Adapted from notes by
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ECE 5317-6351
Microwave Engineering

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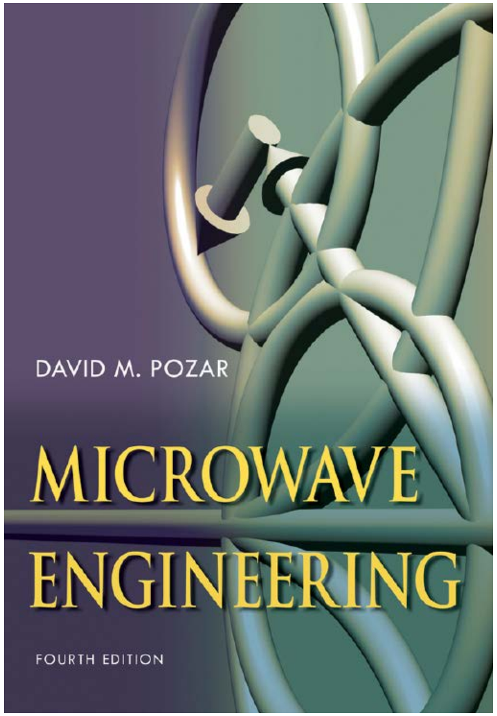
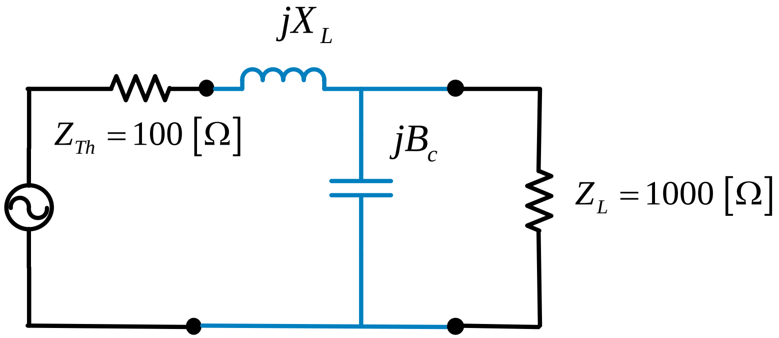
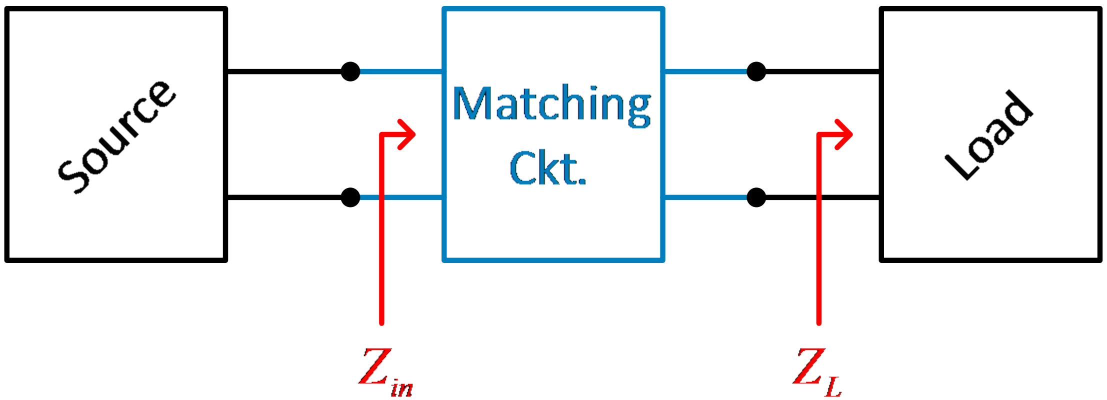
Dept. of ECE



Notes 18
Impedance Matching



Impedance Matching



Impedance matching is used to:

- Maximize power from source to load
- Minimize reflections

We have $Z_L = R_L + jX_L$

We want $Z_{in} = R_{in} + jX_{in}$

↑ (usually zero)

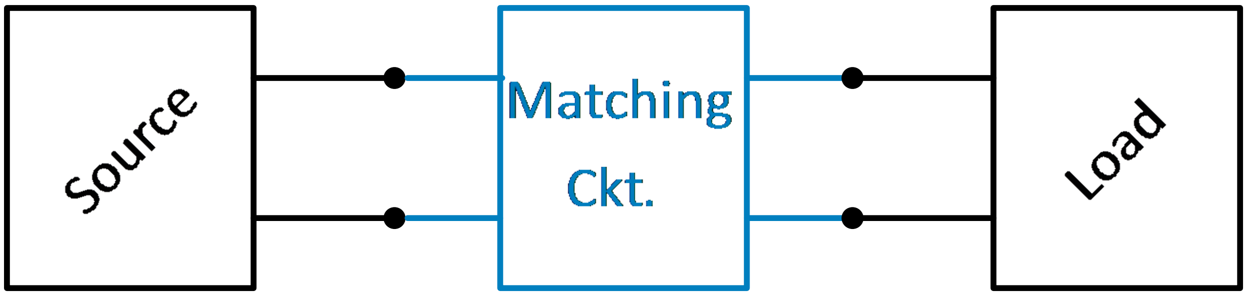
⇓ Two constraints

A matching circuit typically requires at least 2 degrees of freedom.

Considerations:

- Complexity
- Bandwidth
- Implementation
- Adjustability

Impedance Matching (cont.)



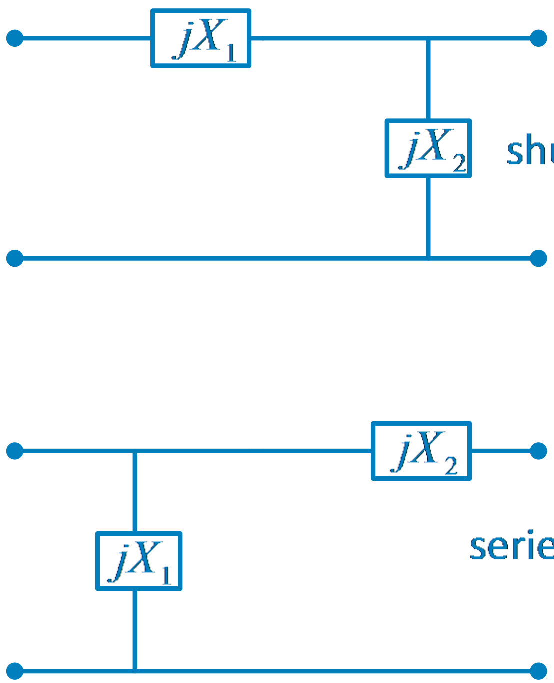
Matching Methods:

1) Lumped element matching circuits
2) Transmission line matching circuits
3) Quarter-wave impedance transformers

Lumped-Element Matching Circuits

Examples

"ell"



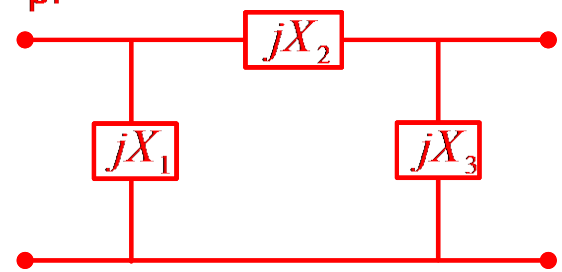
shunt-series

series-shunt

No flexibility

2 elements $\Rightarrow$ 2 degrees of freedom

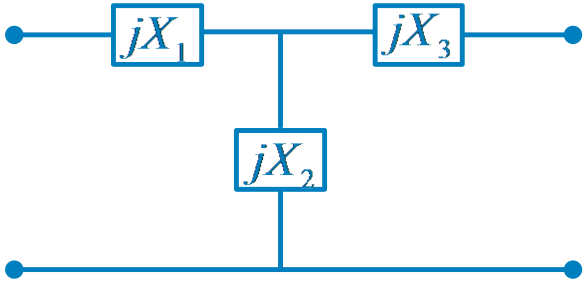
"pi"



One extra degree of freedom

Lumped-Element Matching Circuits (cont.)

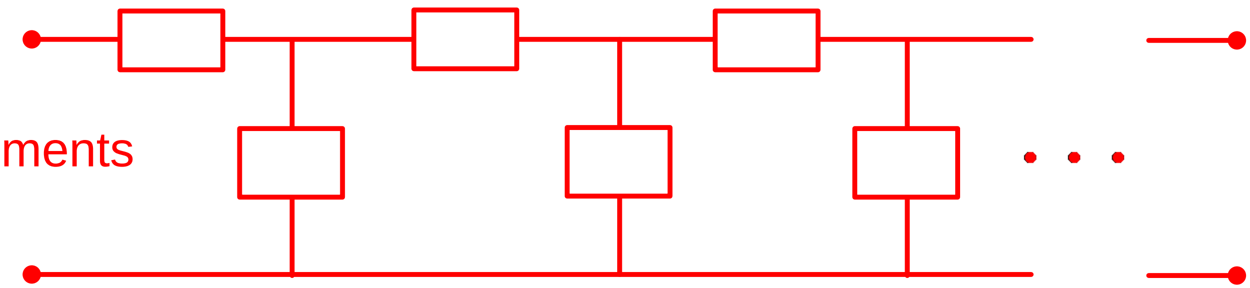
"tee"



One extra degree of freedom

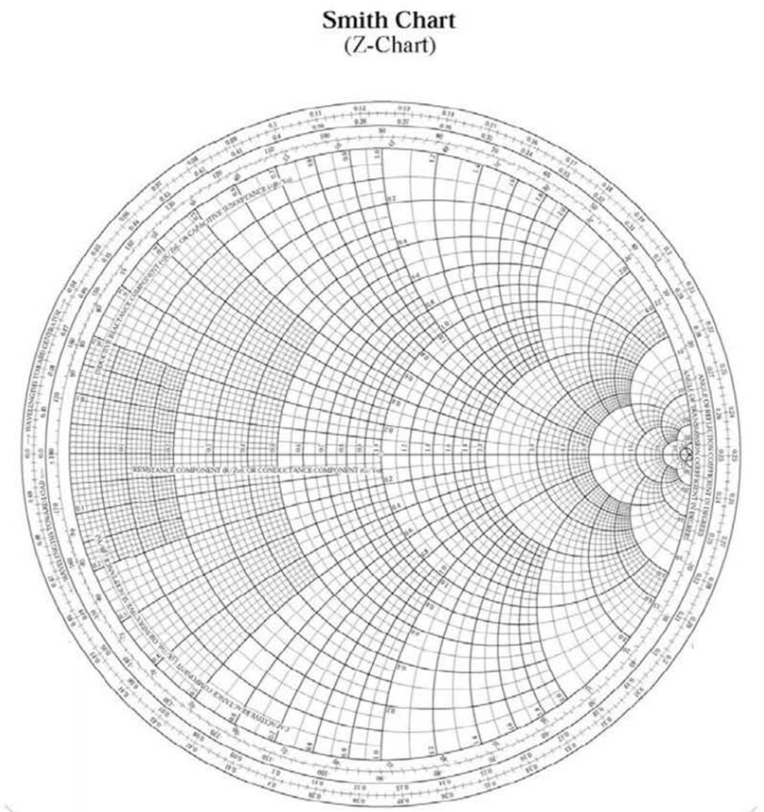
"ladder"

$m+2$ elements



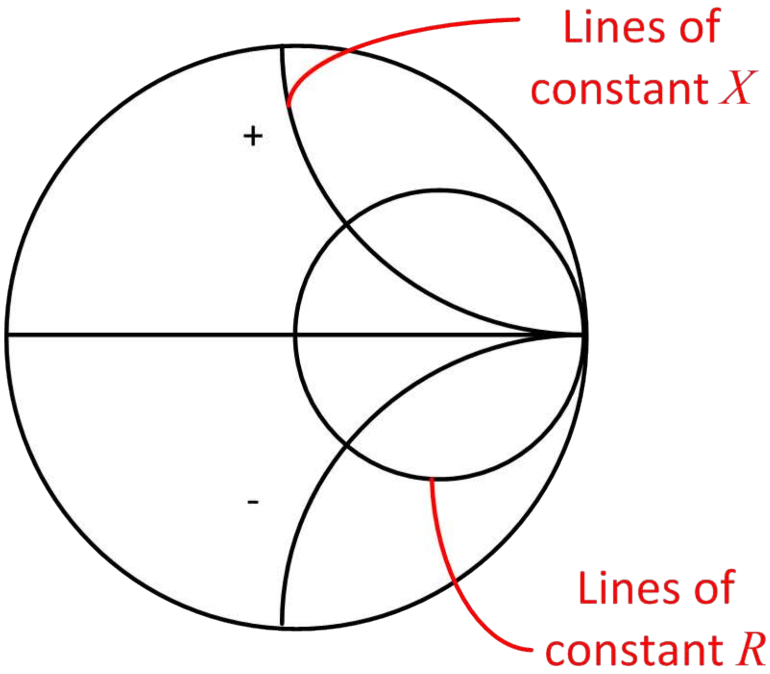
m extra degrees of freedom

Smith Chart Review

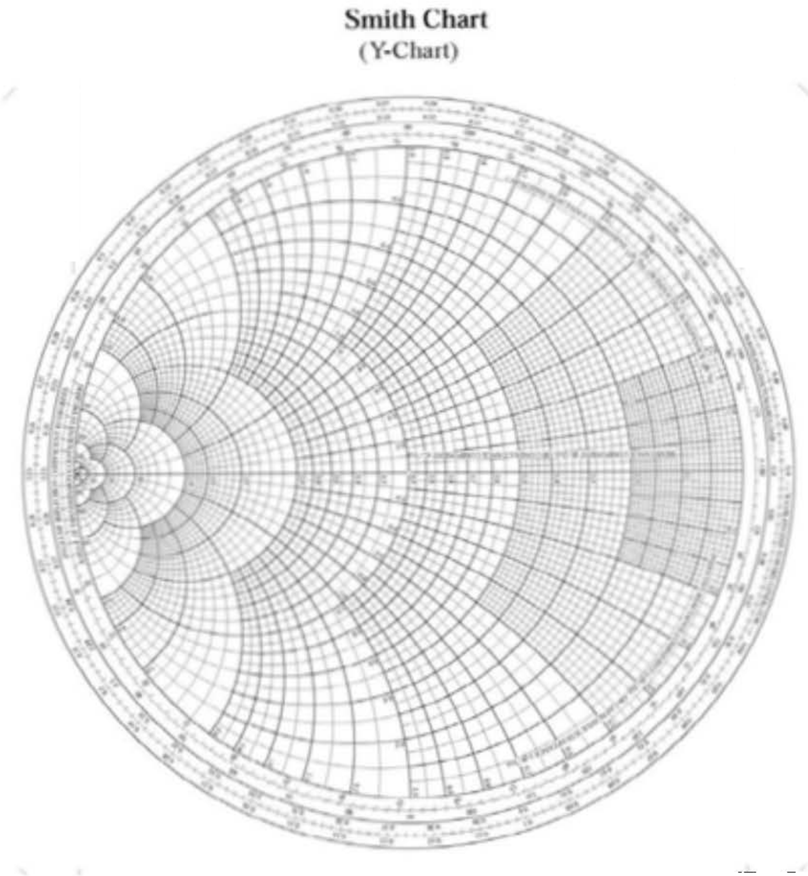


Γ plane

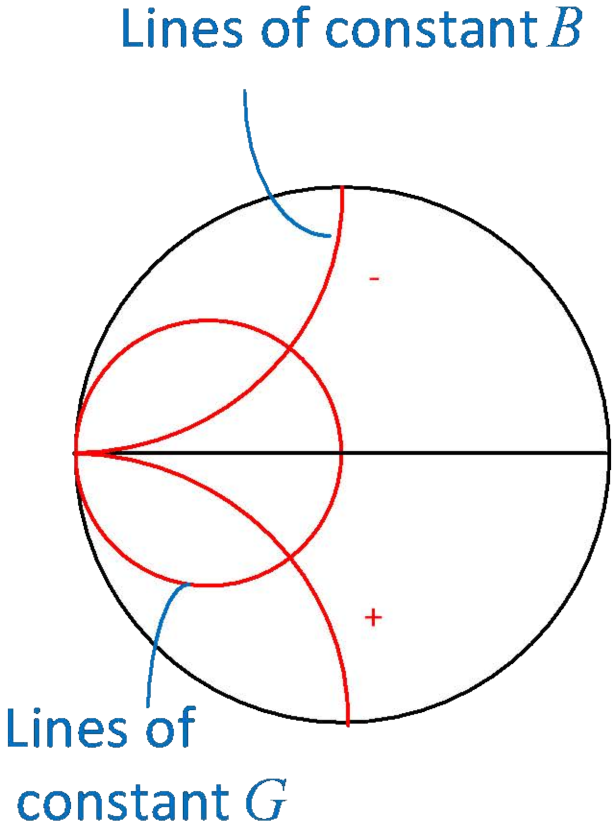
Short-hand version



Smith Charts Review (cont.)

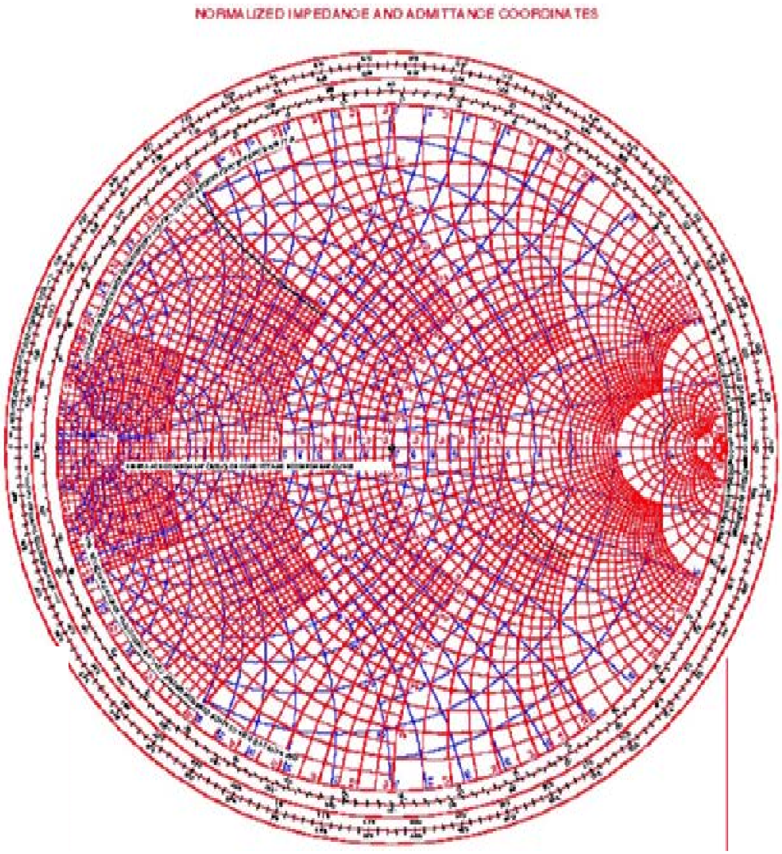


Γ plane

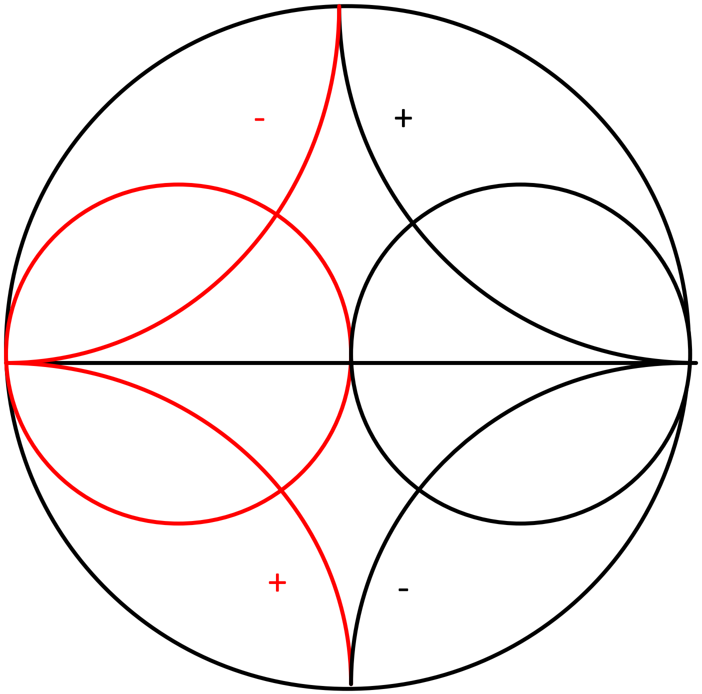


Smith Charts Review (cont.)

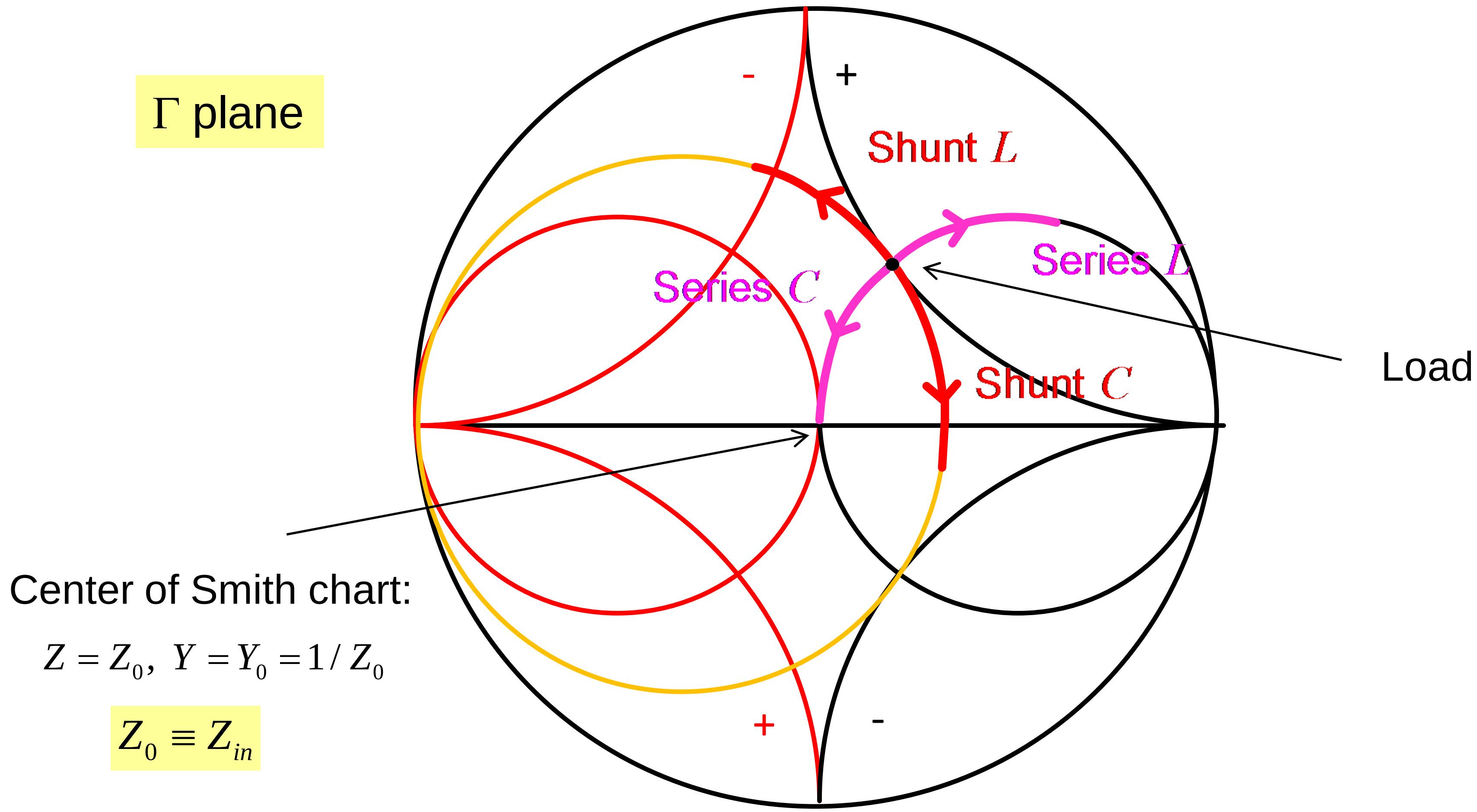
ZY- Chart



Γ plane



Series and Shunt Elements

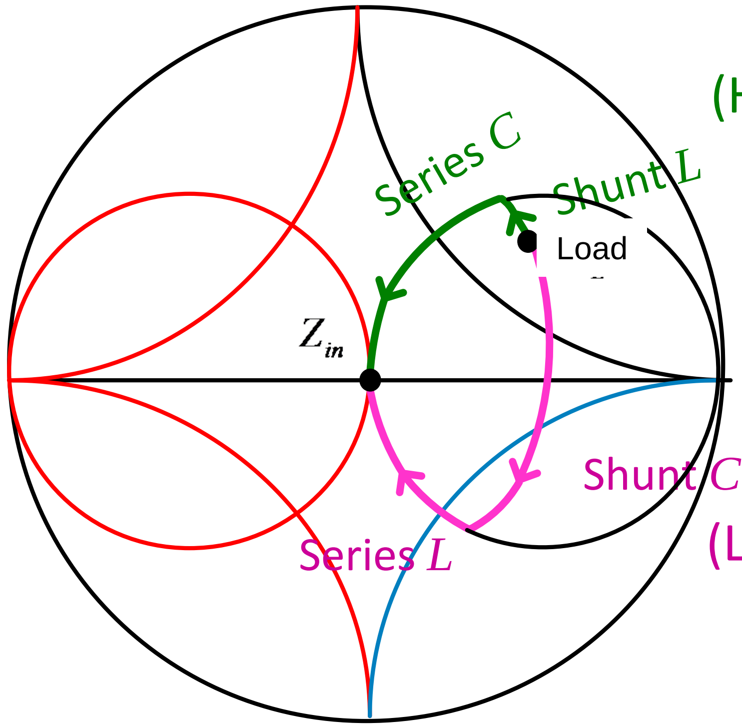


Note:
The Smith chart is not actually being used as a transmission-line calculator but an impedance/admittance calculator. Hence, the normalizing impedance Z_0 is arbitrary. (Usually we choose it to be the desired input resistance R_{in}.)

High Impedance to Low Impedance

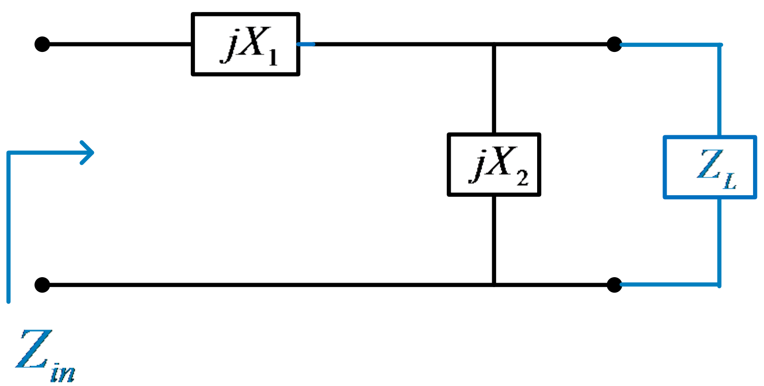
Use when $G_L < Y_{in}$

(The load is outside of the red $G = 1$ circle.)



$Z_0 \equiv Z_{in}$

Shunt-series "ell"



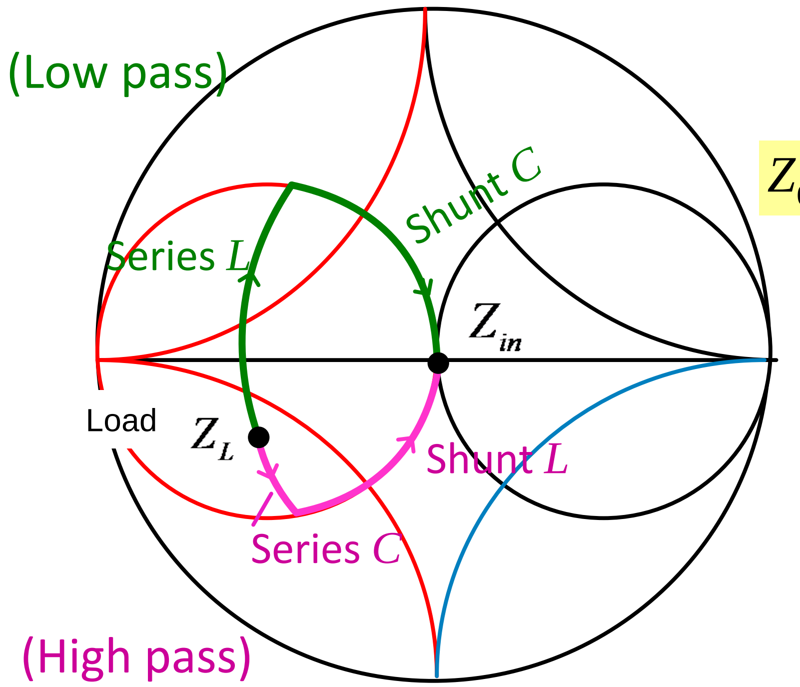
Two possibilities

The shunt element puts us on the $R = 1$ circle; the series element is used to "tune out" the unwanted reactance.

Low Impedance to High Impedance

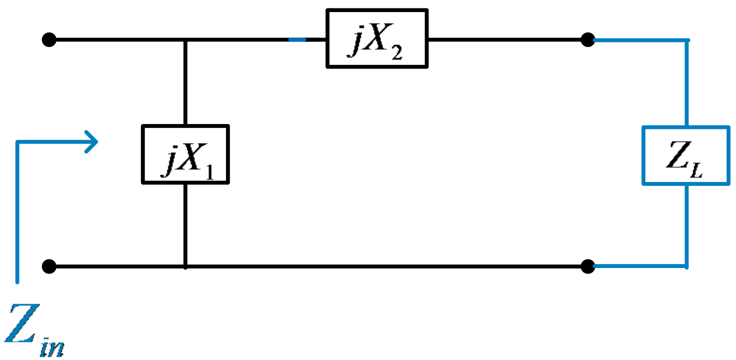
Use when $R_L < R_{in}$

(The load is outside of the black $R = 1$ circle.)



$Z_0 \equiv Z_{in}$

Series-shunt "ell"



Two possibilities

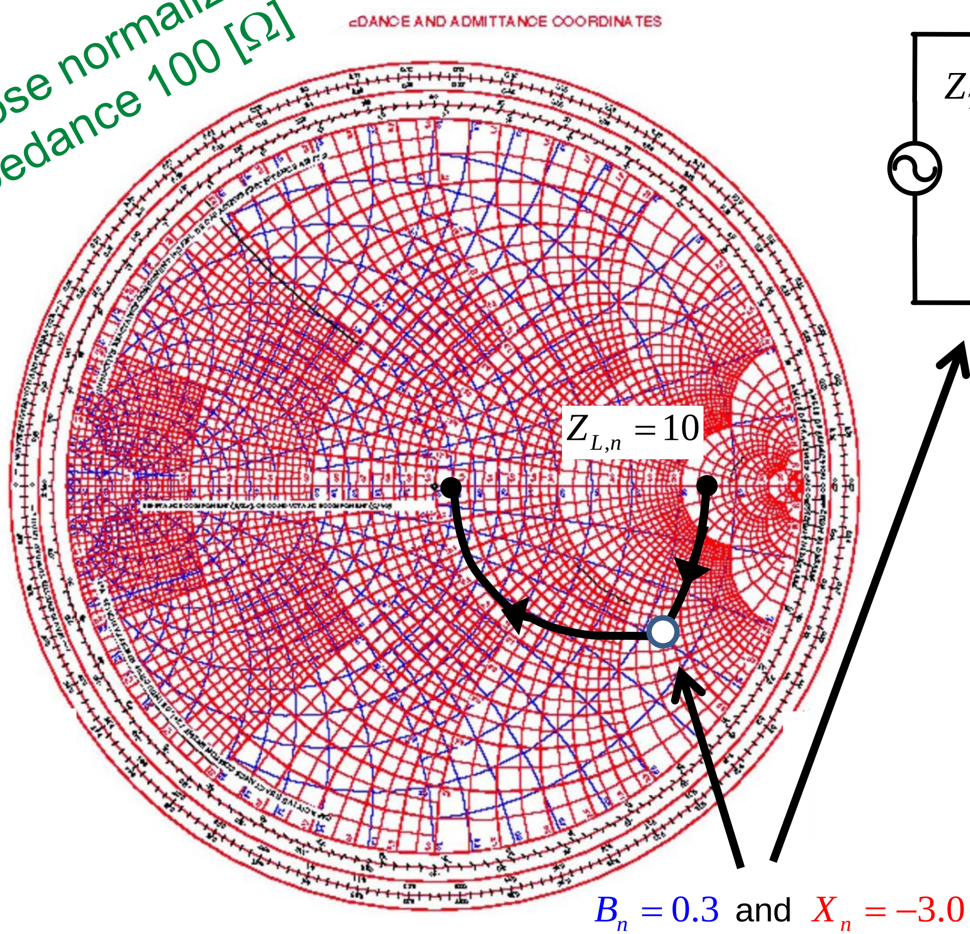
The series element puts us on the $G = 1$ circle; the shunt element is used to "tune out" the unwanted susceptance.

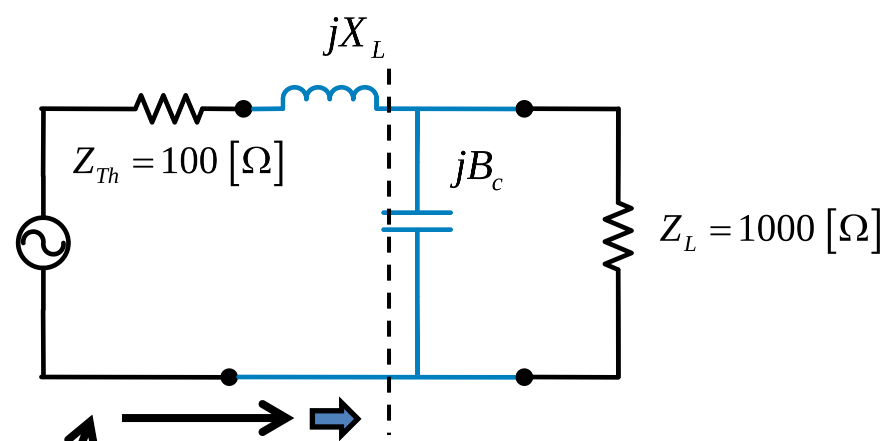
Example

Use high impedance to low impedance matching.

We want $Z_{in} = 100\ [\Omega]$

Choose normalizing impedance 100 [Ω]





$$B_{C,n} = 0.3 \Rightarrow B_C = \frac{0.3}{100[\Omega]} = 3\ [\text{mS}]$$

$$X_C = -1/B_c = -333\ [\Omega] \qquad \left(X_C = -\frac{1}{\omega C}\right)$$

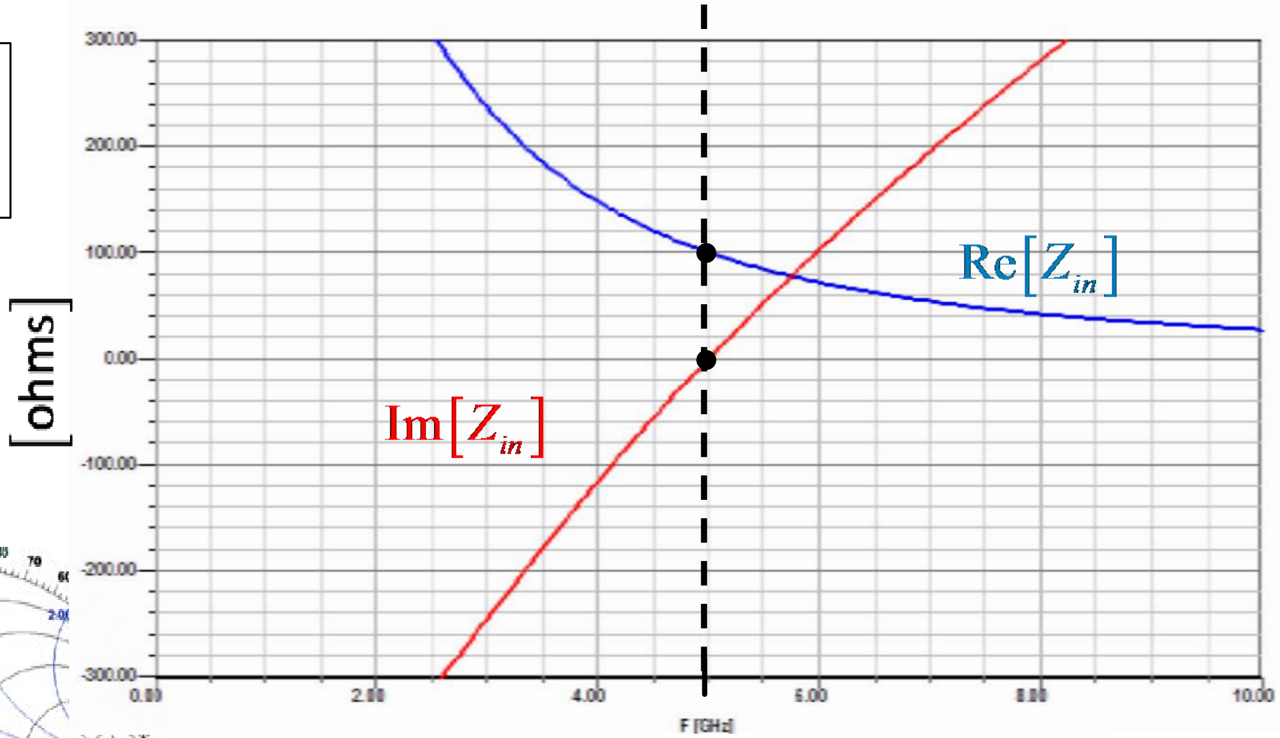
$$X_{L,n} = -X_n = 3 \Rightarrow X_L = 3(100[\Omega])$$

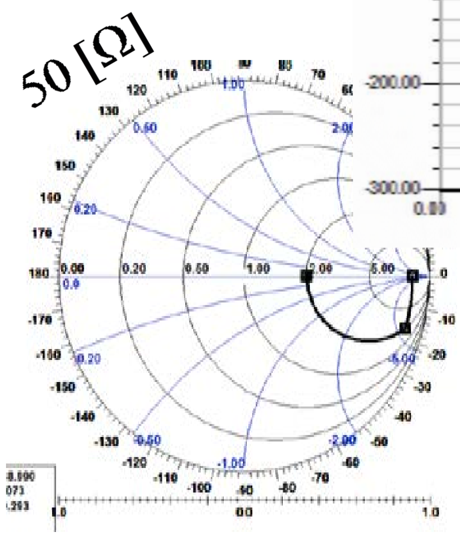
$$X_L = 300\ [\Omega] \qquad (X_L = \omega L)$$

Example (cont.)

Here, the design example was repeated using a 50 [Ω] normalizing impedance.

Note that the final normalized input impedance is 2.0.



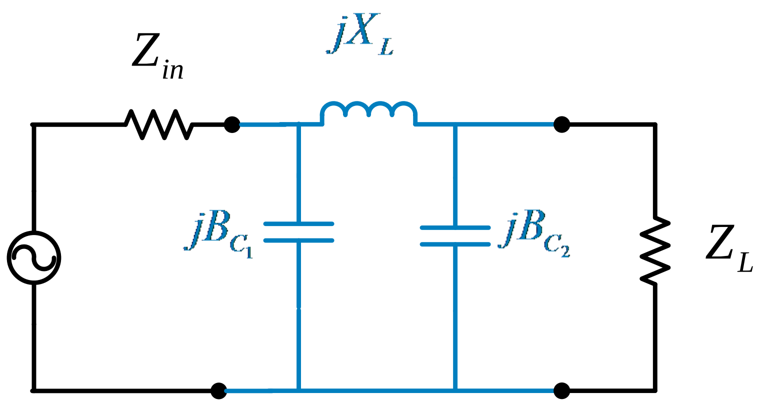


5 GHz design frequency

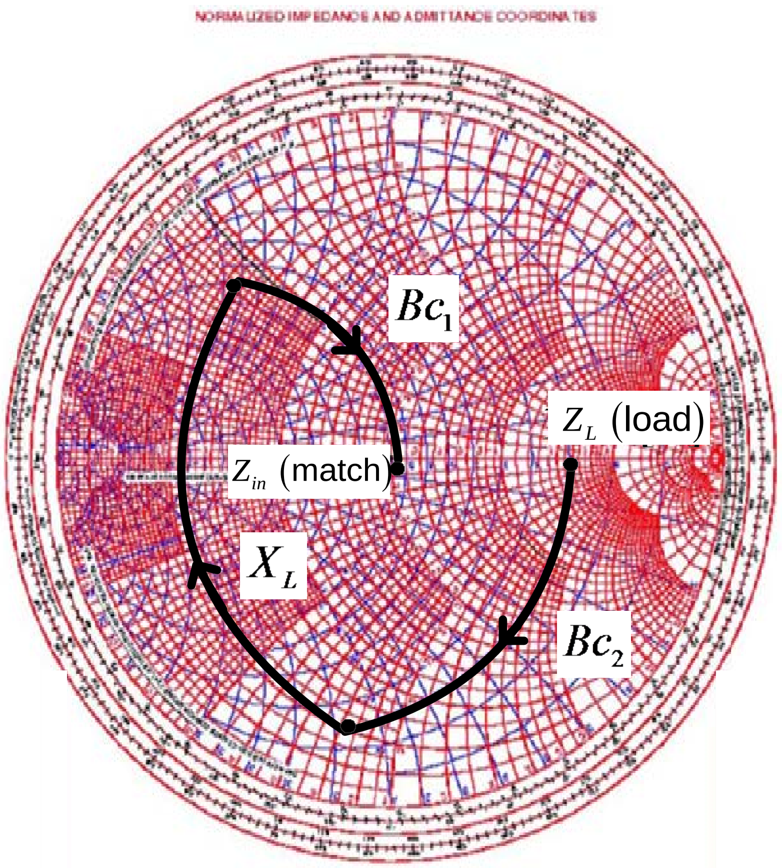
$C = 0.096$ [pF]

$L = 9.55$ [nH]

Matching with a Pi Network



This works for low-high or high-low.



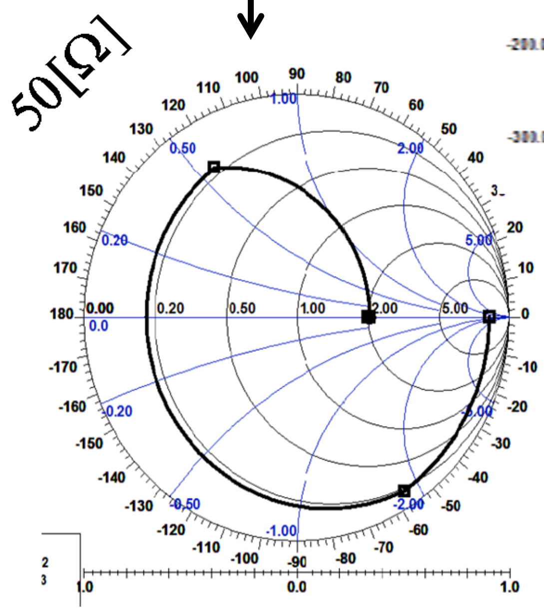
Note:
This solution is *not unique*. Different values for B_{c2} could have been chosen.

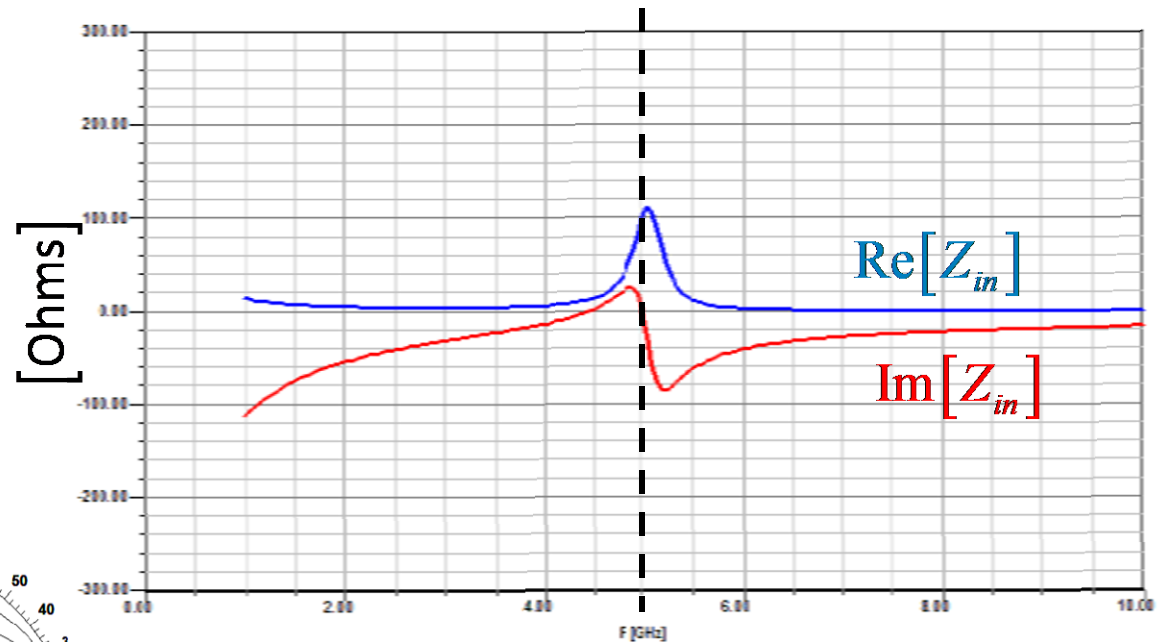
Note:
We could have also used parallel inductors and a series capacitor, or other combinations.

Pi Network Example

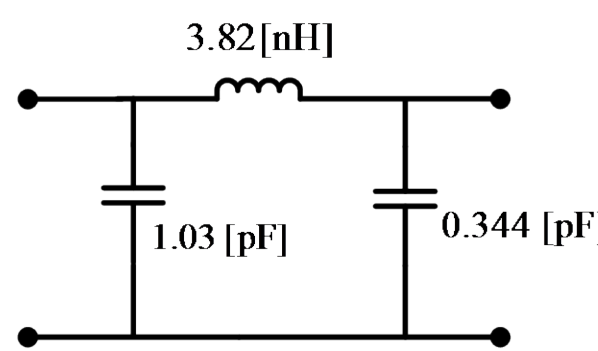
(1000 Ω →100 Ω)

Note that the final normalized input impedance is 2.0.





Design frequency = 5GHz



High Impedance to Low Impedance

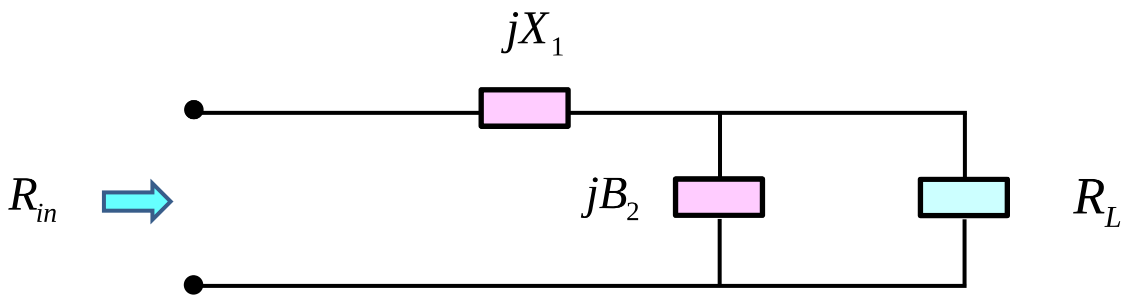
Exact Solution

$R_L > R_{in}$

$G_L = 1/R_L$

$B_2 = -1/X_2$

Shunt-series "ell"



$$\mathrm{Re}\left(\frac{1}{G_L + jB_2}\right) = R_{in}$$

$$\Downarrow$$

$$\frac{G_L}{G_L^2 + B_2^2} = R_{in}$$

$$\frac{G_L^2 + B_2^2}{G_L} = G_{in}$$

$$\Downarrow$$

$$G_L + \frac{B_2^2}{G_L} = G_{in}$$

$$B_2 = \pm\sqrt{G_L\left(G_{in} - G_L\right)}$$

Then we choose:

$$X_1 = -\mathrm{Im}\left(\frac{1}{G_L + jB_2}\right) = \frac{B_2}{G_L^2 + B_2^2}$$

High Impedance to Low Impedance

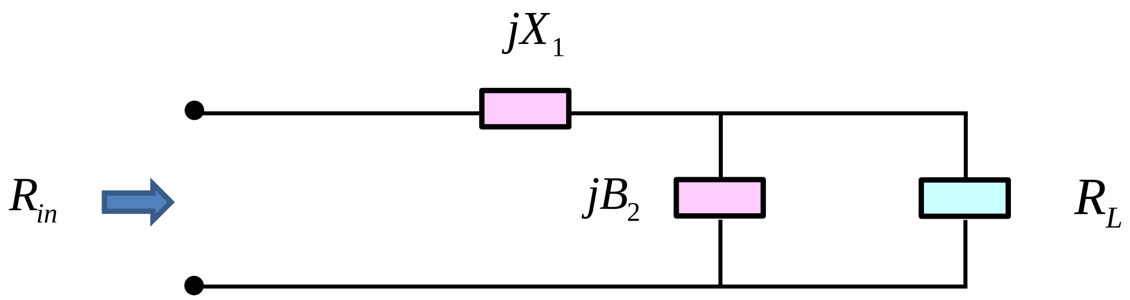
Exact Solution

$R_L > R_{in}$

Summary

$G_L = 1/R_L$

$B_2 = -1/X_2$



$$B_2 = \pm\sqrt{G_L\left(G_{in} - G_L\right)}$$

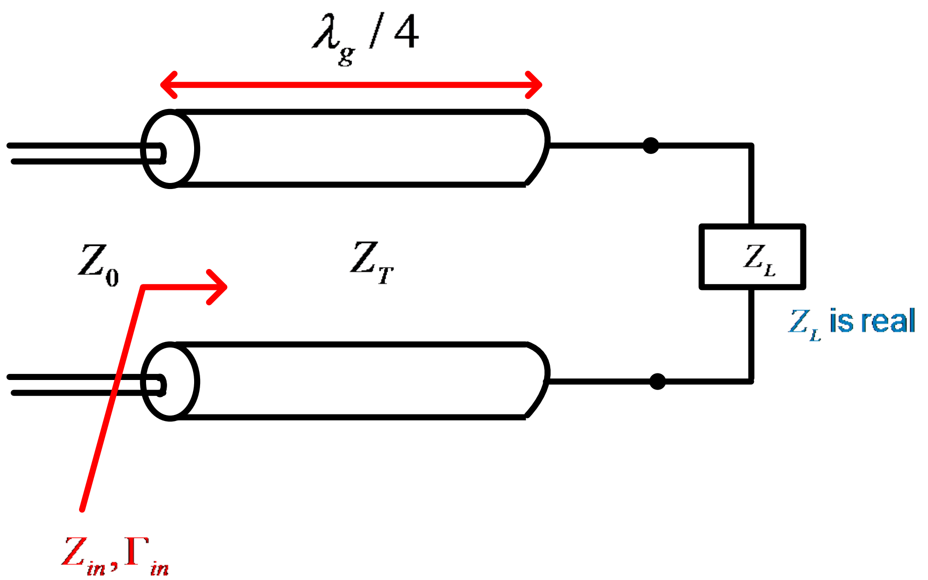
$$X_1 = \frac{B_2}{G_L^2 + B_2^2}$$

Transmission Line Matching

- Single-stub matching
  - This has already been discussed.

- Double-stub matching
  - This is an alternative matching method that gives more flexibility (details omitted – please see the Pozar book).

Quarter-Wave Transformer



$@\ f = f_0$

$$\beta \ell = \frac{2\pi}{\lambda_g}\frac{\lambda_g}{4} = \frac{\pi}{2}$$

$$\Rightarrow Z_{in} = \frac{Z_T^2}{Z_L}$$

$$\Gamma_{in} = 0 \quad \text{when} \quad Z_T = \sqrt{Z_0 Z_L}$$

Only true at f_0 where $\ell = \lambda_g / 4$

Note:
If Z_L is not real, we can always add a reactive load in series or parallel to make it real (or add a length of transmission line between the load and the transformer to get a real impedance).

Quarter-Wave Transformer (cont.)

At a general frequency:

$$Z_{in} = Z_T \left(\frac{Z_L + jZ_T \tan \beta_T \ell}{Z_T + jZ_L \tan \beta_T \ell} \right)$$

$$= Z_T \left(\frac{Z_L + jZ_T t}{Z_T + jZ_L t} \right) \qquad t \equiv \tan \beta_T \ell$$

After some algebra (omitted):

$$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{Z_L - Z_0}{Z_L + Z_0 + j2t\sqrt{Z_0 Z_L}}$$

where we have used $Z_T = \sqrt{Z_0 Z_L}$

Quarter-Wave Transformer (cont.)

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0 + j2t\sqrt{Z_0 Z_L}}$$

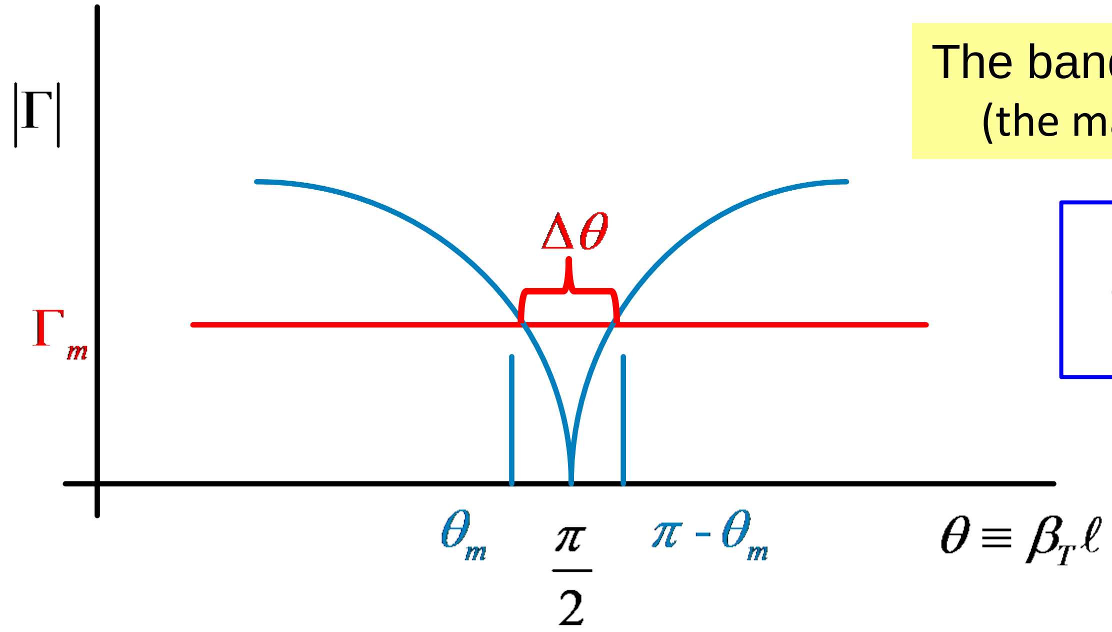
After some more algebra (omitted):

$$|\Gamma| = \frac{1}{\sqrt{1 + \left(\frac{4Z_0 Z_L}{(Z_L - Z_0)^2} \sec^2 \theta \right)}}$$

$$\theta \equiv \beta_T \ell$$

where we used $1 + t^2 = 1 + \tan^2 \beta_T l = \sec^2 \beta_T l = \sec^2 \theta$

Quarter-Wave Transformer (cont.)



The bandwidth is defined by the limit Γ_m (the maximum acceptable value of Γ).

For example, using $\Gamma_m = 1/3$ corresponds to $\text{SWR} = 2.0$. This corresponds to $\Gamma_m = -9.54\ \text{dB}$.

We set:

$$\Gamma_m = \frac{1}{\sqrt{1+\left[\frac{4Z_0Z_L}{(Z_L - Z_0)^2}\sec^2\theta_m\right]}}$$

Bandwidth region of transformer:

$$\Delta\theta = (\beta_{2T} - \beta_{1T})\,\ell = 2\left(\frac{\pi}{2} - \theta_m\right)$$

Solve for θ_m

Solving for θ_m:

$$\theta_m = \cos^{-1}\left(\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\right)\frac{2\sqrt{Z_0Z_L}}{|Z_L - Z_0|}$$

Note: $\cos^{-1}x \in (0,\ \pi/2)$

Quarter-Wave Transformer (cont.)

For TEM lines:

$$\theta = \beta_T \ell = \frac{\pi}{2}\frac{f}{f_0} \quad \Rightarrow \quad f = \theta\left(\frac{2f_0}{\pi}\right)$$

$$\Rightarrow f_m = \theta_m\left(\frac{2f_0}{\pi}\right)$$

$$\Rightarrow \mathrm{BW} = \frac{\Delta f}{f_0} = \frac{2\left(f_0 - f_m\right)}{f_0} = 2 - \frac{2f_m}{f_0} = 2 - \frac{4\theta_m}{\pi}$$

(relative bandwidth)

$$\mathrm{BW} = 2 - \frac{4\theta_m}{\pi}$$

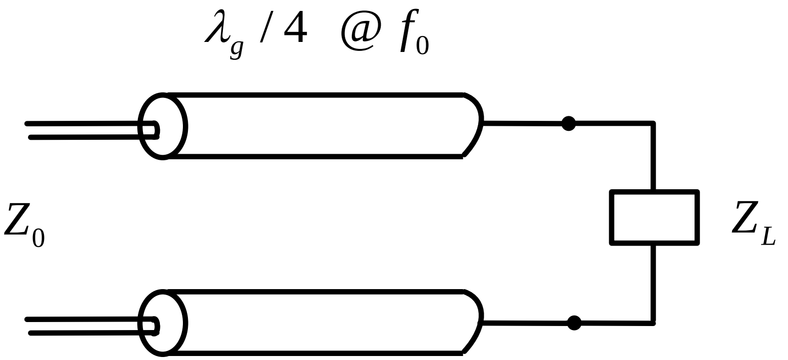
Note:
Multiply by 100 to get BW in percent.

Hence, using θ_m from the previous slide,

$$\mathrm{BW} = 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\;\frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}\right]$$

Quarter-Wave Transformer (cont.)

Summary

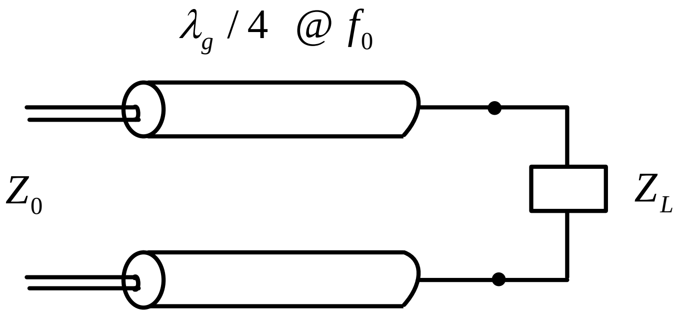


$$\mathrm{BW} = 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\;\frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}\right]$$

Smaller contrast between Z_0 and Z_L $\Rightarrow$ larger BW

Larger contrast between Z_0 and Z_L $\Rightarrow$ smaller BW

Example



Given:

$$Z_L = 100\ [\Omega]$$
$$Z_0 = 50\ [\Omega]$$

We'll illustrate with two choices:

$$\Gamma_m = 1/3 \quad \left(\Gamma_m^{\text{dB}} = -9.54\ [\text{dB}]\right) \quad \Rightarrow \quad \text{BW} = 0.433 \ (43.3\%)$$
$$\Gamma_m = 0.05 \quad \left(\Gamma_m^{\text{dB}} = -26.0\ [\text{dB}]\right) \quad \Rightarrow \quad \text{BW} = 0.060 \ (6.0\%)$$

$$\text{BW} = 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}} \ \frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}\right]$$