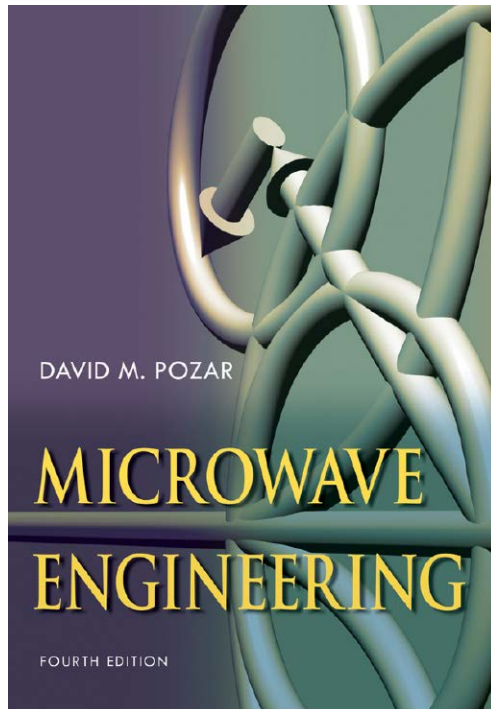


# ECE 5317-6351

# Microwave Engineering

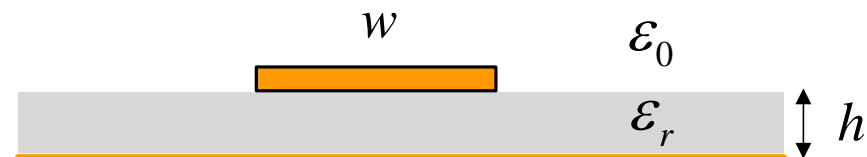
**Fall 2018**

Prof. David R. Jackson  
Dept. of ECE

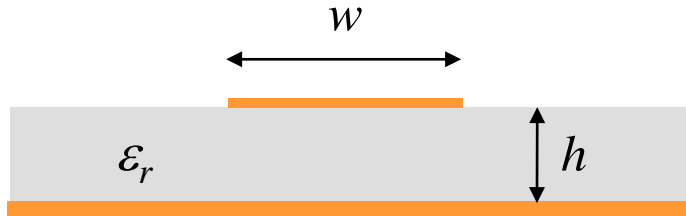


## Notes 11

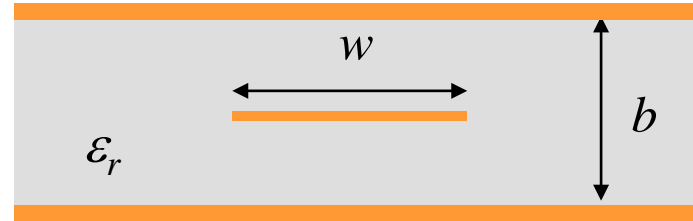
### Waveguiding Structures Part 6: Planar Transmission Lines



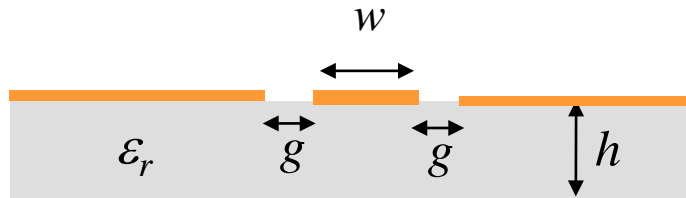
# Planar Transmission Lines



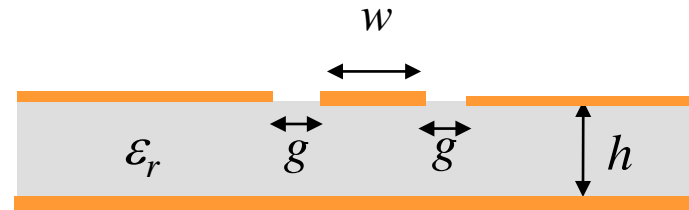
Microstrip



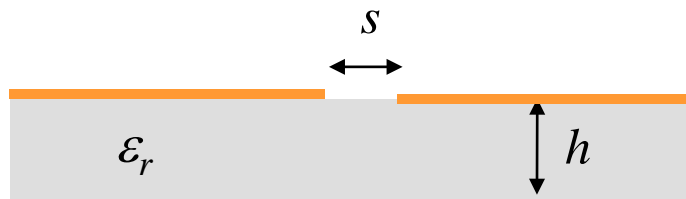
Stripline



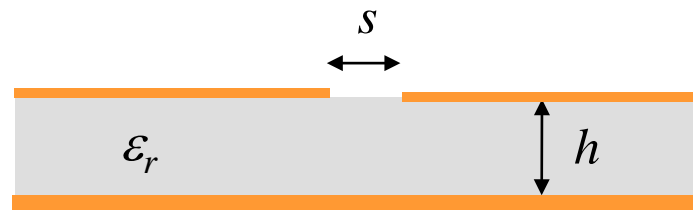
Coplanar Waveguide (CPW)



Conductor-backed CPW

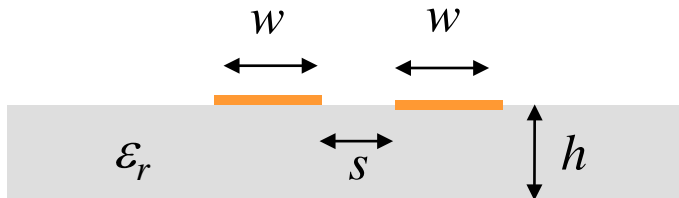


Slotline

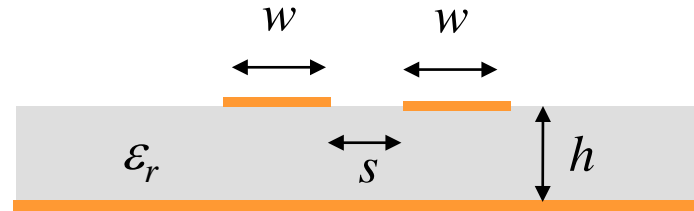


Conductor-backed Slotline

# Planar Transmission Lines (cont.)



Coplanar Strips (CPS)

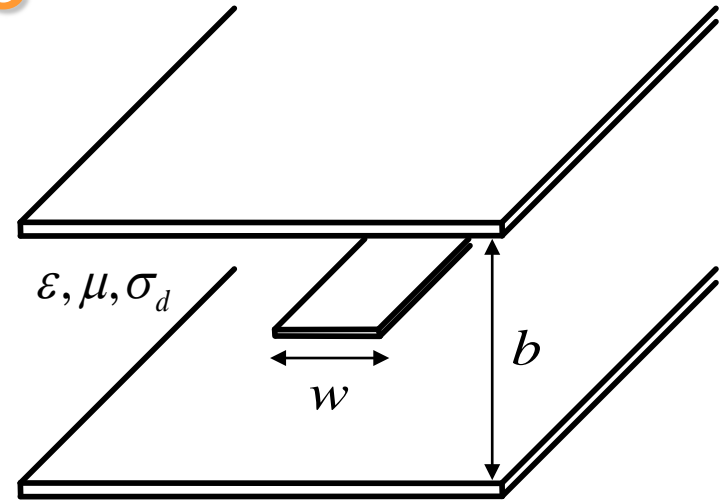


Conductor-backed CPS

- Stripline is a planar version of coax.
- Coplanar strips (CPS) is a planar version of twin lead.

# Stripline

- Common on circuit boards
- Fabricated with two circuit boards
- Homogenous dielectric (perfect TEM mode)

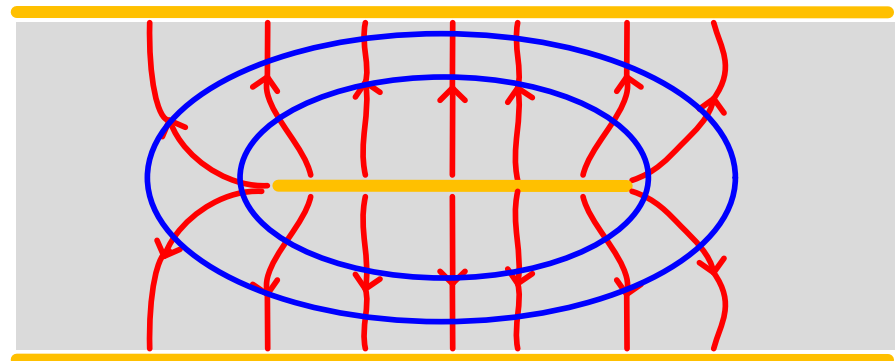


TEM mode

(also TE & TM Modes)

Field structure for TEM mode:

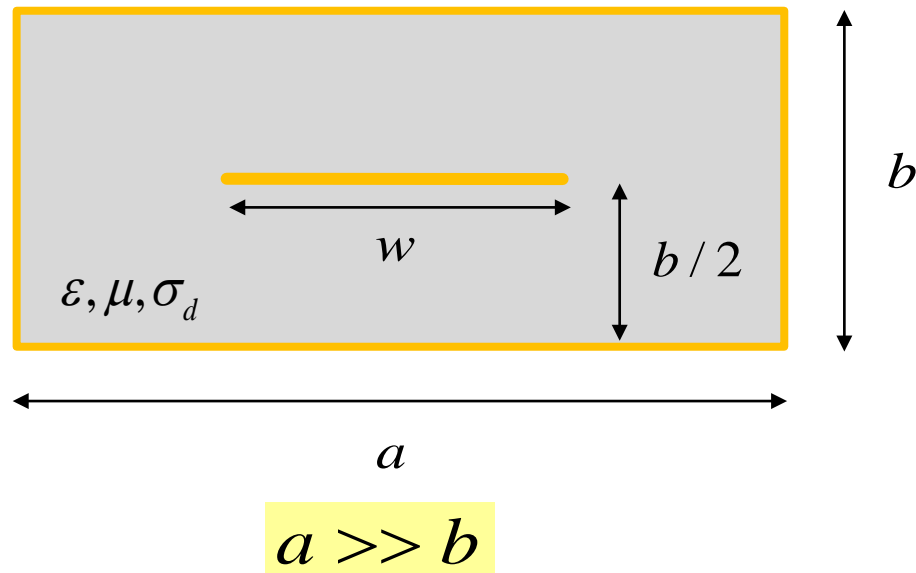
Electric Field —  
Magnetic Field —



# Stripline (cont.)

- Analysis of stripline is not simple.
- TEM mode fields can be obtained from an electrostatic analysis (e.g., conformal mapping).

A closed stripline structure is analyzed in the Pozar book by using an approximate numerical method:



# Stripline (cont.)

Conformal mapping solution (S. Cohn):

Exact solution:

$$Z_0 = \frac{30\pi K(k)}{K(k')}$$

←  $K =$  complete elliptic integral of the first kind

$$K(k) \equiv \int_0^{\pi/2} \frac{1}{\sqrt{1 - k^2 \sin^2 \theta}} d\theta$$

$$k = \operatorname{sech}\left(\frac{\pi w}{2b}\right)$$

$$k' = \tanh\left(\frac{\pi w}{2b}\right)$$

# Stripline (cont.)

Curve fitting this exact solution:

$$Z_0 = \left( \frac{\eta_0}{4\sqrt{\epsilon_r}} \right) \frac{b}{w_e + \frac{\ln(4)}{\pi} b}$$

**Note:**  $\frac{\ln(4)}{\pi} = 0.441$

Effective width

Fringing term

$$\frac{w_e}{b} = \frac{w}{b} - \begin{cases} 0 & ; \text{ for } \frac{w}{b} \geq 0.35 \\ \left( 0.35 - \frac{w}{b} \right)^2 & ; \text{ for } 0.1 \leq \frac{w}{b} \leq 0.35 \end{cases}$$

**Note:**  $Z_0^{\text{ideal}} = \frac{1}{2} \eta \left( \frac{b/2}{w} \right) = \frac{\eta b}{4w} = \frac{\eta_0 b}{4w\sqrt{\epsilon_r}}$

The factor of 1/2 in front is from the parallel combination of two ideal PPWs.

# Stripline (cont.)

Inverting this solution to find  $w$  for given  $Z_0$ :

$$\frac{w}{b} = \begin{cases} X; & \text{for } \sqrt{\epsilon_r} Z_0 \leq 120 \text{ } [\Omega] \\ 0.85 - \sqrt{0.6 - X} & ; \text{ for } \sqrt{\epsilon_r} Z_0 \geq 120 \text{ } [\Omega] \end{cases}$$

$$X \equiv \frac{\eta_0}{4\sqrt{\epsilon_r} Z_0} - \frac{\ln(4)}{\pi}$$



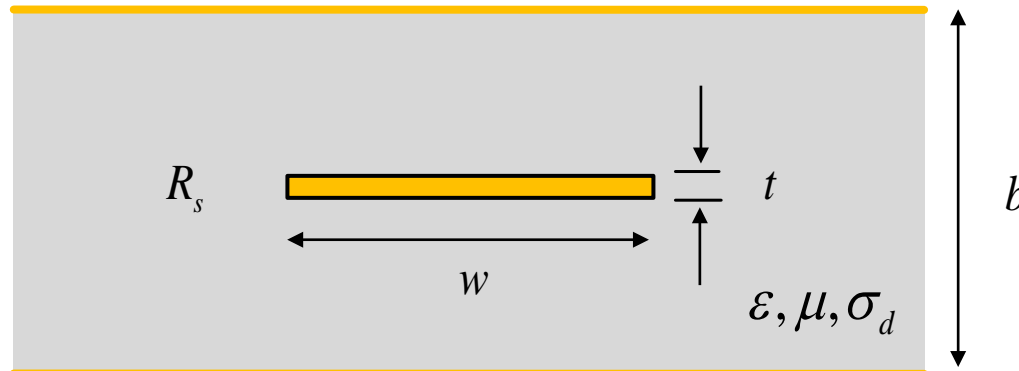
# Stripline (cont.)

## Attenuation

Dielectric Loss:

$$\alpha_d = k'' \approx \frac{k'}{2} \tan \delta \approx \frac{k_0 \sqrt{\epsilon_r}}{2} \tan \delta \quad (\text{TEM formula})$$

$$k = k' - jk'' = \omega \sqrt{\mu_0 \epsilon_c} \quad \epsilon_c = \epsilon - j \frac{\sigma_d}{\omega} \quad \tan \delta = \frac{\epsilon''}{\epsilon'}$$



# Stripline (cont.)

## Conductor Loss:

$$\alpha_c = \begin{cases} (2.7 \times 10^{-3}) \frac{4R_s \epsilon_r Z_0}{\eta_0 (b-t)} A; & \text{for } \sqrt{\epsilon_r} Z_0 \leq 120 [\Omega] \\ 0.16 \left( \frac{R_s}{Z_0 b} \right) B; & \text{for } \sqrt{\epsilon_r} Z_0 \geq 120 [\Omega] \end{cases}$$

where

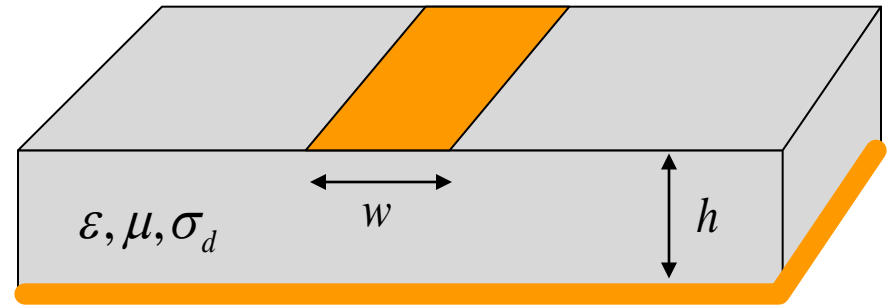
$$R_s = \sqrt{\frac{\omega \mu}{2\sigma}}$$

$$A = 1 + 2 \frac{w}{(b-t)} + \frac{1}{\pi} \left( \frac{b+t}{b-t} \right) \ln \left( \frac{2b-t}{t} \right)$$

$$B = 1 + \frac{b}{\left( \frac{w}{2} + 0.7t \right)} + \left( \frac{1}{2} + 0.414 \frac{t}{w} + \frac{1}{2\pi} \ln \left( 4\pi \frac{w}{t} \right) \right)$$

**Note:** We cannot let  $t \rightarrow 0$  when we calculate the conductor loss.

# Microstrip



- Inhomogeneous dielectric

⇒ No TEM mode

**Note:** Pozar uses  $(W, d)$

TEM mode would require  $k_z = k$  in each region, but  $k_z$  must be unique!

- Requires advanced analysis techniques
- Exact fields are hybrid modes ( $E_z$  and  $H_z$ )

For  $h/\lambda_0 \ll 1$ , the dominant mode is quasi-TEM.

# Microstrip (cont.)

Part of the field lines are in air,  
and part of the field lines are inside the substrate.

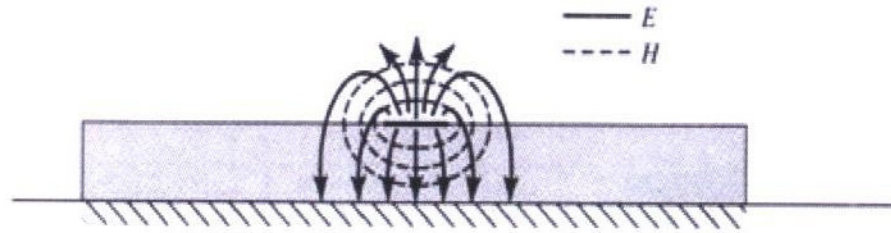


Figure from Pozar book

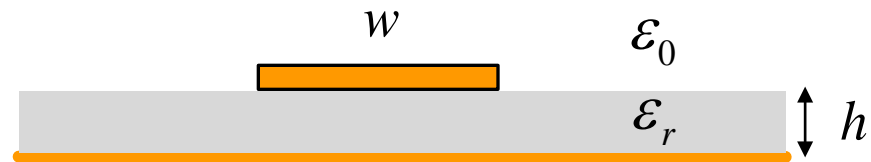
The flux lines get more concentrated in the substrate  
region as the frequency increases.

# Microstrip (cont.)

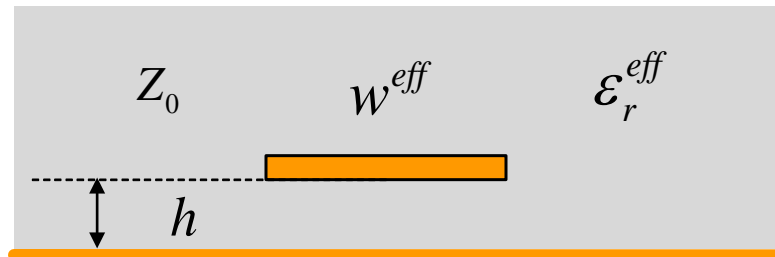
Equivalent TEM problem:

$$\beta \approx k_0 \sqrt{\epsilon_r^{eff}}$$

Actual problem



$$Z_0^{air} : \epsilon_r^{eff} \rightarrow 1$$



Equivalent TEM problem

- The effective permittivity gives the correct phase constant.
- The effective strip width gives the correct  $Z_0$ .

$$Z_0 = Z_0^{air} / \sqrt{\epsilon_r^{eff}}$$

$$\text{(since } Z_0 = \sqrt{L/C}\text{)}$$

# Microstrip (cont.)

Effective permittivity:

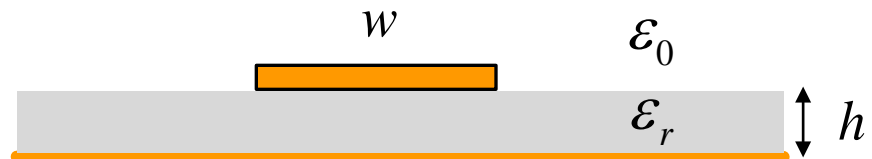
$$\epsilon_r^{eff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left( \frac{1}{\sqrt{1 + 12 \frac{h}{w}}} \right)$$

This formula ignores “dispersion”, i.e., the fact that the effective permittivity is actually a function of frequency.

Limiting cases:

$$w/h \rightarrow 0: \quad \epsilon_r^{eff} \rightarrow \frac{\epsilon_r + 1}{2} \quad (\text{narrow strip})$$

$$w/h \rightarrow \infty: \quad \epsilon_r^{eff} \rightarrow \epsilon_r \quad (\text{wide strip})$$



# Microstrip (cont.)

Characteristic Impedance:

$$Z_0 = \begin{cases} \frac{60}{\sqrt{\epsilon_r^{eff}}} \ln\left(\frac{8h}{w} + \frac{w}{4h}\right); & \text{for } \frac{w}{h} \leq 1 \\ \frac{\eta_0}{\sqrt{\epsilon_r^{eff}} \left(\frac{w}{h} + 1.393 + 0.667 \ln\left(\frac{w}{h} + 1.444\right)\right)} & ; \text{ for } \frac{w}{h} \geq 1 \end{cases}$$

This formula ignores the fact that the characteristic impedance is actually a function of frequency.

# Microstrip (cont.)

Inverting this solution to find  $w$  for a given  $Z_0$ :

$$\frac{w}{h} = \begin{cases} \frac{8e^A}{e^{2A} - 2} ; & \text{for } \frac{w}{h} \leq 2 \\ \frac{2}{\pi} \left[ B - 1 - \ln(2B - 1) + \frac{\epsilon_r - 1}{2\epsilon_r} \left( \ln(B - 1) + 0.39 - \frac{0.61}{\epsilon_r} \right) \right] ; & \text{for } \frac{w}{h} \geq 2 \end{cases}$$

where

$$A = \frac{Z_0}{60} \sqrt{\frac{\epsilon_r + 1}{2}} + \frac{\epsilon_r - 1}{\epsilon_r + 1} \left( 0.33 + \frac{0.11}{\epsilon_r} \right)$$

$$B = \frac{\eta_0 \pi}{2Z_0 \sqrt{\epsilon_r}}$$



# Microstrip (cont.)

## Attenuation

Dielectric loss:

“filling factor”

$$\alpha_d \approx \frac{k_0 \sqrt{\epsilon_r}}{2} \tan \delta \left[ \sqrt{\frac{\epsilon_r}{\epsilon_r^{eff}}} \frac{(\epsilon_r^{eff} - 1)}{(\epsilon_r - 1)} \right]$$

$$\epsilon_r^{eff} \rightarrow 1: \quad \alpha_d \rightarrow 0$$

$$\epsilon_r^{eff} \rightarrow \epsilon_r: \quad \alpha_d \rightarrow \frac{k_0 \sqrt{\epsilon_r}}{2} \tan \delta$$

Conductor loss:

$$\alpha_c \approx \frac{R_s}{Z_0 w} \approx \frac{R_s}{\eta h}$$

$$\left( Z_0 \approx \frac{\eta h}{w} \right)$$

← very crude (“parallel-plate”) approximation

(More accurate formulas are given later.)

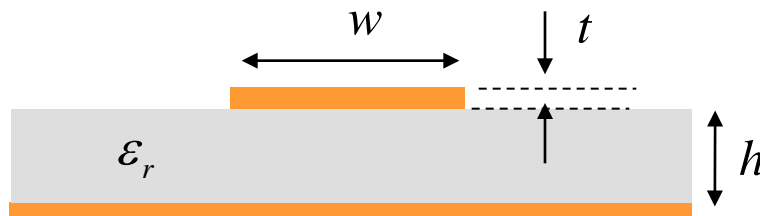
# Microstrip (cont.)

More accurate formulas for characteristic impedance that account for dispersion (frequency variation) and conductor thickness:

$$Z_0(f) = Z_0(0) \left( \frac{\epsilon_r^{\text{eff}}(f) - 1}{\epsilon_r^{\text{eff}}(0) - 1} \right) \sqrt{\frac{\epsilon_r^{\text{eff}}(0)}{\epsilon_r^{\text{eff}}(f)}}$$

$$Z_0(0) = \frac{\eta_0}{\sqrt{\epsilon_r^{\text{eff}}(0) \left[ (w'/h) + 1.393 + 0.667 \ln \left( (w'/h) + 1.444 \right) \right]}} \quad (w/h \geq 1)$$

$$w' = w + \frac{t}{\pi} \left( 1 + \ln \left( \frac{2h}{t} \right) \right)$$



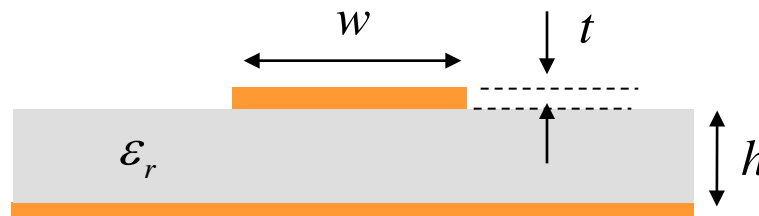
# Microstrip (cont.)

where

$$\epsilon_r^{eff}(f) = \left( \sqrt{\epsilon_r^{eff}(0)} + \frac{\sqrt{\epsilon_r} - \sqrt{\epsilon_r^{eff}(0)}}{1 + 4F^{-1.5}} \right)^2 \quad (w/h \geq 1)$$

$$\epsilon_r^{eff}(0) = \frac{\epsilon_r + 1}{2} + \left( \frac{\epsilon_r - 1}{2} \right) \left( \frac{1}{\sqrt{1 + 12(h/w)}} \right) - \left( \frac{\epsilon_r - 1}{4.6} \right) \left( \frac{t/h}{\sqrt{w/h}} \right)$$

$$F = 4 \left( \frac{h}{\lambda_0} \right) \sqrt{\epsilon_r - 1} \left( 0.5 + \left( 1 + 0.868 \ln \left( 1 + \frac{w}{h} \right) \right)^2 \right)$$



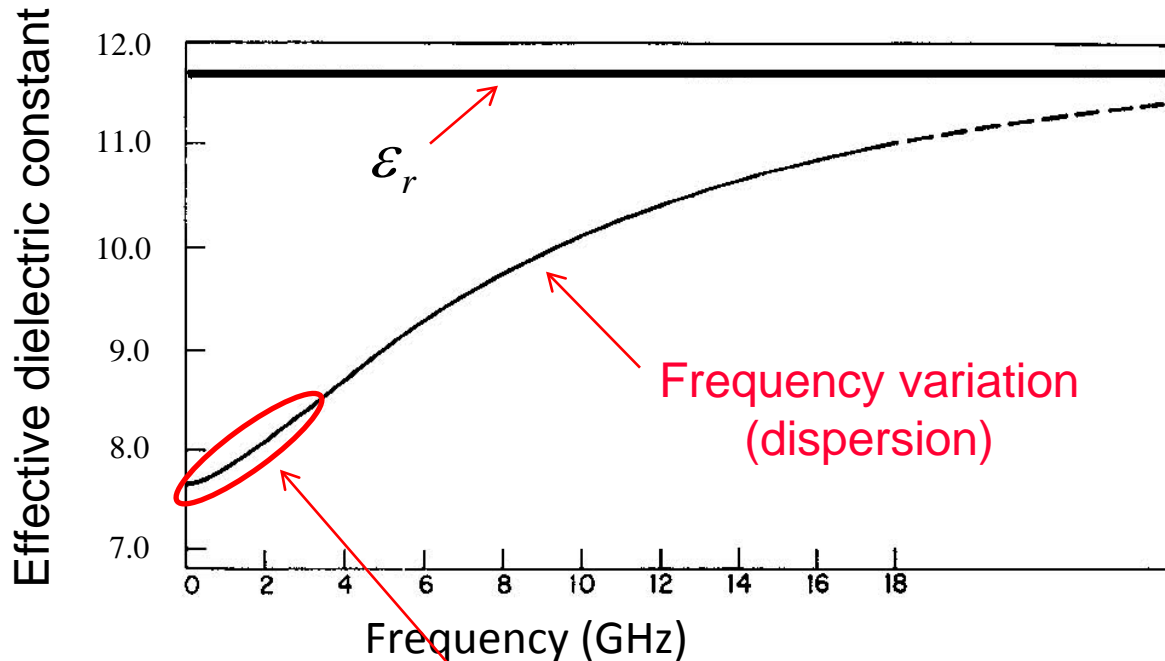
As  $f \rightarrow 0$ :

$$\epsilon_r^{eff}(f) \rightarrow \epsilon_r^{eff}(0)$$

As  $f \rightarrow \infty$ :

$$\epsilon_r^{eff}(f) \rightarrow \epsilon_r$$

# Microstrip (cont.)



$$\epsilon_r^{eff} = \left( \frac{\beta}{k_0} \right)^2$$

$$v_p = \frac{c}{\sqrt{\epsilon_r^{eff}}}$$

"A frequency-dependent solution for microstrip transmission lines," E. J. Denlinger, *IEEE Trans. Microwave Theory and Techniques*, Vol. 19, pp. 30-39, Jan. 1971.

**Note:**  
The phase velocity is a function of frequency, which causes pulse distortion.

**Note:**  
The flux lines get more concentrated in the substrate region as the frequency increases.

Quasi-TEM region

# Microstrip (cont.)

More accurate formulas for conductor attenuation:

$$\frac{1}{2\pi} < \frac{w}{h} \leq 2$$

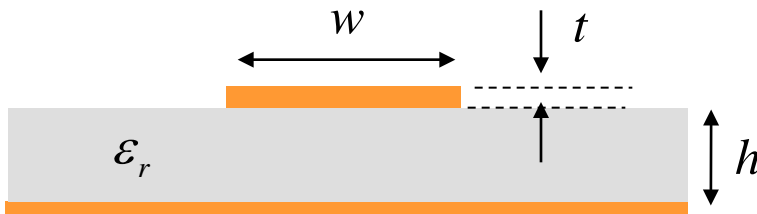
$$\alpha_c = \left( \frac{R_s}{hZ_0} \right) \left( \frac{1}{2\pi} \right) \left[ 1 - \left( \frac{w'}{4h} \right)^2 \right] \left[ 1 + \frac{h}{w'} + \frac{h}{\pi w'} \left( \ln \left( \frac{2h}{t} \right) - \frac{t}{h} \right) \right]$$

$$\frac{w}{h} \geq 2$$

$$\alpha_c = \left( \frac{R_s}{hZ_0} \right) \left[ \frac{w'}{h} + \frac{2}{\pi} \ln \left( 2\pi e \left( \frac{w'}{2h} + 0.94 \right) \right) \right]^{-2} \left[ \frac{w'}{h} + \frac{w' / (\pi h)}{\frac{w'}{2h} + 0.94} \right] \left[ 1 + \frac{h}{w'} + \frac{h}{\pi w'} \left( \ln \left( \frac{2h}{t} \right) - \frac{t}{h} \right) \right]$$

This is the number  $e = 2.71828$  multiplying the term in parenthesis.

$$w' = w + \frac{t}{\pi} \left( 1 + \ln \left( \frac{2h}{t} \right) \right)$$



# Microstrip (cont.)

Note about conductor attenuation:

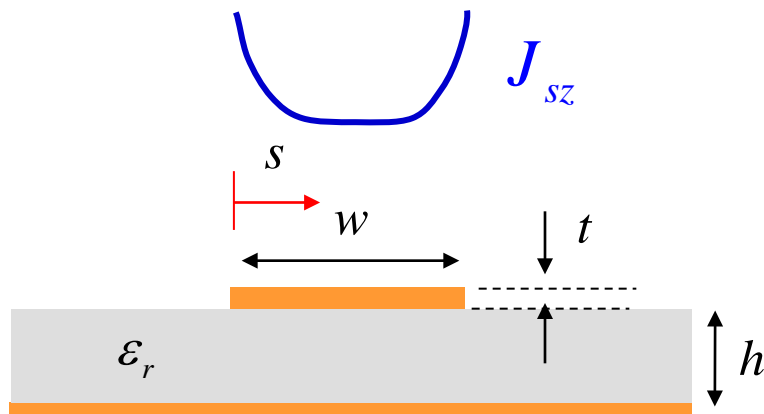
It is necessary to assume a nonzero conductor thickness in order to accurately calculate the conductor attenuation.

The perturbational method predicts an infinite attenuation if a zero thickness is assumed.

$$t = 0: \quad J_{sz} \propto \frac{1}{\sqrt{s}} \text{ as } s \rightarrow 0$$

$$\alpha_c = \frac{P_l(0)}{2P_0}$$

$$P_l(0) = \frac{R_s}{2} \int_{C_1+C_2} |J_s|^2 \Big|_{z=0} d\ell$$



**Practical note:**

A standard metal thickness for PCBs is 0.7 [mils] (17.5 [μm]), called “half-ounce copper”.

1 mil = 0.001 inch

# TXLINE

This is a public-domain software for calculating the properties of some common planar transmission lines.

<http://www.awrcorp.com/products/optional-products/tx-line-transmission-line-calculator>



NI AWR Design Environment

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Downloads and Support

# TXLINE (cont.)



## TX-LINE: Transmission Line Calculator

**TX-LINE\*** software is a FREE and easy-to-use Windows-based interactive transmission line calculator for the analysis and synthesis of transmission line structures.

[Register and Download Your FREE Copy of TX-LINE Software Today!](#)

TX-LINE software enables users to enter either physical or electrical characteristics for common transmission mediums:

Microstrip

Stripline

Coplanar waveguide (WG)

Grounded coplanar WG

Slotline

Learn more: [TX-LINE Software Video Demonstration \(3 minutes\)](#)

*\*Note: TX-LINE software is embedded within NI AWR Design Environment and can be launched from the "Tools" menu.*



# Microstrip (cont.)

## REFERENCES

L. G. Maloratsky, *Passive RF and Microwave Integrated Circuits*, Elsevier, 2004.

I. Bahl and P. Bhartia, *Microwave Solid State Circuit Design*, Wiley, 2003.

R. A. Pucel, D. J. Masse, and C. P. Hartwig, "Losses in Microstrip," *IEEE Trans. Microwave Theory and Techniques*, pp. 342-350, June 1968.

R. A. Pucel, D. J. Masse, and C. P. Hartwig, "Corrections to 'Losses in Microstrip'," *IEEE Trans. Microwave Theory and Techniques*, Dec. 1968, p. 1064.