Adapted from notes by
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ECE 5317-6351
Microwave Engineering

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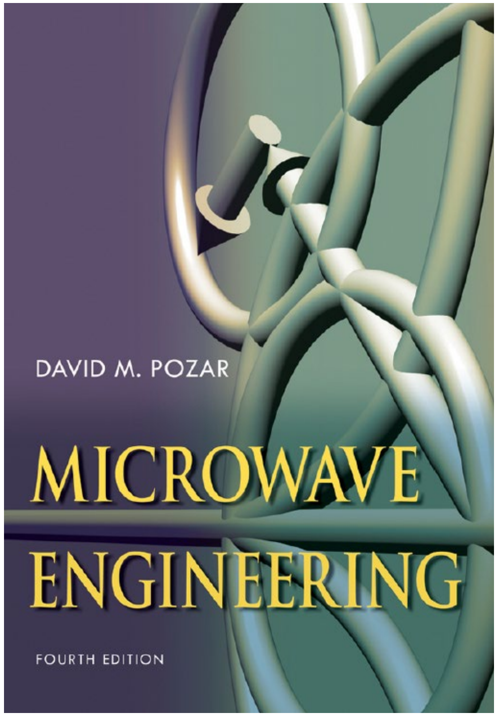
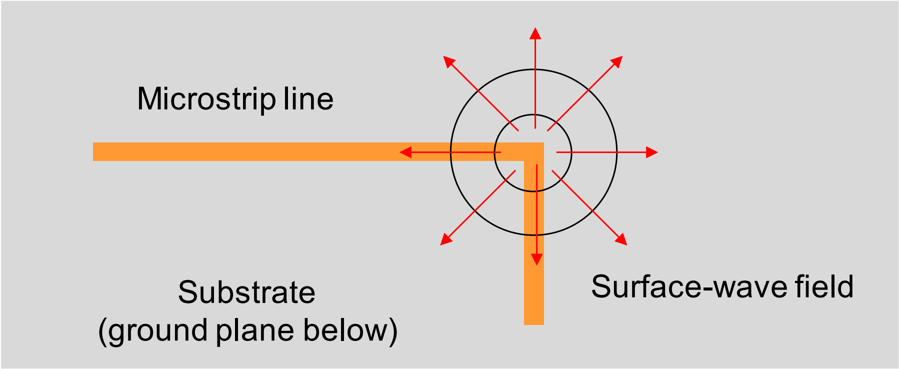
Dept. of ECE

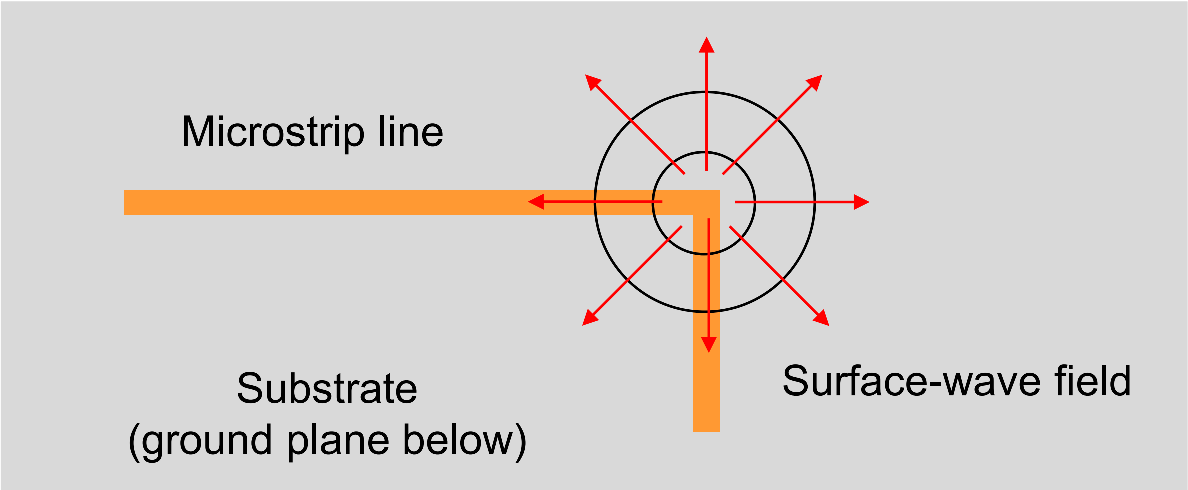


Notes 14
Surface Waves



Grounded Dielectric Slab

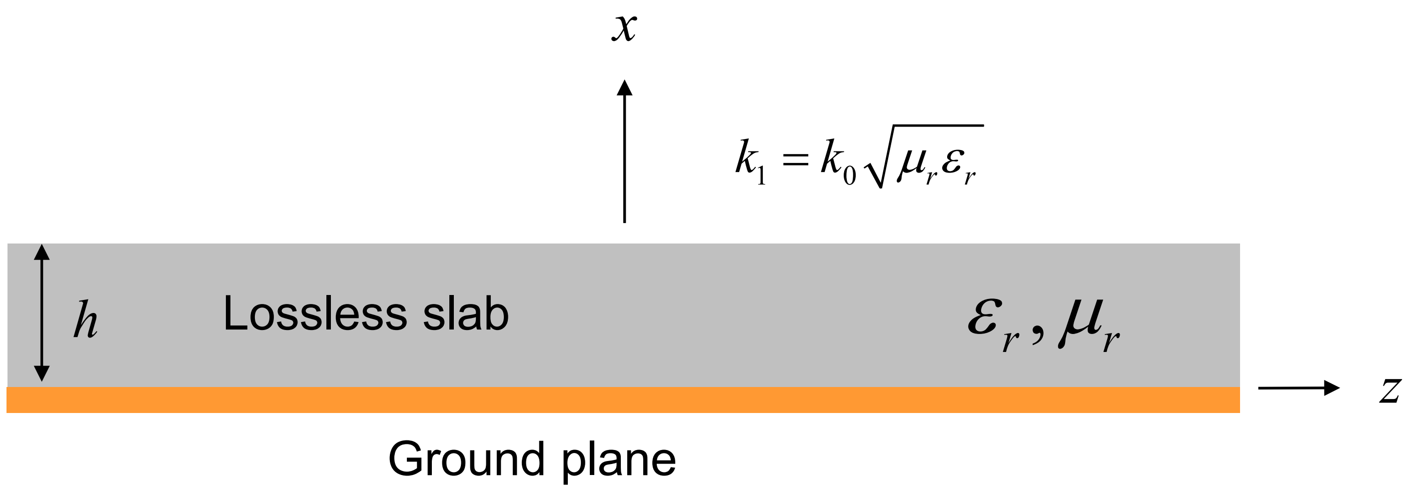
Discontinuities on planar transmission lines such as microstrip will radiate surface-wave fields.



It is important to understand these waves.

Note:
Surface waves can also be used as a propagation mechanism for microwave and millimeter-wave frequencies. (The physics is similar to that of a fiber-optic guide.)

Grounded Dielectric Slab



Goal: Determine the modes of propagation and their wavenumbers.

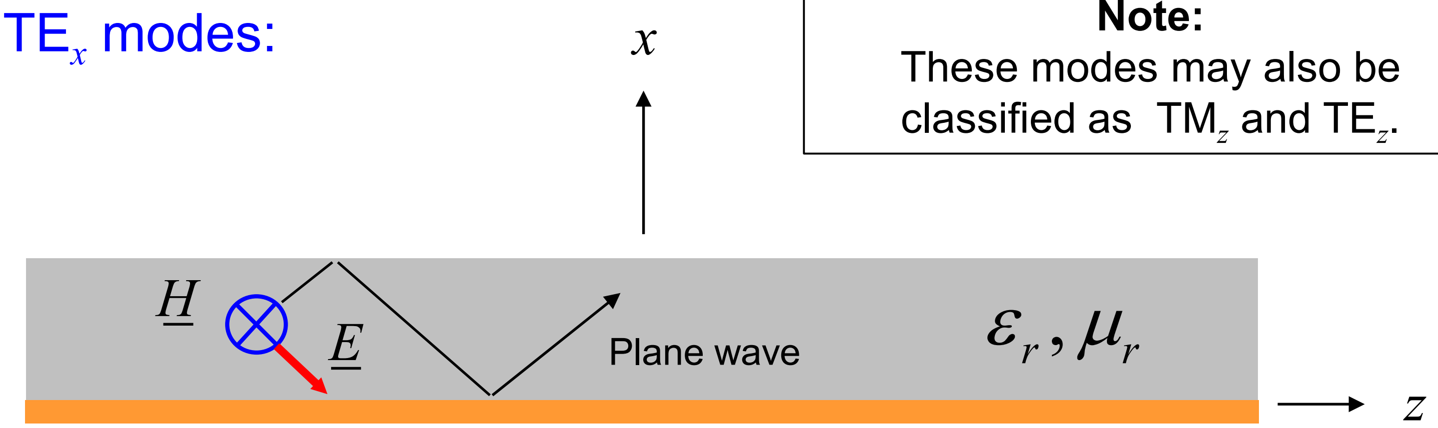
Assumption: There is no variation of the fields in the y direction, and propagation is along the z direction.

Dielectric Slab

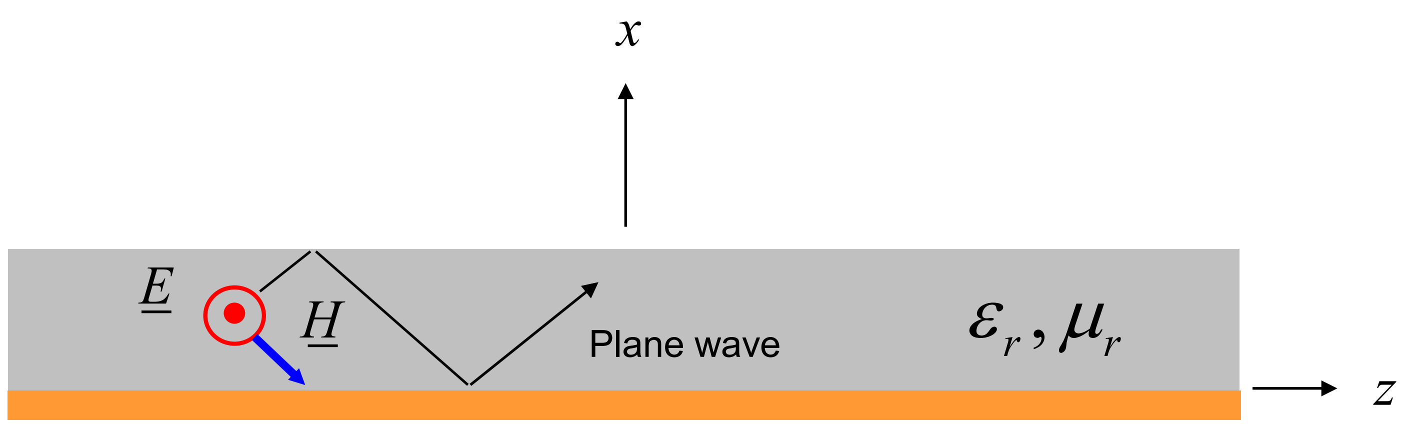
TM$_x$ & TE$_x$ modes:

Note:
These modes may also be classified as TM$_z$ and TE$_z$.

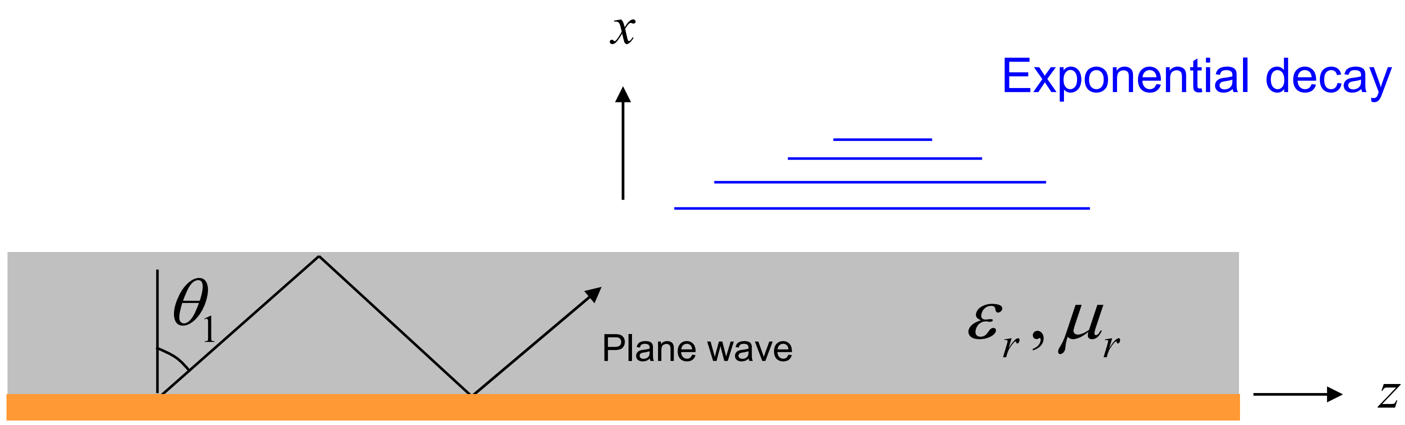
TM$_x$



TE$_x$



Surface Wave



$$\theta_1 > \theta_c$$

The internal angle is larger than the critical angle, so there is exponential decay in the air region.

$$k_z = k_1 \sin\theta_1 > k_0 \quad \left(k_1 \sin\theta_c = k_0\right)$$

The surface wave is a "slow wave": $k_z > k_0 \;\Rightarrow\; v_p < c$

Hence $k_{x0} = \left(k_0^2 - k_z^2\right)^{1/2} = -j\sqrt{k_z^2 - k_0^2} = -j\alpha_{x0}$

TM$_x$ Solution

Assume TM$_x$:

$$E_x\left(x,z\right)=e^{-jk_z z}e_x\left(x\right)$$

$$\frac{\partial^2 E_x}{\partial x^2}+\frac{\partial E_x}{\partial y^2}+\frac{\partial^2 E_x}{\partial z^2}+k^2 E_x=0$$

$$\Rightarrow \quad \frac{\partial^2 E_x}{\partial x^2}+\left(k^2-k_z^2\right)E_x=0$$

so

$$\frac{d^2 e_x\left(x\right)}{dx^2}+\left(k^2-k_z^2\right)e_x\left(x\right)=0$$

TM_x Solution (cont.)

$$\frac{d^2 e_x(x)}{dx^2} + \left(k^2 - k_z^2\right) e_x(x) = 0 \qquad k = k_0 \text{ or } k_1$$

Denote

$$k_{x0} = \left(k_0^2 - k_z^2\right)^{1/2} = -j\sqrt{k_z^2 - k_0^2} = -j\alpha_{x0}$$

$$k_{x1} = \sqrt{k_1^2 - k_z^2}$$

Then we have:

$$x \geq h \qquad \frac{d^2 e_x(x)}{dx^2} - \alpha_{x0}^2 \, e_x(x) = 0$$

$$x \leq h \qquad \frac{d^2 e_x(x)}{dx^2} + k_{x1}^2 e_x(x) = 0$$

TM_x Solution (cont.)

Applying boundary conditions at the ground plane, we have:

$$x \le h \qquad E_{x1} = e^{-jk_z z} \cos(k_{x1} x)$$

$$x \ge h \qquad E_{x0} = A e^{-jk_z z} e^{-\alpha_{x0} x}$$

Note: on PEC

$$\frac{\partial E_x}{\partial x} = 0$$

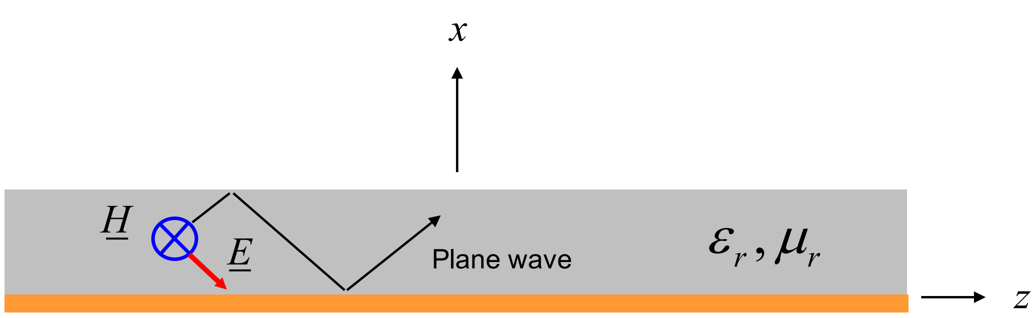
This follows since

$$\nabla \cdot \underline{E} = 0$$

$$\Rightarrow \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$$

$$\Rightarrow \frac{\partial E_x}{\partial x} = -(-jk_z) E_z$$

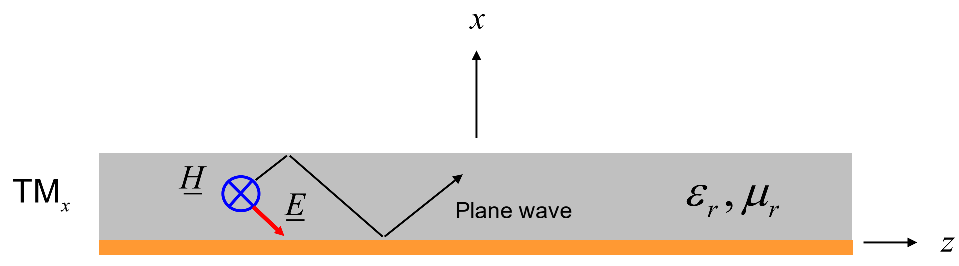
TM_x



Boundary Conditions

BC 1) $D_{x0} = D_{x1} \quad @\, x = h$

$\Rightarrow E_{x0} = \varepsilon_r E_{x1}$



BC 2) $E_{z0} = E_{z1} \quad @\, x = h$

$$\Rightarrow \frac{\partial E_{x0}}{\partial x} = \frac{\partial E_{x1}}{\partial x}$$

$$E_{x0} = Ae^{-jk_z z} e^{-\alpha_{x0} x}$$

$$E_{x1} = e^{-jk_z z} \cos(k_{x1} x)$$

Recall: $\dfrac{\partial E_x}{\partial x} = jk_z E_z$

Boundary Conditions (cont.)

These two BC equations yield:

BC 1) $$A e^{-\alpha_{x0} h} = \varepsilon_r \cos(k_{x1} h)$$

BC 2) $$-\alpha_{x0} A e^{-\alpha_{x0} h} = (-k_{x1}) \sin(k_{x1} h)$$

Divide second by first:

$$-\alpha_{x0} = \frac{1}{\varepsilon_r}(-k_{x1}) \tan(k_{x1} h)$$

or

$$\alpha_{x0}\, \varepsilon_r = k_{x1} \tan(k_{x1} h)$$

$$E_{x0} = A e^{-jk_z z} e^{-\alpha_{x0} x}$$

$$E_{x1} = e^{-jk_z z} \cos(k_{x1} x)$$

$$E_{x0} = \varepsilon_r E_{x1}$$

$$\frac{\partial E_{x0}}{\partial x} = \frac{\partial E_{x1}}{\partial x}$$

Final Result: TM_x

This may be written as:

$$\varepsilon_r \sqrt{k_z^2 - k_0^2} = \sqrt{k_1^2 - k_z^2} \tan\left[\sqrt{k_1^2 - k_z^2}\, h\right]$$

This is a transcendental equation for the unknown wavenumber k_z.

Final Result: TE$_x$

Omitting the derivation, the final result for TE$_x$ modes is:

$$\sqrt{k_z^2 - k_0^2} = -\frac{1}{\mu_r}\sqrt{k_1^2 - k_z^2}\,\cot\left(\sqrt{k_1^2 - k_z^2}\,h\right)$$

This is a transcendental equation for the unknown wavenumber k_z.

Graphical Solution for SW Modes

Consider TM$_x$: $$\alpha_{x0}\varepsilon_r = k_{x1}\tan(k_{x1}h)$$

or

$$\alpha_{x0}h = \frac{1}{\varepsilon_r}(k_{x1}h)\tan(k_{x1}h)$$

Define:

$$u \equiv k_{x1}h$$

$$v \equiv \alpha_{x0}h$$

Then

$$v = \frac{1}{\varepsilon_r}u\tan u$$

Graphical Solution (cont.)

We can develop another equation by relating u and v:

$$u = h\sqrt{k_1^{\ 2} - k_z^2}$$

$$v = h\sqrt{k_z^{\ 2} - k_0^2}$$

Hence

$$\left.\begin{aligned} u^2 &= h^2\,(k_1^2 - k_z^2) \\ v^2 &= h^2\,(k_z^2 - k_0^2) \end{aligned}\right\} \quad \text{Add}$$

$$\begin{aligned} u^2 + v^2 &= h^2(k_1^2 - k_0^2) \\ &= (k_0 h)^2(n_1^2 - 1) \end{aligned}$$

where

$$n_1 \equiv k_1 / k_0 = \sqrt{\varepsilon_r \mu_r}$$

Graphical Solution (cont.)

Define

$$R^2 \equiv (k_0 h)^2 (n_1^{\,2} - 1)$$

or

$$R = (k_0 h)\sqrt{n_1^{\,2} - 1}$$

Note:
R is proportional to frequency.

Then

$$u^2 + v^2 = R^2$$

Graphical Solution (cont.)

Summary for TM$_x$ Case

$$v = \frac{1}{\varepsilon_r} u \tan u$$

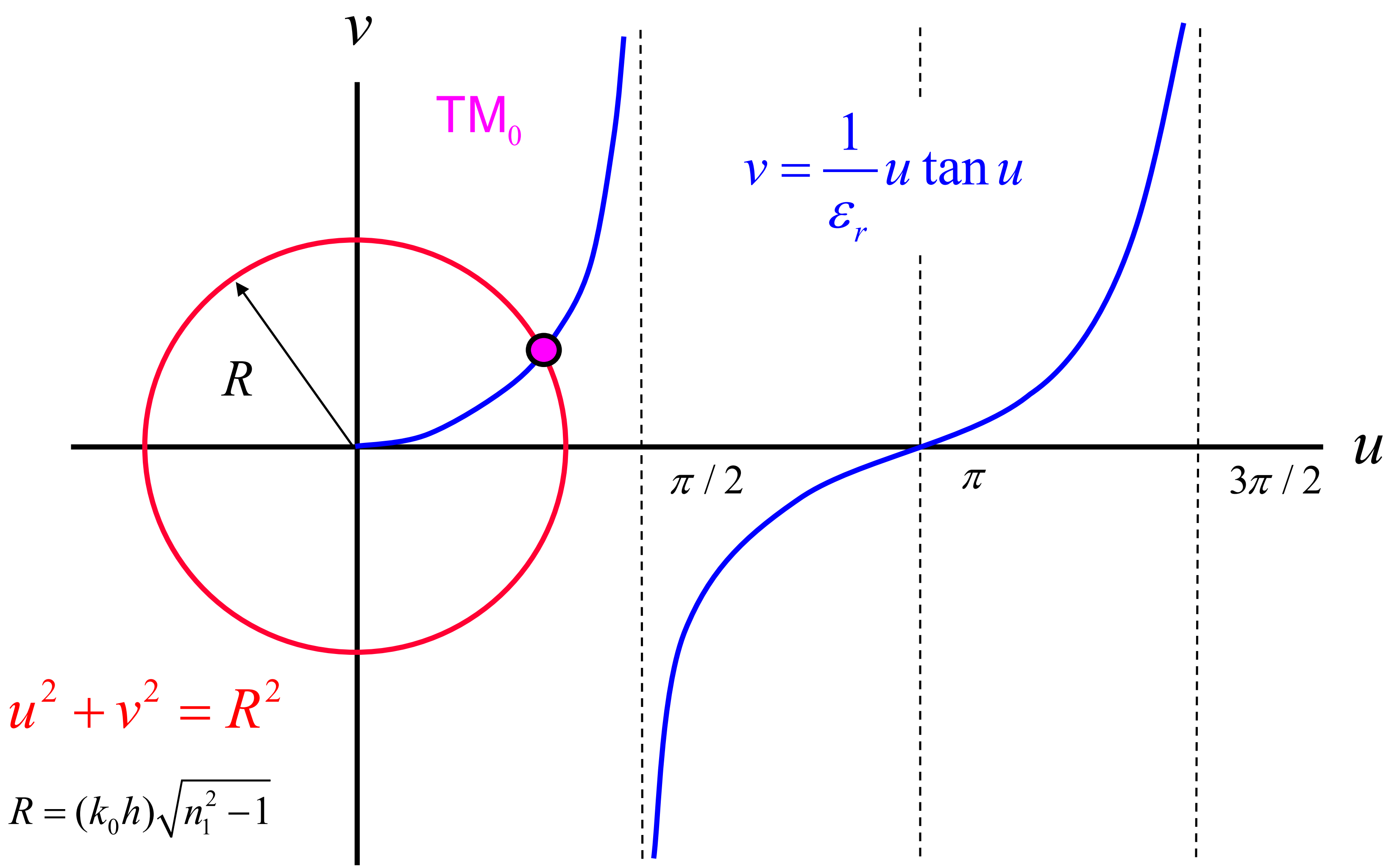
$$u^2 + v^2 = R^2$$

$$u \equiv k_{x1} h = h\sqrt{k_1^2 - k_z^2}$$

$$v \equiv \alpha_{x0} h = h\sqrt{k_z^2 - k_0^2}$$

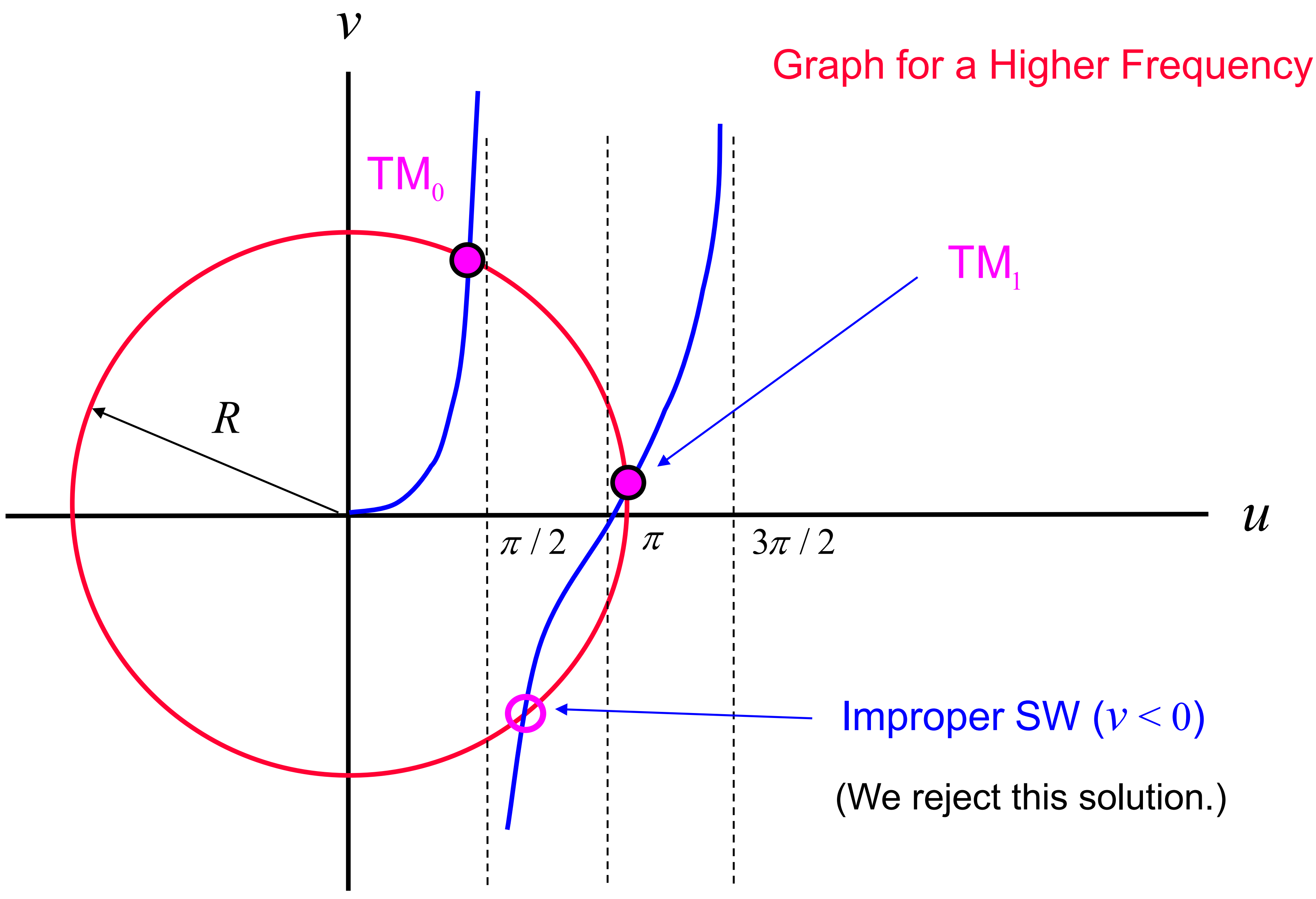
$$R = (k_0 h)\sqrt{n_1^{\,2} - 1}$$

Graphical Solution (cont.)



Note: The TM$_0$ mode exists at all frequencies (no cutoff frequency).

Graphical Solution (cont.)



Proper vs. Improper

$$v = \alpha_{x0} h$$

If $v > 0$: "proper SW" (fields decrease in x direction)

If $v < 0$: "improper SW" (fields increase in x direction)

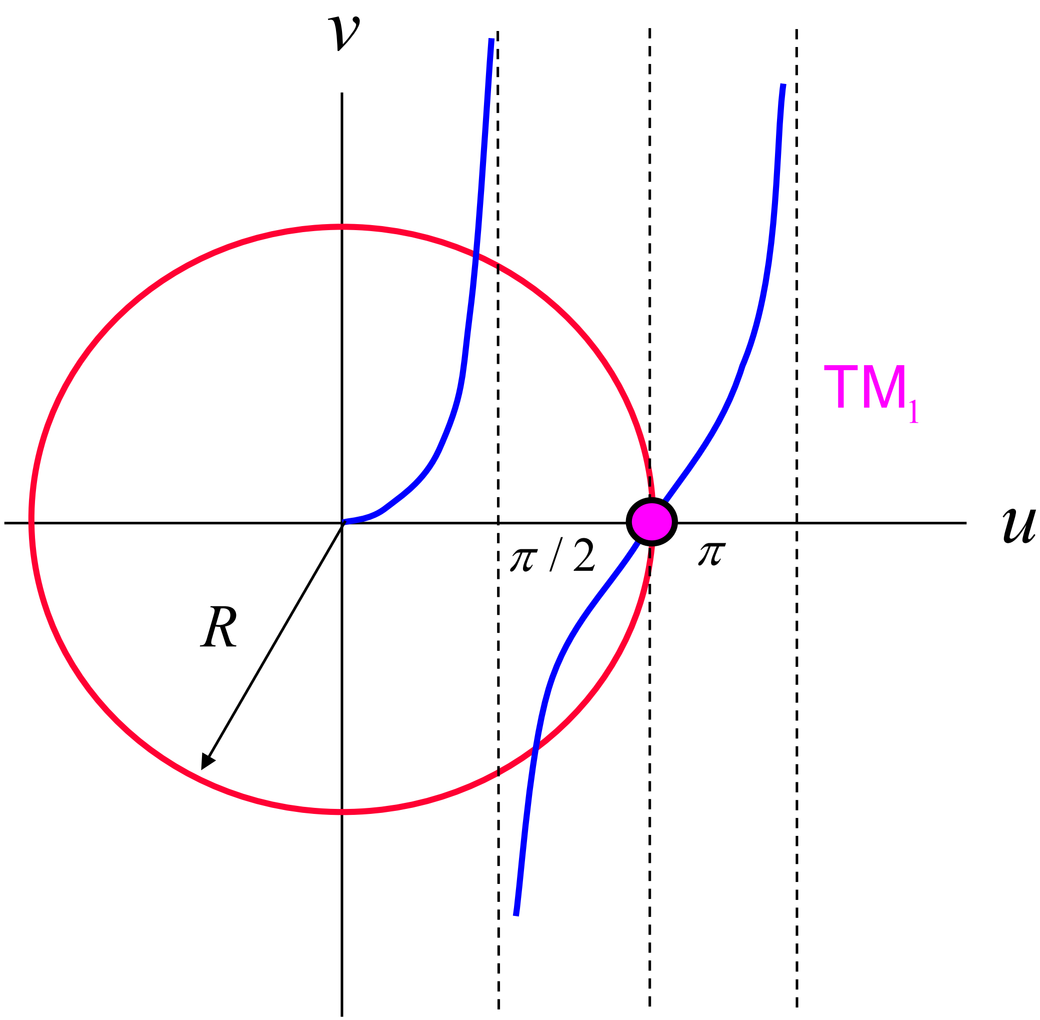
Cutoff frequency: The transition between a proper and improper mode.

Note:
The definition of cutoff frequency for this type of structure (an open structure) is different from that for a closed waveguide structure (e.g., rectangular waveguide) (where $k_z = 0$ at the cutoff frequency).

Cutoff frequency: TM_1 mode:

$$\boxed{\begin{array}{l} v = 0 \\ u = \pi \end{array}} \Rightarrow R = \pi$$

TM_1 Cutoff Frequency



TM_1: $R = \pi$

$$k_0 h \sqrt{n_1^{\,2} - 1} = \pi$$

$$\Rightarrow \frac{h}{\lambda_0} = \frac{1/2}{\sqrt{n_1^2 - 1}}$$

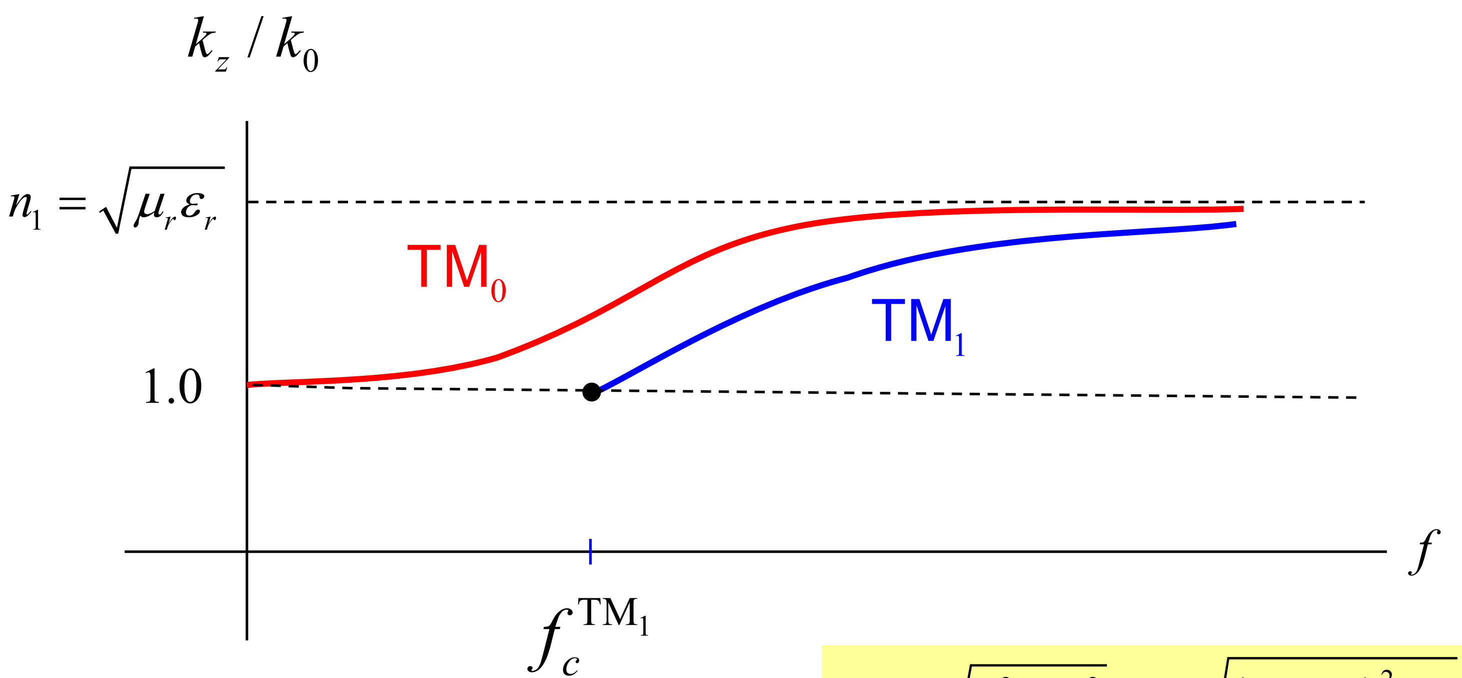
For other TM_n modes:

$$TM_n : \quad \frac{h}{\lambda_0} = \frac{n/2}{\sqrt{n_1^2 - 1}}$$

$$n = 0, 1, 2, \ldots$$

TM_0 Mode

The TM_0 mode has no cutoff frequency (it can propagate at any frequency):



$$\alpha_{x0} = \sqrt{k_z^2 - k_0^2} = k_0 \sqrt{\left(k_z / k_0\right)^2 - 1}$$

Note:
The lower the frequency, the slower the field decays away from the interface. As high frequency the wave decays very quickly since $v \to \infty$.

TM_0 Mode (cont.)

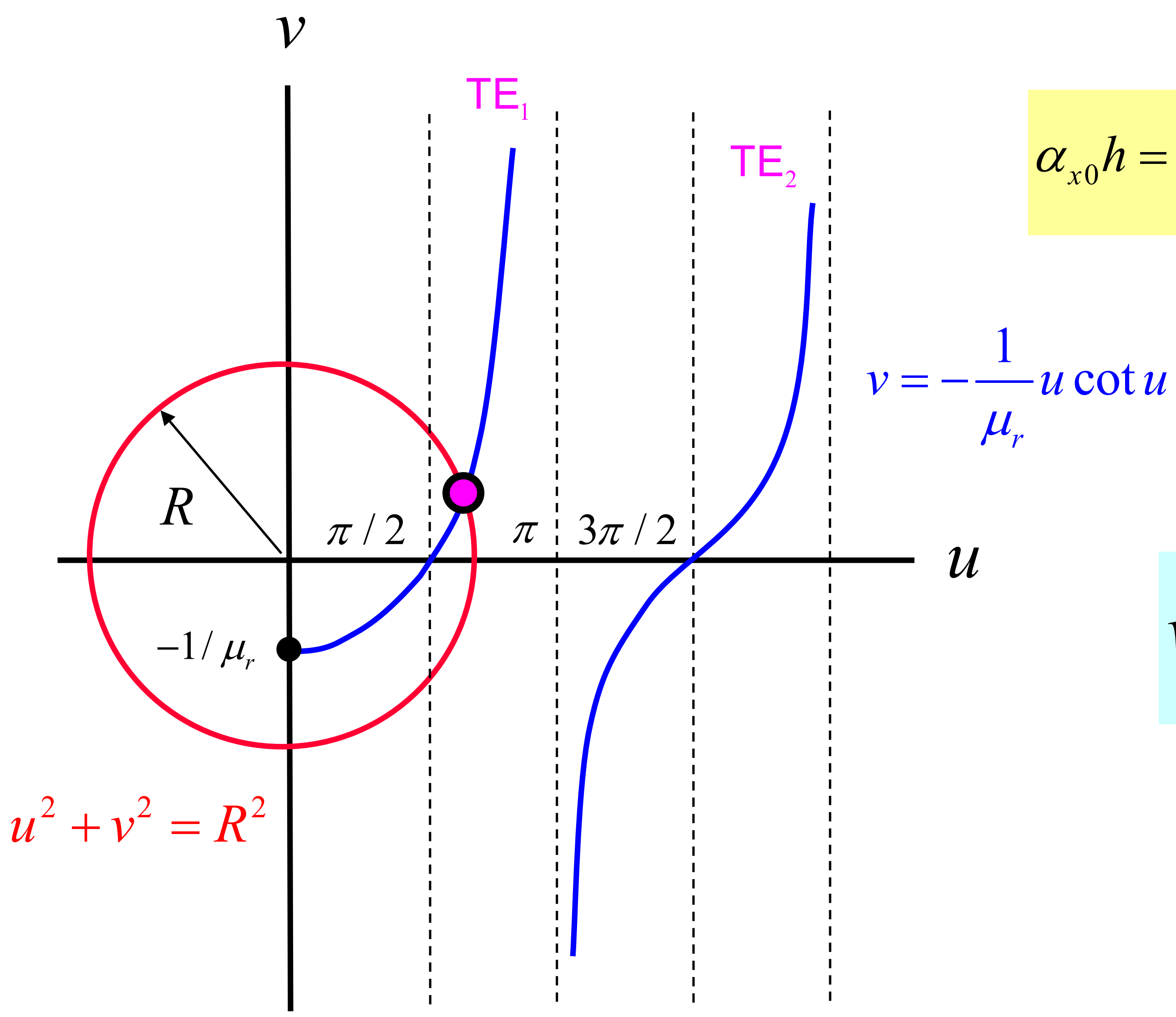
Approximate CAD Formula

After making some approximations to the transcendental equation, valid for low frequency, we have the following approximate result for the TM_0 mode (derivation omitted):

$$\beta_{TM_0} \approx k_0 \left[1 + \frac{(k_0 h)^2 (n_1^2 - 1)^2}{\varepsilon_r^2} \right]^{1/2}$$

$$k_0 h << 1$$

TE_x Modes



$$\alpha_{x0} h = -\frac{1}{\mu_r}\left(k_{x1} h\right)\cot\left(k_{x1} h\right)$$

Hence

$$v = -\frac{1}{\mu_r} u \cot u$$

$$u \equiv k_{x1} h$$
$$v \equiv \alpha_{x0} h$$

TE_x Modes (cont.)

No TE_0 mode ($f_c = 0$). The lowest TE_x mode is the TE_1 mode.

TE_1 cut-off frequency at $R = \pi / 2$:

$$(k_0 h)\sqrt{n_1^2 - 1} = \frac{\pi}{2}$$

$$\Rightarrow \quad \frac{h}{\lambda_0} = \frac{1/4}{\sqrt{n_1^2 - 1}}$$

The TE_1 mode will start to propagate when the substrate thickness is roughly 1/4 of a dielectric wavelength.

In general, we have

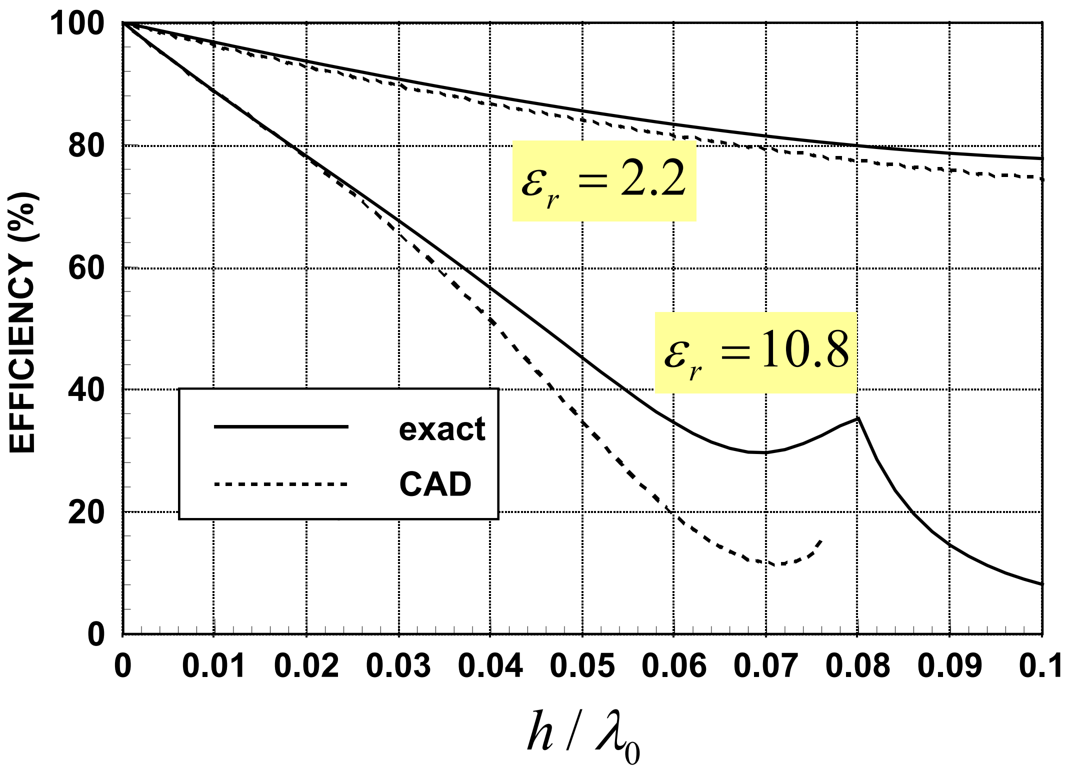
TE_n:

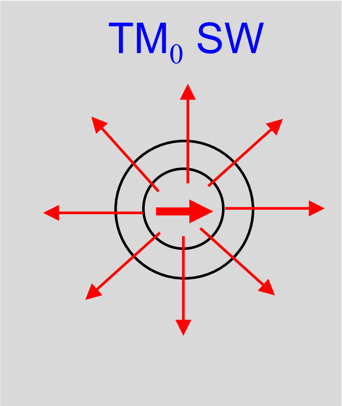
$$\frac{h}{\lambda_0} = \frac{(2n-1)/4}{\sqrt{n_1^{\,2} - 1}}$$

$$n = 1, 2, 3, \ldots\ldots\ldots$$

TE$_x$ Modes (cont.)

Here we examine the radiation efficiency e_r of a small electric dipole placed on top of the substrate (which could model a microstrip antenna, or a bend on a microstrip line).

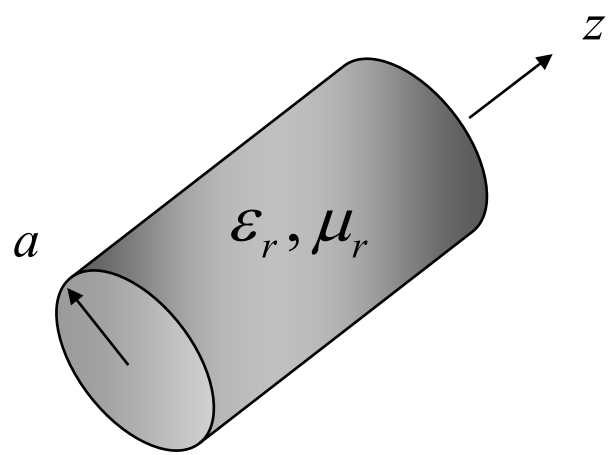




$$e_r \equiv \frac{P_{sp}}{P_{sp} + P_{sw}}$$

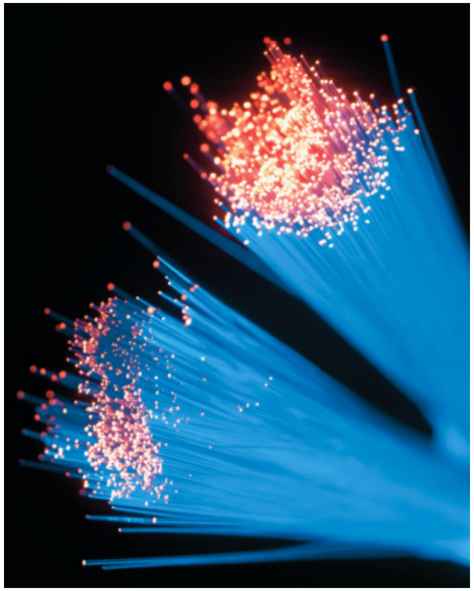
P_{sp} = power radiated into space $\quad$ P_{sw} = power launched into surface wave

Dielectric Rod



The physics is similar to that of the TM_0 surface wave on a grounded substrate.

This serves as a model for a <u>single-mode</u> fiber-optic cable.



Fiber-Optic Guide

Two types of fiber-optic guides:

1) Single-mode fiber

This fiber carries a single mode (HE_{11}). This requires the fiber diameter to be on the order of a wavelength. It has less loss, dispersion, and signal distortion than multimode fiber. It is often used for long-distances (e.g., greater than 1 km).

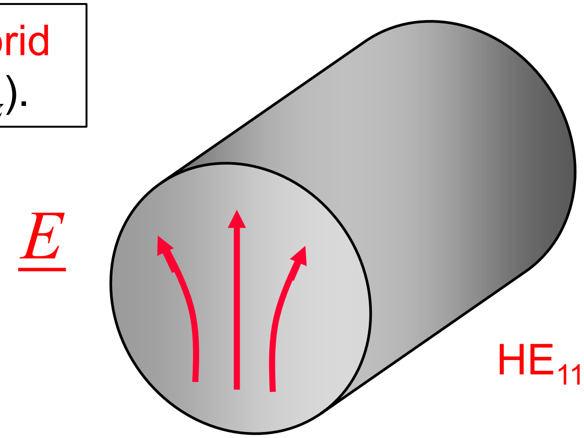
2) Multi-mode fiber

This fiber has a diameter that is large relative to a wavelength (e.g., 10 wavelengths). It operates on the principle of total internal reflection (critical-angle effect). It can handle more power than the single-mode fiber, and is less expensive, it but has more loss and dispersion.

Dielectric Rod: Single Mode Fiber

Dominant mode (lowest cutoff frequency): $\mathrm{HE}_{11} \quad (f_c = 0)$

The dominant mode is a hybrid mode (it has both E_z and H_z).



$$\alpha_{\rho 0} = \sqrt{k_z^2 - k_0^2}$$

HE_{11} mode on single-mode fiber

Note:
The notation HE means that the mode is hybrid, and has both E_z and H_z, although H_z is stronger. (For an EH mode, E_z would be stronger.)

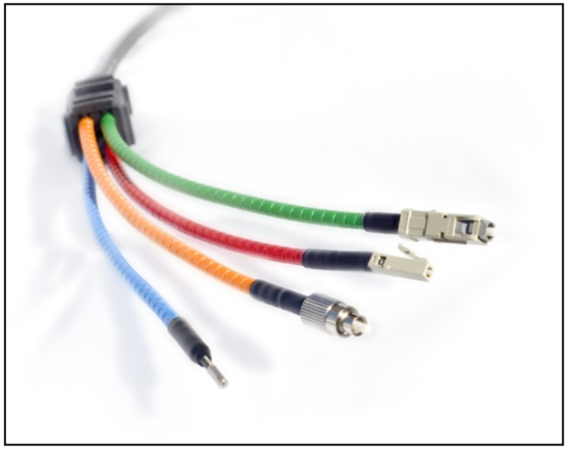
The field shape is somewhat similar to the TE_{11} circular waveguide mode.

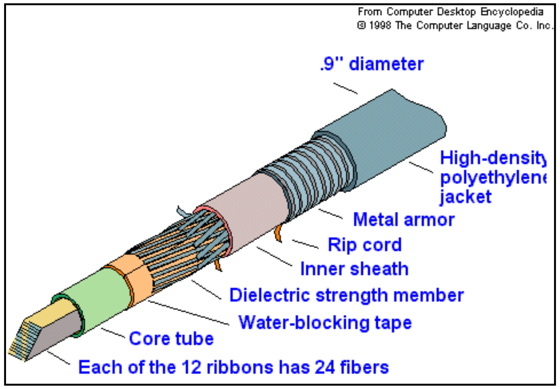
The physical properties of the fields are similar to those of the TM_0 surface wave on a slab. (For example, at low frequency the field is more loosely bound to the rod.)

Single Mode Fiber (cont.)

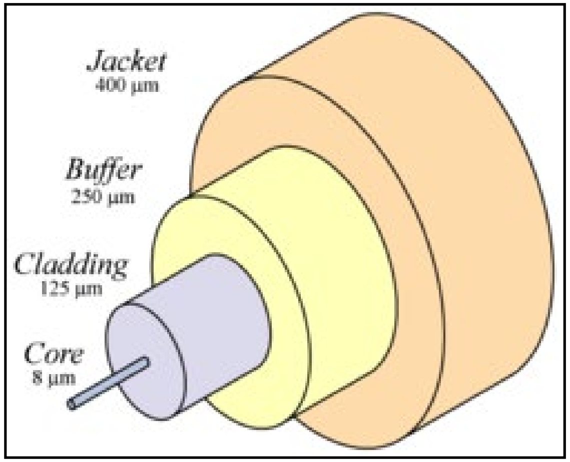
http://en.wikipedia.org/wiki/Optical_fiber

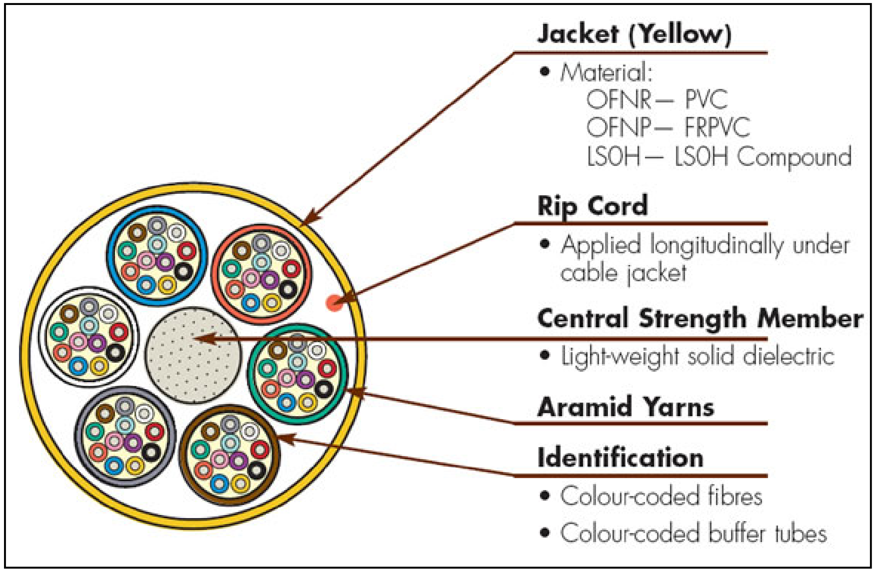
What they look like in practice:



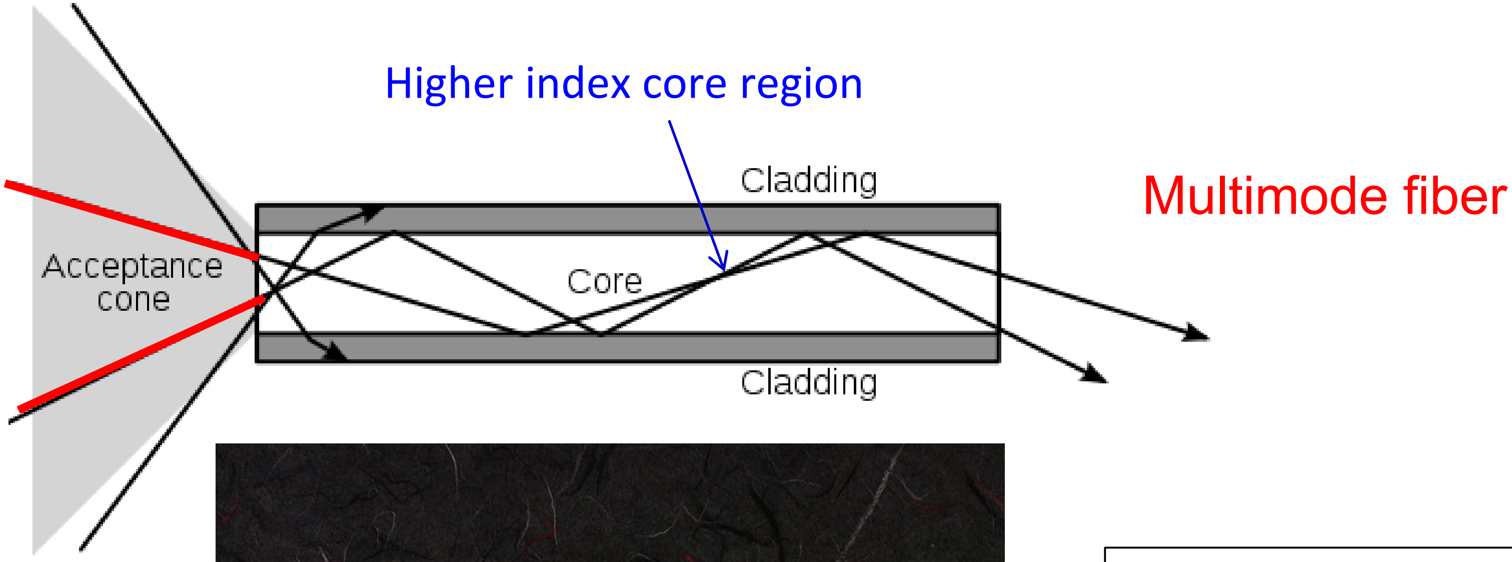


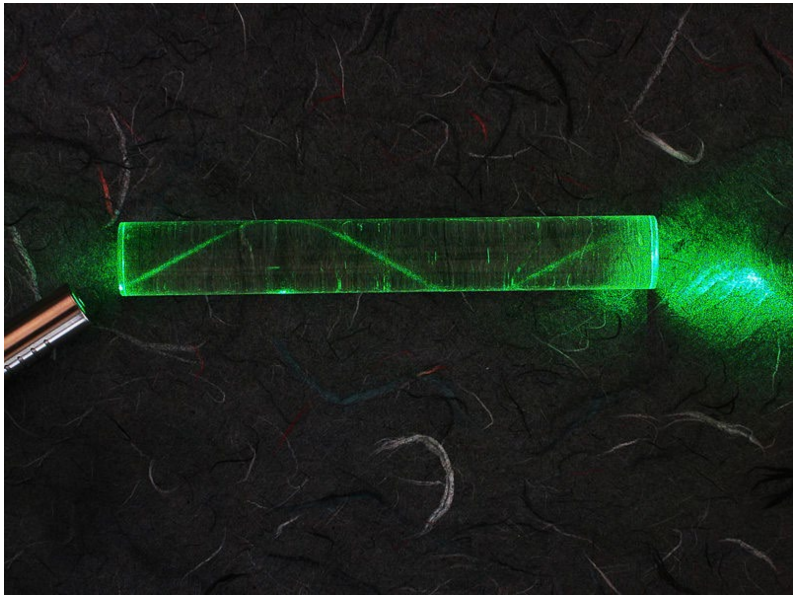
Single-mode fiber





Multimode Fiber





A laser bouncing down an acrylic rod, illustrating the total internal reflection of light in a multi-mode optical fiber

A multimode fiber can be explained using geometrical optics and internal reflection.

The "ray" of light is actually a superposition of many waveguide modes (hence the name "multimode").

http://en.wikipedia.org/wiki/Optical_fiber