, which the approximation follows nicely.

Example 2

See Figure 2.

Summary

Sketching straight-line approximations of Bode plots is a matter of following the straight-forward approach outlined here, and the result is a plot that is very close to the actual frequency response of the circuit. Things do become more complicated when the poles and zeros are close to each other, particularly when making the phase plot. Remember that the long-term goal is to become comfortable with making these plots so we can see the connection between components in a circuit and the effects on the frequency response. In that sense, relying on pure memorization of the process is a good first step, but will not get you all of the way.

Also, note that the straight-line approximation reveals a lot more about the transfer function of the circuit. Compare the plot in Figure 3, which does not have any straight lines, to that in Figure 1. The transition points, especially in the phase response, are easier to see in the approximation.

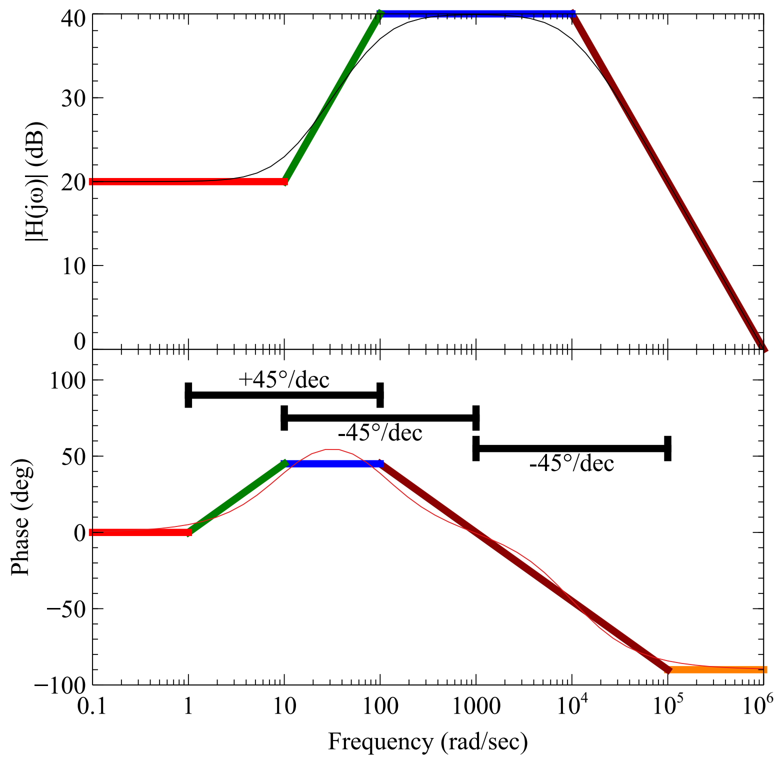

Figure 1: The graph for Example 1.

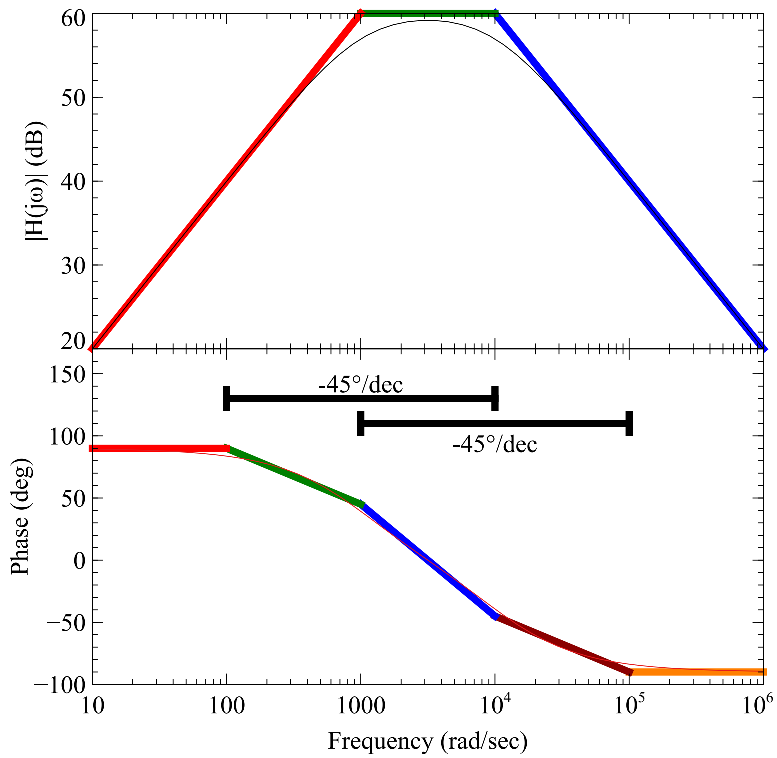

Figure 2: The graph for Example 2.

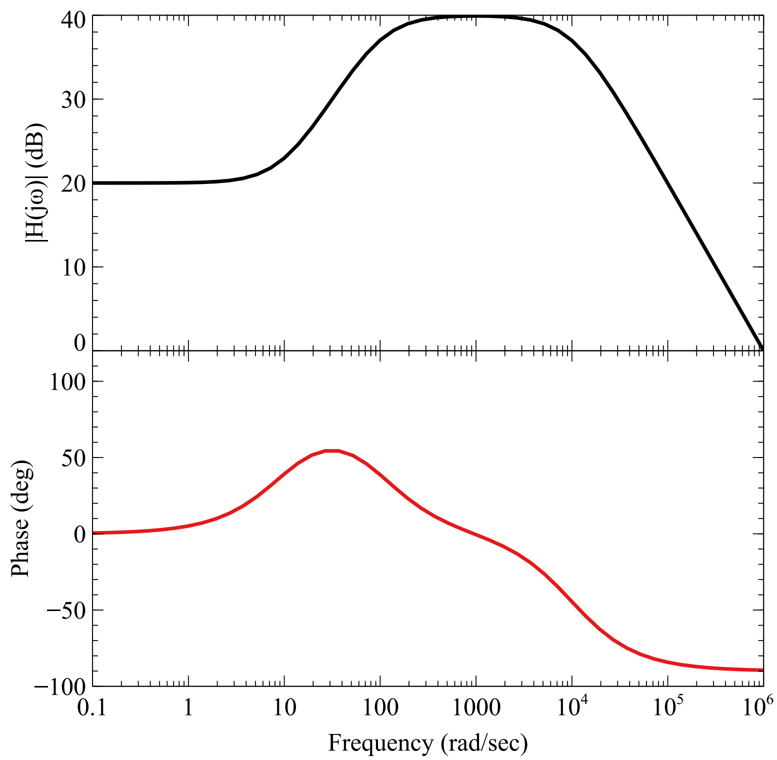

Figure 3: The graph for Example 2 plotted without the straight-line approximation. Note that the transition points are not easy to identify.