Adapted from notes by
Prof. Jeffery T. Williams

# ECE 5317-6351
# Microwave Engineering

**Fall 2019**

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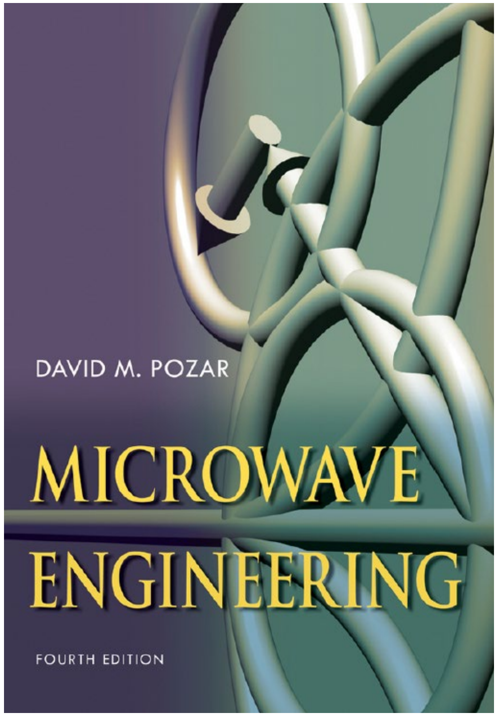
Dept. of ECE




## Notes 21

## Quadrature Coupler and Rat-Race Coupler

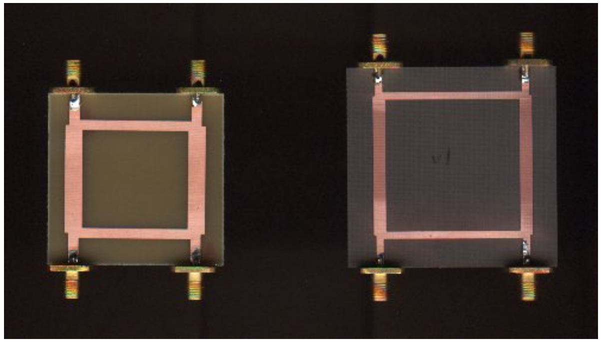
# Quadrature (90$^o$) Coupler

"A quadrature coupler is one in which the input is split into two signals (usually with a goal of equal magnitudes) that are 90 degrees apart in phase. Types of quadrature couplers include branchline couplers (also known as quadrature hybrid* couplers), Lange couplers and overlay couplers."

Taken from "Microwaves 101"

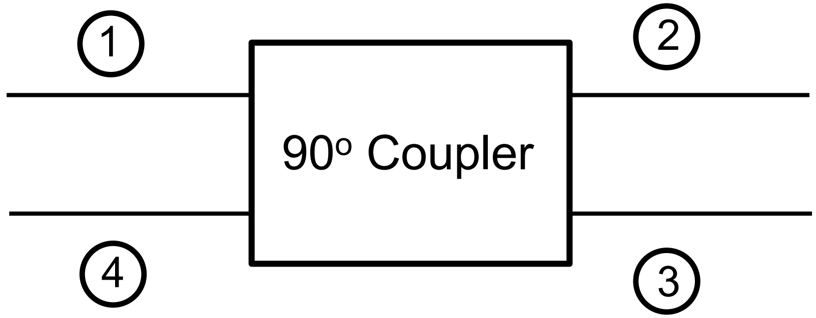
http://www.microwaves101.com/encyclopedia/Quadrature_couplers.cfm

This coupler is very useful for obtaining circular polarization:
There is a 90$^o$ phase difference between ports 2 and 3.

**Note:**
The term "hybrid" denotes the fact that there is an equal (3 dB) power split to the output ports.

# Quadrature Coupler (cont.)

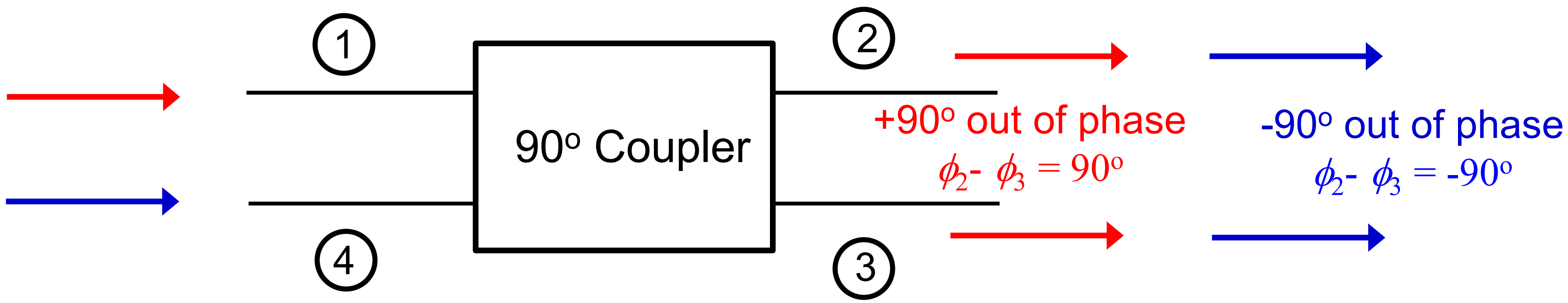



$$[S] = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

- The quadrature hybrid is a lossless 4-port (the $S$ matrix is unitary ).
- All four ports are matched.
- The device is reciprocal (the $S$ matrix is symmetric.)
- Port 4 is isolated from port 1, and ports 2 and 3 are isolated from each other.

# Quadrature Coupler (cont.)

The quadrature coupler is usually used as a splitter:




**Note:** A matched load is usually placed on port 4.

- The signal from port 1 splits evenly between ports 2 and 3, with a 90° phase difference.

$$S_{21} = jS_{31}$$

Can be used to produce right-handed circular polarization.

- The signal from port 4 splits evenly between ports 2 and 3, with a -90° phase difference.

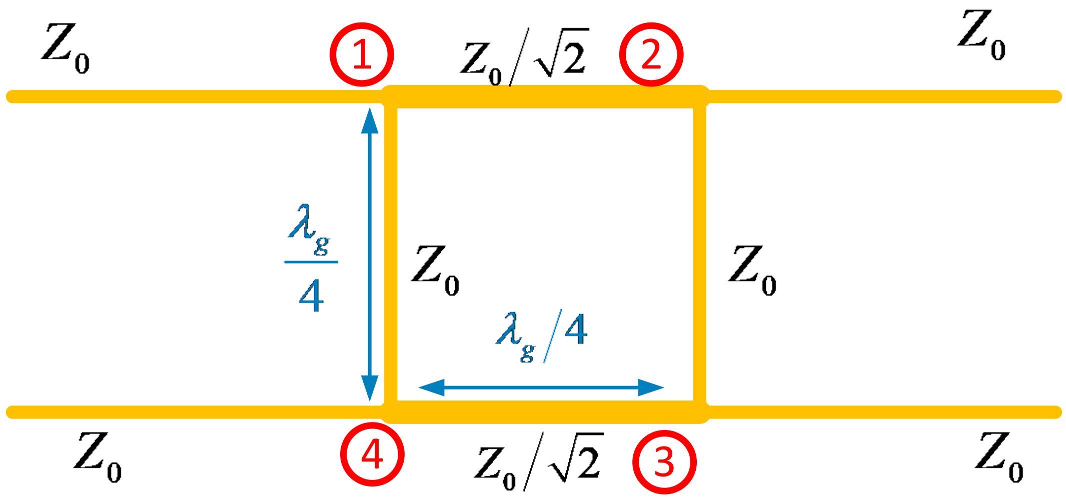
$$S_{24} = -jS_{34}$$

Can be used to produce left-handed circular polarization.

# Quadrature Coupler (cont.)

## Branch-Line Coupler

A microstrip realization of a quadrature hybrid (branch-line coupler) is shown here.



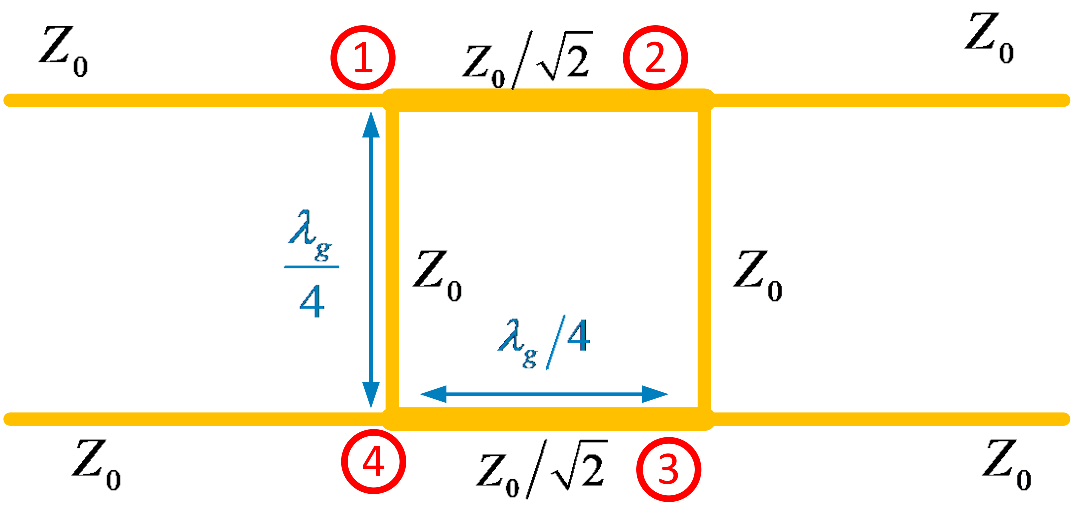

**Notes:**

- We only need to study what happens when we excite port 1, since the structure is physically symmetric.
- We use even/odd mode analysis (exciting ports 1 and 4) to figure out what happens when we excite port 1.

**An analysis of the branch-line coupler is given in the Appendix.**

# Quadrature Coupler (cont.)

## Summary




$$[S] = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

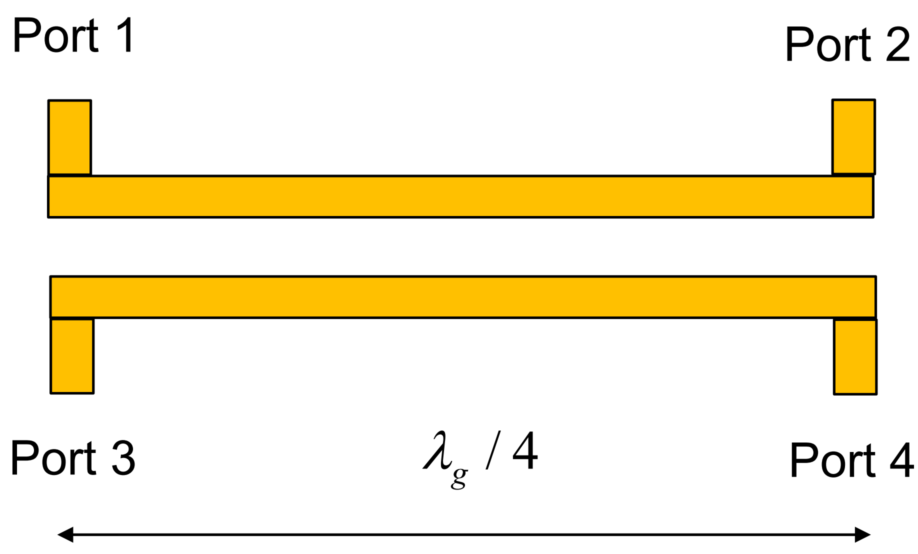
The input power to port 1 divides evenly between ports 2 and 3, with ports 2 and 3 being 90° out of phase.

**Note:** A matched load is usually placed on port 4.

# Quadrature Coupler (cont.)

A coupled-line coupler is one that uses coupled lines (microstrip, stripline) with no direct connection between all of the ports.

Please see the Pozar book for more details.




This coupler has a 90° phase difference between the output ports (ports 2 and 3), and can be used to obtain an equal (-3 dB) power split or another split ratio.

# Quadrature Coupler (cont.)

Circularly-polarized microstrip antennas can be fed with a 90$^{o}$ coupler.

One feed port produces RHCP, the other feed port produced LHCP.

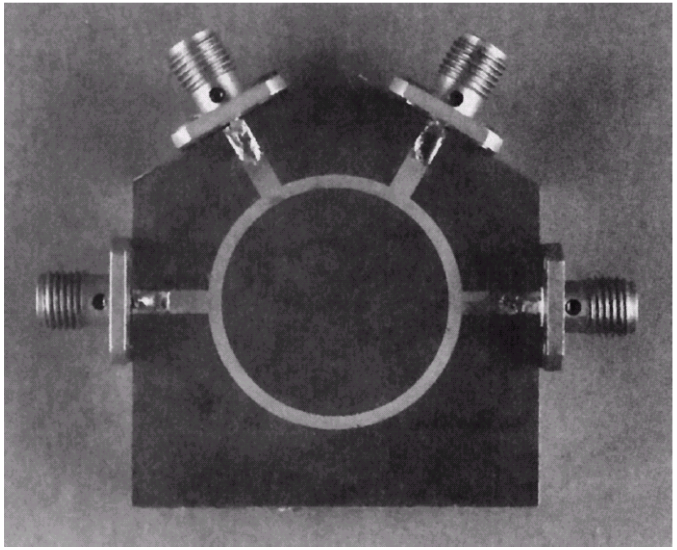
**Note:** This is a better way (higher bandwidth) to get CP than with a simple 90$^{o}$ delay line.

# Rat-Race Ring Coupler (180$^o$ Coupler)

"Applications of rat-race couplers are numerous, and include mixers and phase shifters. The rat-race gets its name from its circular shape, shown below."

Taken from "Microwaves 101"

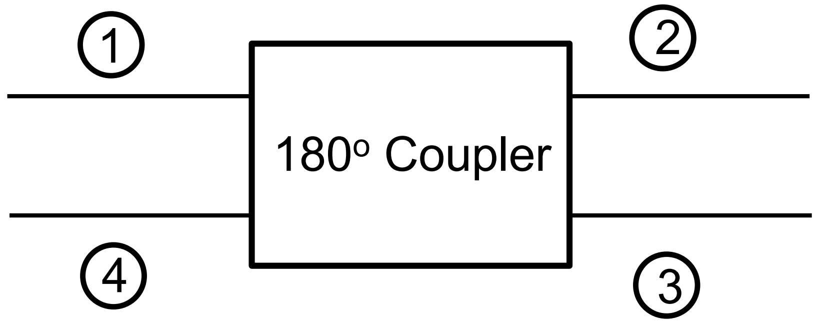
http://www.microwaves101.com/encyclopedia/ratrace_couplers.cfm

Photograph of a microstrip ring coupler

Courtesy of M. D. Abouzahra, MIT Lincoln Laboratory

# Rat-Race Coupler (cont.)

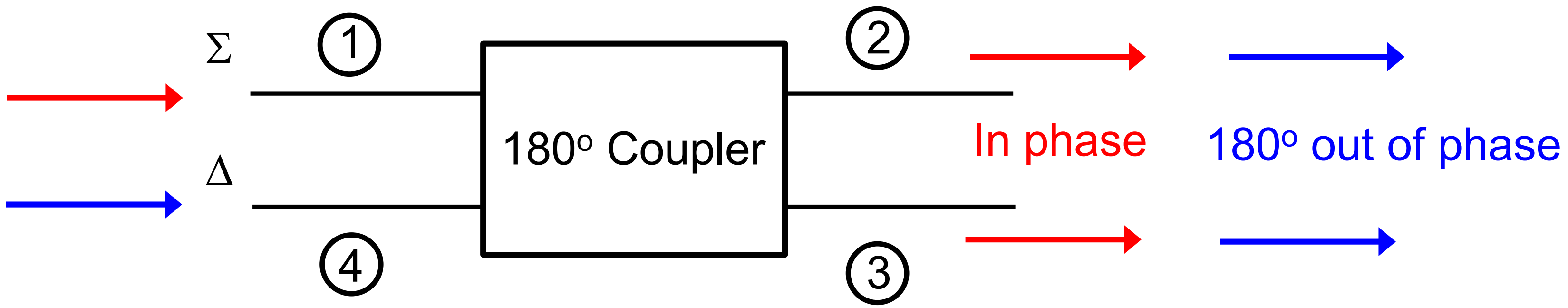



$$[S] = \frac{-j}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

- The rat race is a lossless 4-port (the $S$ matrix is unitary).
- All four ports are matched.
- The device is reciprocal (the $S$ matrix is symmetric).
- Port 4 is isolated from port 1, and ports 2 and 3 are isolated from each other.

# Rat-Race Coupler (cont.)

The rat race can be used as a splitter:




**Note:** A matched load is usually placed on port 4.

- The signal from the "sum port" Σ (port 1) splits evenly between ports 2 and 3, in phase. This could be used as a power splitter (alternative to Wilkinson).

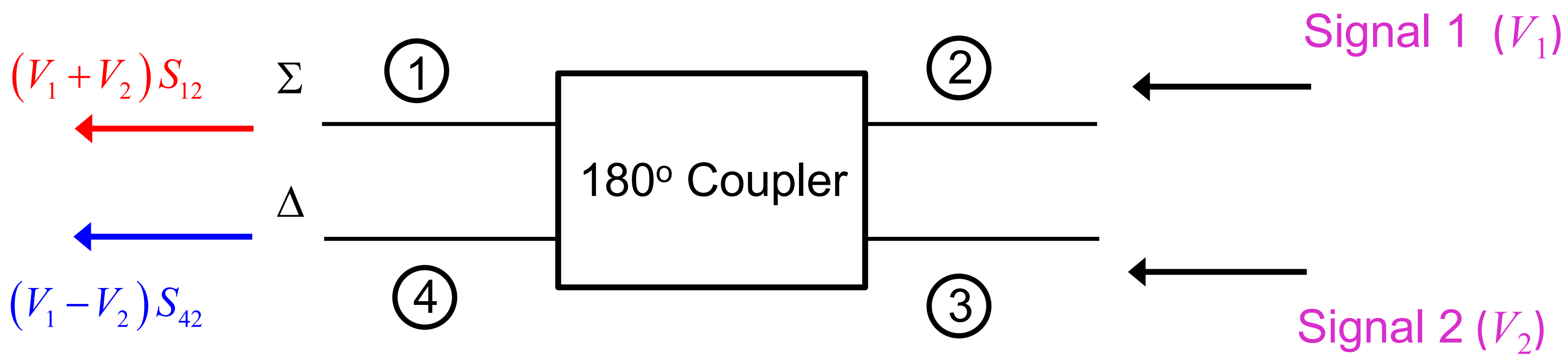
$$S_{21} = S_{31}$$

- The signal from the "difference port" Δ (port 4) splits evenly between ports 1 and 2, 180° out of phase. This could be used as a balun.

$$S_{24} = -S_{34}$$

# Rat-Race Coupler (cont.)

The rat race can be used as a combiner:




- The signal from the sum port Σ (port 1) is the sum of the input signals 1 and 2.

$$S_{12} = S_{13}$$

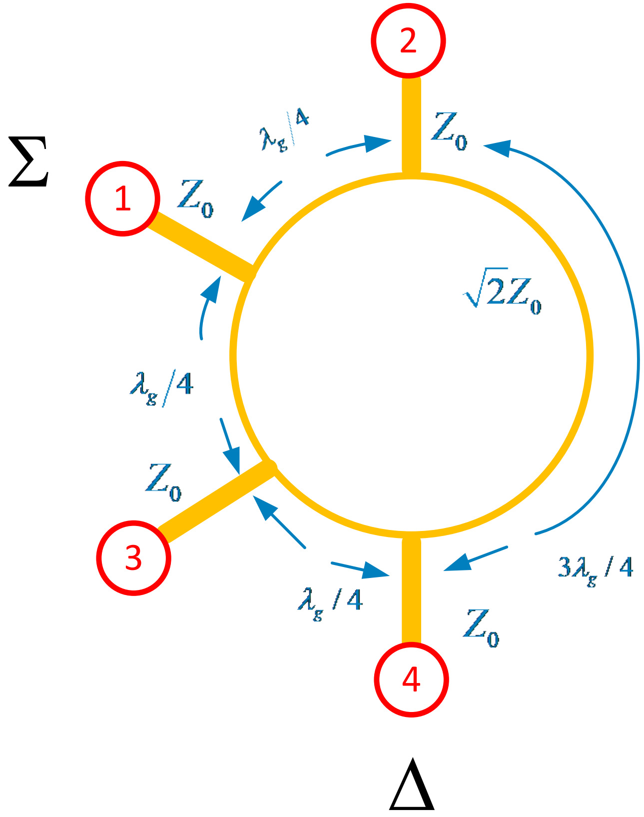
- The signal from the difference port Δ (port 4) is the difference of the input signals 1 and 2.

$$S_{42} = -S_{43}$$

# Rat-Race Coupler (cont.)

A microstrip realization is shown here.

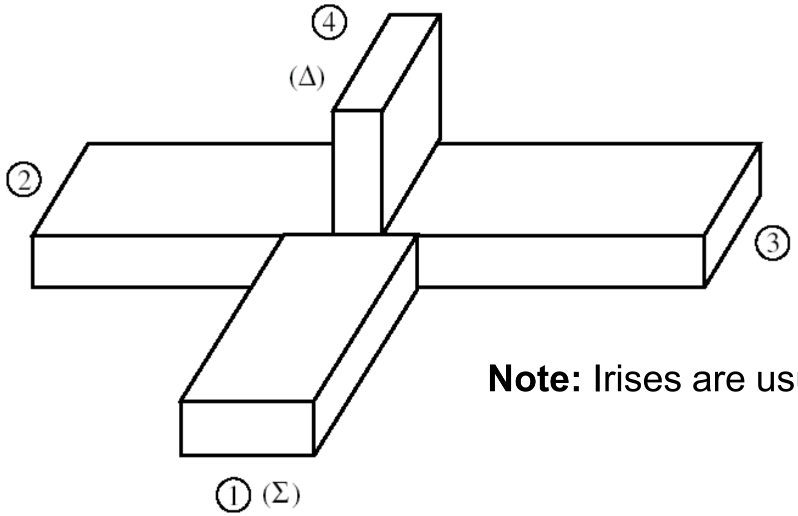
$$[S]=\frac{-j}{\sqrt{2}}\begin{bmatrix}0 & 1 & 1 & 0\\ 1 & 0 & 0 & -1\\ 1 & 0 & 0 & 1\\ 0 & -1 & 1 & 0\end{bmatrix}$$




**An analysis of the rat-race coupler is given in the Appendix.**

# Magic T

A waveguide realization of a 180° coupler is shown here, called a "Magic T."




"Magic T"

**Note:** Irises are usually used to obtain matching at the ports.

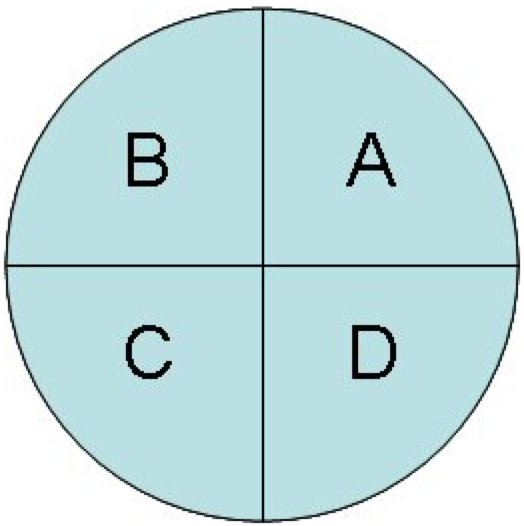
$$[S] = \frac{-j}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

IEEE Microwave Theory and Techniques Society

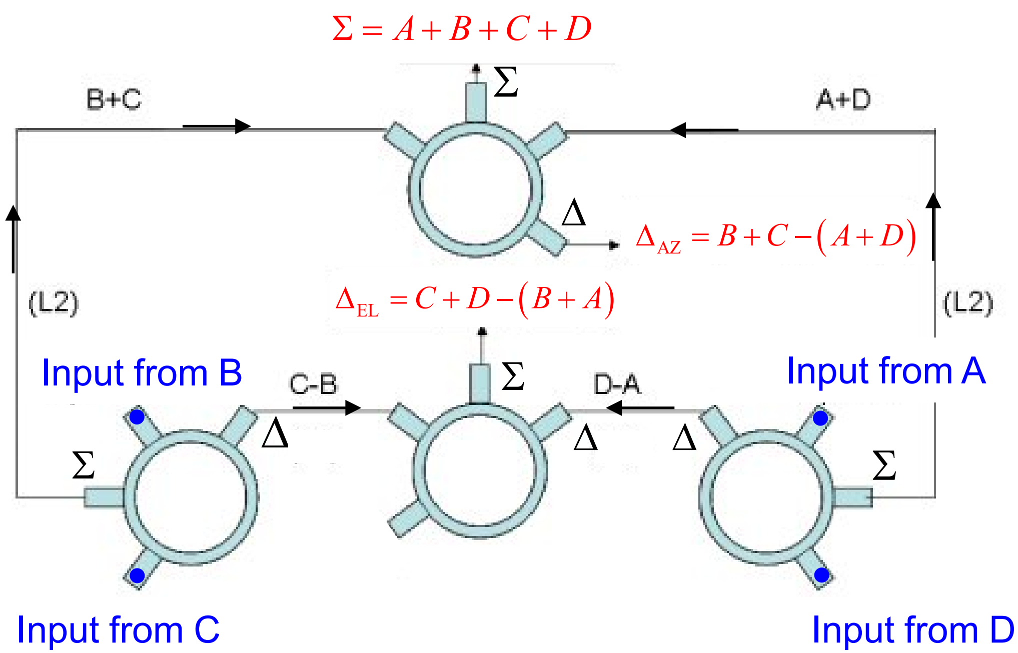
Note the logo!

# Monopulse Radar

Rat-Race couplers are often used in monpulse radar.




Four antennas

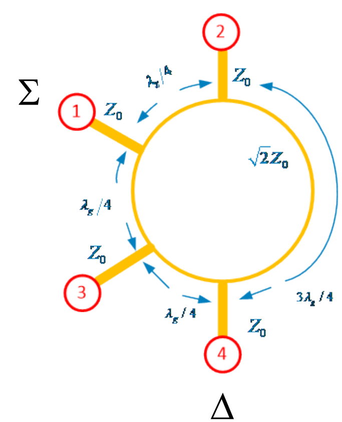



$$\Sigma = A + B + C + D$$

$$\Delta_{AZ} = (B + C) - (A + D)$$

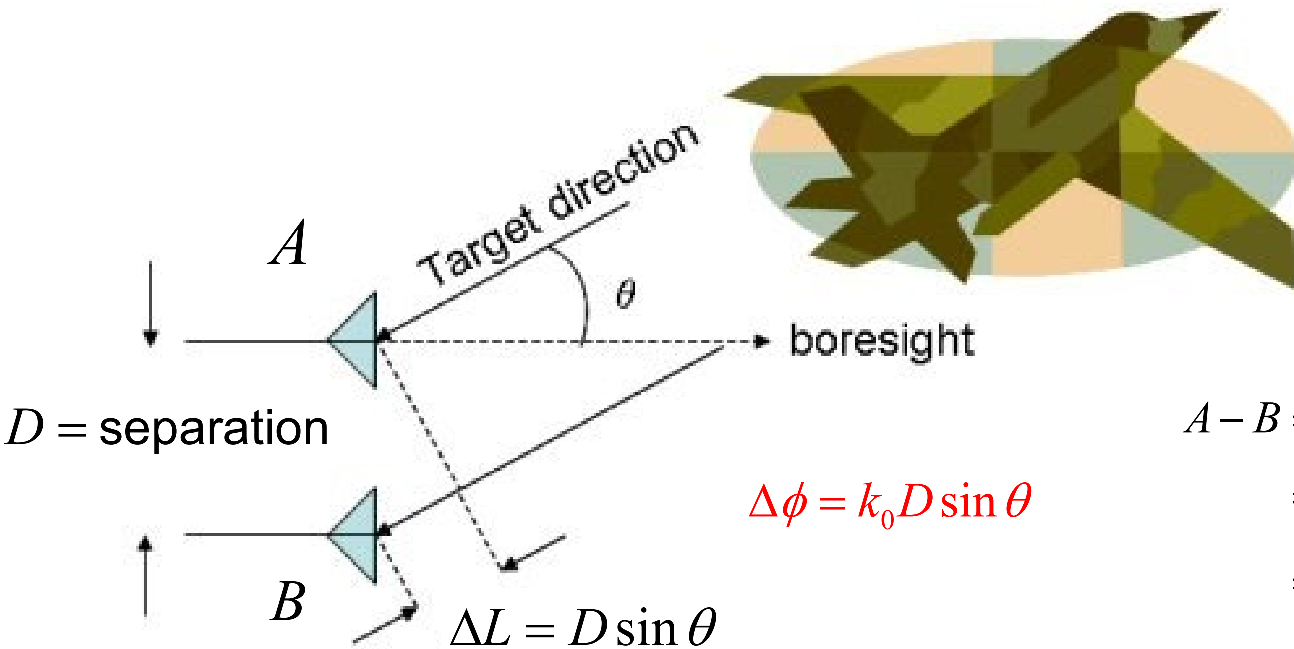
$$\Delta_{EL} = (C + D) - (B + A)$$

Rat-Race:




https://www.microwaves101.com/encyclopedias/monopulse-comparator-networks

# Monopulse Radar (cont.)




$$\Delta\phi = k_0 D \sin\theta$$

$$\begin{aligned} A - B &= 1 - e^{-j\Delta\phi} \\ &= e^{-j\Delta\phi/2}\left(e^{+j\Delta\phi/2} - e^{-j\Delta\phi/2}\right) \\ &= e^{-j\Delta\phi/2}\left(2j\sin\left(\Delta\phi/2\right)\right) \end{aligned}$$

The difference signals are used to determine the azimuth and elevation of the target.

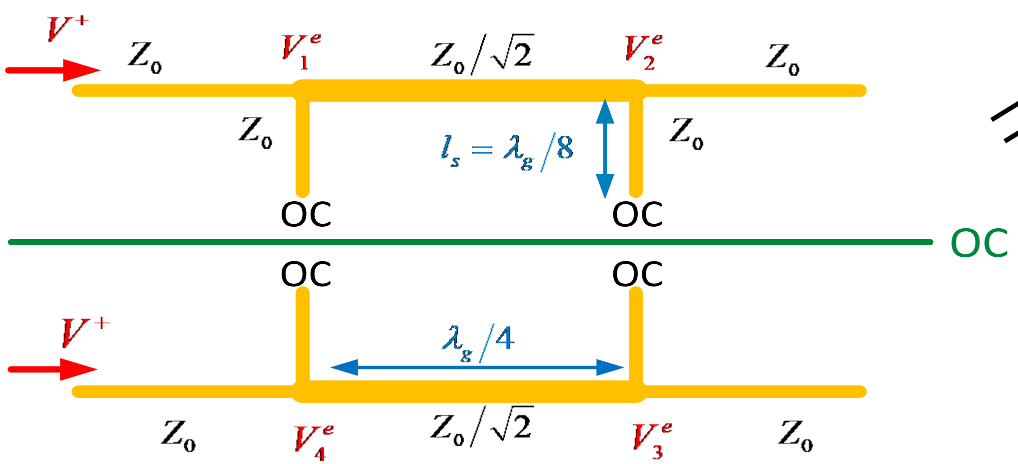
The difference between the two antenna signals maps into the phase difference $\Delta\phi$, which maps into the angle $\theta$.

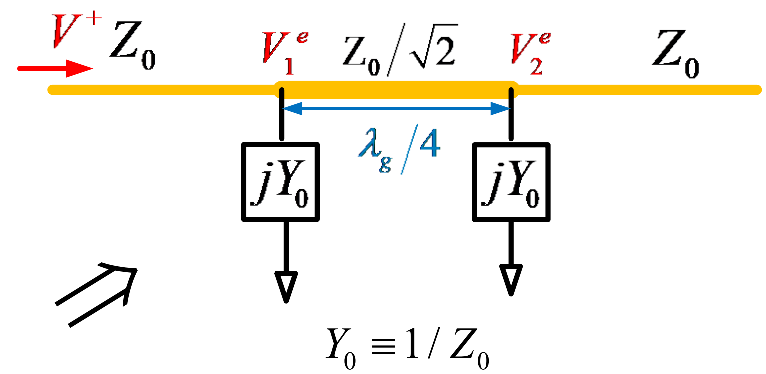
https://www.microwaves101.com/encyclopedias/monopulse-comparator-networks

# Appendix A

**Here we analyze the quadrature coupler.**

Port 1 Excitation
"even" analysis




$$V_3^e = V_2^e$$

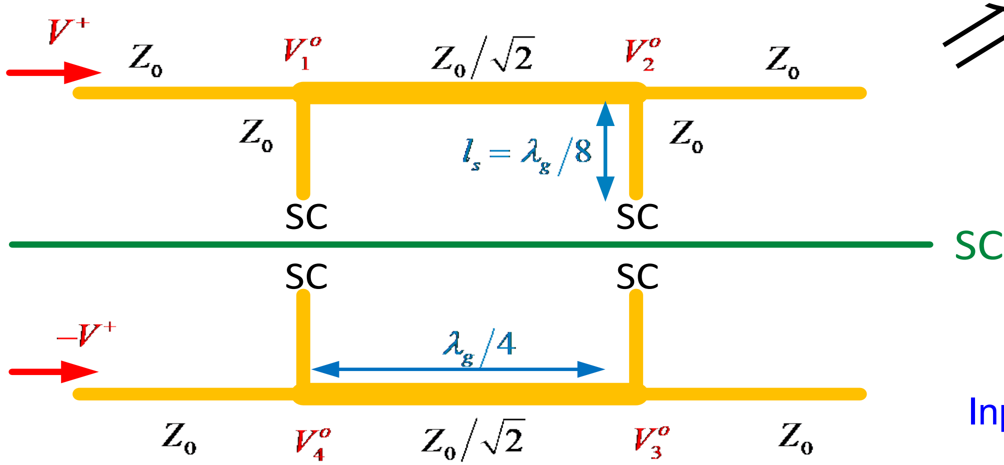
$$V_4^e = V_1^e$$

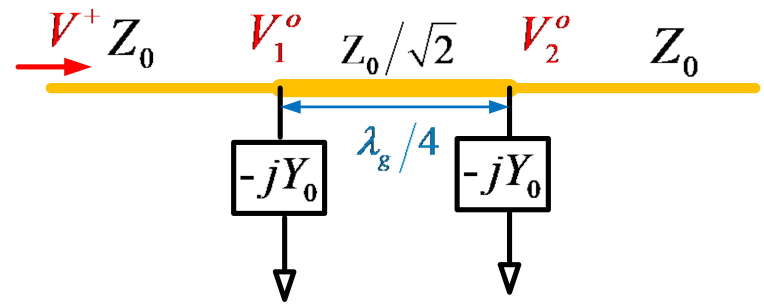
Input admittance of open-circuited stub:

$$\begin{aligned} Y_s &= jY_0 \tan(\beta_s l_s) \\ &= jY_0 \tan(\pi/4) \\ &= jY_0 \end{aligned}$$

# Appendix A (cont.)

Port 1 Excitation
"odd" problem




$$Y_0 \equiv 1/Z_0$$

$$V_3^o = -V_2^o$$

$$V_4^o = -V_1^o$$

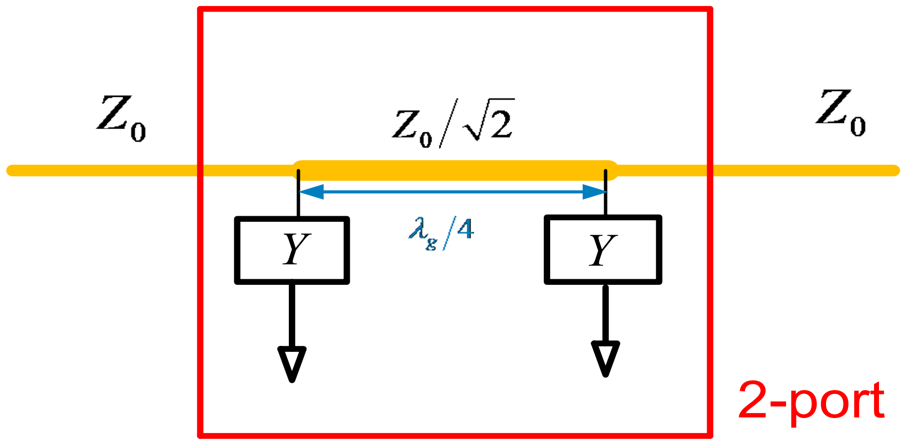
Input admittance of short-circuited stub:

$$\begin{aligned} Y_s &= -jY_0 \cot(\beta_s l_s) \\ &= -jY_0 \cot(\pi/4) \\ &= -jY_0 \end{aligned}$$

# Appendix A (cont.)

Consider the general case:

$$Y_s = \pm jY_0 \quad \begin{pmatrix} + \text{ for even} \\ - \text{ for odd} \end{pmatrix}$$




In general:

$$[ABCD]_Y = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

Shunt load on line

$$[ABCD]_{\frac{\lambda}{4}} = \begin{bmatrix} 0 & \dfrac{jZ_0}{\sqrt{2}} \\ \dfrac{j\sqrt{2}}{Z_0} & 0 \end{bmatrix}$$

Quarter-wave line

$$\left[ABCD^{line}\right] = \begin{bmatrix} \cos(\beta\ell) & jZ_0^{line}\sin(\beta\ell) \\ \left(j/Z_0^{line}\right)\sin(\beta\ell) & D = \cos(\beta\ell) \end{bmatrix}$$

Here: $Z_0^{line} = Z_0/\sqrt{2}$
$\beta\ell = \pi/2$

$$\Rightarrow [ABCD] = [ABCD]_Y [ABCD]_{\frac{\lambda}{4}} [ABCD]_Y$$

# Appendix A (cont.)

Hence we have:

$$[ABCD] = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \begin{bmatrix} 0 & \dfrac{jZ_0}{\sqrt{2}} \\ \dfrac{j\sqrt{2}}{Z_0} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \begin{bmatrix} \dfrac{jZ_0 Y}{\sqrt{2}} & \dfrac{jZ_0}{\sqrt{2}} \\ \dfrac{j\sqrt{2}}{Z_0} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{jZ_0 Y}{\sqrt{2}} & \dfrac{jZ_0}{\sqrt{2}} \\ \dfrac{jZ_0 Y^2}{\sqrt{2}} + \dfrac{j\sqrt{2}}{Z_0} & \dfrac{jZ_0 Y}{\sqrt{2}} \end{bmatrix}$$

$$Y = \pm jY_0 = \pm \frac{j}{Z_0}$$ ( + for even, - for odd )

# Appendix A (cont.)

Continuing with the algebra, we have:

$$[ABCD]=\frac{1}{\sqrt{2}}\begin{bmatrix} jZ_0\left(\pm\dfrac{j}{Z_0}\right) & jZ_0 \\ jZ_0\left(\pm\dfrac{j}{Z_0}\right)^2+\dfrac{j2}{Z_0} & jZ_0\left(\pm\dfrac{j}{Z_0}\right) \end{bmatrix}$$

$$=\frac{1}{\sqrt{2}}\begin{bmatrix} j(\pm j) & jZ_0 \\ -j\left(\dfrac{1}{Z_0}\right)+\dfrac{j2}{Z_0} & j(\pm j) \end{bmatrix}$$

$$=\frac{1}{\sqrt{2}}\begin{bmatrix} \mp 1 & jZ_0 \\ j\left(\dfrac{1}{Z_0}\right) & \mp 1 \end{bmatrix}$$

# Appendix A (cont.)

Hence we have:

$$[ABCD]_{\substack{e\\0}} = \frac{1}{\sqrt{2}} \begin{bmatrix} \mp 1 & jZ_0 \\ j\left(\frac{1}{Z_0}\right) & \mp 1 \end{bmatrix}$$

Convert this to $S$ parameters (use Table 4.2 in Pozar):

$$[S]_{\substack{e\\0}} = \begin{bmatrix} 0 & \frac{\mp 1 - j}{\sqrt{2}} \\ \frac{\mp 1 - j}{\sqrt{2}} & 0 \end{bmatrix}$$

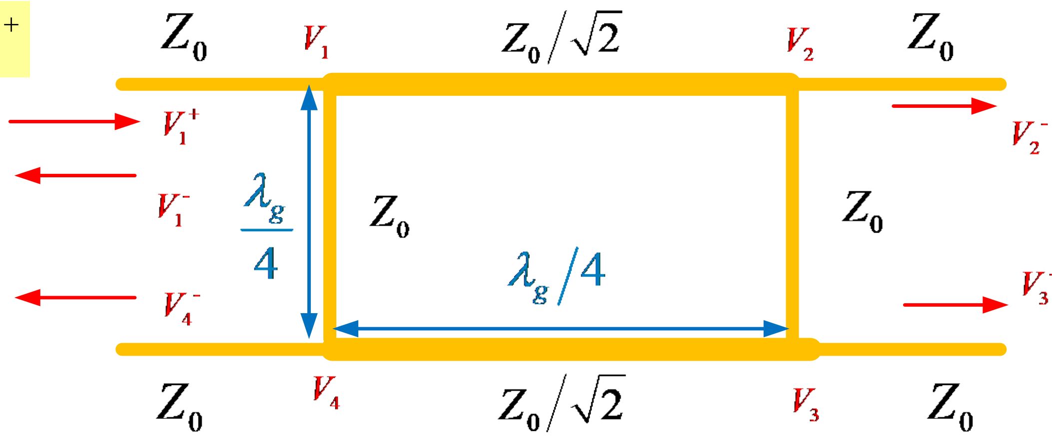
**Note:**
We are describing a two-port device here, in the even and odd mode cases.

This is a 2×2 matrix, not a 4×4 matrix.

# Appendix A (cont.)

Adding even and odd mode cases together:

$$V_1^+ = V^+ + V^+$$




$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{a_2=a_3=a_4=0} \Rightarrow S_{11} = \frac{V_1^{-e} + V_1^{-o}}{V^+ + V^+} = \frac{V_1^{-e} + V_1^{-o}}{2V^+} = \frac{1}{2}\left(\frac{V_1^{-e}}{V^+} + \frac{V_1^{-o}}{V^+}\right)$$

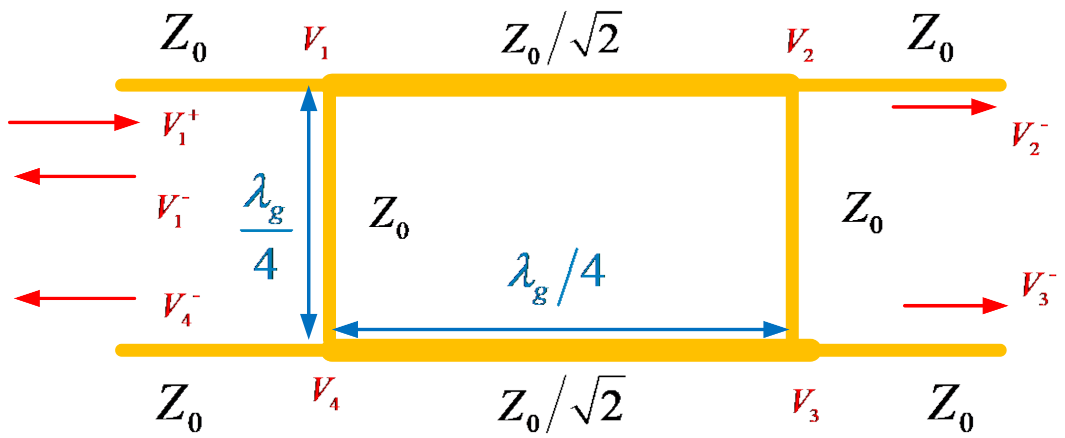
$$= \frac{1}{2}\left(S_{11}^e + S_{11}^o\right) = 0 + 0$$

Hence $S_{11} = 0$

By symmetry: $S_{11} = S_{22} = S_{33} = S_{44} = 0$

# Appendix A (cont.)

$$V_1^+ = V^+ + V^+$$




$$S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{a_2=a_3=a_4=0} \Rightarrow$$

$$S_{21} = \frac{V_2^{-e} + V_2^{-o}}{V^+ + V^+} = \frac{V_2^{-e} + V_2^{-o}}{2V^+} = \frac{1}{2}\left(S_{21}^e + S_{21}^o\right)$$

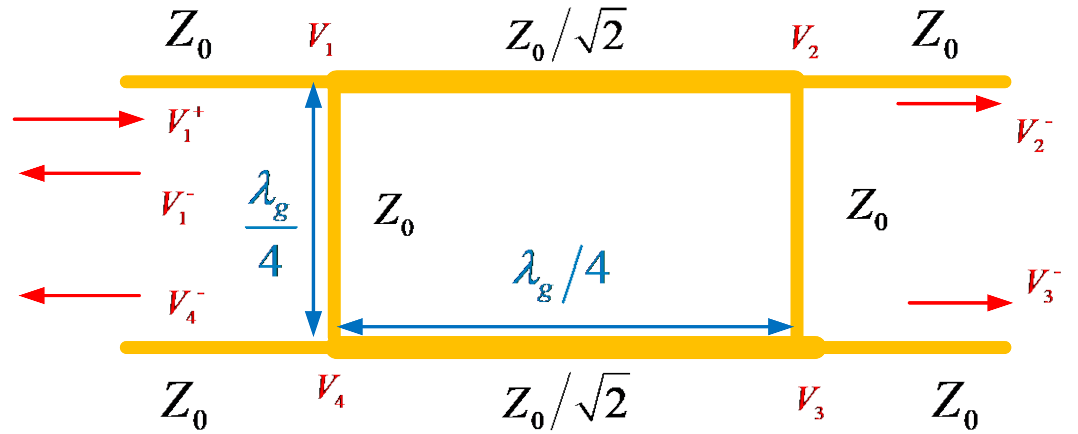
$$= \frac{1}{2}\left[\left(\frac{-1-j}{\sqrt{2}}\right) + \left(\frac{1-j}{\sqrt{2}}\right)\right]$$

$$= \frac{-j}{\sqrt{2}}$$

By symmetry and reciprocity: $$S_{21} = S_{12} = S_{43} = S_{34} = \frac{-j}{\sqrt{2}}$$

# Appendix A (cont.)

$$V_1^+ = V^+ + V^+$$




$$S_{31} = \left.\frac{V_3^-}{V_1^+}\right|_{a_2=a_3=a_4=0} \Rightarrow$$

$$S_{31} = \frac{V_3^{-e} + V_3^{-o}}{V^+ + V^+} = \frac{V_3^{-e} + V_3^{-o}}{2V^+} = \frac{V_2^{-e} - V_2^{-o}}{2V^+} = \frac{1}{2}\left(S_{21}^e - S_{21}^o\right)$$

$$= \frac{1}{2}\left[\left(\frac{-1-j}{\sqrt{2}}\right) - \left(\frac{1-j}{\sqrt{2}}\right)\right]$$

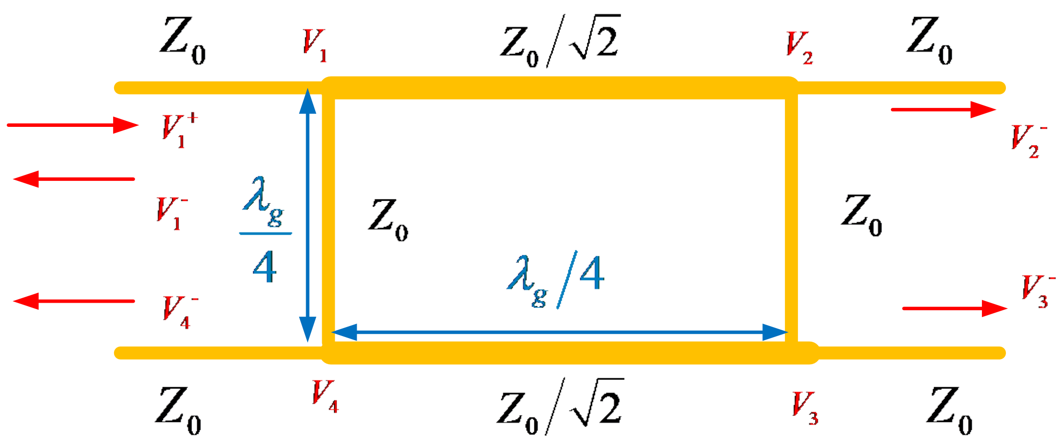
$$= \frac{-1}{\sqrt{2}}$$

By symmetry and reciprocity:

$$S_{31} = S_{13} = S_{24} = S_{42} = \frac{-1}{\sqrt{2}}$$

# Appendix A (cont.)

$$V_1^+ = V^+ + V^+$$

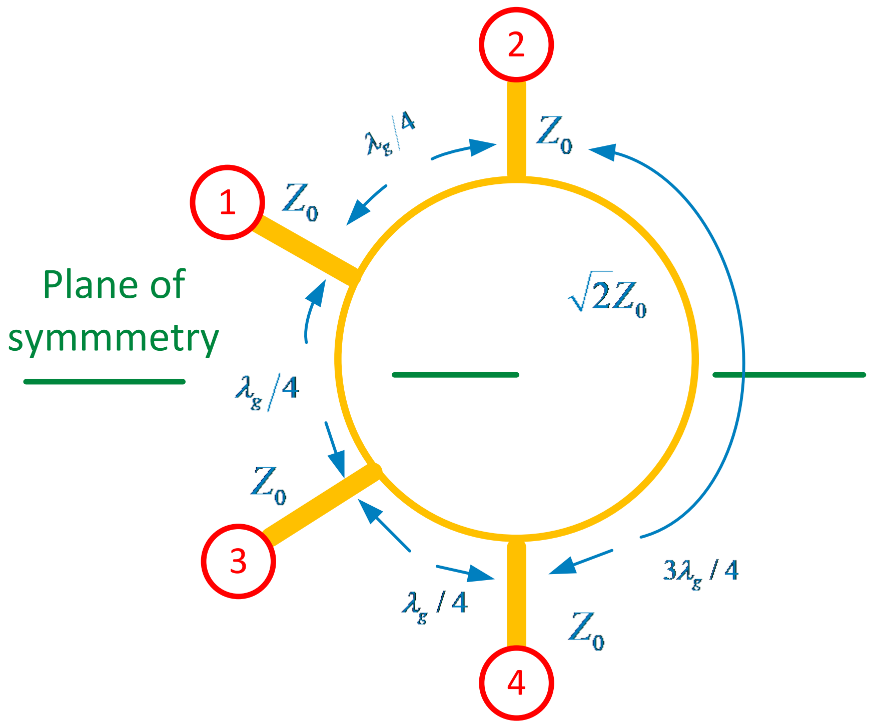



$$S_{41} = \left.\frac{V_4^-}{V_1^+}\right|_{a_2=a_3=a_4=0} \Rightarrow S_{41} = \frac{V_4^{-e}+V_4^{-o}}{V^+ + V^+} = \frac{V_4^{-e}+V_4^{-o}}{2V^+} = \frac{V_1^{-e}-V_1^{-o}}{2V^+} = \frac{1}{2}\left(S_{11}^e - S_{11}^o\right) = 0$$

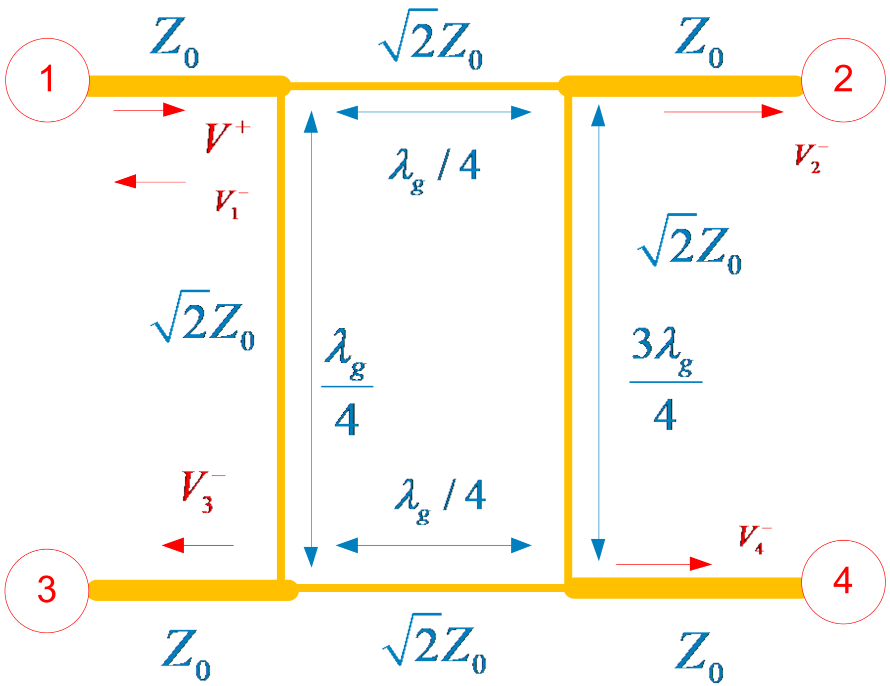
By symmetry and reciprocity: $S_{41} = S_{14} = S_{23} = S_{32} = 0$

# Appendix B

**Here we analyze the Rat-Race Ring coupler.**



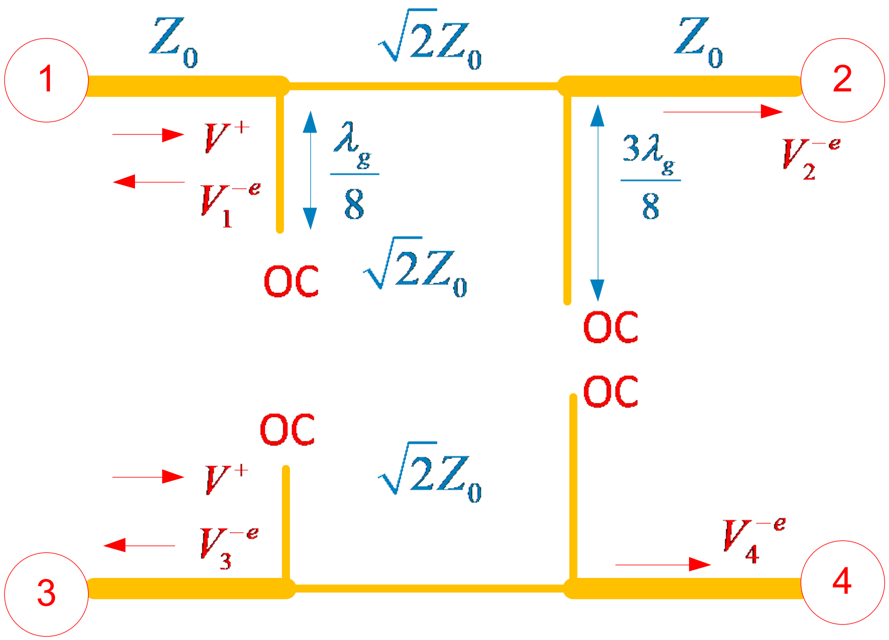

Layout

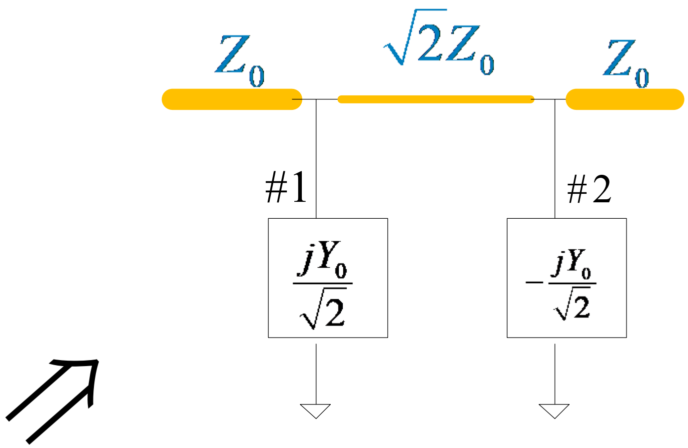



Schematic

# Appendix B (cont.)

Port 1 Excitation
"even" problem




$$Y_0 = 1/Z_0$$

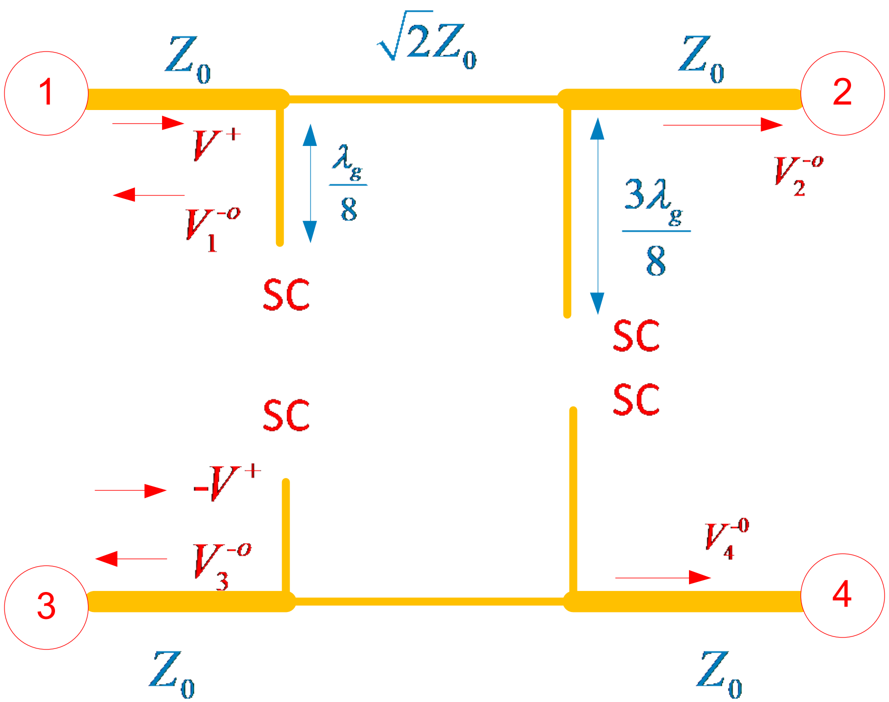
$$Y_{0s} = Y_0/\sqrt{2}$$

$$\begin{aligned} Y_{s1} &= jY_{0s1}\tan(\beta_s l_{s1}) \\ &= j\left(Y_0/\sqrt{2}\right)\tan(\pi/4) \\ &= jY_0/\sqrt{2} \end{aligned}$$

$$\begin{aligned} Y_{s2} &= jY_{0s2}\tan(\beta_s l_{s2}) \\ &= j\left(Y_0/\sqrt{2}\right)\tan(3\pi/4) \\ &= -jY_0/\sqrt{2} \end{aligned}$$

# Appendix B (cont.)

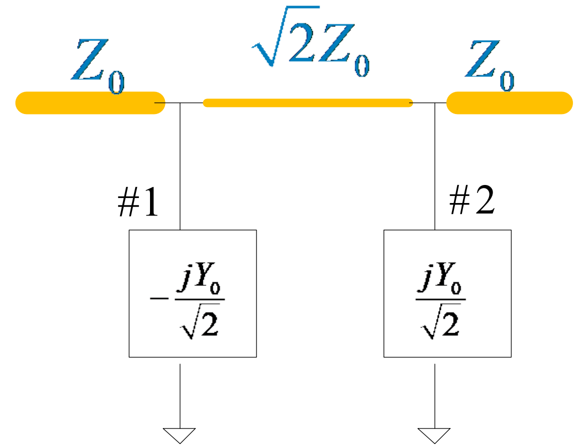
## Port 1 Excitation
## "odd" problem




$$Y_0 = 1/Z_0$$

$$Y_{0s} = Y_0/\sqrt{2}$$

$\Rightarrow$




$$\begin{aligned} Y_{s1} &= -jY_{0s1}\cot(\beta_s l_s) \\ &= -j\left(Y_0/\sqrt{2}\right)\cot(\pi/4) \\ &= -jY_0/\sqrt{2} \end{aligned}$$

$$\begin{aligned} Y_{s1} &= -jY_{0s2}\cot(\beta_s l_s) \\ &= -j\left(Y_0/\sqrt{2}\right)\cot(3\pi/4) \\ &= jY_0/\sqrt{2} \end{aligned}$$

# Appendix B (cont.)

Proceeding as for the 90° coupler, we have:

$$\left[ABCD\right]_{0}^{e} = \begin{bmatrix} 1 & 0 \\ \dfrac{\pm jY_0}{\sqrt{2}} & 1 \end{bmatrix} \begin{bmatrix} 0 & j\sqrt{2}Z_0 \\ j\dfrac{1}{\sqrt{2}Z_0} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \mp\dfrac{jY_0}{\sqrt{2}} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ \dfrac{\pm jY_0}{\sqrt{2}} & 1 \end{bmatrix} \begin{bmatrix} \pm 1 & j\sqrt{2}Z_0 \\ j\dfrac{1}{\sqrt{2}Z_0} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \pm 1 & j\sqrt{2}Z_0 \\ j\dfrac{\sqrt{2}}{Z_0} & \mp 1 \end{bmatrix}$$

# Appendix B (cont.)

Converting from the *ABCD* matrix to the *S* matrix, we have:

$$[ABCD]_{\substack{e\\0}} = \begin{bmatrix} \pm 1 & j\sqrt{2}Z_0 \\ j\dfrac{\sqrt{2}}{Z_0} & \mp 1 \end{bmatrix}$$

**Note:**
We are describing a two-port device here, in the even and odd mode cases.

This is a 2×2 matrix, not a 4×4 matrix.

Use Table 4.2 in Pozar

$$[S]_{\substack{e\\0}} = \frac{-j}{\sqrt{2}} \begin{bmatrix} \pm 1 & 1 \\ 1 & \mp 1 \end{bmatrix}$$

# Appendix B (cont.)

For the $S$ parameters coming from port 1 excitation, we then have:

$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{a_2=a_3=a_4=0}$$

$$S_{11} = \frac{V_1^{-e} + V_1^{-o}}{2V^+} = \frac{1}{2}\left(S_{11}^e + S_{11}^o\right)$$

$$= \frac{1}{2}\left(\frac{-j}{\sqrt{2}} + \frac{j}{\sqrt{2}}\right)$$

$$= 0$$

$$S_{11} = S_{33} = 0$$

(symmetry)

$$S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{a_2=a_3=a_4=0}$$

$$S_{21} = \frac{V_2^{-e} + V_2^{-o}}{2V^+} = \frac{1}{2}\left(S_{21}^e + S_{21}^o\right)$$

$$= \frac{1}{2}\left(\frac{-j}{\sqrt{2}} + \frac{-j}{\sqrt{2}}\right)$$

$$= \frac{-j}{\sqrt{2}}$$

$$S_{21} = S_{12} = S_{34} = S_{43} = \frac{-j}{\sqrt{2}}$$

(symmetry and reciprocity)

# Appendix B (cont.)

$$S_{31} = \left.\frac{V_3^-}{V_1^+}\right|_{a_2=a_3=a_4=0}$$

$$S_{31} = \frac{V_3^{-e} + V_3^{-o}}{2V^+} = \frac{1}{2}\left(S_{11}^e - S_{11}^o\right)$$

$$= \frac{1}{2}\left(\frac{-j}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right)$$

$$= \frac{-j}{\sqrt{2}}$$

$$S_{31} = S_{13} = -\frac{j}{\sqrt{2}}$$

(symmetry)

Similarly, exciting port 2, and using symmetry and reciprocity, we have the following results (derivation omitted):

$$S_{22} = S_{44} = 0$$

$$S_{23} = S_{32} = S_{14} = S_{41} = 0$$

$$S_{24} = S_{42} = \frac{j}{\sqrt{2}}$$