Notes 17
S-Parameter Measurements
S-Parameter Measurements

S-parameters are typically measured, at microwave frequencies, with a network analyzer (NA).

These instruments have found wide, almost universal, application since the mid to late 1970’s.

- Vector* network analyzer: Magnitudes and phases of the $S$ parameters are measured.
- Scalar network analyzer: Only the magnitudes of the $S$-parameters are measured.

Most NA’s measure 2-port parameters. Some measure 4 and 6 ports.

* The $S$ parameters are really complex numbers, not vectors, but this is the customary name. There is an analogy between complex numbers and 2D vectors.
A Vector Network Analyzer (VNA) is usually used to measure $S$ parameters.

Note:
If there are more than 2 ports, we measure different pairs of ports separately with a 2-port VNA.
$S$-Parameter Measurements (cont.)

Vector Network Analyzer

Port 1 \hspace{1cm} Measurement plane 1

$a_1 \downarrow \hspace{1cm} b_1 \uparrow$

Device under test (DUT)

Measurement plane 2

$a_2 \downarrow \hspace{1cm} b_2 \uparrow$

Port 2

Test cables
Error boxes contain effects of test cables, connectors, couplers,…

We want to measure $[S]$ for DUT
$S$-Parameter Measurements (cont.)

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}^{\text{MEAS}} = \begin{bmatrix}
A & B \\
C & D
\end{bmatrix}^A \begin{bmatrix}
A & B \\
C & D
\end{bmatrix} \begin{bmatrix}
A & B \\
C & D
\end{bmatrix}^B
\]

Embedded inside measured ABCD matrix

De-embedded

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix} = \left(\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}^A \right)^{-1} \begin{bmatrix}
A & B \\
C & D
\end{bmatrix}^{\text{MEAS}} \left(\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}^B \right)^{-1}
\]
Assume error boxes are reciprocal (symmetric matrices)

We need to "calibrate" to find $[S^A]$ and $[S^B]$.

If $[S^A]$ and $[S^B]$ are known $\Rightarrow$ we can extract $[S]$ from measurements.

This is called “de-embedding”.

$S$-Parameter Measurements (cont.)
Calibration

“Short, open, match” calibration procedure

Error box $\alpha$

Connect

SC

$\Gamma_L = -1$

Short

OC

$\Gamma_L = 1$

Open

Match

$\Gamma_L = 0$

Calibration loads

These loads are connected to the end of the cable from the VNA.

\[ S_{11_{SC}}^m = S_{11}^\alpha - \frac{(S_{21}^\alpha)^2}{1 + S_{22}^\alpha} \]

\[ S_{11_{OC}}^m = S_{11}^\alpha + \frac{(S_{21}^\alpha)^2}{1 - S_{22}^\alpha} \]

\[ S_{11_{match}}^m = S_{11}^\alpha \]

3 measurements:

\[ (S_{11_{SC}}^m, S_{11_{OC}}^m, S_{11_{match}}^m) \]

3 unknowns:

\[ (S_{11}^\alpha, S_{21}^\alpha, S_{22}^\alpha) \]

Recall from Notes 16:

\[ \Gamma_{in} = S_{11} + \frac{S_{21} S_{12} \Gamma_L}{1 - \Gamma_L S_{22}} \]
“Thru-Reflect-Line (TRL)” calibration procedure

This is an improved calibration method that involves three types of connections:

1) The “thru” connection, in which port 1 is directly connected to port 2.

2) The “reflect” connection, in which a load with an (ideally) large (but not necessarily precisely known) reflection coefficient is connected.

3) The “line” connection, in which a length of matched transmission line (with an unknown length) is connected between ports 1 and 2.

The advantage of the TRL calibration is that it does not require precise short, open, and matched loads.

This method is discussed in the Pozar book (pp. 193-196).
In microwave engineering, discontinuities are often represented by pi or tee networks.

Sometimes the pi or tee network reduces to a single series or shunt element.

For waveguide systems, the TEN is used to represent the waveguide.
Discontinuities: Rectangular Waveguide

- Inductive iris or strip
- Capacitive iris or strip
- Resonant iris
Discontinuities: RWG (cont.)

E plane step

\[ Z_{01} \quad \Rightarrow \quad Z_{02} \]

H plane step

\[ Z_{01} \quad \Rightarrow \quad Z_{02} \]
Discontinuities: Microstrip

Note:
For a good equivalent circuit, the element values are fairly stable over a wide range of frequencies.
Z-Parameter Extraction

Assume a reciprocal and symmetrical waveguide or transmission-line discontinuity.

Examples

We want to find $Z_1$ and $Z_2$ to model the discontinuity.

Note:
We could also use a pi network if we wish.
The $Z_2$ element is split in two:
Z-Parameter Extraction (cont.)

Assume that we place a short or an open along the plane of symmetry.

\[ Z_1 = Z_L^{\text{SC}} , \quad Z_2 = \frac{1}{2} (Z_L^{\text{OC}} - Z_L^{\text{SC}}) \]
The short or open can be realized by using odd-mode or even-mode excitation.

Even/odd-mode analysis is very useful in analyzing devices (e.g., using HFSS).
**Z-Parameter Extraction (cont.)**

**Odd mode voltage waves**

\[
Z_L^{SC} = Z_0 \left( \frac{1 + S_{11}^{SC}}{1 - S_{11}^{SC}} \right)
\]

**Even mode voltage waves**

\[
Z_L^{OC} = Z_0 \left( \frac{1 + S_{11}^{OC}}{1 - S_{11}^{OC}} \right)
\]
Hence we have:

\[ Z_1 = Z_0 \left( \frac{1 + S_{11}^{SC}}{1 - S_{11}^{SC}} \right) \]

\[ Z_2 = \frac{1}{2} \left( Z_0 \left( \frac{1 + S_{11}^{OC}}{1 - S_{11}^{OC}} \right) - Z_0 \left( \frac{1 + S_{11}^{SC}}{1 - S_{11}^{SC}} \right) \right) \]
De-embedding of a Line Length

We wish to know the reflection coefficient of a 1-port device under test (DUT), but the DUT is not assessable directly – it has an extra length of transmission line connected to it (whose length may not be known).

\[ S_{11}^{MEAS} = S_{11}^{DUT} e^{-j2\beta L} \]

Recall: \[ S'_{ij} = S_{ij} e^{-\gamma_i l_i} e^{-\gamma_j l_j} \]

\[ l_i = l_j = L \]
\[ \gamma_i = \gamma_j = \gamma = j\beta \]

Replace DUT with short circuit \( S_{11}^{DUT} \rightarrow -1 \) \[ \Rightarrow S_{11}^{MEAS, SC} = -e^{-j2\beta L} = -1 / e^{+j2\beta L} \]

\[ S_{11}^{DUT} = S_{11}^{MEAS,DUT} e^{+j2\beta L} \]

\[ S_{11} = S_{11}^{MEAS,DUT} \left( \frac{-1}{S_{11}^{MEAS,SC}} \right) \]