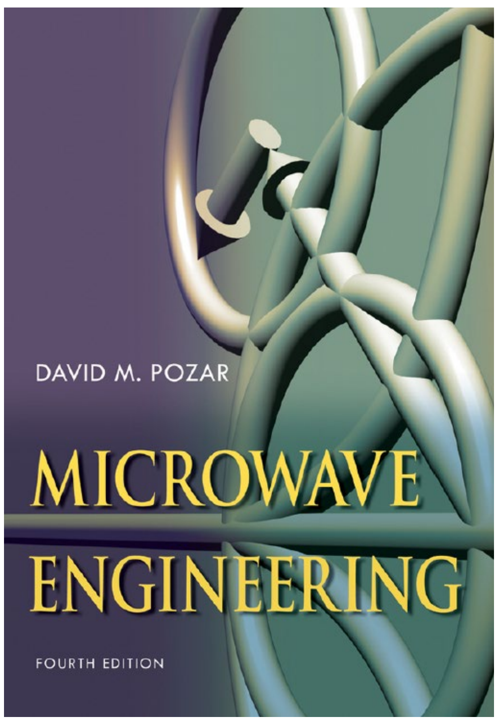
Adapted from notes by
Prof. Jeffery T. Williams

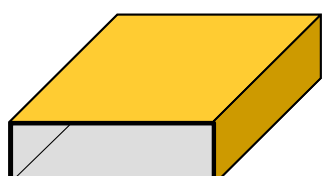
ECE 5317-6351
Microwave Engineering

Fall 2019

Prof. David R. Jackson
Dept. of ECE

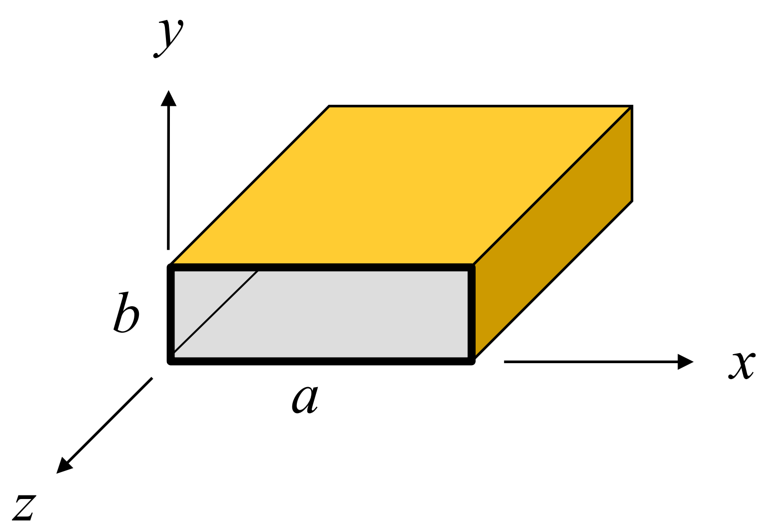


Notes 12
Waveguiding Structures Part 7: Transverse Equivalent Network (TEN)

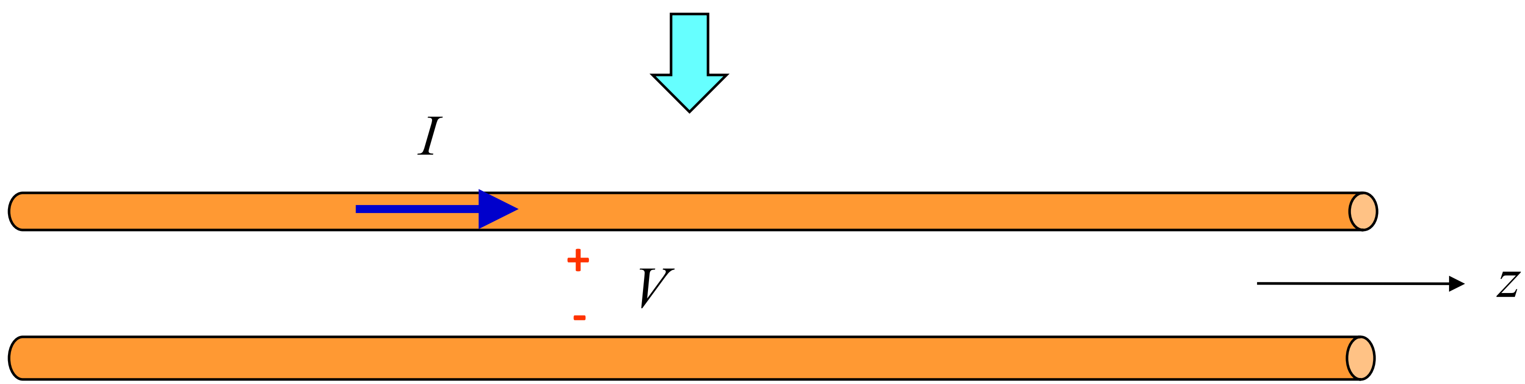


Waveguide Transmission Line Model

Our goal is to come up with a transmission line model for a waveguide mode.



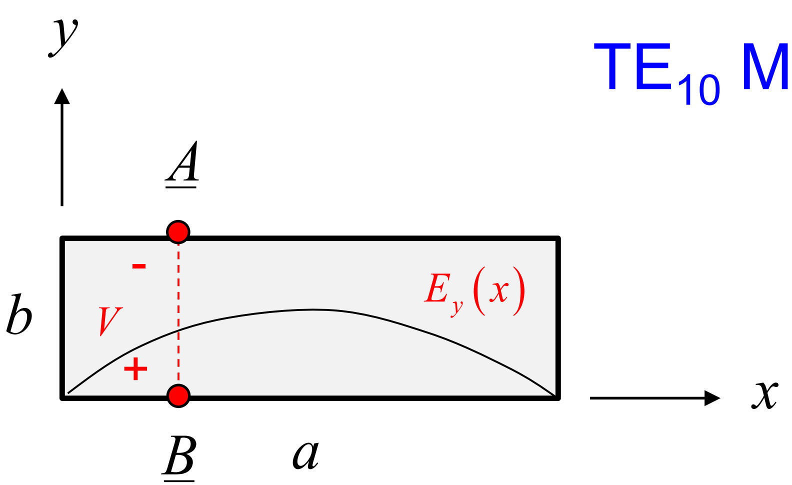
The waveguide mode is not a TEM mode, but it can be modeled as a wave on a transmission line.



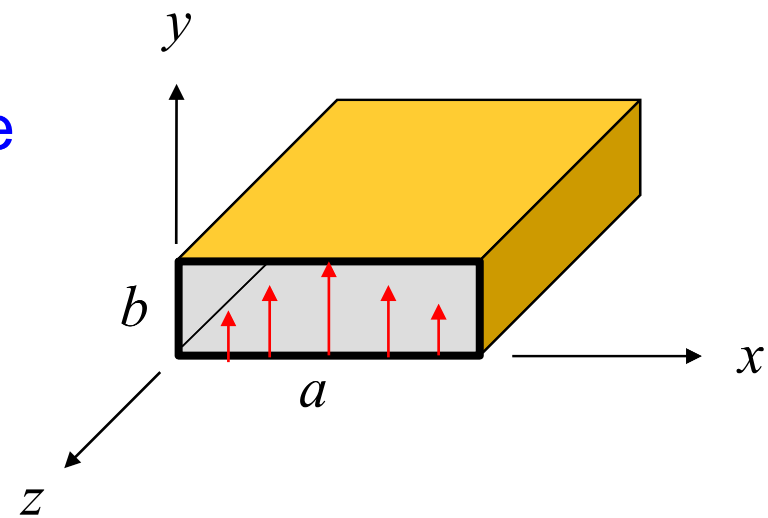
"Transverse Equivalent Network" model of TE_{10} waveguide mode

Waveguide Transmission Line Model (cont.)

For a waveguide mode, voltage and current are not uniquely defined.



TE$_{10}$ Mode



$$E_y = E_{10} \sin\left(\frac{\pi}{a}x\right) e^{-jk_z z}$$

$$V(x,z) = V_{AB}(x,z) = \int_{\underline{A}}^{\underline{B}} \underline{E} \cdot d\underline{r} = \int_0^b E_y \, dy = E_{10} b \sin\left(\frac{\pi}{a}x\right) e^{-jk_z z} = V_0 \sin\left(\frac{\pi}{a}x\right) e^{-jk_z z}$$

The voltage depends on x! $\quad V_0 \equiv E_{10} b = V(a/2, 0)$

Waveguide Transmission Line Model (cont.)

Examine the transverse (x, y) fields of a waveguide mode:

Modal amplitudes

$$\underline{E}_t(x,y,z) = \underline{e}_t(x,y)\left(A^+ e^{-jk_z z} + A^- e^{+jk_z z}\right)$$

$$\underline{H}_t(x,y,z) = \underline{h}_t(x,y)\left(A^+ e^{-jk_z z} - A^- e^{+jk_z z}\right)$$

$$\underline{h}_t(x,y) = \frac{1}{Z_w}(\underline{\hat{z}} \times \underline{e}_t)$$

Note:
The shape function $\underline{e}_t(x,y)$ has an arbitrary amplitude normalization.

Note: The minus sign above arises from:

$$\underline{H}_t^{\pm} = \pm\frac{1}{Z_w}(\underline{\hat{z}} \times \underline{E}_t^{\pm})$$

Wave impedance

$$Z_w = Z_{TE} \text{ or } Z_{TM}$$

$$Z_{TE} \equiv \frac{\omega\mu}{k_z} \qquad Z_{TM} = \frac{k_z}{\omega\varepsilon_c}$$

Waveguide Transmission Line Model (cont.)

Introduce a defined voltage into the field equations:

$$V_0^{\pm} \equiv C_1 A^{\pm}$$

We may use whatever definition of voltage we wish here.
(In other words, C_1 is arbitrary.)

We then have:

$$\underline{E}_t(x,y,z) = \frac{1}{C_1}\underline{e}_t(x,y)\left(V_0^{+} e^{-jk_z z} + V_0^{-} e^{+jk_z z}\right)$$

$$\underline{H}_t(x,y,z) = \frac{1}{C_1}\underline{h}_t(x,y)\left(V_0^{+} e^{-jk_z z} - V_0^{-} e^{+jk_z z}\right)$$

Waveguide Transmission Line Model (cont.)

Next, introduce a characteristic impedance (having an arbitrary value) into the equations:

$$\underline{H}_t(x,y,z) = \frac{Z_0}{C_1}\underline{h}_t(x,y)\left(\frac{V_0^+}{Z_0}e^{-jk_z z} - \frac{V_0^-}{Z_0}e^{+jk_z z}\right)$$

$$\Downarrow$$

$$\underline{H}_t(x,y,z) = \frac{1}{C_2}\underline{h}_t(x,y)\left(\frac{V_0^+}{Z_0}e^{-jk_z z} - \frac{V_0^-}{Z_0}e^{+jk_z z}\right)$$

where

$$C_2 \equiv \frac{C_1}{Z_0}$$

Waveguide Transmission Line Model (cont.)

Summary of Fields

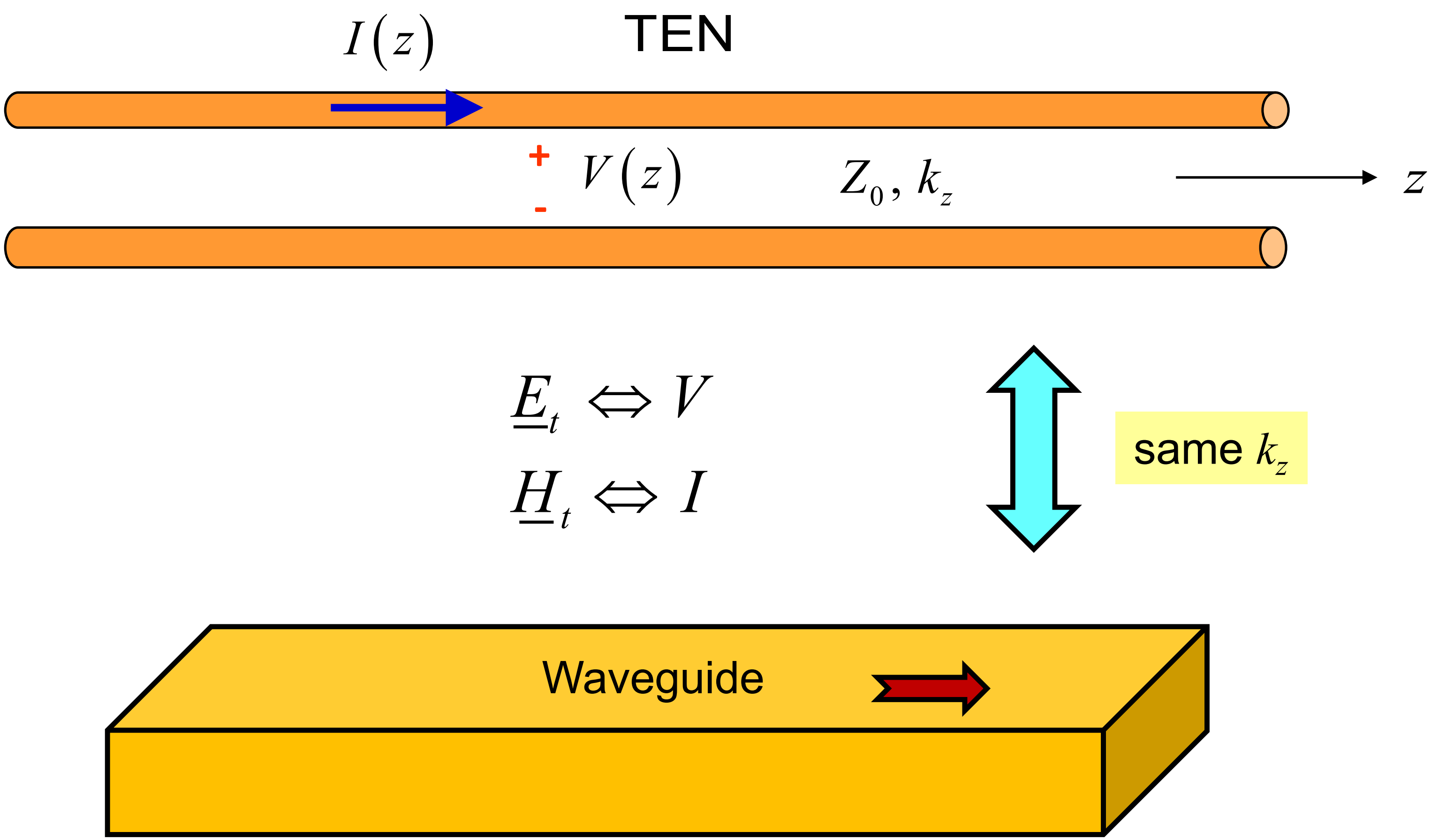
$$\underline{E}_t(x,y,z) = \frac{1}{C_1}\underline{e}_t(x,y)\overbrace{\left(V_0^+ e^{-jk_z z} + V_0^- e^{+jk_z z}\right)}^{V(z)}$$

$$\underline{H}_t(x,y,z) = \frac{1}{C_2}\underline{h}_t(x,y)\overbrace{\left(\frac{V_0^+}{Z_0} e^{-jk_z z} - \frac{V_0^-}{Z_0} e^{+jk_z z}\right)}^{I(z)}$$

The z dependence of the transverse fields behaves like voltage and current on a transmission line.

Waveguide Transmission Line Model (cont.)

The transmission-line model is called the Transverse Equivalent Network (TEN) model of the waveguide.



Waveguide Transmission Line Model (cont.)

Power flow down the waveguide (complex power):

$$P^{\mathrm{WG}}(z) = \frac{1}{2}\int_S \left(\underline{E}_t \times \underline{H}_t^{\,*}\right)\cdot \underline{\hat{z}}\, dS$$

$$= \frac{1}{2} V(z) I^*(z) \left[\frac{1}{C_1 C_2^*} \int_S \left(\underline{e}_t(x,y) \times \underline{h}_t^*(x,y) \right) \cdot \underline{\hat{z}}\, dS \right]$$

$$P^{\mathrm{WG}}(z) = P^{\mathrm{TEN}}(z) \left[\frac{1}{C_1 C_2^*} \int_S \left(\underline{e}_t(x,y) \times \underline{h}_t^*(x,y) \right) \cdot \underline{\hat{z}}\, dS \right]$$

↑

Complex power flowing down the TEN transmission line.

Waveguide Transmission Line Model (cont.)

Assume we <u>choose</u> to have:

$$P^{\mathrm{WG}}(z) = P^{\mathrm{TEN}}(z)$$

Then we have the following constraint:

$$C_1 C_2^* = \int_S \left(\underline{e}_t(x,y) \times \underline{h}_t^*(x,y) \right) \cdot \underline{\hat{z}}\, dS$$

It is not necessary to make this assumption of equal powers, but it is a useful choice that can be made (we will adopt this choice).

Waveguide Transmission Line Model (cont.)

Summary of Constants
(assuming equal powers)

$$\frac{C_1}{C_2} = Z_0$$

$$C_1 C_2^* = \int_S \left(\underline{e}_t(x,y) \times \underline{h}_t^*(x,y) \right) \cdot \underline{\hat{z}} \, dS$$

Once we pick Z_0, the constants are determined.

The most common choice: $Z_0 = Z_w$

Waveguide Transmission Line Model (cont.)

Summary

$$\underline{E}_t(x,y,z) = \frac{1}{C_1}\underline{e}_t(x,y)\overbrace{\left(V_0^+ e^{-jk_z z} + V_0^- e^{+jk_z z}\right)}^{V(z)}$$

$$\underline{H}_t(x,y,z) = \frac{1}{C_2}\underline{h}_t(x,y)\overbrace{\left(\frac{V_0^+}{Z_0} e^{-jk_z z} - \frac{V_0^-}{Z_0} e^{+jk_z z}\right)}^{I(z)}$$

$$\frac{C_1}{C_2} = Z_0 \qquad Z_0 = Z_w \; (\text{most common choice})$$

$$C_1 C_2^* = \int_S \left(\underline{e}_t(x,y) \times \underline{h}_t^*(x,y)\right) \cdot \underline{\hat{z}}\, dS$$

$$Z_{TE} \equiv \frac{\omega\mu}{k_z}$$

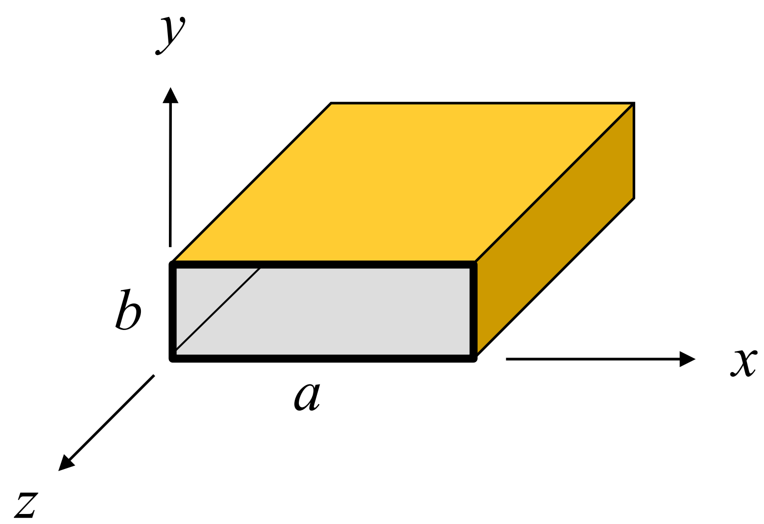
$$\underline{h}_t(x,y) = \frac{1}{Z_w}(\underline{\hat{z}} \times \underline{e}_t) \qquad Z_w = Z_{TE} \text{ or } Z_{TM}$$

$$Z_{TM} = \frac{k_z}{\omega\varepsilon_c}$$

TE_{10} Mode of Rectangular Waveguide

We make the following choices:

- Choose $Z_0 = Z_{TE}$
- Assume power equality



$$\frac{C_1}{C_2} = Z_{TE}$$

$$Z_{TE} = \frac{\omega \mu_0}{k_z^{10}}$$

$$C_1 C_2^* = \int_S \left(\underline{e}_t(x,y) \times \underline{h}_t^*(x,y) \right) \cdot \underline{\hat{z}} \, dS$$

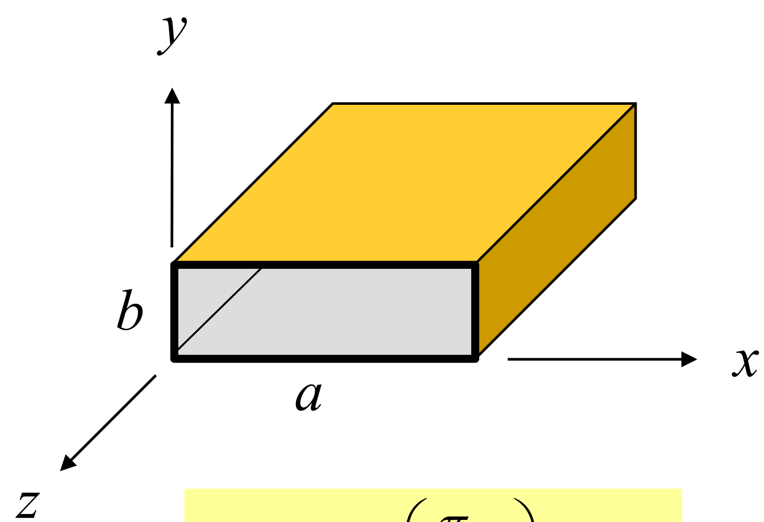
$$k_z^{10} = \sqrt{k^2 - \left(\frac{\pi}{a} \right)^2}$$

$$k = \omega \sqrt{\mu \varepsilon_c}$$

TE_{10} Mode (cont.)

Calculate this term:

$$C_1 C_2^* = \int_S \left(\underline{e}_t(x,y) \times \underline{h}_t^*(x,y) \right) \cdot \underline{\hat{z}}\, dS$$



$$\underline{e}_t = \underline{\hat{y}} \sin\left(\frac{\pi}{a} x\right)$$

$$\underline{h}_t = -\underline{\hat{x}} \frac{1}{Z_{TE}} \sin\left(\frac{\pi}{a} x\right)$$

$$Z_{TE} = \frac{\omega\mu}{k_z^{10}}$$

so

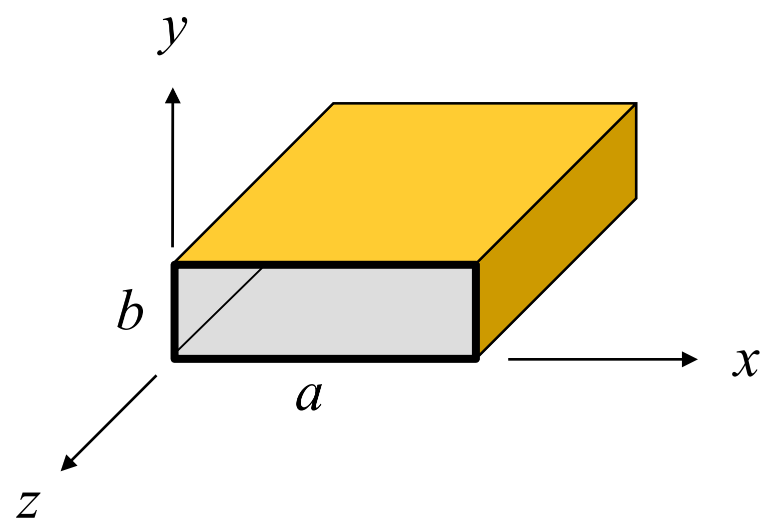
$$C_1 C_2^* = \int_S \frac{1}{Z_{TE}^*} \sin^2\left(\frac{\pi x}{a}\right) dS$$

$$= \frac{1}{Z_{TE}^*} \int_0^a \int_0^b \sin^2\left(\frac{\pi x}{a}\right) dy\, dx$$

$$= \frac{1}{Z_{TE}^*} \left(\frac{ab}{2}\right)$$

Note:
The transverse shape function $\underline{e}_t$ has been chosen (arbitrarily) to have a unit amplitude at the center of the waveguide.

TE_{10} Mode (cont.)



We have:

$$C_1 C_2^* = \frac{1}{Z_{TE}^*}\left(\frac{ab}{2}\right)$$

$$\frac{C_1}{C_2} = Z_{TE}$$

Take the conjugate of the second one and then multiply the two equations together.

$$\Rightarrow \quad |C_1|^2 = \frac{ab}{2}$$

Note:
The solution is unique to within a phase term (we choose the phase to be zero here).

Solution:

$$C_1 = \sqrt{\frac{ab}{2}}$$

$$C_2 = \frac{1}{Z_{TE}}\sqrt{\frac{ab}{2}}$$

$$\underline{e}_t = \underline{\hat{y}}\sin\left(\frac{\pi}{a}x\right)$$

$$\underline{h}_t = -\underline{\hat{x}}\frac{1}{Z_{TE}}\sin\left(\frac{\pi}{a}x\right)$$

$$Z_{TE} = \frac{\omega\mu}{k_z^{10}}$$

TE_{10} Mode (cont.)

Recall:

$$\underline{E}_t(x,y,z) = \frac{1}{C_1}\underline{e}_t(x,y)\overbrace{\left(V_0^+ e^{-jk_z z} + V_0^- e^{+jk_z z}\right)}^{V(z)}$$

$$\underline{H}_t(x,y,z) = \frac{1}{C_2}\underline{h}_t(x,y)\overbrace{\left(\frac{V_0^+}{Z_0} e^{-jk_z z} - \frac{V_0^-}{Z_0} e^{+jk_z z}\right)}^{I(z)}$$

$$C_1 = \sqrt{\frac{ab}{2}}$$

$$C_2 = \frac{1}{Z_{TE}}\sqrt{\frac{ab}{2}}$$

$$\underline{e}_t = \underline{\hat{y}}\sin\left(\frac{\pi}{a}x\right)$$

$$\underline{h}_t = -\underline{\hat{x}}\frac{1}{Z_{TE}}\sin\left(\frac{\pi}{a}x\right)$$

$$Z_{TE} = \frac{\omega\mu}{k_z^{10}}$$

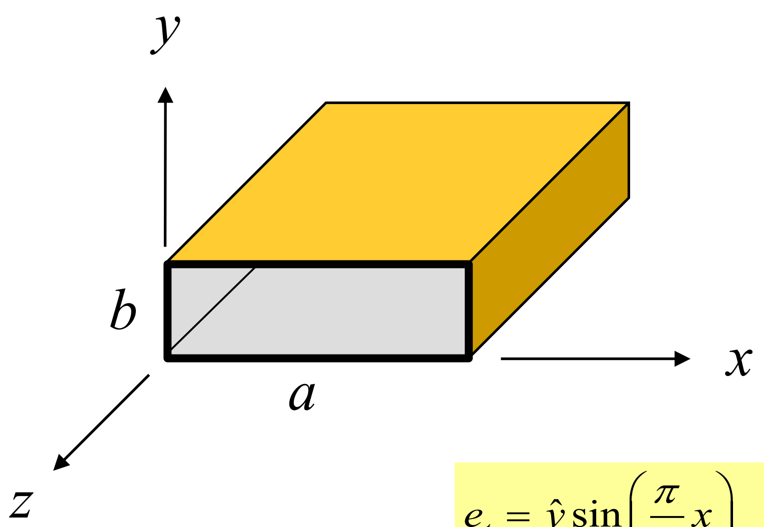
Hence:

$$\underline{E}_t(x,y,z) = \sqrt{\frac{2}{ab}}\left(\underline{\hat{y}}\sin\left(\frac{\pi}{a}x\right)\right)\overbrace{\left(V_0^+ e^{-jk_z z} + V_0^- e^{+jk_z z}\right)}^{V(z)}$$

$$\underline{H}_t(x,y,z) = \cancel{Z}_{TE}\sqrt{\frac{2}{ab}}\left(-\underline{\hat{x}}\frac{1}{\cancel{Z}_{TE}}\sin\left(\frac{\pi}{a}x\right)\right)\overbrace{\left(\frac{V_0^+}{Z_{TE}} e^{-jk_z z} - \frac{V_0^-}{Z_{TE}} e^{+jk_z z}\right)}^{I(z)}$$

TE$_{10}$ Mode (cont.)

Summary for TE$_{10}$ mode



$$C_1 = \sqrt{\frac{ab}{2}}$$

$$C_2 = \frac{1}{Z_{TE}^{10}}\sqrt{\frac{ab}{2}}$$

$$Z_{TE}^{10} = \frac{\omega\mu}{k_z^{10}}$$

$$k_z^{10} = \sqrt{k^2 - \left(\frac{\pi}{a}\right)^2}$$

$$\underline{e}_t = \underline{\hat{y}}\sin\left(\frac{\pi}{a}x\right)$$

$$\underline{h}_t = -\underline{\hat{x}}\frac{1}{Z_{TE}}\sin\left(\frac{\pi}{a}x\right)$$

$$Z_{TE} = \frac{\omega\mu}{k_z^{10}}$$

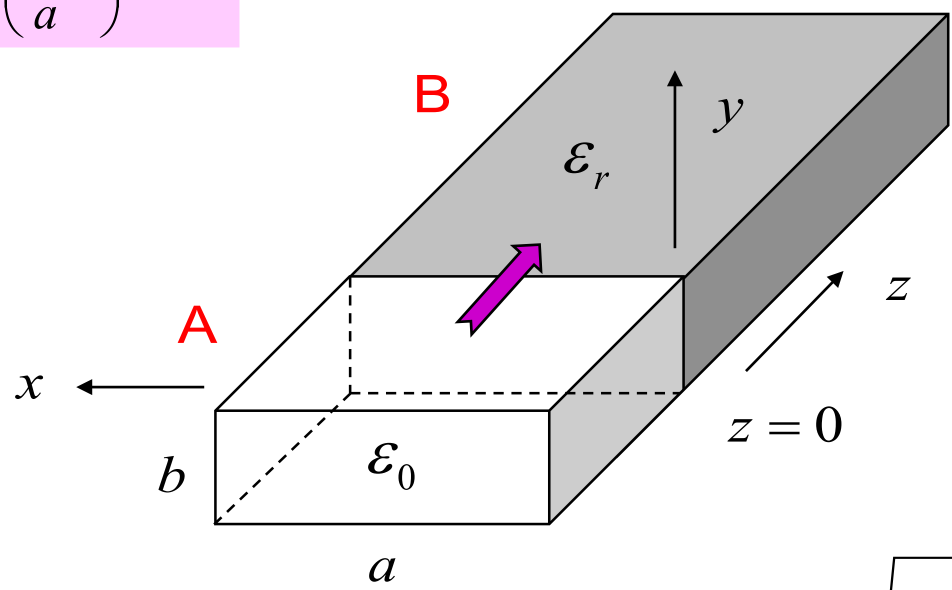
$$\underline{E}_t(x,y,z) = \underline{\hat{y}}\sqrt{\frac{2}{ab}}\sin\left(\frac{\pi}{a}x\right)\overbrace{\left(V_0^+ e^{-jk_z z} + V_0^- e^{+jk_z z}\right)}^{V(z)}$$

$$\underline{H}_t(x,y,z) = -\underline{\hat{x}}\sqrt{\frac{2}{ab}}\sin\left(\frac{\pi}{a}x\right)\overbrace{\left(\frac{V_0^+}{Z_{TE}} e^{-jk_z z} - \frac{V_0^-}{Z_{TE}} e^{+jk_z z}\right)}^{I(z)}$$

Example: Waveguide Discontinuity

For a 1 [V/m] (field at the center of the guide) incident TE_{10} mode in guide A, find the TE_{10} mode fields in both guides, and the reflected and transmitted powers.

$$E_y^{inc} = \sin\left(\frac{\pi}{a}x\right)e^{-jk_{za}z}$$



$$\mu_a = \mu_b = \mu_0$$

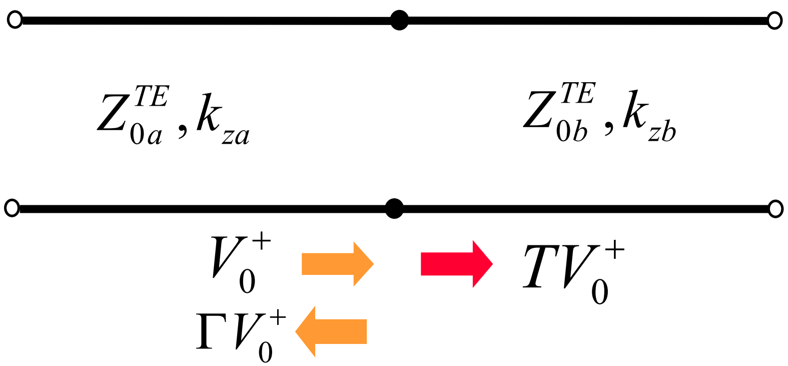
$$a = 2.2856 \text{ cm}$$
$$b = 1.016 \text{ cm}$$
$$\varepsilon_r = 2.54$$
$$f = 10 \text{ GHz}$$

$$k_{za} = \sqrt{k_0^2 - \left(\frac{\pi}{a}\right)^2} = 158.0 \; [\text{rad/m}]$$

$$k_{zb} = \sqrt{\varepsilon_r k_0^2 - \left(\frac{\pi}{a}\right)^2} = 304.1 \; [\text{rad/m}]$$

Example (cont.)

TEN



$$\underline{e}_t = \underline{\hat{y}} \sin\left(\frac{\pi}{a} x\right)$$

$$\underline{h}_t = -\underline{\hat{x}} \frac{1}{Z_{TE}} \sin\left(\frac{\pi}{a} x\right)$$

$$Z_{TE} = \frac{\omega \mu_0}{k_z^{10}}$$

Convention:

- Choose $Z_0 = Z_{TE}$
- Assume power equality

$$C_1 = \sqrt{\frac{ab}{2}}$$

$$C_2 = \frac{1}{Z_{TE}} \sqrt{\frac{ab}{2}}$$

Note:
C_1 is the same for both guides, but C_2 is different.

$$Z_{0a} = Z_{TE}^a = \frac{\omega \mu}{k_{za}} = 499.7 \;\; [\Omega]$$

$$Z_{0b} = Z_{TE}^b = \frac{\omega \mu}{k_{zb}} = 259.6 \;\; [\Omega]$$

$$\underline{E}_t^{inc}(x, y, z) = \underline{e}_t(x, y)\left(A^+ e^{-jk_z z}\right)$$

$A^+ = 1$ (since $\underline{e}_t(x, y)$ already has 1 [V/m])

$$\Rightarrow \quad V_0^+ = C_1 A^+ = C_1 = \sqrt{\frac{ab}{2}}$$

Example (cont.)

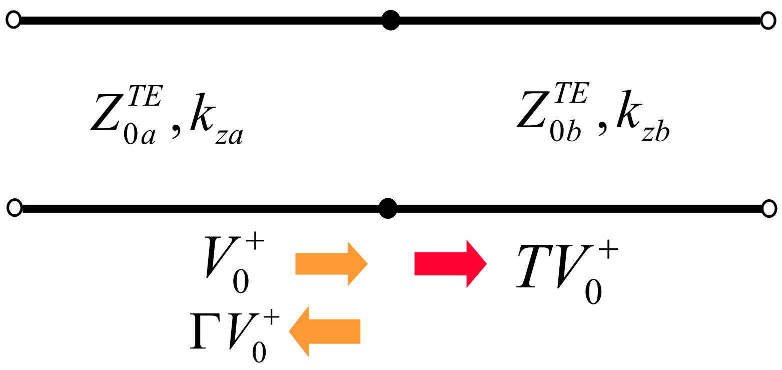
$$V_a(z) = V_0^+ \left(e^{-jk_{za}z} + \Gamma e^{+jk_{za}z} \right)$$

$$V_b(z) = V_0^+ \, T e^{-jk_{zb}z}$$

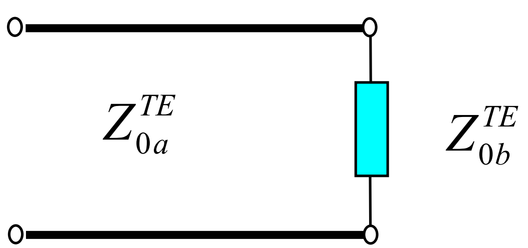
$$I_a(z) = \frac{V_0^+}{Z_{0a}} \left(e^{-jk_{za}z} - \Gamma e^{+jk_{za}z} \right)$$

$$I_b(z) = \frac{V_0^+ T}{Z_{0b}} e^{-jk_{zb}z}$$

TEN



Equivalent reflection problem:



$$\Gamma = \frac{Z_{0b} - Z_{0a}}{Z_{0b} + Z_{0a}} = -0.316$$

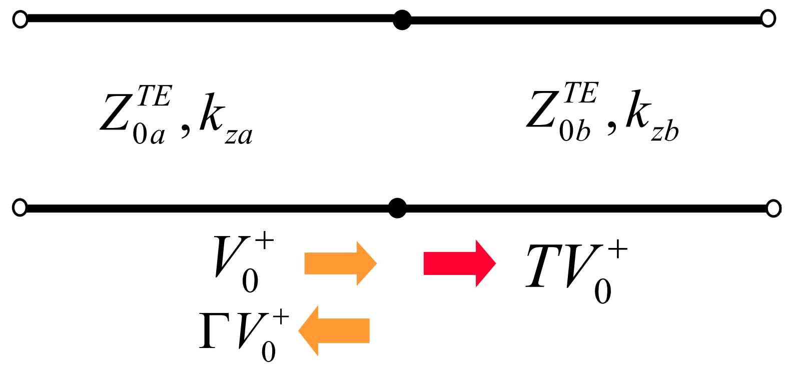
$$T = 1 + \Gamma = 0.684$$

Note:
The above TEN enforces the continuity of voltage and current at the junction, and hence the tangential electric and magnetic fields are automatically continuous in the WG problem.

Example (cont.)

Hence we have:

TEN



$$V_a(z) = \sqrt{\frac{ab}{2}}\left(e^{-jk_{za}z} + (-0.316)e^{+jk_{za}z}\right)$$

$$V_b(z) = \sqrt{\frac{ab}{2}}(0.684)e^{-jk_{zb}z}$$

$$I_a(z) = \sqrt{\frac{ab}{2}}\frac{1}{Z_{0a}}\left(e^{-jk_{za}z} - (-0.316)e^{+jk_{za}z}\right)$$

$$I_b(z) = \sqrt{\frac{ab}{2}}\frac{(0.684)}{Z_{0b}}e^{-jk_{zb}z}$$

$$k_{za} = 158.0 \;[\text{rad/m}]$$

$$k_{zb} = 304.1 \;[\text{rad/m}]$$

$$\underline{e}_t = \underline{\hat{y}}\sin\left(\frac{\pi}{a}x\right)$$

$$\underline{h}_t = -\underline{\hat{x}}\frac{1}{Z_{TE}}\sin\left(\frac{\pi}{a}x\right)$$

$$Z_{TE} = \frac{\omega\mu}{k_z^{10}}$$

Recall that for the TE_{10} mode:

$$\underline{E}_t(x,y,z) = \frac{1}{C_1}\underline{e}_t(x,y)V(z)$$

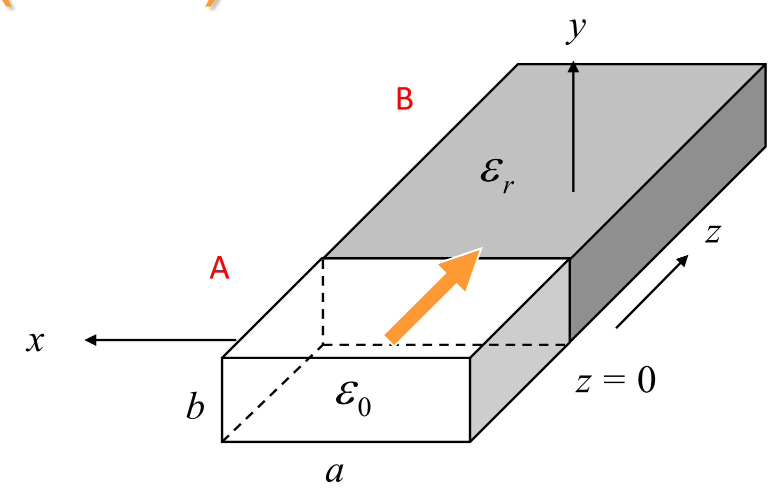
$$\underline{H}_t(x,y,z) = \frac{1}{C_2}\underline{h}_t(x,y)I(z)$$

$$C_1 = \sqrt{\frac{ab}{2}}$$

$$C_2 = \frac{1}{Z_{TE}}\sqrt{\frac{ab}{2}}$$

Example (cont.)

Hence, for the waveguide problem we have the fields as:



$$\underline{E}_{ta}(x,y,z) = \frac{1}{C_1}\underline{e}_t(x,y)\sqrt{\frac{ab}{2}}\left(e^{-jk_{za}z} + (-0.316)\,e^{+jk_{za}z}\right)$$

$$\underline{H}_{ta}(x,y,z) = \frac{1}{C_2}\underline{h}_t(x,y)\sqrt{\frac{ab}{2}}\frac{1}{Z_{0a}}\left(e^{-jk_{za}z} - (-0.316)\,e^{+jk_{za}z}\right)$$

$$\underline{E}_{tb}(x,y,z) = \frac{1}{C_1}\underline{e}_t(x,y)\sqrt{\frac{ab}{2}}(0.684)\,e^{-jk_{zb}z}$$

$$\underline{H}_{tb}(x,y,z) = \frac{1}{C_2}\underline{h}_t(x,y)\sqrt{\frac{ab}{2}}\frac{1}{Z_{0b}}(0.684)\,e^{-jk_{zb}z}$$

Example (cont.)

Substituting in for C_1 and $\underline{e}_t$, we have (guide A):

$$\underline{E}_{ta}(x,y,z) = \frac{1}{C_1}\underline{e}_t(x,y)\sqrt{\frac{ab}{2}}\left(e^{-jk_{za}z} + (-0.316)\,e^{+jk_{za}z}\right)$$

$$\underline{E}_{ta}(x,y,z) = \frac{1}{\sqrt{\frac{ab}{2}}}\left[\underline{\hat{y}}\sin\left(\frac{\pi}{a}x\right)\right]\sqrt{\frac{ab}{2}}\left(e^{-jk_{za}z} + (-0.316)\,e^{+jk_{za}z}\right)$$

$$C_1 = \sqrt{\frac{ab}{2}}$$

$$C_2 = \frac{1}{Z_{TE}}\sqrt{\frac{ab}{2}}$$

$$\underline{e}_t = \underline{\hat{y}}\sin\left(\frac{\pi}{a}x\right)$$

$$\underline{h}_t = -\underline{\hat{x}}\frac{1}{Z_{TE}}\sin\left(\frac{\pi}{a}x\right)$$

$$Z_{TE} = \frac{\omega\mu}{k_z^{10}}$$

Example (cont.)

Substituting in for C_2 and $\underline{h}_t$, we have (guide A):

$$\underline{H}_{ta}(x,y,z) = \frac{1}{C_2}\,\underline{h}_t(x,y)\sqrt{\frac{ab}{2}}\,\frac{1}{Z_{0a}}\left(e^{-jk_{za}z} - (-0.316)\,e^{+jk_{za}z}\right)$$

$$Z_{0a} = Z_a^{TE}$$

$$\underline{H}_{ta}(x,y,z) = \frac{1}{\dfrac{1}{Z_a^{TE}}\sqrt{\dfrac{ab}{2}}}\left[-\underline{\hat{x}}\,\frac{1}{Z_a^{TE}}\sin\left(\frac{\pi}{a}x\right)\right]\sqrt{\frac{ab}{2}}\,\frac{1}{Z_a^{TE}}\left(e^{-jk_{za}z} - (-0.316)\,e^{+jk_{za}z}\right)$$

Example (cont.)

Substituting in for C_1 and $\underline{e}_t$, we have (guide B):

$$\underline{E}_{tb}(x,y,z) = \frac{1}{C_1}\underline{e}_t(x,y)\sqrt{\frac{ab}{2}}(0.684)e^{-jk_{zb}z}$$

$$\underline{E}_{tb}(x,y,z) = \frac{1}{\sqrt{\frac{ab}{2}}}\left[\underline{\hat{y}}\sin\left(\frac{\pi}{a}x\right)\right]\sqrt{\frac{ab}{2}}(0.684)e^{-jk_{zb}z}$$

Example (cont.)

Substituting in for C_2 and $\underline{h}_t$, we have (guide B):

$$\underline{H}_{tb}(x,y,z)=\frac{1}{C_2}\underline{h}_t(x,y)\sqrt{\frac{ab}{2}}\frac{1}{Z_{0b}}(0.684)e^{-jk_{zb}z}$$

$$\Big\Downarrow \qquad Z_{0b}=Z_b^{TE}$$

$$\underline{H}_{tb}(x,y,z)=\frac{1}{\frac{1}{Z_b^{TE}}\sqrt{\frac{ab}{2}}}\left[-\underline{\hat{x}}\frac{1}{Z_b^{TE}}\sin\left(\frac{\pi}{a}x\right)\right]\sqrt{\frac{ab}{2}}\frac{1}{Z_b^{TE}}(0.684)e^{-jk_{zb}z}$$

Example (cont.)

Summary of Fields

$$\underline{E}_{ta}(x,y,z) = \left[\underline{\hat{y}}\sin\left(\frac{\pi}{a}x\right)\right]\left(e^{-jk_{za}z} + (-0.316)\,e^{+jk_{za}z}\right)$$

$$\underline{H}_{ta}(x,y,z) = \left[-\underline{\hat{x}}\frac{1}{Z_a^{TE}}\sin\left(\frac{\pi}{a}x\right)\right]\left(e^{-jk_{za}z} - (-0.316)\,e^{+jk_{za}z}\right)$$

$$\underline{E}_{tb}(x,y,z) = \left[\underline{\hat{y}}\sin\left(\frac{\pi}{a}x\right)\right](0.684)\,e^{-jk_{zb}z}$$

$$\underline{H}_{tb}(x,y,z) = \left[-\underline{\hat{x}}\frac{1}{Z_b^{TE}}\sin\left(\frac{\pi}{a}x\right)\right](0.684)\,e^{-jk_{zb}z}$$

$$Z_a^{TE} = 499.7\ [\Omega]$$
$$Z_b^{TE} = 259.6\ [\Omega]$$

$$k_{za} = 158.0\ [\text{rad/m}]$$
$$k_{zb} = 304.1\ [\text{rad/m}]$$

Example (cont.)

Power Calculations: **Note:** In this example, Z_0 and Γ are real.

Recall: $V_0^+ \equiv C_1 A^+ = C_1(1) = C_1 = \sqrt{\dfrac{ab}{2}}$

$$P_a^{inc} = \mathrm{Re}\left(\frac{1}{2} V_0^+ I_0^{+*}\right) = \mathrm{Re}\left(\frac{1}{2}\left|V_0^+\right|^2 \frac{1}{Z_{0a}^*}\right) = \frac{1}{2}\left|\sqrt{\frac{ab}{2}}\right|^2 \frac{1}{Z_a^{TE}}$$

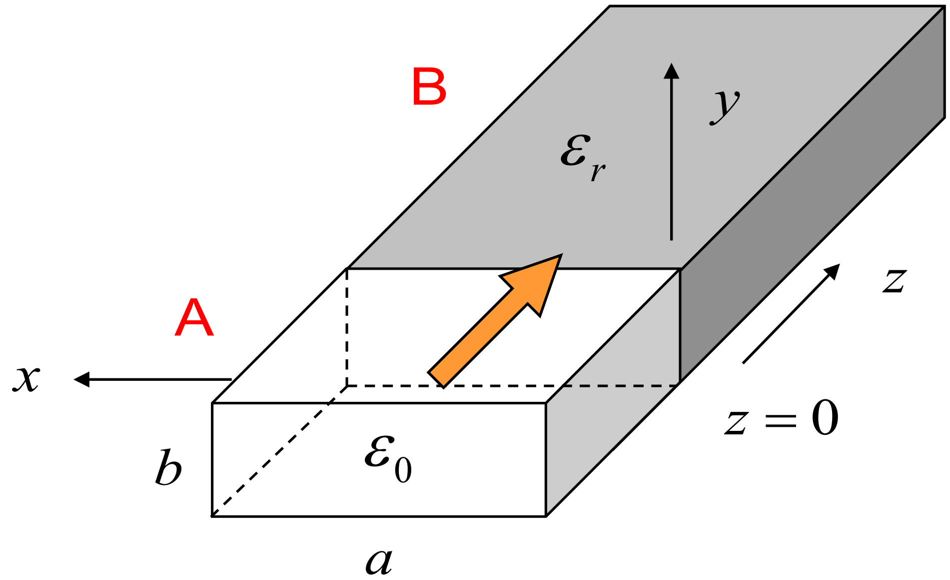
$$P_a^{ref} = \mathrm{Re}\left(\frac{1}{2} V_0^+ I_0^{+*}\right)\left|\Gamma\right|^2 = \frac{1}{2}\left|\sqrt{\frac{ab}{2}}\right|^2 \frac{1}{Z_a^{TE}}\left|\Gamma\right|^2$$

$$P_b^{trans} = \mathrm{Re}\left(\frac{1}{2} V_0^+ I_0^{+*}\right)\left(1-\left|\Gamma\right|^2\right) = \frac{1}{2}\left|\sqrt{\frac{ab}{2}}\right|^2 \frac{1}{Z_a^{TE}}\left(1-\left|\Gamma\right|^2\right)$$

Example (cont.)

Final Results:

$$P_a^{inc} = 1.161 \; [\text{mW}]$$
$$P_a^{ref} = 0.116 \; [\text{mW}]$$
$$P_b^{trans} = 1.045 \; [\text{mW}]$$



$$a = 2.2856 \text{ cm}$$
$$b = 1.016 \text{ cm}$$
$$\varepsilon_r = 2.54$$
$$f = 10 \text{ GHz}$$

Note: 90.1% of the incident power is transmitted.

Quarter-Wave Transformer in Waveguide

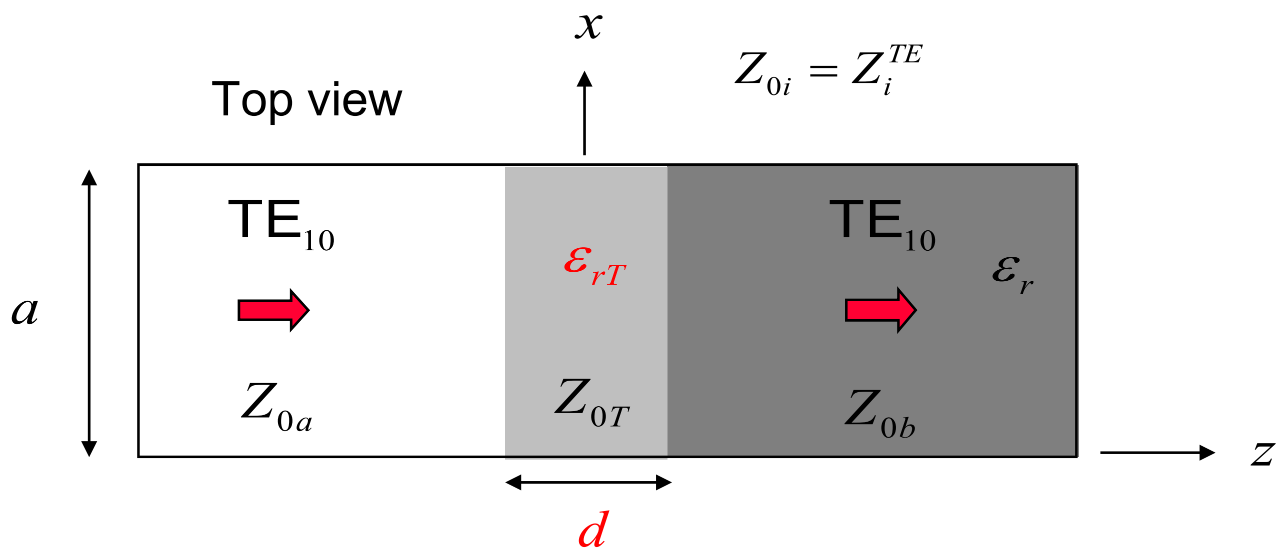
A quarter-wave transformer is shown here.

$$Z_{0T} = \sqrt{Z_{0a} Z_{0b}}$$

$$d = \lambda_{gT} / 4$$

Goal:

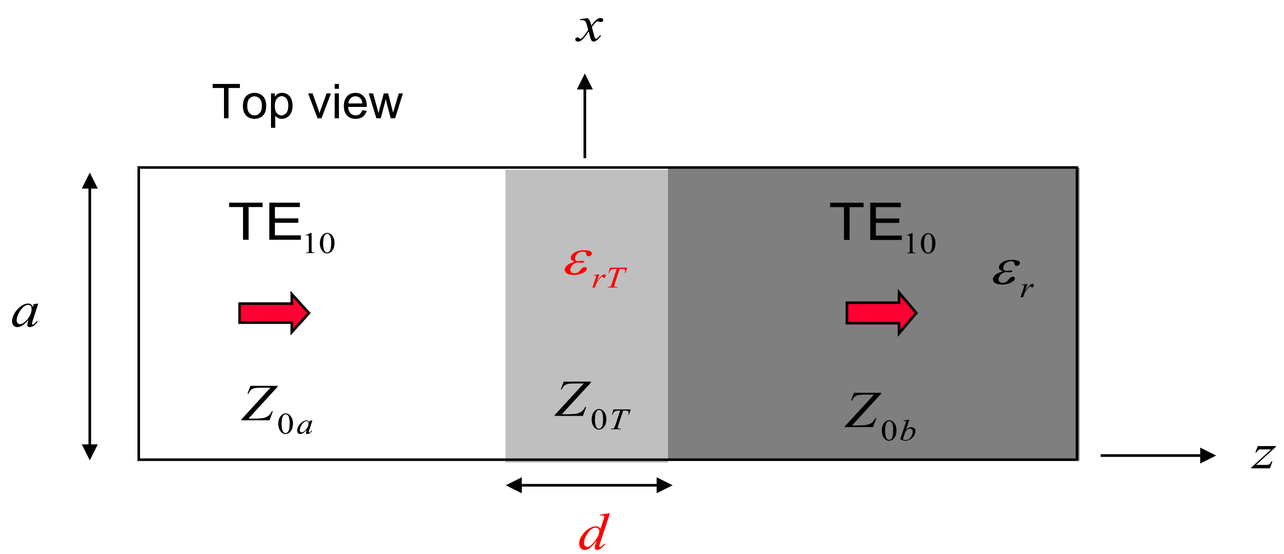
Determine : d, ε_T



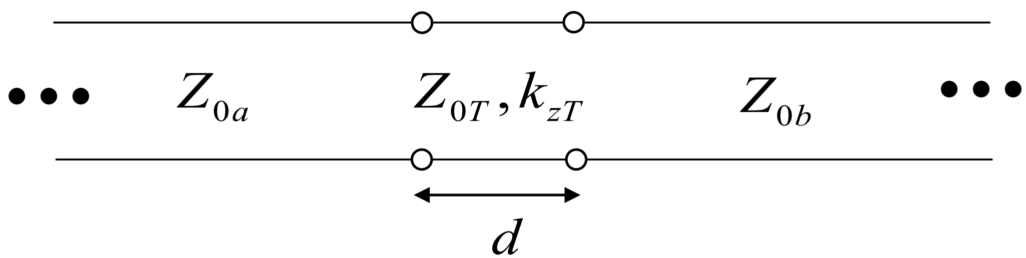
Now 100% of the incident power is now transmitted.

Quarter-Wave Transformer in Waveguide (cont.)

Waveguide problem



TEN



Quarter-Wave Transformer in Waveguide (cont.)

Design recipe:

1) Determine Z_{0T}: $Z_{0T} = \sqrt{Z_{0a} Z_{0b}}$

2) Determine k_{zT}: $Z_{0T} = \omega \mu_0 / k_{zT}$

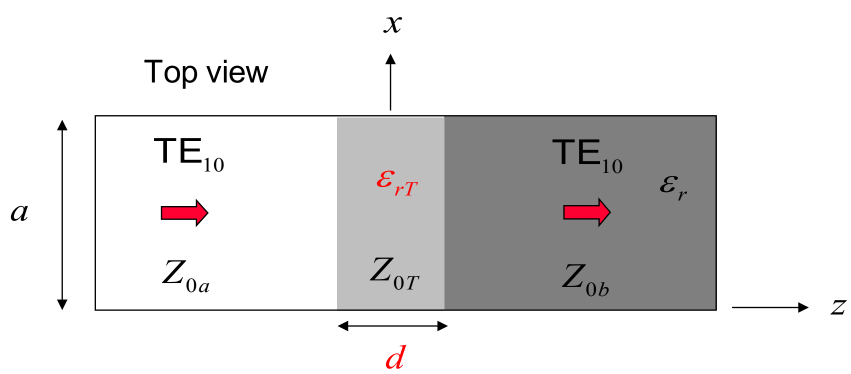
3) Determine k_T: $k_{zT} = \sqrt{k_T^2 - \left(\frac{\pi}{a}\right)^2}$

4) Determine ε_{rT}: $k_T = k_0 \sqrt{\varepsilon_{rT}}$

5) Determine β_T: $\beta_T = k_{zT}$

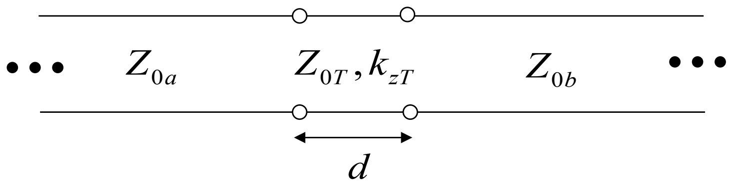
6) Determine λ_{gT}: $\lambda_{gT} = 2\pi / \beta_T$

7) Determine d: $d = \lambda_{gT} / 4$



$$Z_{0T} = \sqrt{Z_{0a} Z_{0b}}$$

$$d = \lambda_{gT} / 4$$



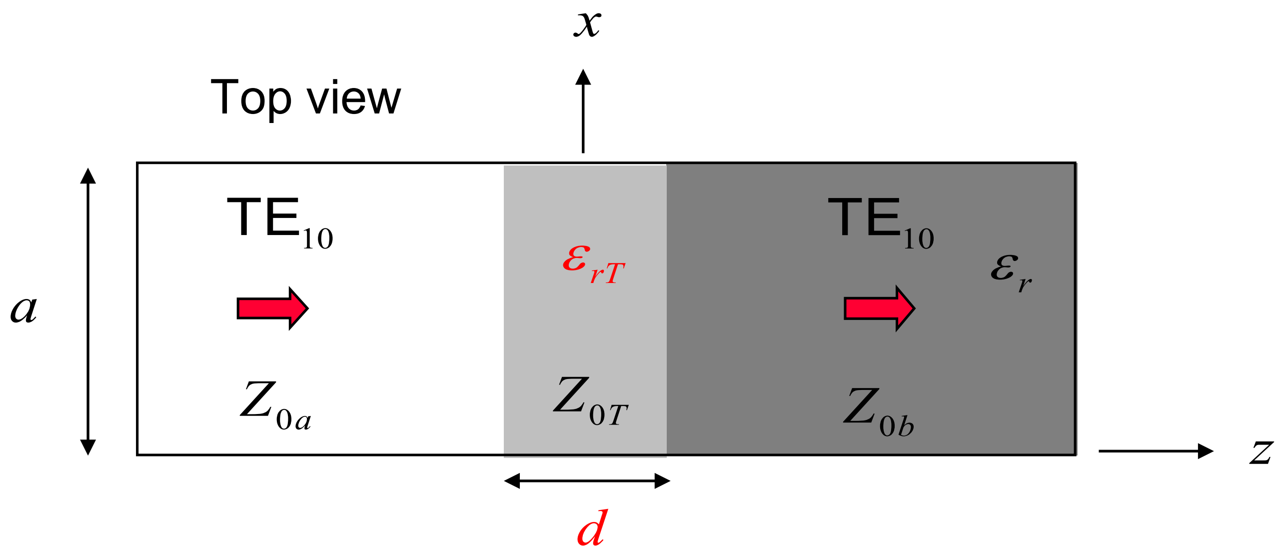
Quarter-Wave Transformer in Waveguide (cont.)

Example:

$$a = 2.2856 \text{ cm}$$
$$b = 1.016 \text{ cm}$$
$$\varepsilon_r = 2.54$$
$$f = 10 \text{ GHz}$$

Results:

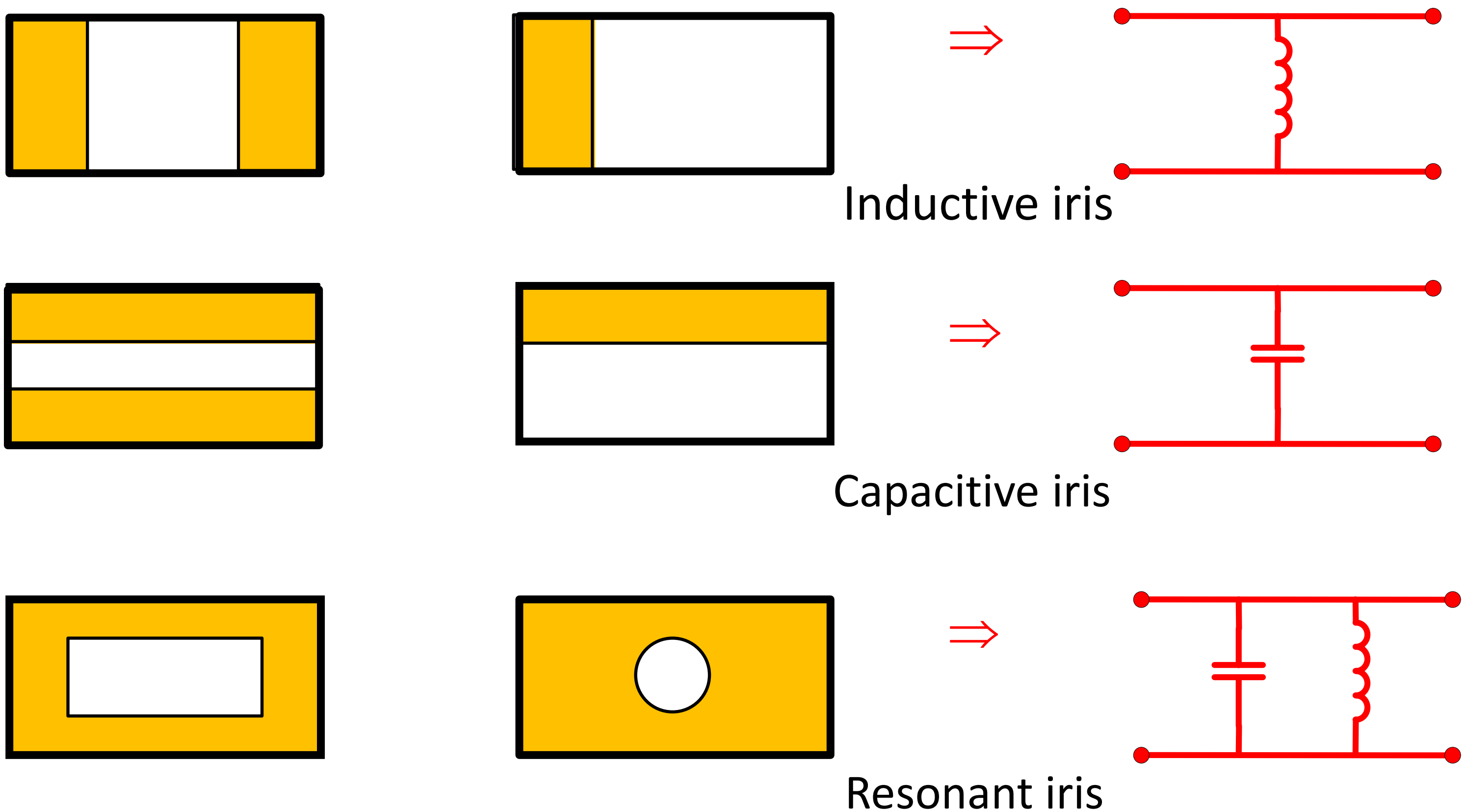
$$\varepsilon_{rT} = 1.53$$
$$d = 0.716 \text{ cm}$$



Matching Elements in Waveguide

Rectangular Waveguide
(end view)

Note:
Planar discontinuities are modeled as purely shunt elements.

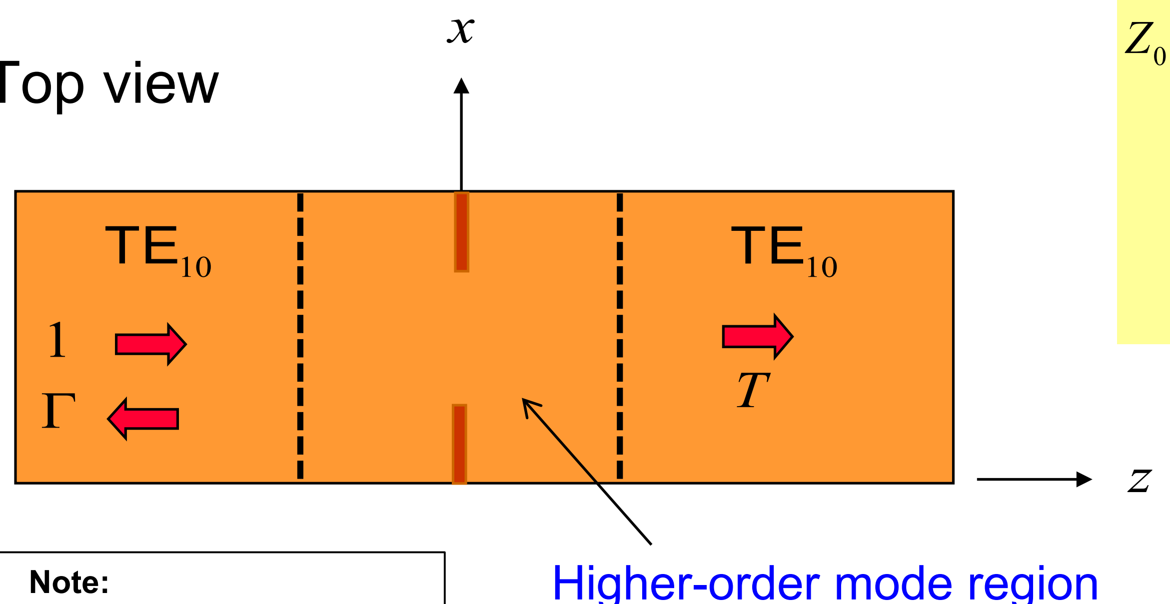


The equivalent circuit gives us the correct reflection and transmission of the TE_{10} mode.

Matching Elements in Waveguide (cont.)

Inductive iris in air-filled waveguide

Top view



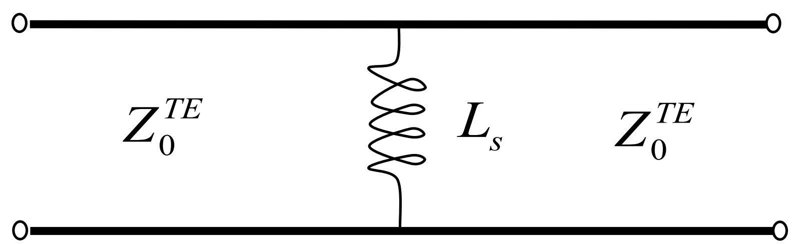
$$Z_0 = Z_{TE}^{10} = \frac{\omega\mu_0}{k_z^{10}} = \frac{\eta_0}{\sqrt{1-\left(\dfrac{\pi}{k_0 a}\right)^2}}$$

Note:
The shunt inductor models the effects of the iris and gives the amplitudes of the TE$_{10}$ mode correctly everywhere, but the TEN model does not tell us how strong the higher-order modes are.

Because the element is a shunt discontinuity, we have

$$T = 1 + \Gamma$$

TEN Model



Matching Elements in Waveguide (cont.)

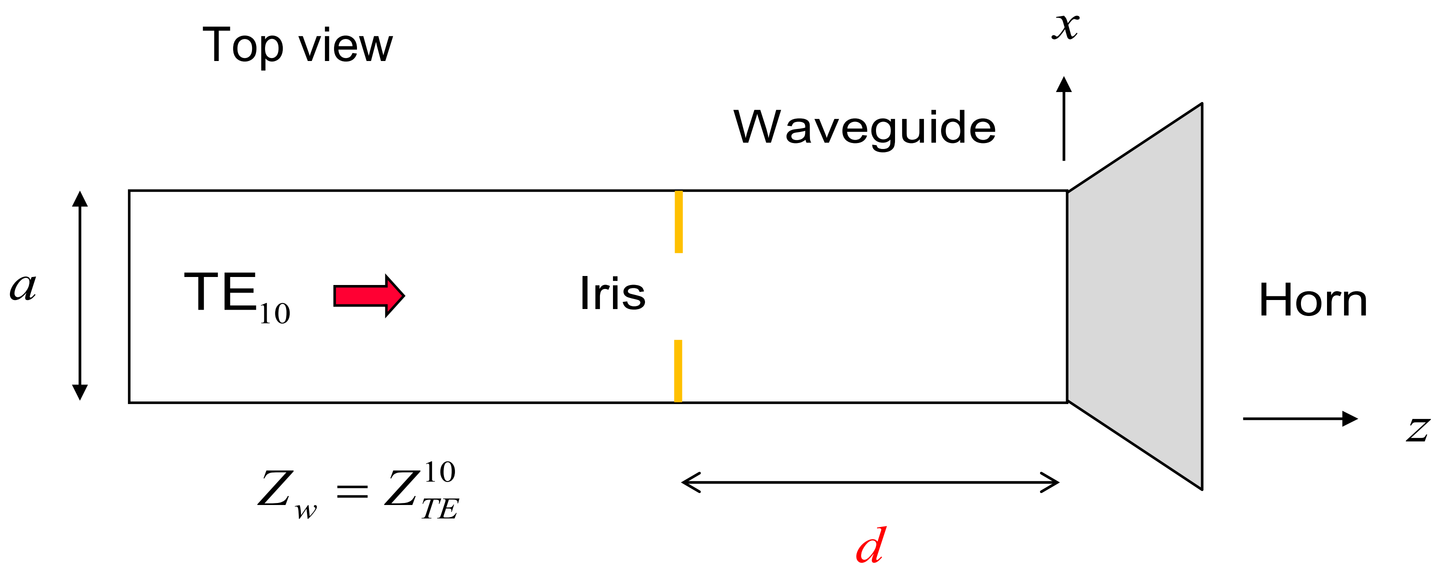
Much more information can be found in the following reference:

N. Marcuvitz*, Waveguide Handbook*, Peter Perigrinus, Ltd. (on behalf of the Institute of Electrical Engineers), 1986.

- Equivalent circuits for many types of discontinuities
- Accurate CAD formulas for many of the discontinuities
- Graphical results for many of the cases
- Sometimes, measured results

Using Irises for Matching

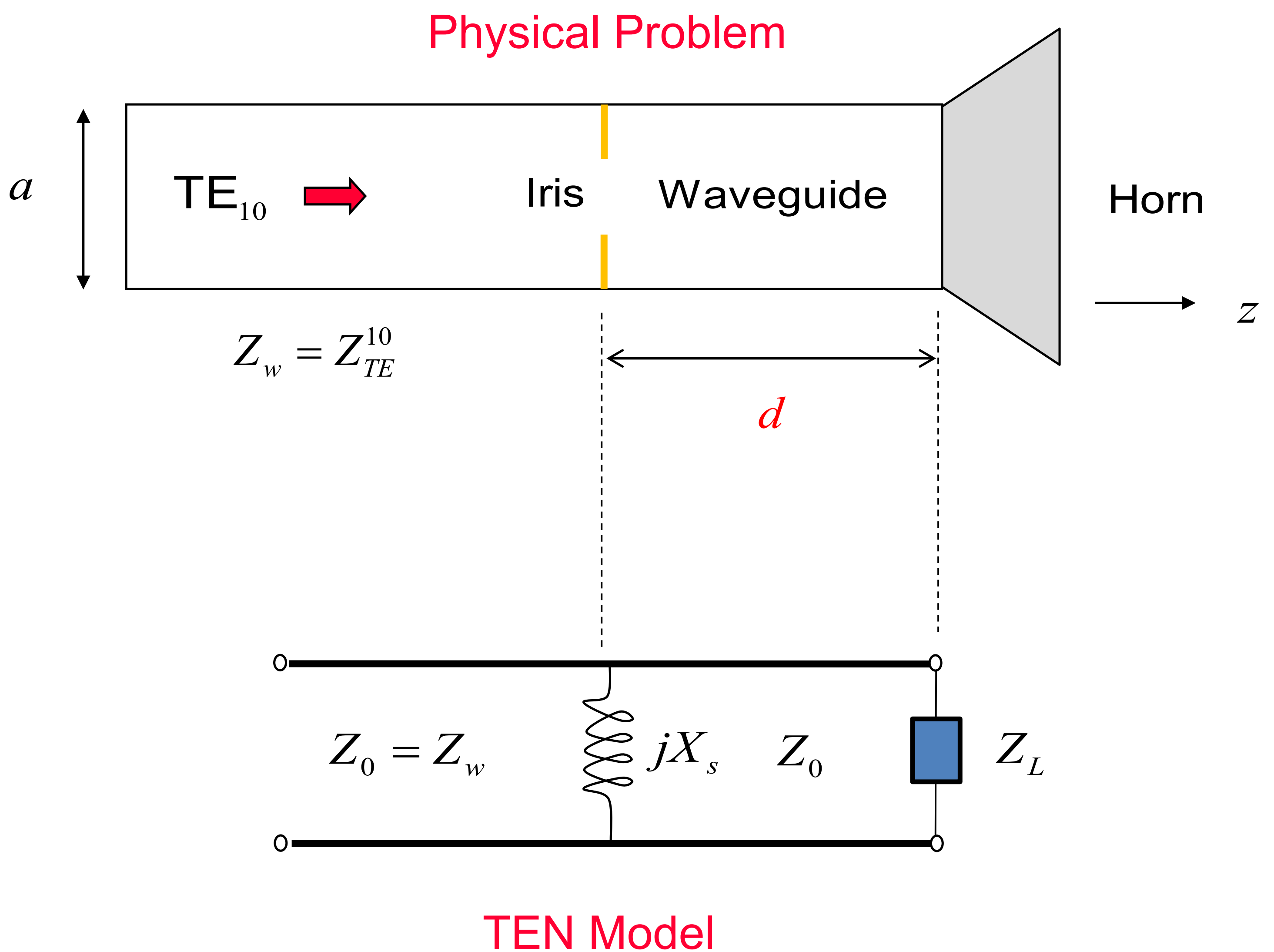
An iris is shown here being used for matching to a load (illustrated for a horn antenna load).



An inductive iris is shown being used for matching.

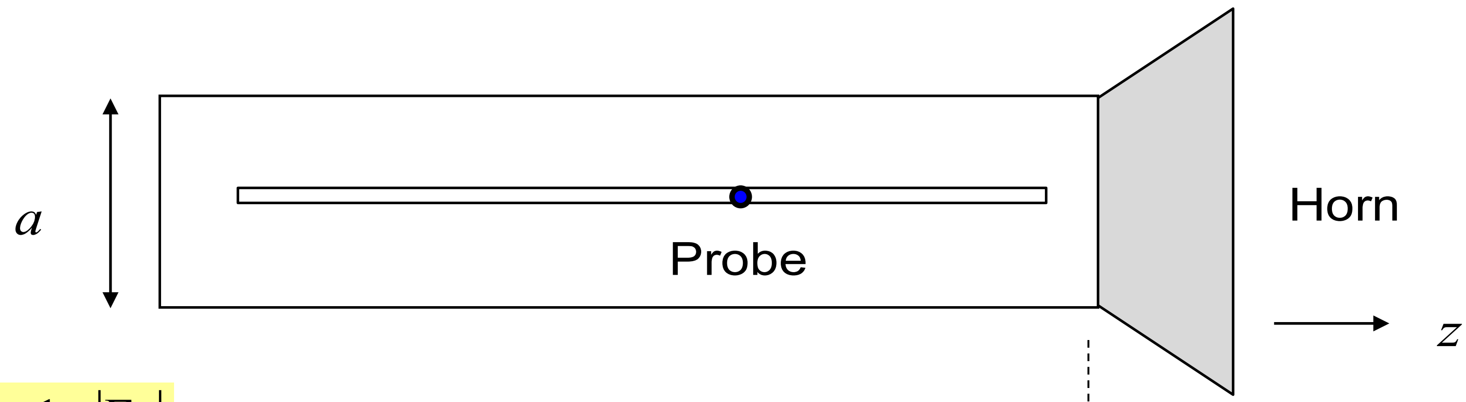
Using Irises for Matching (cont.)

The TEN is shown



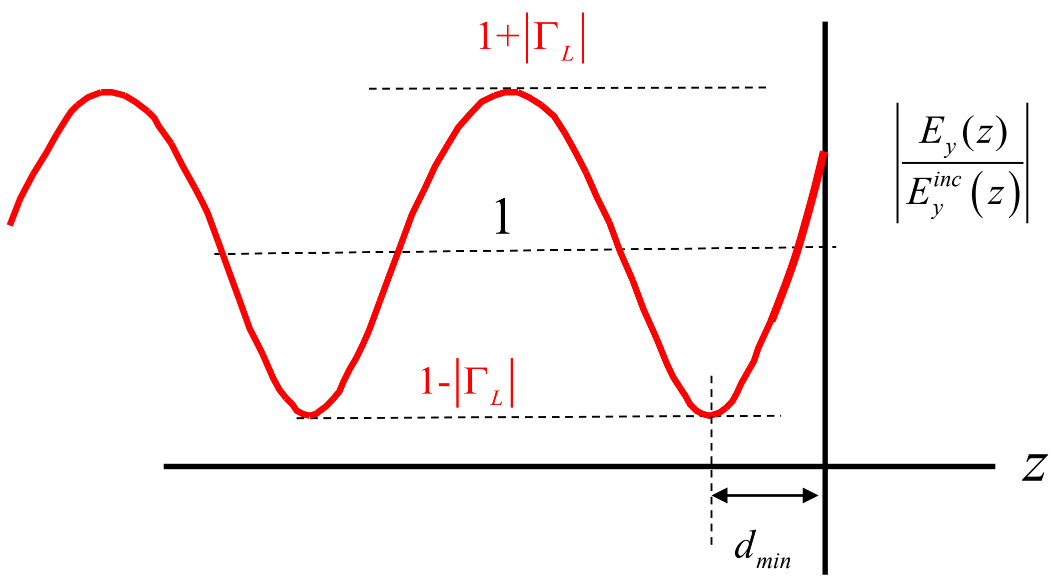
Using Irises for Matching (cont.)

A field probing can be used to determine the unknown load impedance Z_L.



$$\text{SWR} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

$$|\Gamma_L| = \frac{\text{SWR}-1}{\text{SWR}+1}$$

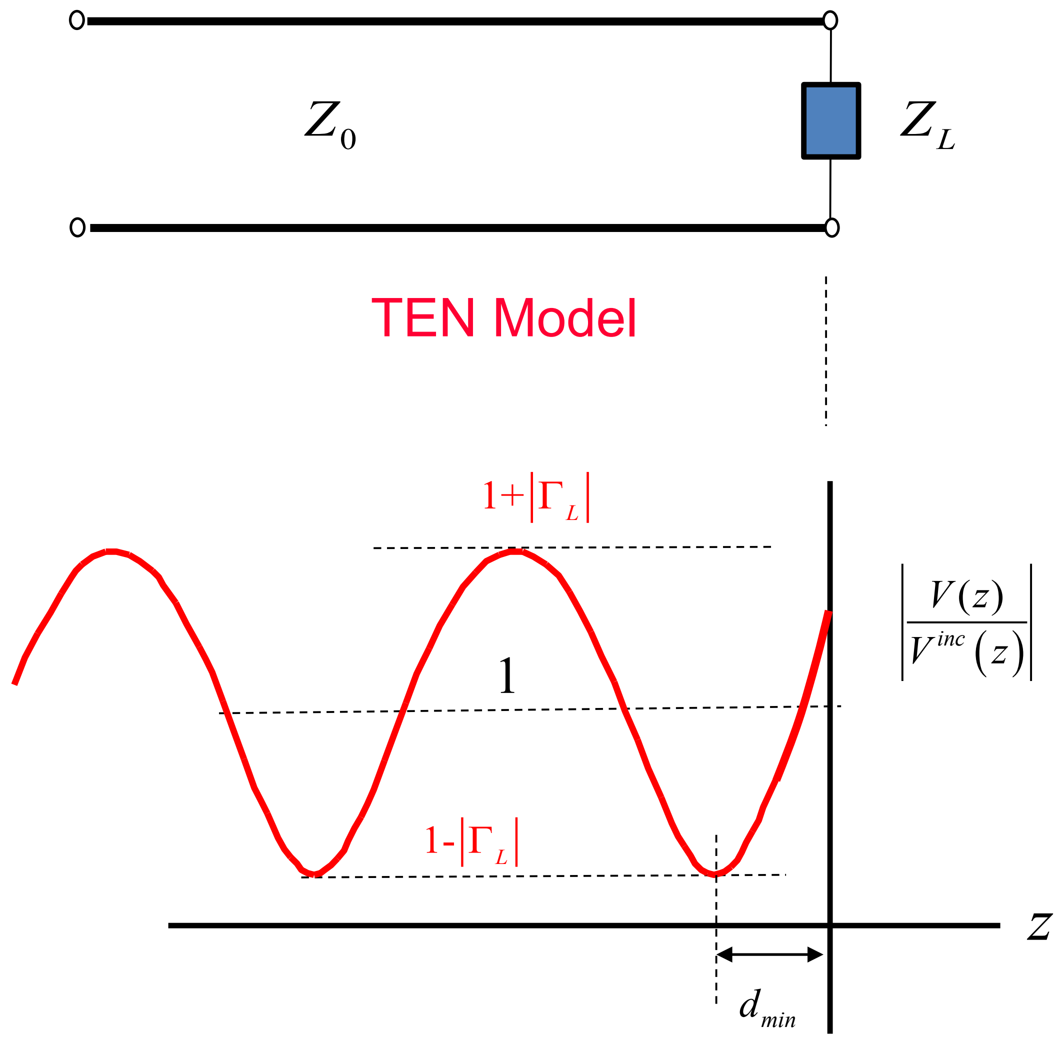


Using Irises for Matching (cont.)

A field probing can be used to determine the unknown load impedance Z_L.

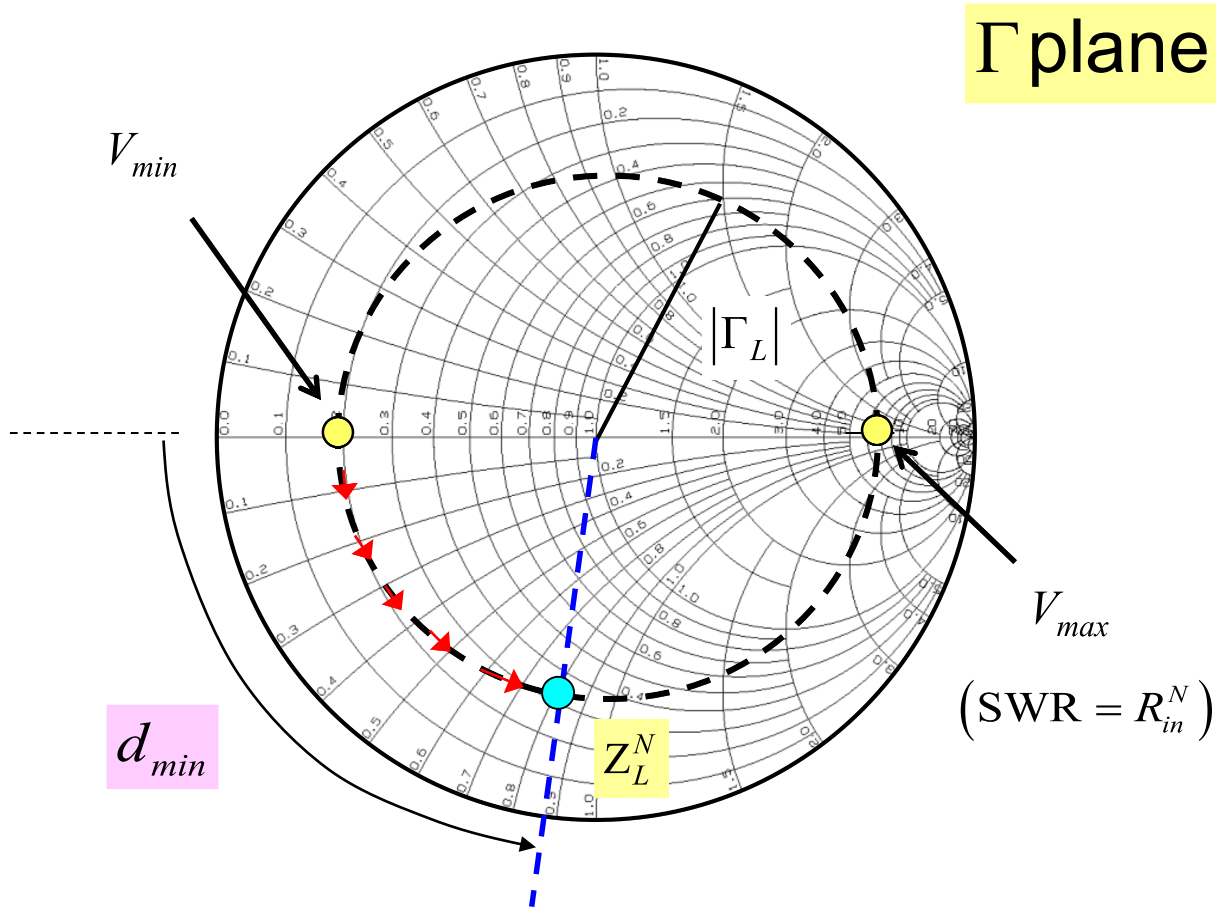
$$\text{SWR} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

$$|\Gamma_L| = \frac{\text{SWR}-1}{\text{SWR}+1}$$



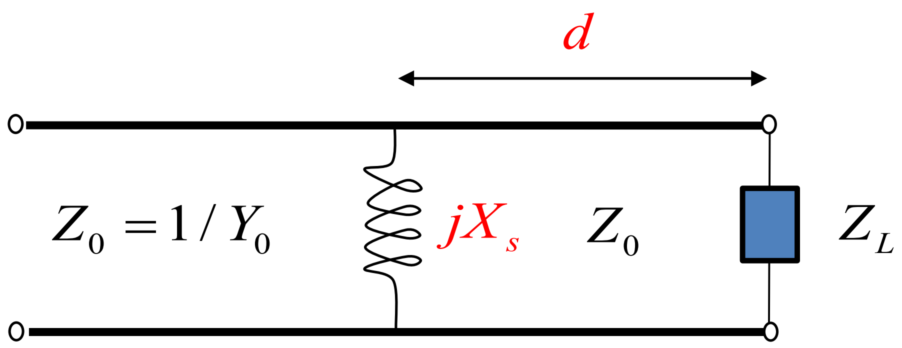
Using Irises for Matching (cont.)

A Smith chart can be used to find the unknown load impedance.



Using Irises for Matching (cont.)

A Smith chart can also be used to find the location of the shunt susceptance and its value.



$$Z_s = jX_s$$

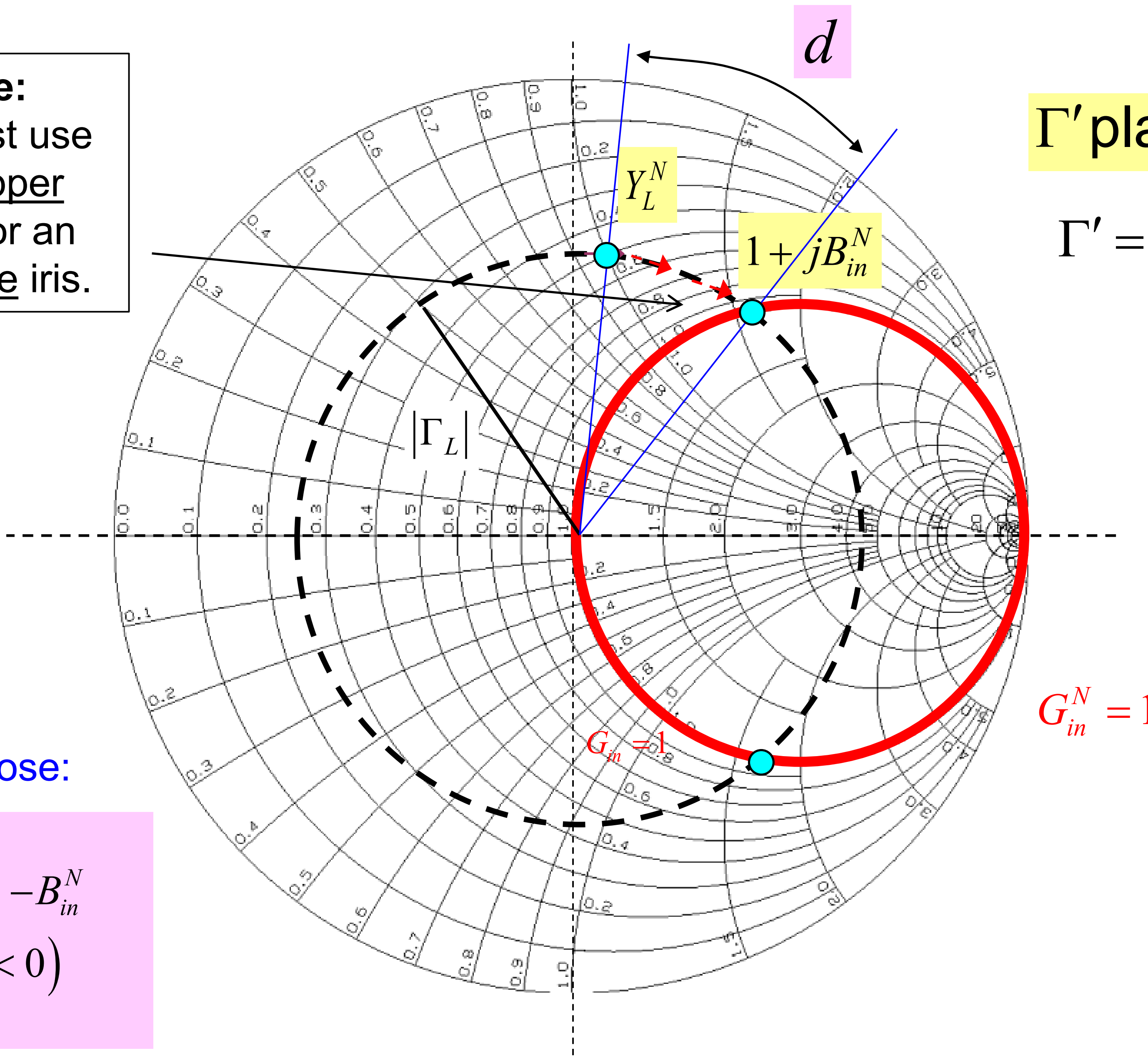
$$Y_s = jB_s$$

$$B_s^N \equiv \frac{B_s}{Y_0}$$

$$\text{Note}: \; B_s = -\frac{1}{X_s}$$

Using Irises for Matching (cont.)

Note:
We must use this <u>upper</u> point for an <u>inductive</u> iris.



Γ' plane

$$\Gamma' = -\Gamma$$

Choose:

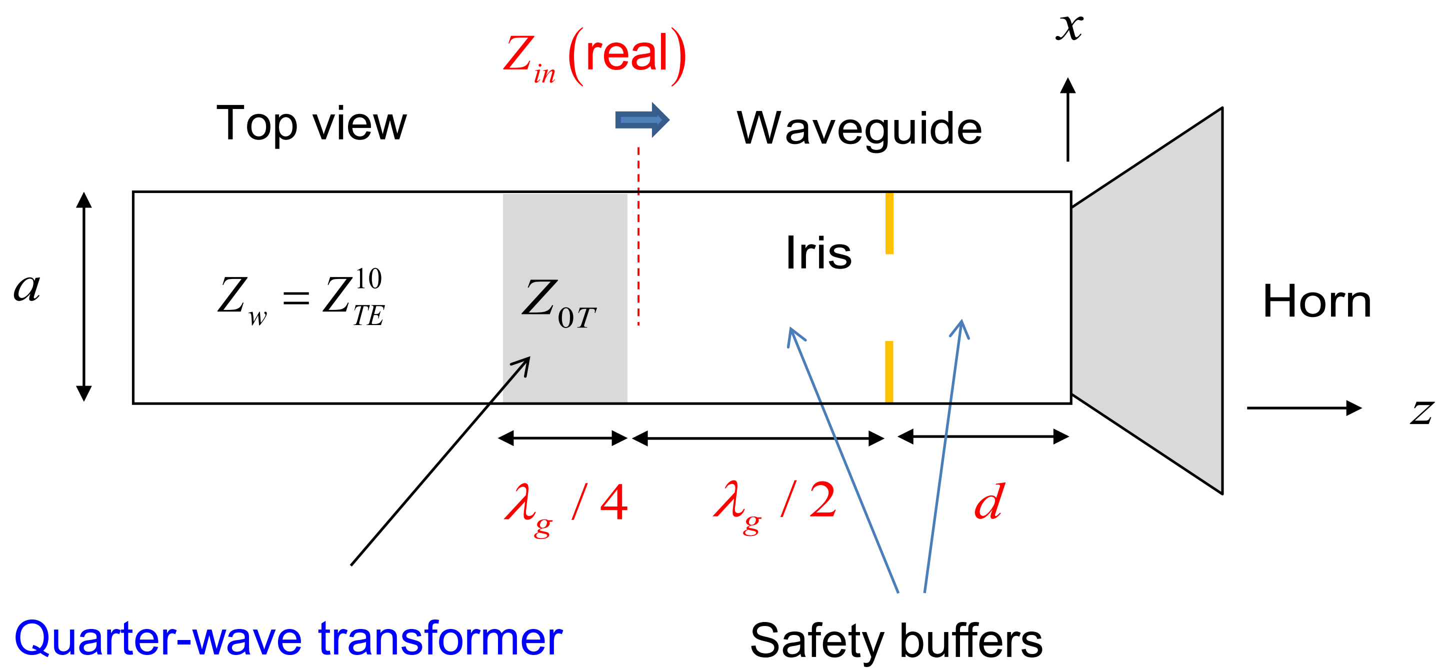
$$B_s^N = -B_{in}^N$$
$$\left(B_s^N < 0\right)$$

Using Irises for Matching (cont.)

A quarter-wave transformer matching scheme is shown here.

d is arbitrary

(The arbitrary distance d gives us a "safety buffer" from the horn discontinuity, to allow the higher-order modes to decay. The extra $\lambda_g/2$ is also a safety buffer.)



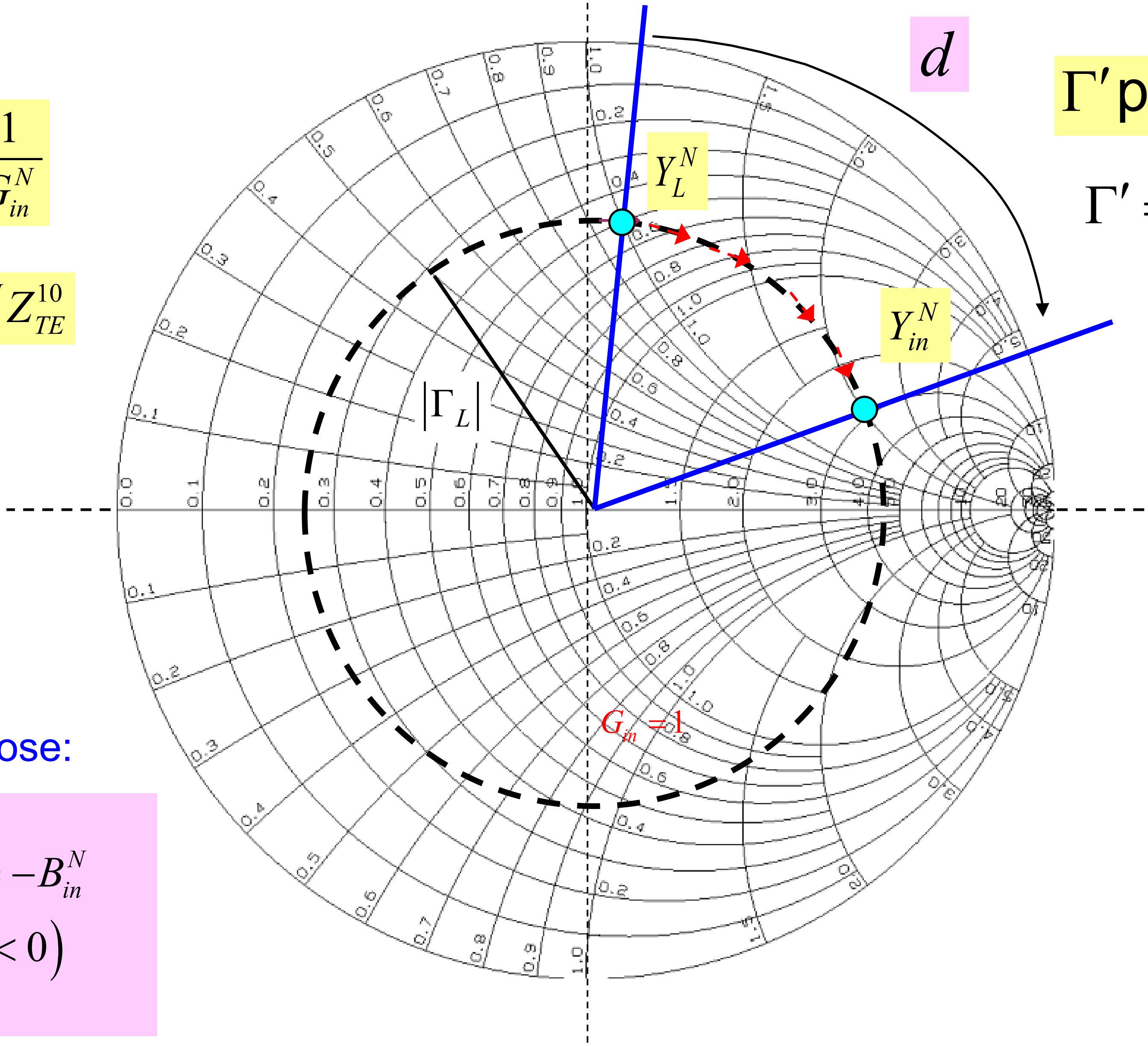
$$Z_{0T} = \sqrt{Z_{TE}^{10} Z_{in}}$$

The iris is used to cancel the input susceptance at $z = -d$.

Using Irises for Matching (cont.)

$$Z_{in}^{N} = \frac{1}{G_{in}^{N}}$$

$$Z_{in} = Z_{in}^{N} Z_{TE}^{10}$$



Choose:

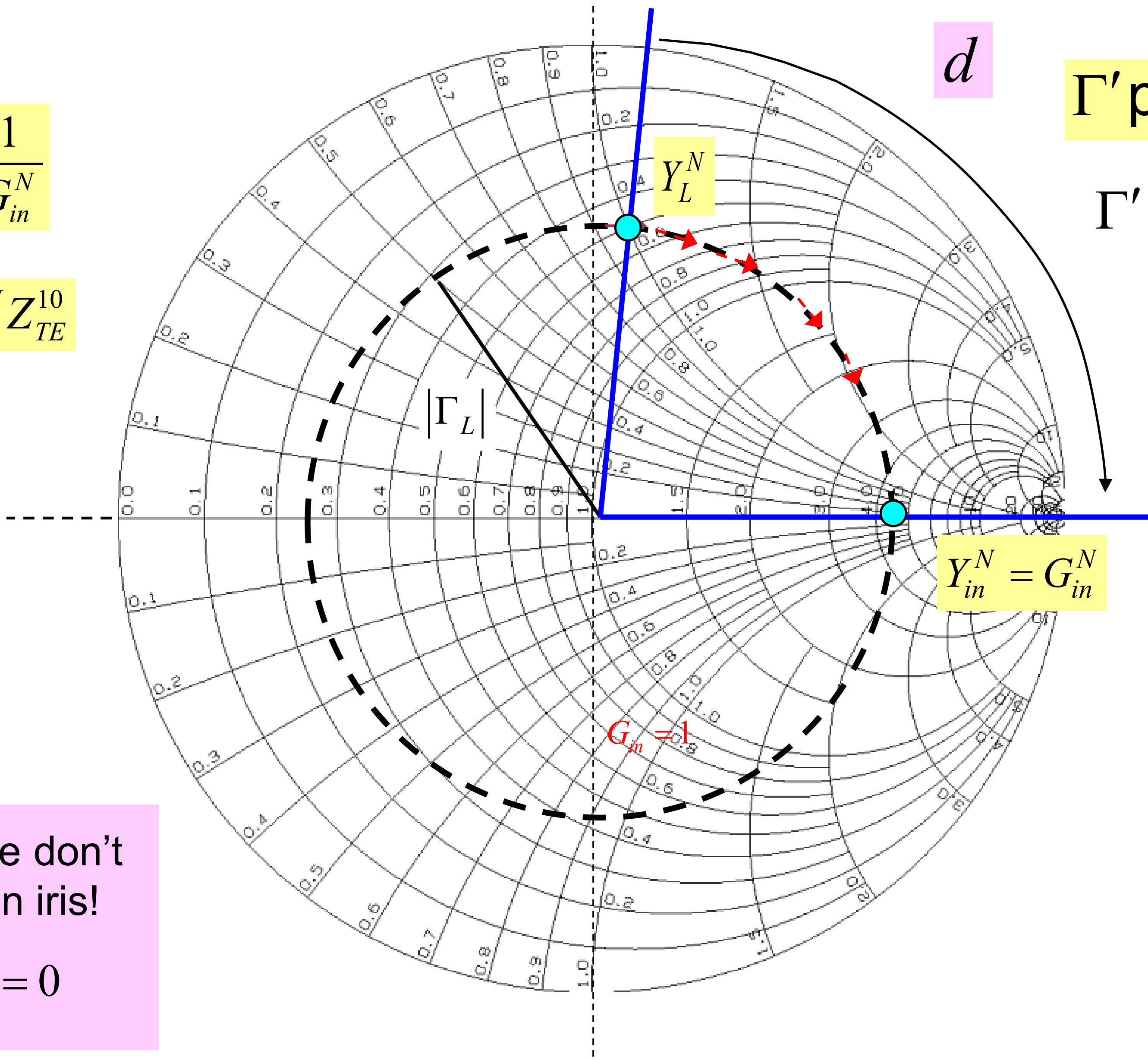
$$B_s^N = -B_{in}^N$$

$$\left(B_s^N < 0\right)$$

Using Irises for Matching (cont.)

$$Z_{in}^{N} = \frac{1}{G_{in}^{N}}$$

$$Z_{in} = Z_{in}^{N} Z_{TE}^{10}$$



Now we don't need an iris!

$$B_s^N = 0$$