Adapted from notes by
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ECE 5317-6351
Microwave Engineering

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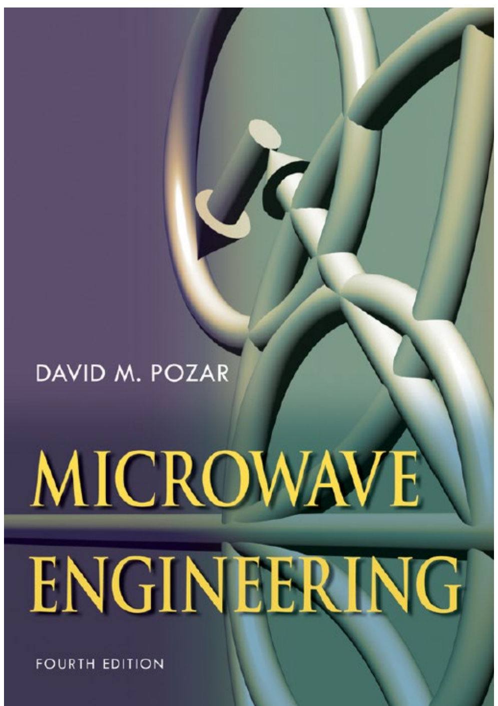
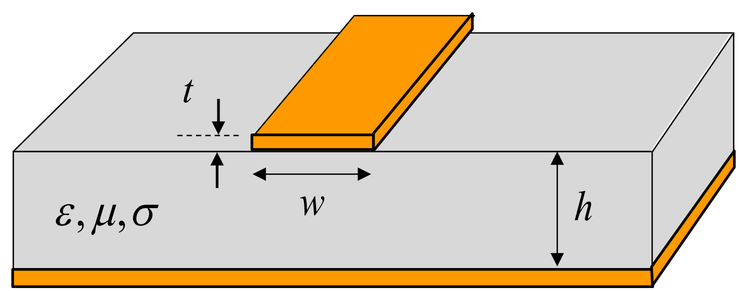
Dept. of ECE

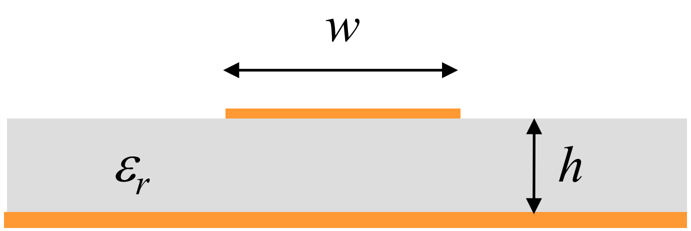


Notes 11

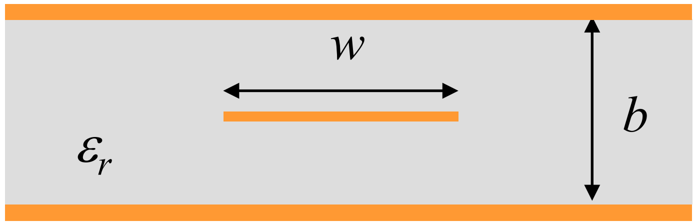
Waveguiding Structures Part 6: Planar Transmission Lines



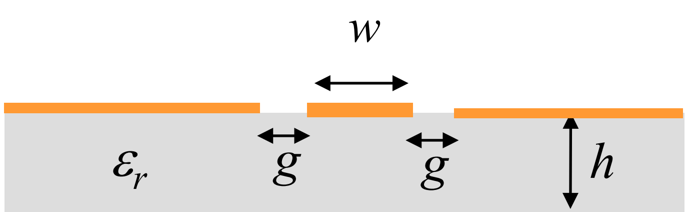
Planar Transmission Lines



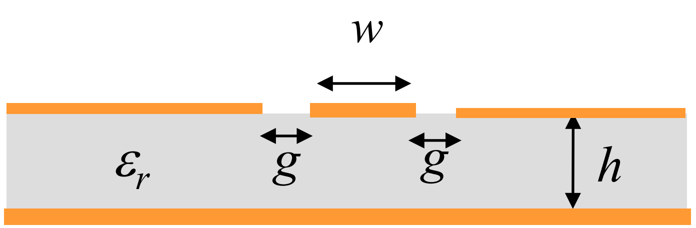
Microstrip



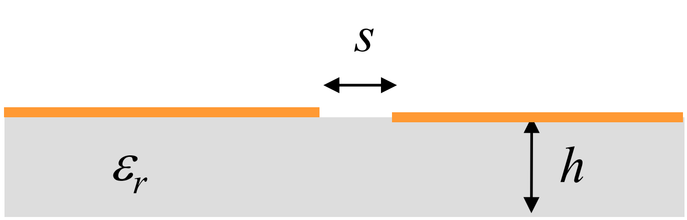
Stripline



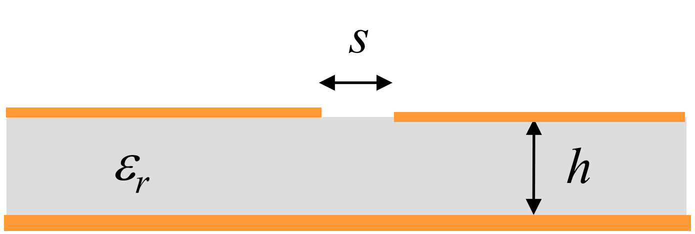
Coplanar Waveguide (CPW)



Conductor-backed CPW

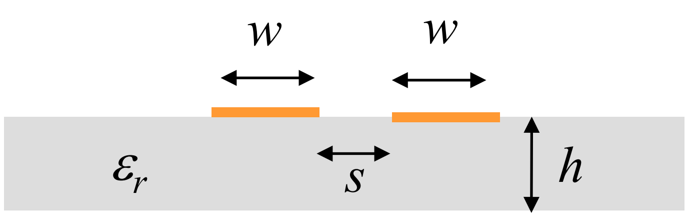


Slotline

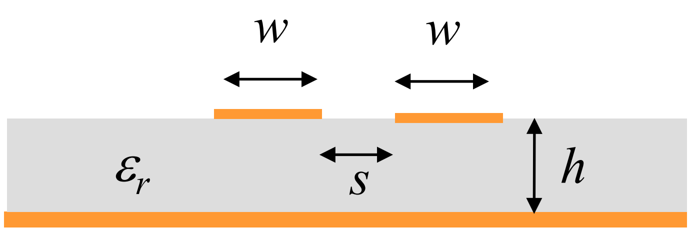


Conductor-backed Slotline

Planar Transmission Lines (cont.)



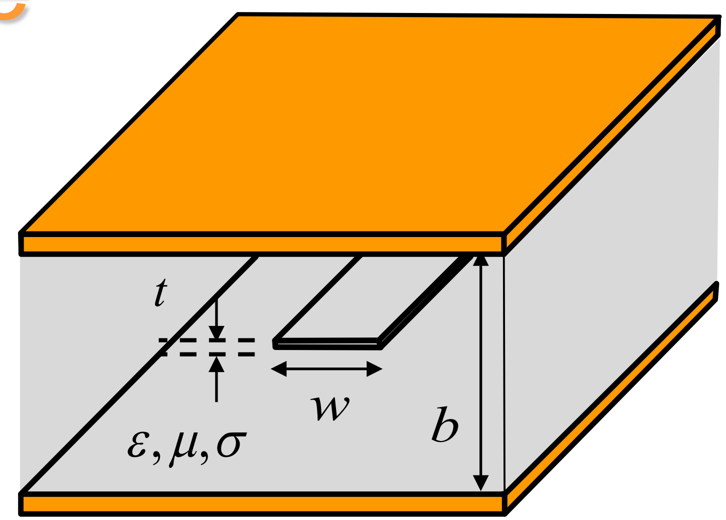
Coplanar Strips (CPS)



Conductor-backed CPS

- Stripline is a planar version of coax.
- Coplanar strips (CPS) is a planar version of twin lead.

Stripline



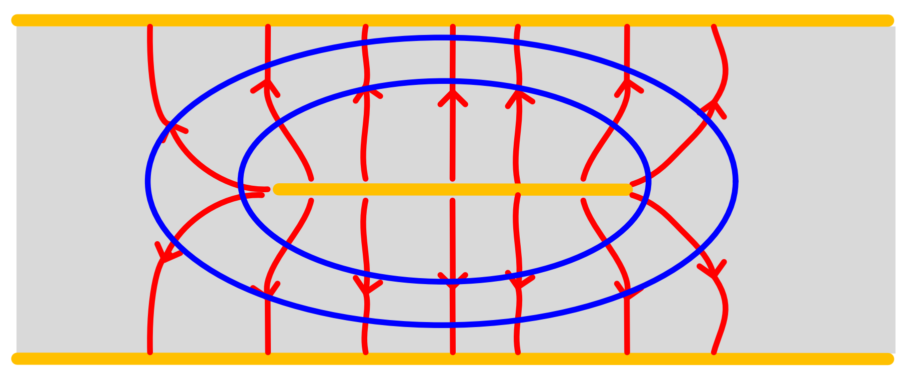
- Common on circuit boards
- Fabricated with two circuit boards
- Homogenous dielectric
  (perfect TEM mode*)

TEM mode

(also TE & TM Modes at high frequency)

* The mode is a <u>perfect</u> TEM mode if there is no conductor loss.

Field structure for TEM mode:

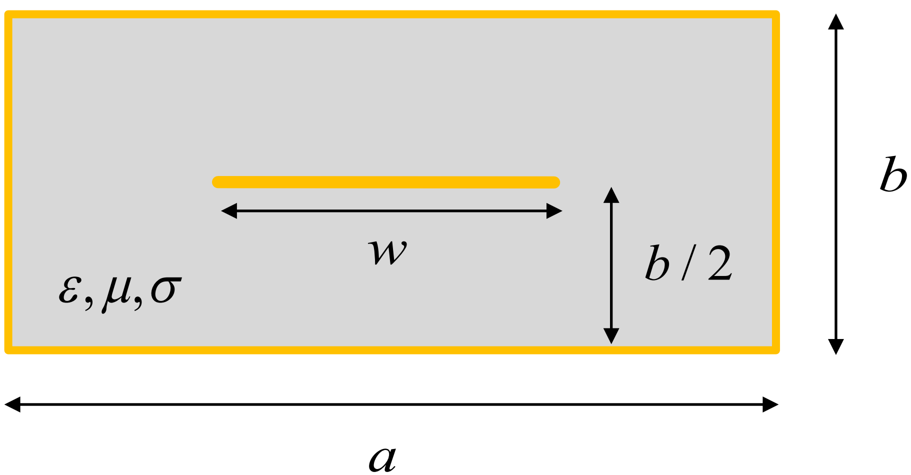


Electric Field —
Magnetic Field —

Stripline (cont.)

- Analysis of stripline is not simple.
- TEM mode fields can be obtained from an electrostatic analysis (e.g., conformal mapping).

A closed stripline structure is analyzed in the Pozar book by using an approximate numerical method:



$a >> b$ (to simulate stripline)

Stripline (cont.)

Conformal Mapping Solution (R. H. T. Bates)

Exact solution (for $t = 0$):

$$Z_0 = \frac{30\pi K(k)}{K(k')}$$

K = complete elliptic integral of the first kind

$$K(k) \equiv \int_0^{\pi/2} \frac{1}{\sqrt{1 - k^2 \sin^2\theta}} d\theta$$

$$k = \operatorname{sech}\left(\frac{\pi w}{2b}\right)$$

$$k' = \tanh\left(\frac{\pi w}{2b}\right)$$

R. H. T. Bates, "The characteristic impedance of the shielded slab line," *IEEE Trans. Microwave Theory and Techniques*, vol. 4, pp. 28-33, Jan. 1956.

Stripline (cont.)

Curve fitting this exact solution:

$$Z_0 = \left(\frac{\eta_0}{4\sqrt{\varepsilon_r}}\right)\frac{b}{w_e + \frac{\ln(4)}{\pi}b}$$

Note : $\frac{\ln(4)}{\pi} = 0.441$

Effective width

Fringing term

$$\frac{w_e}{b} = \frac{w}{b} - \begin{cases} 0 & ; \text{ for } \frac{w}{b} \geq 0.35 \\ \left(0.35 - \frac{w}{b}\right)^2 & ; \text{ for } 0.1 \leq \frac{w}{b} \leq 0.35 \end{cases}$$

Note: $Z_0^{\text{ideal}} = \frac{1}{2}\eta\left(\frac{b/2}{w}\right) = \frac{\eta b}{4w} = \frac{\eta_0 b}{4w\sqrt{\varepsilon_r}}$

The factor of 1/2 in front is from the parallel combination of two ideal PPWs.

Stripline (cont.)

Inverting this solution to find w for given Z_0:

$$\frac{w}{b}=\begin{cases} X; \quad \text{for} \quad \sqrt{\varepsilon_r}Z_0 \le 120 \ [\Omega] \\ \\ 0.85-\sqrt{0.6-X} \quad ; \quad \text{for} \ \sqrt{\varepsilon_r}Z_0 \ge 120 \ [\Omega] \end{cases}$$

$$X \equiv \frac{\eta_0}{4\sqrt{\varepsilon_r}Z_0}-\frac{\ln(4)}{\pi}$$

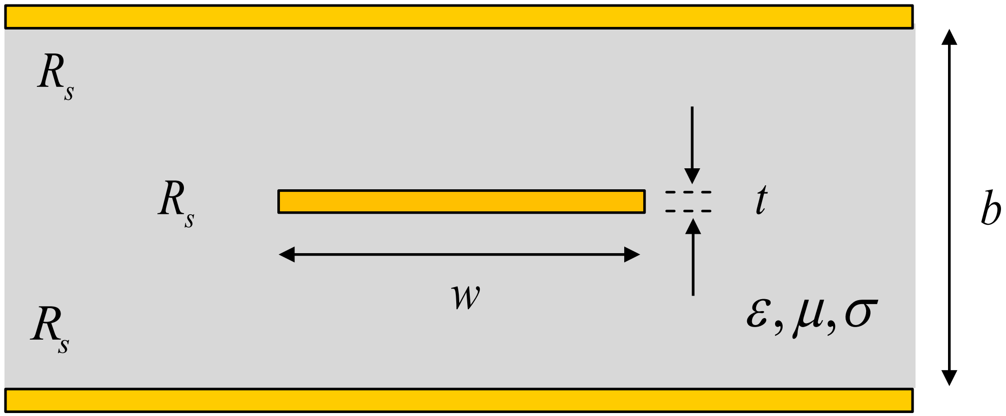
Stripline (cont.)

Attenuation

Dielectric Loss:

$$\alpha_d = k'' \approx \frac{k'}{2}\tan\delta_d \approx \frac{k_0\sqrt{\varepsilon_r}}{2}\tan\delta_d$$ (TEM formula)

$$k = k' - jk'' = \omega\sqrt{\mu_0\varepsilon_c} \qquad \varepsilon_c = \varepsilon - j\frac{\sigma}{\omega} \qquad \tan\delta_d = \frac{\varepsilon_c''}{\varepsilon_c'}$$



Stripline (cont.)

Conductor Loss:

$$\alpha_c \approx \begin{cases} (2.7\times 10^{-3})\dfrac{4R_s\varepsilon_r Z_0}{\eta_0(b-t)}A\,; & \text{for } \sqrt{\varepsilon_r}Z_0 \le 120\ [\Omega] \quad \text{(wider strips)} \\[2ex] 0.16\left(\dfrac{R_s}{Z_0 b}\right)B\,; & \text{for } \sqrt{\varepsilon_r}Z_0 \ge 120\ [\Omega] \quad \text{(narrower strips)} \end{cases}$$

$$R_s = \sqrt{\frac{\omega\mu}{2\sigma}}$$

where

$\sigma =$ conductivity of metal

$$A = 1 + 2\frac{w}{(b-t)} + \frac{1}{\pi}\left(\frac{b+t}{b-t}\right)\ln\left(\frac{2b-t}{t}\right)$$

$$B = 1 + \frac{b}{\left(\dfrac{w}{2}+0.7t\right)} + \left(\frac{1}{2} + 0.414\frac{t}{w} + \frac{1}{2\pi}\ln\left(4\pi\frac{w}{t}\right)\right)$$

Note: We cannot let $t \to 0$ when we calculate the conductor loss.

Stripline (cont.)

Note about conductor attenuation:

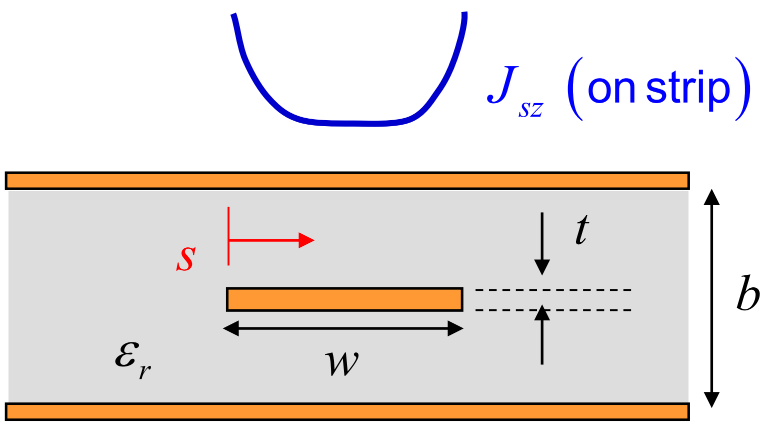
It is necessary to assume a nonzero conductor thickness in order to accurately calculate the conductor attenuation.

The perturbational method predicts an infinite attenuation if a zero thickness is assumed.

$$\alpha_c = \frac{P_l(0)}{2P_0}$$

$$t = 0: \qquad J_{sz} \propto \frac{1}{\sqrt{s}} \text{ as } s \to 0$$

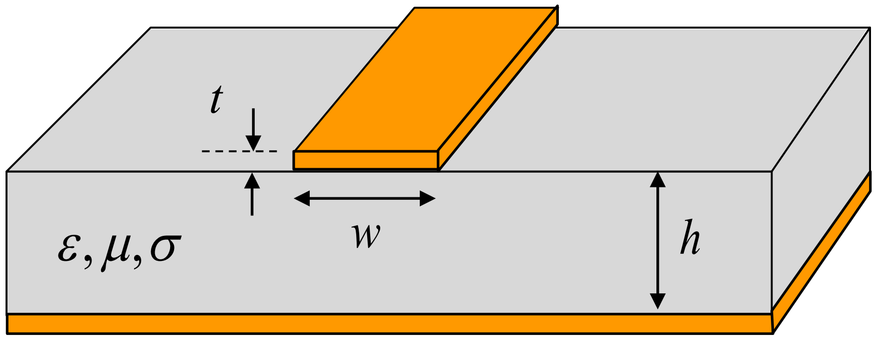
$$P_l(0) = \frac{R_s}{2} \int_{C_1+C_2} \left| \underline{J}_s \right|^2 \Big|_{z=0} d\ell \quad \to \quad \infty$$



Practical note:
A standard metal thickness for PCBs is 0.7 [mils] (17.5 [μm]), called "half-ounce copper".

1 mil = 0.001 inch

Microstrip



- Inhomogeneous dielectric

$\Rightarrow$ No TEM mode

Note: Pozar uses (W, d)

TEM mode would require $k_z = k$ in each region, but k_z must be unique!

- Requires advanced analysis techniques
- Exact fields are hybrid modes (E_z and H_z)

For $h/\lambda_0 << 1$, the dominant mode is quasi-TEM.

Microstrip (cont.)

Part of the field lines are in air,
and part of the field lines are inside the substrate.

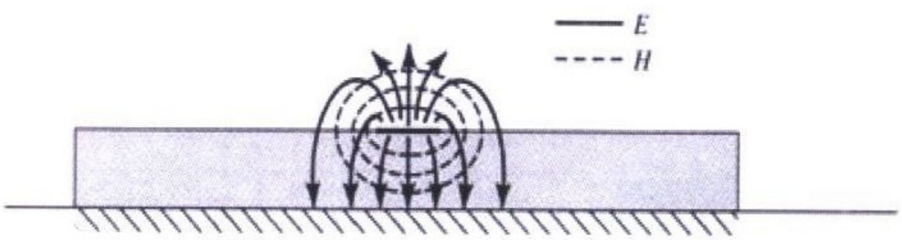


Figure from Pozar book

Note:
The flux lines get more concentrated in the substrate region as the frequency increases.

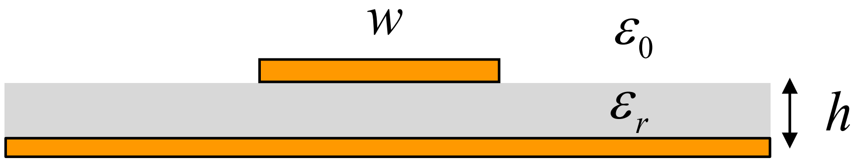
Microstrip (cont.)

Equivalent TEM problem:

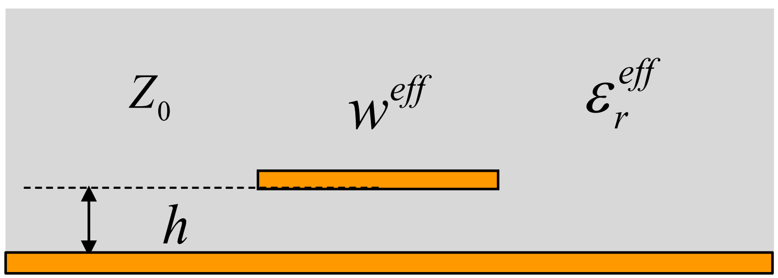
$$\beta \approx k_0 \sqrt{\varepsilon_r^{eff}}$$

$$\Rightarrow \quad \varepsilon_r^{eff} = \left(\frac{\beta}{k_0}\right)^2$$

Actual problem



- The effective permittivity gives the correct phase constant.
- The effective strip width gives the correct Z_0.



Equivalent TEM problem

$$Z_0 = Z_0^{air} / \sqrt{\varepsilon_r^{eff}}$$

$$Z_0^{air} : \quad \varepsilon_r^{eff} \to 1$$

$$\left(\text{since } \; Z_0 = \sqrt{L/C}\right)$$

Microstrip (cont.)

Effective permittivity:

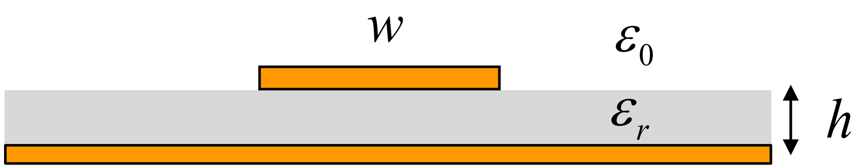
$$\varepsilon_r^{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left(\frac{1}{\sqrt{1 + 12\dfrac{h}{w}}}\right)$$

Note:
This formula ignores "dispersion", i.e., the fact that the effective permittivity is actually a function of frequency.

Limiting cases:

$w/h \to 0:$ $\quad \varepsilon_r^{eff} \to \dfrac{\varepsilon_r + 1}{2}$ (narrow strip)

$w/h \to \infty:$ $\quad \varepsilon_r^{eff} \to \varepsilon_r$ (wide strip)



Microstrip (cont.)

Characteristic Impedance:

$$Z_0 = \begin{cases} \dfrac{60}{\sqrt{\varepsilon_r^{eff}}} \ln\left(\dfrac{8h}{w} + \dfrac{w}{4h}\right) ; & \text{for } \dfrac{w}{h} \le 1 \\[2ex] \dfrac{\eta_0}{\sqrt{\varepsilon_r^{eff}}\left(\dfrac{w}{h} + 1.393 + 0.667 \ln\left(\dfrac{w}{h} + 1.444\right)\right)} ; & \text{for } \dfrac{w}{h} \ge 1 \end{cases}$$

Note:
This formula ignores the fact that the characteristic impedance is actually a function of frequency.

Microstrip (cont.)

Inverting this solution to find w for a given Z_0:

$$\frac{w}{h} = \begin{cases} \dfrac{8e^{A}}{e^{2A}-2} \;;\; \text{for } \dfrac{w}{h} \le 2 \\ \dfrac{2}{\pi}\left[B-1-\ln(2B-1)+\dfrac{\varepsilon_r-1}{2\varepsilon_r}\left(\ln\left(B-1\right)+0.39-\dfrac{0.61}{\varepsilon_r}\right)\right] \;;\; \text{for } \dfrac{w}{h} \ge 2 \end{cases}$$

where

$$A = \frac{Z_0}{60}\sqrt{\frac{\varepsilon_r+1}{2}}+\frac{\varepsilon_r-1}{\varepsilon_r+1}\left(0.33+\frac{0.11}{\varepsilon_r}\right)$$

$$B = \frac{\eta_0 \pi}{2Z_0\sqrt{\varepsilon_r}}$$

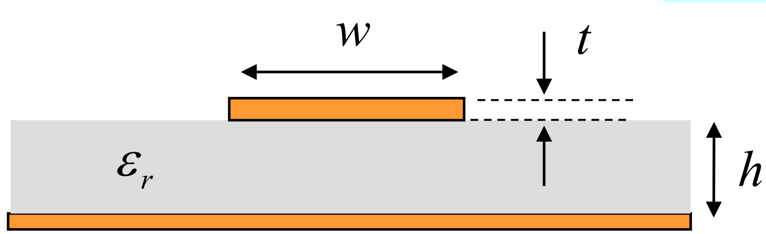
Microstrip (cont.)

More accurate formulas for characteristic impedance that account for dispersion (frequency variation) and conductor thickness:

$$Z_0(f) = Z_0(0)\left(\frac{\varepsilon_r^{eff}(f)-1}{\varepsilon_r^{eff}(0)-1}\right)\sqrt{\frac{\varepsilon_r^{eff}(0)}{\varepsilon_r^{eff}(f)}}$$

$$Z_0(0) = \frac{\eta_0}{\sqrt{\varepsilon_r^{eff}(0)}\left[(w'/h)+1.393+0.667\ln\left((w'/h)+1.444\right)\right]} \qquad (w/h \geq 1)$$

$$w' = w + \frac{t}{\pi}\left(1+\ln\left(\frac{2h}{t}\right)\right)$$



Microstrip (cont.)

where

$$\varepsilon_r^{eff}(f) = \left(\sqrt{\varepsilon_r^{eff}(0)} + \frac{\sqrt{\varepsilon_r} - \sqrt{\varepsilon_r^{eff}(0)}}{1 + 4F^{-1.5}} \right)^2 \qquad (w/h \geq 1)$$

$$\varepsilon_r^{eff}(0) = \frac{\varepsilon_r + 1}{2} + \left(\frac{\varepsilon_r - 1}{2}\right)\left(\frac{1}{\sqrt{1 + 12(h/w)}}\right) - \left(\frac{\varepsilon_r - 1}{4.6}\right)\left(\frac{t/h}{\sqrt{w/h}}\right)$$

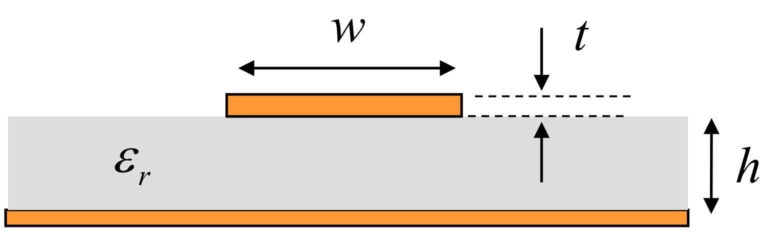
$$F = 4\left(\frac{h}{\lambda_0}\right)\sqrt{\varepsilon_r - 1}\left(0.5 + \left(1 + 0.868\ln\left(1 + \frac{w}{h}\right)\right)^2\right)$$

As $f \to 0$:

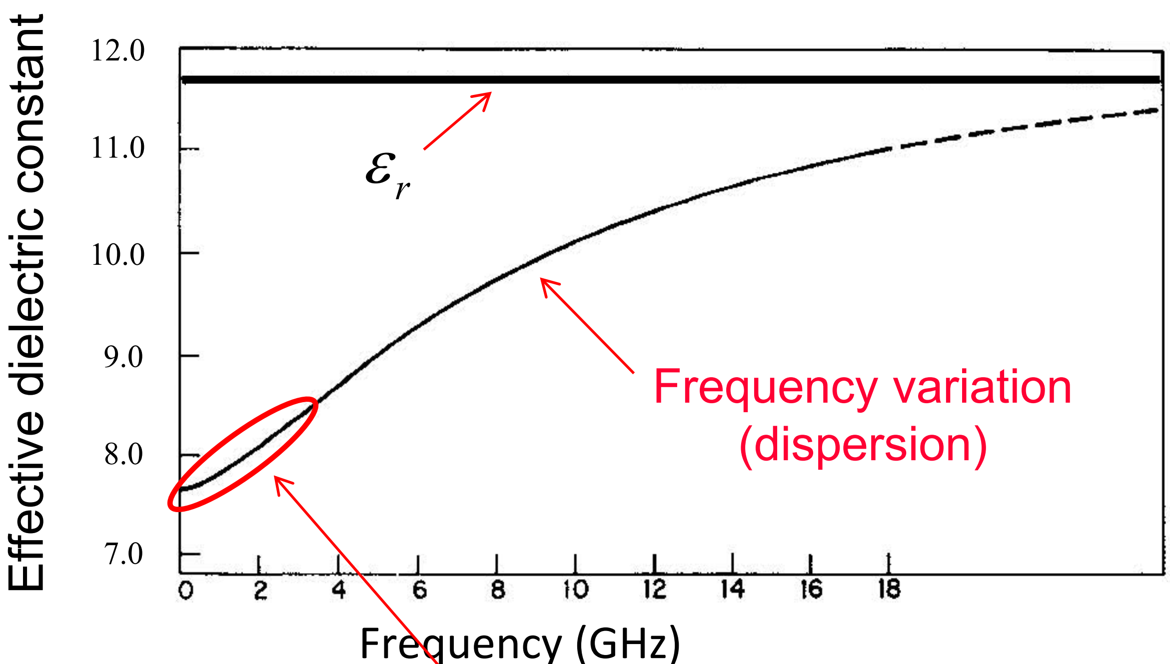
$\varepsilon_r^{eff}(f) \to \varepsilon_r^{eff}(0)$

As $f \to \infty$:

$\varepsilon_r^{eff}(f) \to \varepsilon_r$



Microstrip (cont.)



Parameters: $\varepsilon_r = 11.7,\ w/h = 0.96,\ h = 0.317\ \text{cm}$

$$\varepsilon_r^{eff} = \left(\frac{\beta}{k_0}\right)^2$$

$$v_p = \frac{c}{\sqrt{\varepsilon_r^{eff}}}$$

"A frequency-dependent solution for microstrip transmission lines," E. J. Denlinger, *IEEE Trans. Microwave Theory and Techniques,* Vol. 19, pp. 30-39, Jan. 1971.

Note:
The flux lines get more concentrated in the substrate region as the frequency increases.

Note:
The phase velocity is a function of frequency, which causes pulse distortion.

Quasi-TEM region

Microstrip (cont.)

Attenuation

Dielectric loss:

"filling factor"

$$\alpha_d \approx \frac{k_0\sqrt{\varepsilon_r}}{2}\tan\delta_d\left[\sqrt{\frac{\varepsilon_r}{\varepsilon_r^{eff}}}\,\frac{\left(\varepsilon_r^{eff}-1\right)}{\left(\varepsilon_r-1\right)}\right]$$

$$\varepsilon_r^{eff}\to 1: \quad \alpha_d \to 0$$

$$\varepsilon_r^{eff}\to \varepsilon_r: \quad \alpha_d \to \frac{k_0\sqrt{\varepsilon_r}}{2}\tan\delta_d$$

Conductor loss:

$$\alpha_c \approx \frac{R_s}{Z_0 w} \approx \frac{R_s}{\eta h}$$

$$\left(Z_0 \approx \frac{\eta h}{w}\right)$$

← very crude ("parallel-plate") approximation

(More accurate formulas are given on next slide.)

Microstrip (cont.)

More accurate formulas for <u>conductor attenuation</u>:

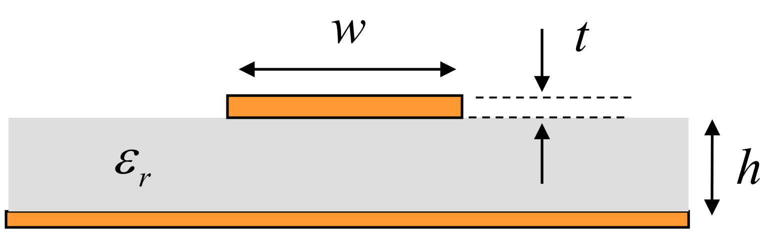
$$\frac{1}{2\pi} < \frac{w}{h} \le 2$$

$$\alpha_c = \left(\frac{R_s}{hZ_0}\right)\left(\frac{1}{2\pi}\right)\left[1-\left(\frac{w'}{4h}\right)^2\right]\left[1+\frac{h}{w'}+\frac{h}{\pi w'}\left(\ln\left(\frac{2h}{t}\right)-\frac{t}{h}\right)\right]$$

$$\frac{w}{h} \ge 2$$

$$\alpha_c = \left(\frac{R_s}{hZ_0}\right)\left[\frac{w'}{h}+\frac{2}{\pi}\ln\left(2\pi e\left(\frac{w'}{2h}+0.94\right)\right)\right]^{-2}\left[\frac{w'}{h}+\frac{w'/(\pi h)}{\frac{w'}{2h}+0.94}\right]\left[1+\frac{h}{w'}+\frac{h}{\pi w'}\left(\ln\left(\frac{2h}{t}\right)-\frac{t}{h}\right)\right]$$

This is the number $e = 2.71828$ multiplying the term in parenthesis.



$$w' = w + \frac{t}{\pi}\left(1+\ln\left(\frac{2h}{t}\right)\right)$$

Microstrip (cont.)

REFERENCES

L. G. Maloratsky, Passive RF and Microwave Integrated Circuits, Elsevier, 2004.

I. Bahl and P. Bhartia, Microwave Solid State Circuit Design, Wiley, 2003.

R. A. Pucel, D. J. Masse, and C. P. Hartwig, "Losses in Microstrip," *IEEE Trans. Microwave Theory and Techniques*, pp. 342-350, June 1968.

R. A. Pucel, D. J. Masse, and C. P. Hartwig, "Corrections to 'Losses in Microstrip'," *IEEE Trans. Microwave Theory and Techniques*, Dec. 1968, p. 1064.

TXLINE

This is a public-domain software for calculating the properties of some common planar transmission lines.

https://www.awr.com/software/options/tx-line