Adapted from notes by
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ECE 5317-6351
Microwave Engineering

Fall 2019

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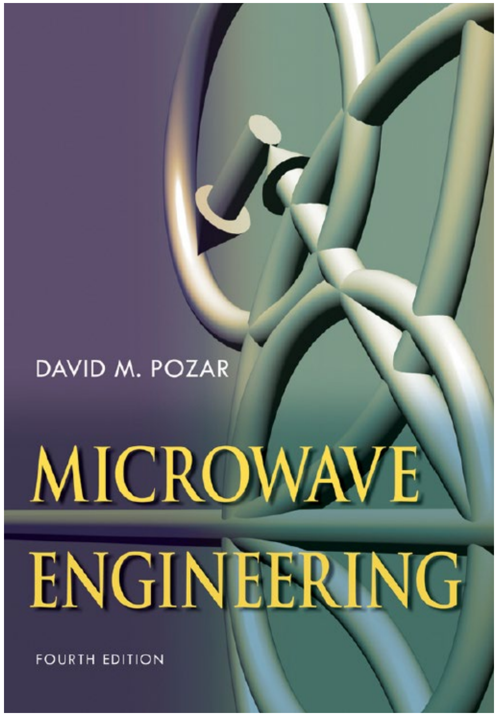
Dept. of ECE



Notes 23

Filter Design Part 2:
General Filter Design

General Filter Design

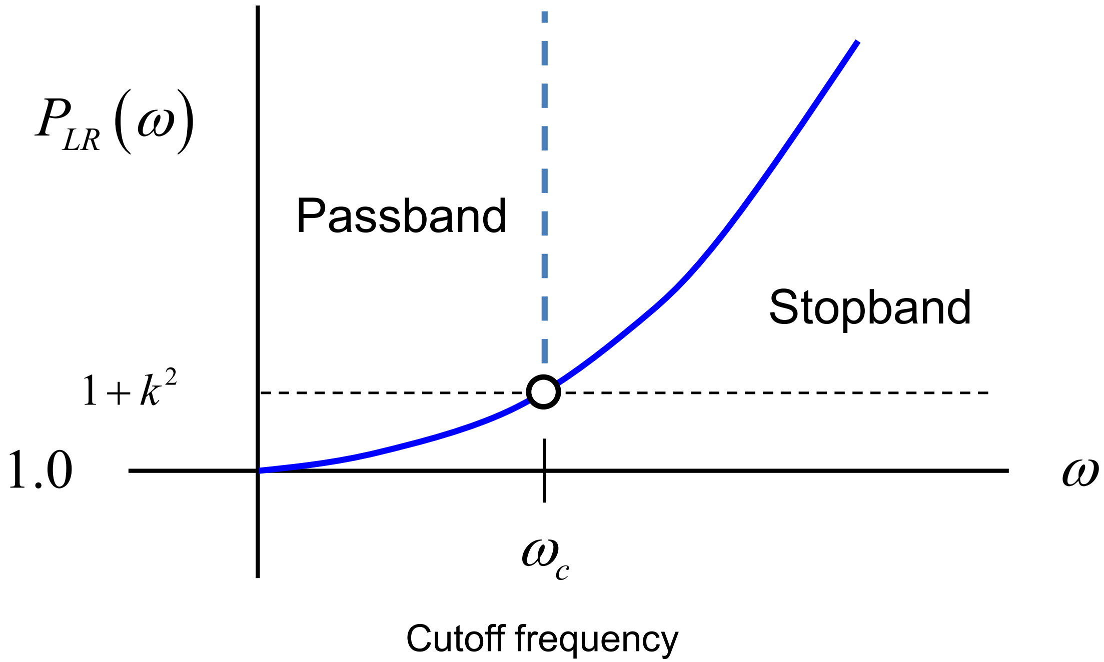
In this set of notes we examine a general method for designing filters of arbitrary order.

Recipe:

- Start with a normalized low-pass "prototype" design ($R_0 = 1,\ \omega_c = 1$).
- De-normalize to get a low-pass design with a specified ($R_0,\ \omega_c$).
- Use frequency transformations to convert the normalized low-pass to a high-pass, bandpass, or bandstop design.

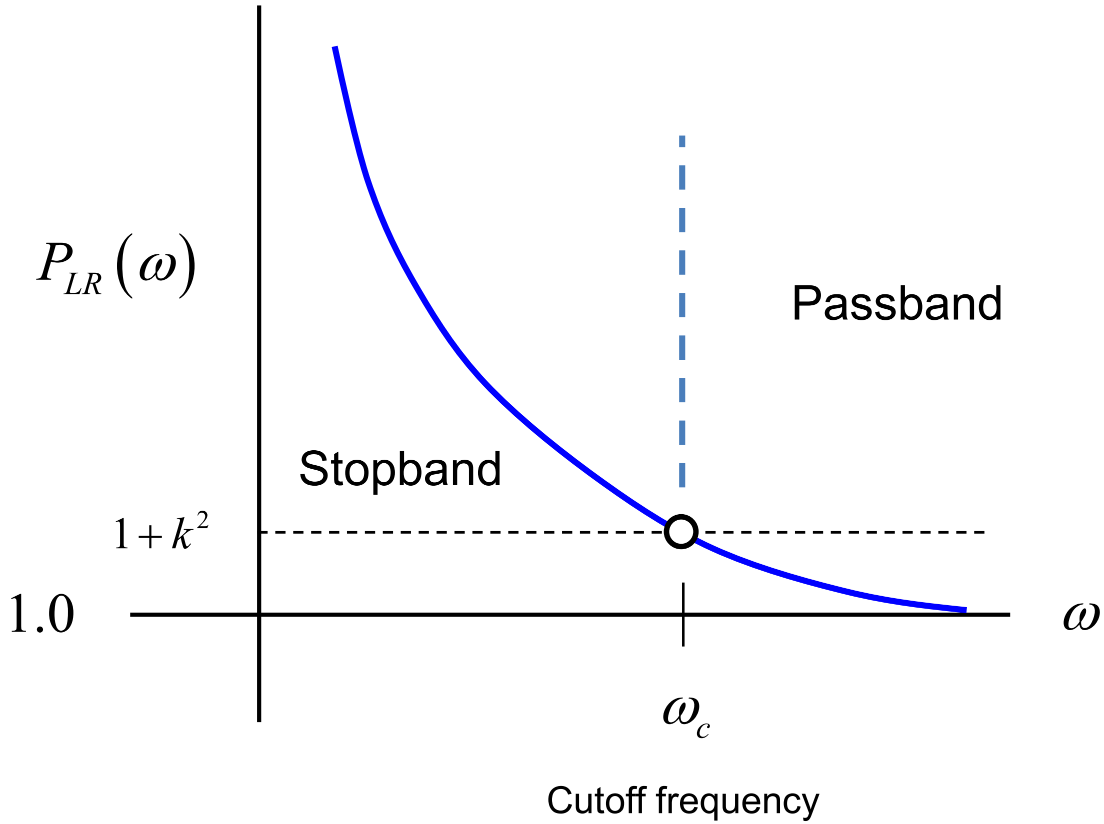
Filter Types

Low-pass



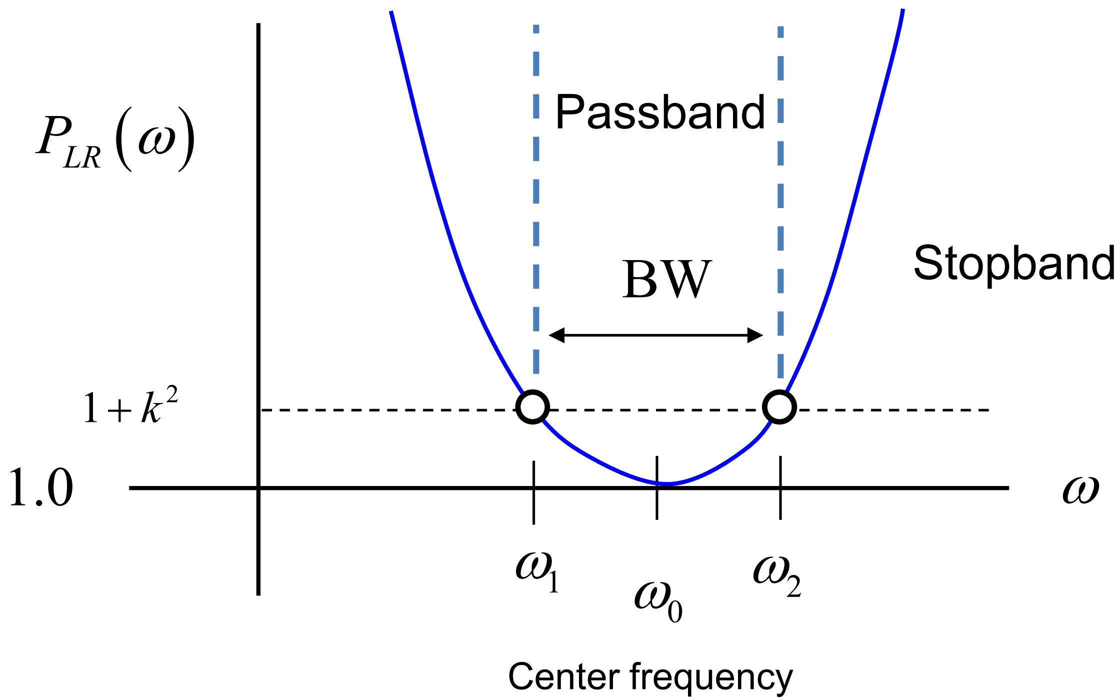
Filter Types (cont.)

High-pass



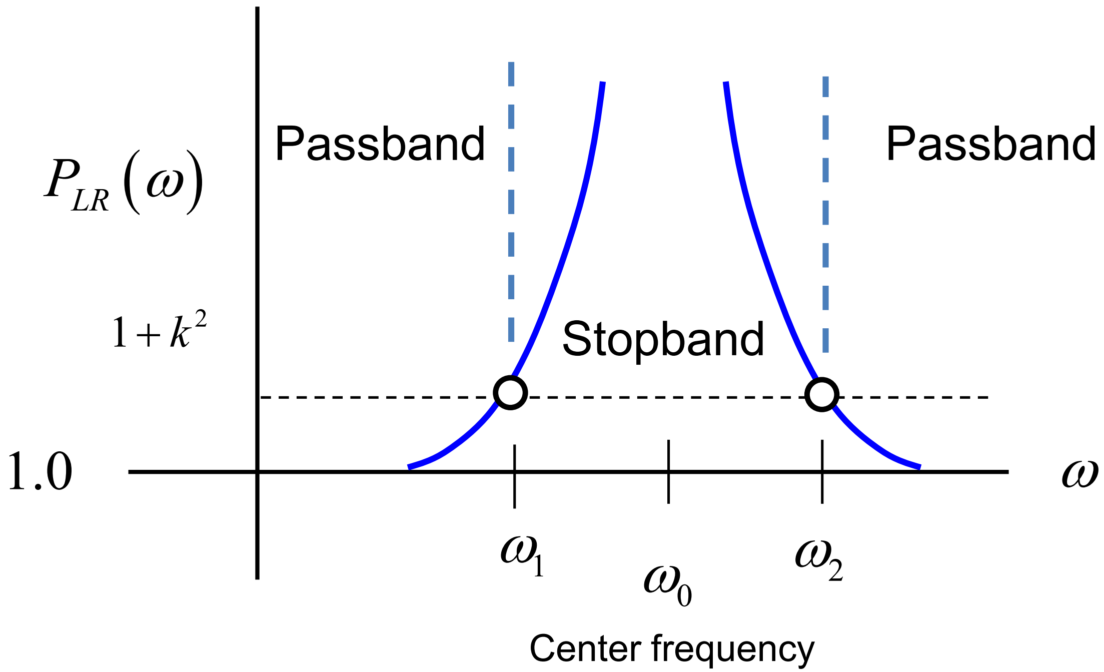
Filter Types (cont.)

Bandpass



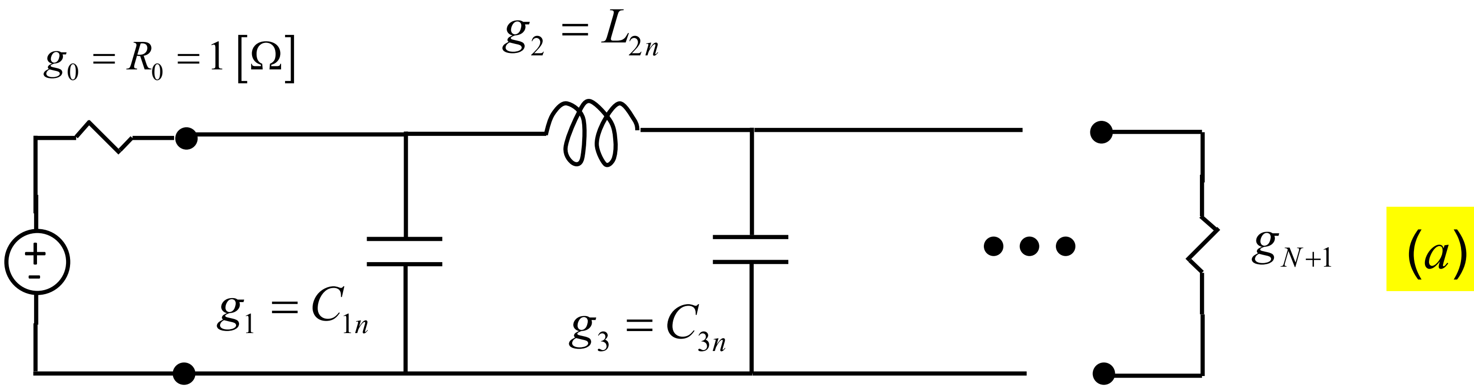
Filter Types (cont.)

Bandstop



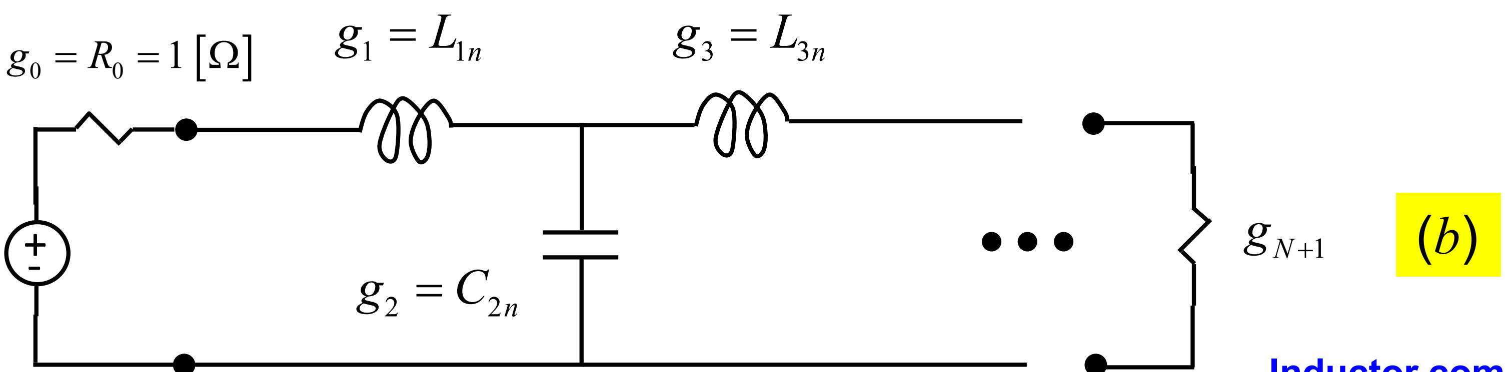
General Filter Design (cont.)

Consider a general normalized low-pass filter ladder network:



Capacitor comes first

Note: The last element can be either a series inductor or a parallel capacitor in designs (a) and (b).



Inductor comes first

Note: Both forms (a and b) have the same frequency response (for the same N).

General Filter Design (cont.)

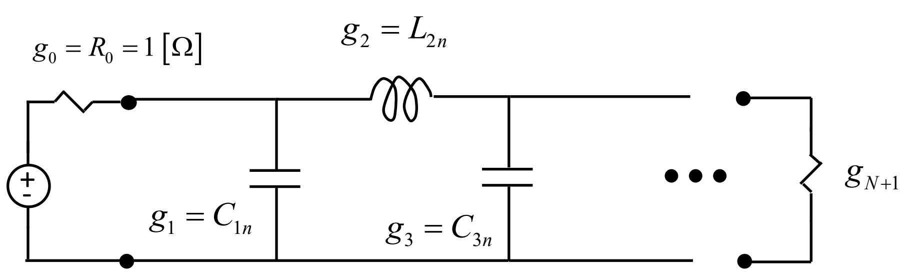
Notation:

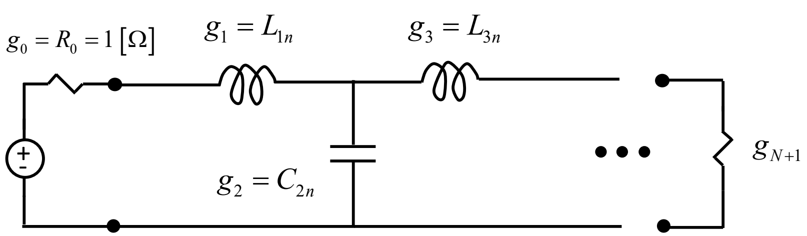
g_0 = normalized generator resistance

$$g_k = \begin{cases} \text{normalized inductance for series inductor} \\ \text{normalized capacitance for parallel capacitor} \end{cases}$$

$$g_{N+1} = \begin{cases} \text{normalized load resistance if } g_N \text{ is a shunt capacitance} \\ \text{normalized load conductance if } g_N \text{ is a series inductance} \end{cases}$$

Note: In most cases, $g_{N+1} = 1.0$ (load resistance R_L = source resistance R_s).



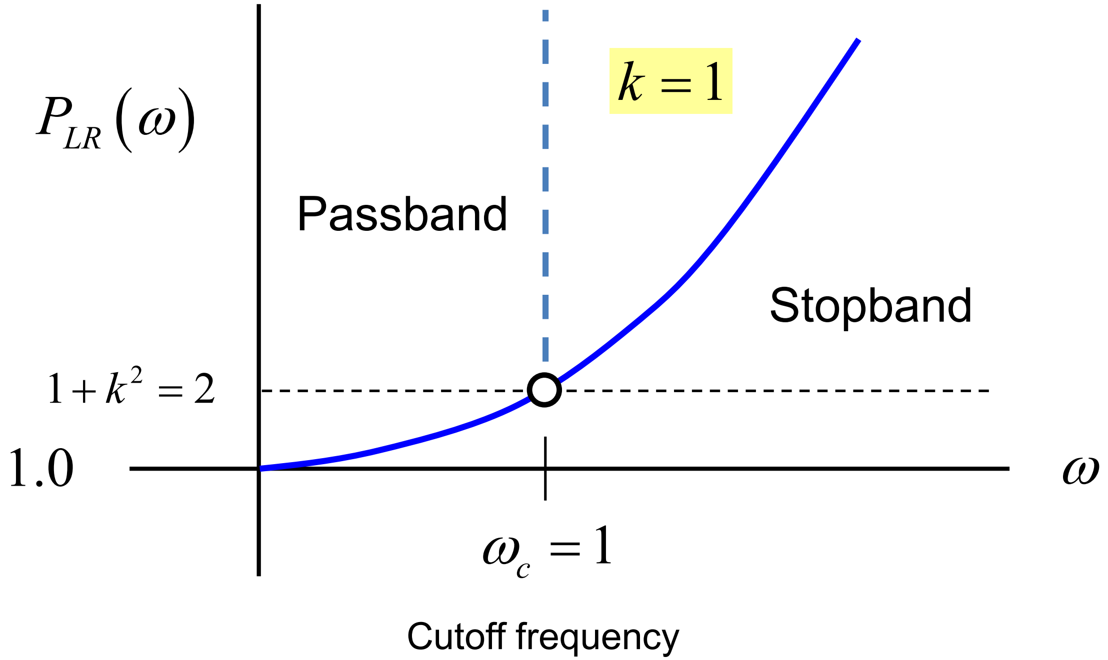


Butterworth Behavior

Insertion Loss : $\mathrm{IL}(\omega) \equiv 10\log_{10} P_{LR}(\omega)$

$$\mathrm{IL}(\omega_c) = 3\text{ dB}$$

Normalized loss-pass prototype



Butterworth Design Table

TABLE 8.3 Element Values for Maximally Flat Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, $N = 1$ to 10)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

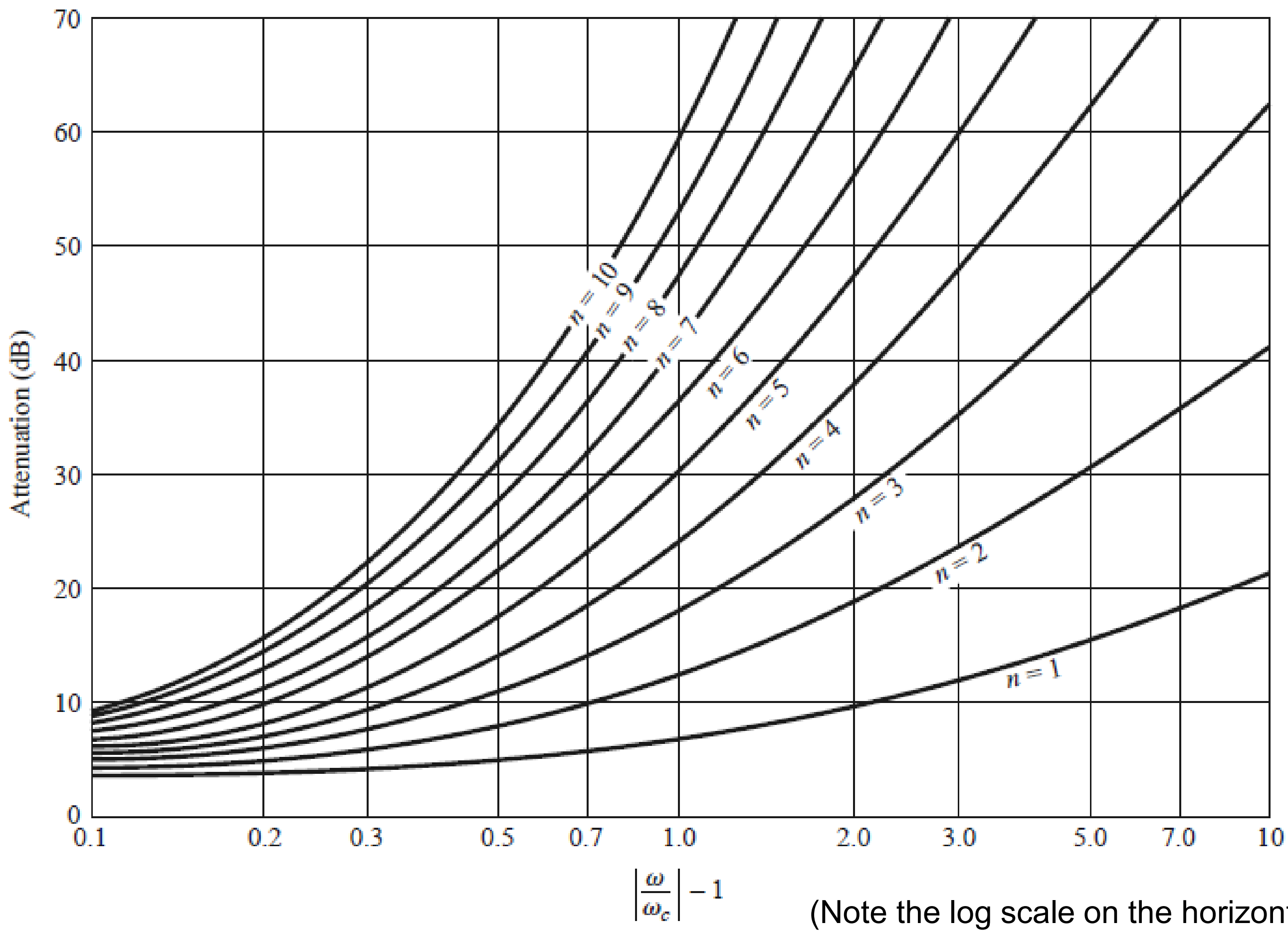
$k = 1$

Note : $R_L = R_0$

Source: Reprinted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House, Dedham, Mass., 1980, with permission.

Butterworth Attenuation Plot

Attenuation (Insertion Loss)



(Note the log scale on the horizontal axis.)

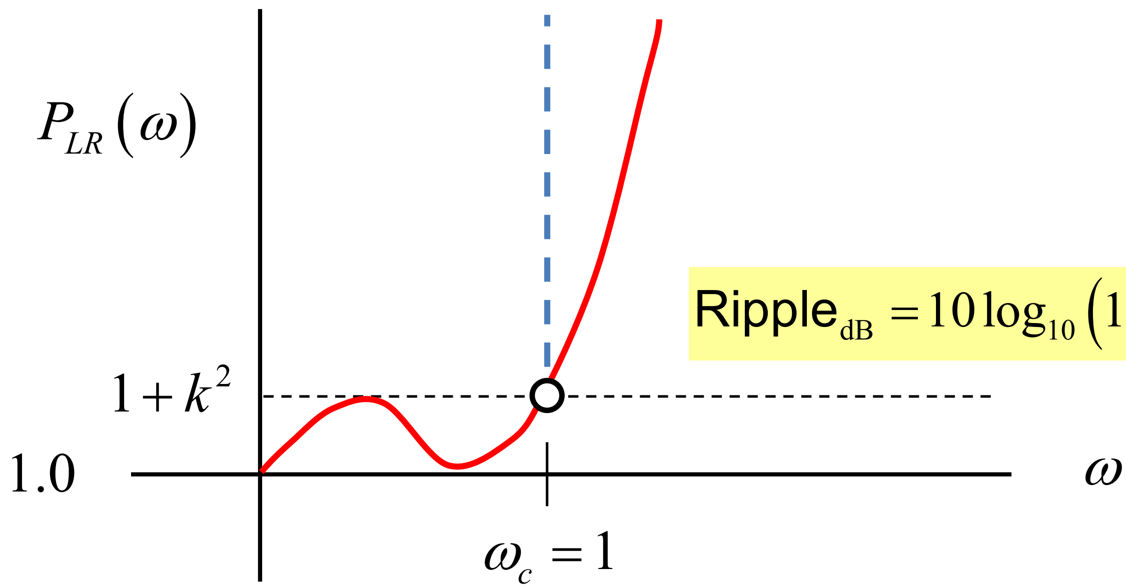
FIGURE 8.26 Attenuation versus normalized frequency for maximally flat filter prototypes.

Adapted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House, Dedham, Mass., 1980, with permission.

Chebyshev Behavior

Insertion Loss : $\mathrm{IL}(\omega) \equiv 10\log_{10} P_{LR}(\omega)$

$$\mathrm{IL}(\omega_c) = 10\log_{10}\left(1+k^2\right)\ \mathrm{dB}$$



$\mathrm{Ripple}_{\mathrm{dB}} = 10\log_{10}\left(1+k^2\right)$

Chebyshev Design Table

0.5 dB ripple

TABLE 8.4 Element Values for Equal-Ripple Low-Pass Filter Prototypes ($g_0 = 1, \omega_c = 1, N = 1$ to 10, 0.5 dB and 3.0 dB ripple)

0.5 dB Ripple

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	0.6986	1.0000									
2	1.4029	0.7071	1.9841								
3	1.5963	1.0967	1.5963	1.0000							
4	1.6703	1.1926	2.3661	0.8419	1.9841						
5	1.7058	1.2296	2.5408	1.2296	1.7058	1.0000					
6	1.7254	1.2479	2.6064	1.3137	2.4758	0.8696	1.9841				
7	1.7372	1.2583	2.6381	1.3444	2.6381	1.2583	1.7372	1.0000			
8	1.7451	1.2647	2.6564	1.3590	2.6964	1.3389	2.5093	0.8796	1.9841		
9	1.7504	1.2690	2.6678	1.3673	2.7239	1.3673	2.6678	1.2690	1.7504	1.0000	
10	1.7543	1.2721	2.6754	1.3725	2.7392	1.3806	2.7231	1.3485	2.5239	0.8842	1.9841

Note : N has to be odd for $R_L = R_0$.

$$\text{Ripple}_{\text{dB}} = 10\log_{10}\left(1+k^2\right)$$

Chebyshev Design Table

3.0 dB ripple

TABLE 8.4 Element Values for Equal-Ripple Low-Pass Filter Prototypes ($g_0 = 1, \omega_c = 1, N = 1$ to 10, 0.5 dB and 3.0 dB ripple)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	1.9953	1.0000									
2	3.1013	0.5339	5.8095								
3	3.3487	0.7117	3.3487	1.0000							
4	3.4389	0.7483	4.3471	0.5920	5.8095						
5	3.4817	0.7618	4.5381	0.7618	3.4817	1.0000					
6	3.5045	0.7685	4.6061	0.7929	4.4641	0.6033	5.8095				
7	3.5182	0.7723	4.6386	0.8039	4.6386	0.7723	3.5182	1.0000			
8	3.5277	0.7745	4.6575	0.8089	4.6990	0.8018	4.4990	0.6073	5.8095		
9	3.5340	0.7760	4.6692	0.8118	4.7272	0.8118	4.6692	0.7760	3.5340	1.0000	
10	3.5384	0.7771	4.6768	0.8136	4.7425	0.8164	4.7260	0.8051	4.5142	0.6091	5.8095

Source: Reprinted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House, Dedham, Mass.,1980, with permission.

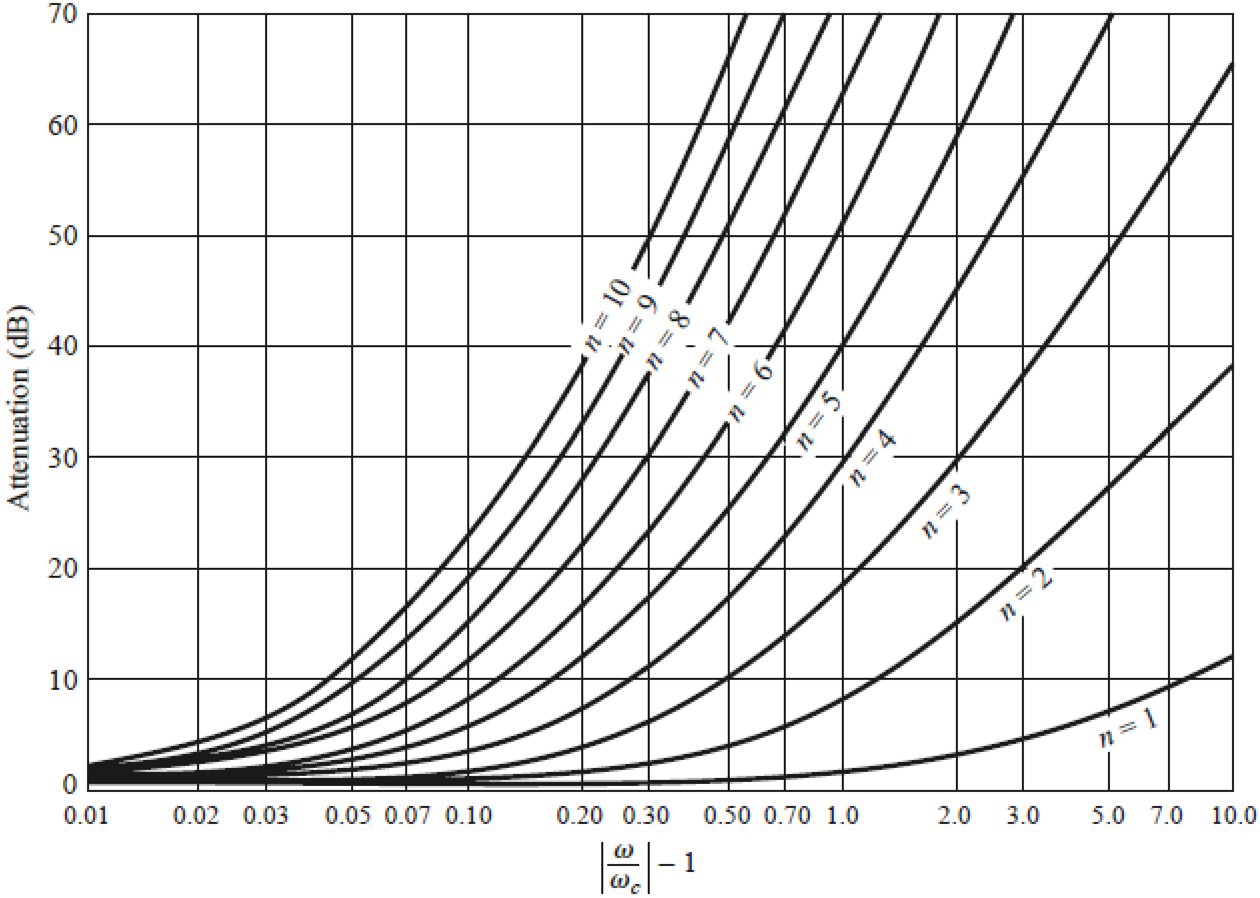
Note : N has to be odd for $R_L = R_0$.

$$\text{Ripple}_{\text{dB}} = 10\log_{10}\left(1+k^2\right)$$

Chebyshev Attenuation Plot

Attenuation (Insertion Loss)

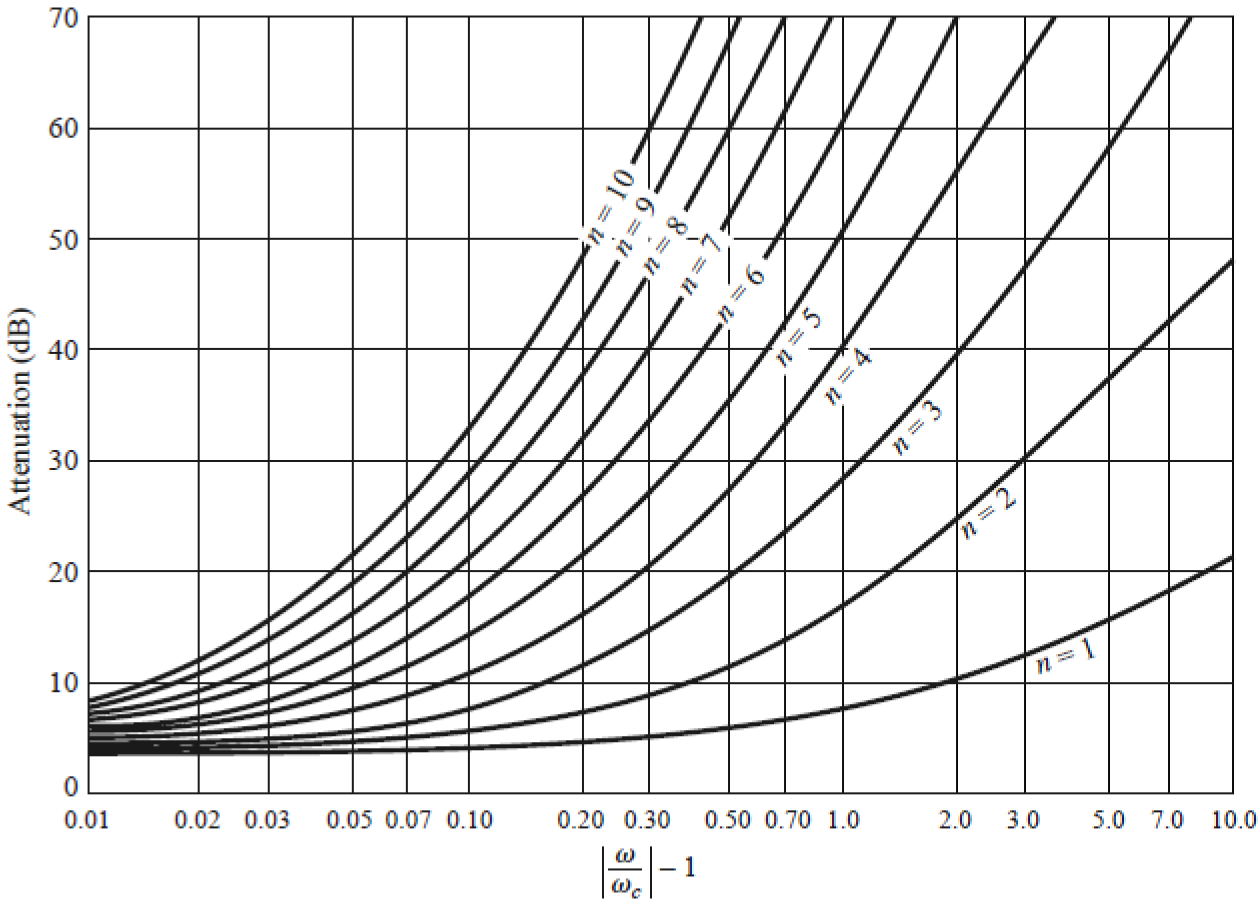
0.5 dB ripple



Chebyshev Attenuation Plot (cont.)

Attenuation (Insertion Loss)

3.0 dB ripple



Linear Phase Design Table

(Minimal Pulse Distortion)

TABLE 8.5 Element Values for Maximally Flat Time Delay Low-Pass Filter Prototypes ($g_0 = 1, \omega_c = 1, N = 1$ to 10)

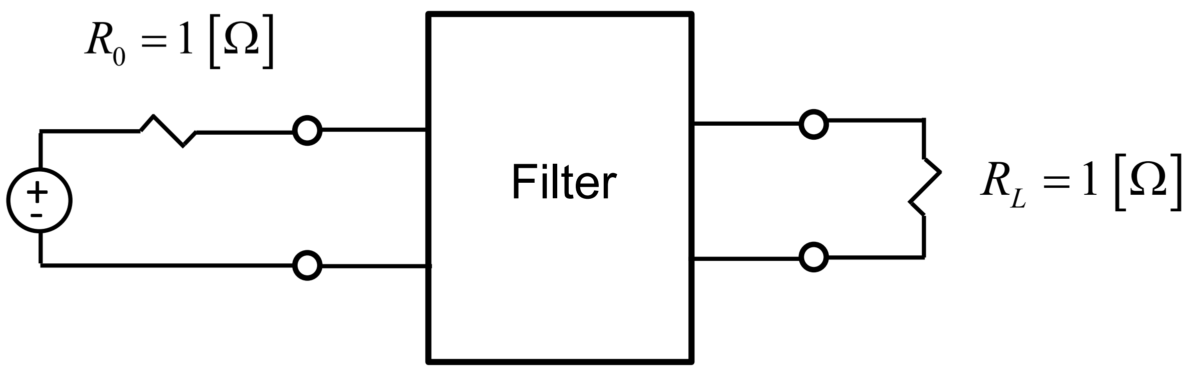
N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.5774	0.4226	1.0000								
3	1.2550	0.5528	0.1922	1.0000							
4	1.0598	0.5116	0.3181	0.1104	1.0000						
5	0.9303	0.4577	0.3312	0.2090	0.0718	1.0000					
6	0.8377	0.4116	0.3158	0.2364	0.1480	0.0505	1.0000				
7	0.7677	0.3744	0.2944	0.2378	0.1778	0.1104	0.0375	1.0000			
8	0.7125	0.3446	0.2735	0.2297	0.1867	0.1387	0.0855	0.0289	1.0000		
9	0.6678	0.3203	0.2547	0.2184	0.1859	0.1506	0.1111	0.0682	0.0230	1.0000	
10	0.6305	0.3002	0.2384	0.2066	0.1808	0.1539	0.1240	0.0911	0.0557	0.0187	1.0000

Source: Reprinted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House, Dedham, Mass., 1980, with permission.

Note : $R_L = R_0$

Example

Design a normalized low-pass Butterworth filter for a matched load with an attenuation greater than 15 dB when $\omega / \omega_c > 1.5$.



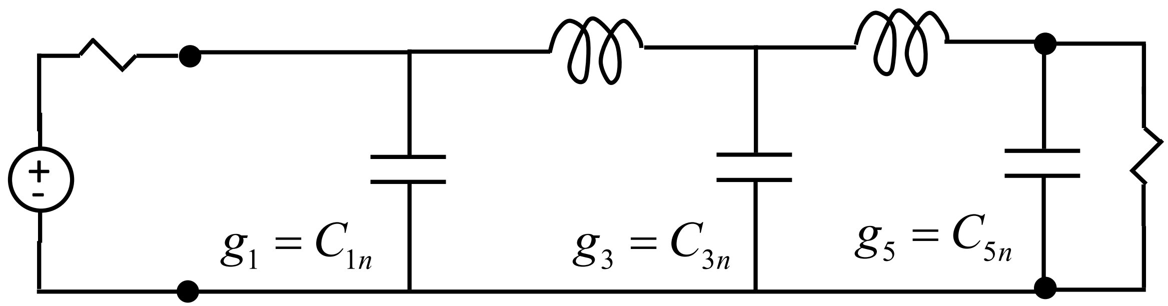
From attenuation plot: $N > 4$

Choose: $N = 5$

Choose "a" design

Example (cont.)

Proposed filter layout:



Recall: $g_{N+1} = \begin{cases} \text{normalized load resistance if } g_N \text{ is a shunt capacitance} \\ \text{normalized load conductance if } g_N \text{ is a series inductance} \end{cases}$

From Table:

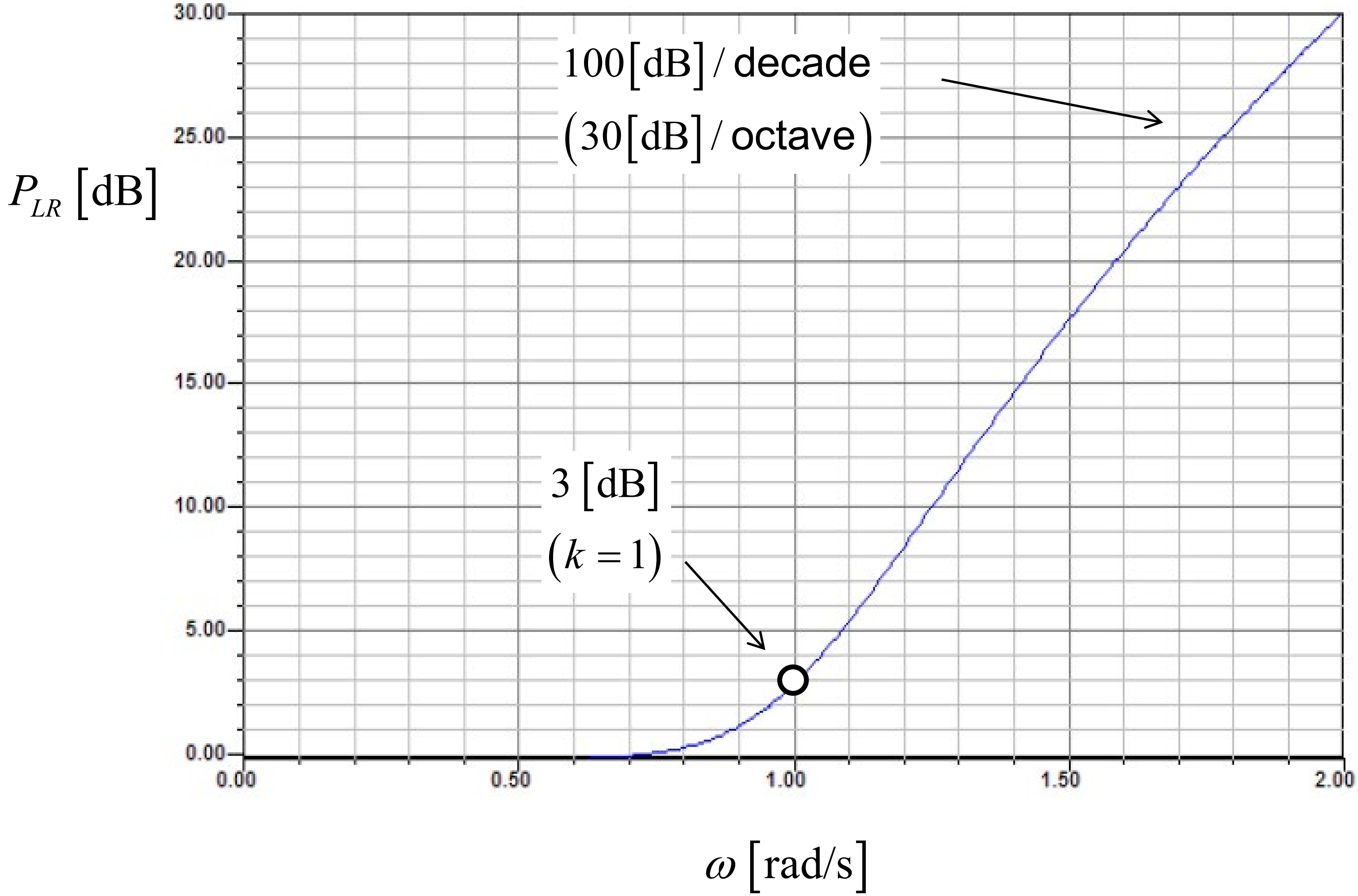
$$g_1 = 0.618$$
$$g_2 = 1.618$$
$$g_3 = 2.000$$
$$g_4 = 1.618$$
$$g_5 = 0.618$$

Hence:

$$C_{1n} = 0.618\ [\mathrm{F}]$$
$$L_{2n} = 1.618\ [\mathrm{H}]$$
$$C_{3n} = 2.000\ [\mathrm{F}]$$
$$L_{4n} = 1.618\ [\mathrm{H}]$$
$$C_{5n} = 0.618\ [\mathrm{F}]$$

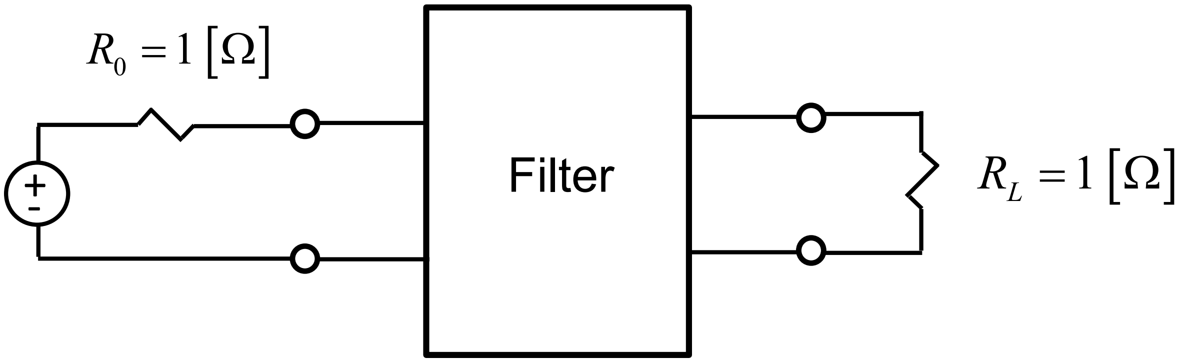
Example (cont.)

Results from Ansys Designer



Example

Design a normalized Chebyshev low-pass filter for a matched load with 3.0 dB of ripple in the passband and an attenuation greater than 15 dB when $\omega / \omega_c > 1.5$.



Note: N has to be odd when $R_L = R_0$.

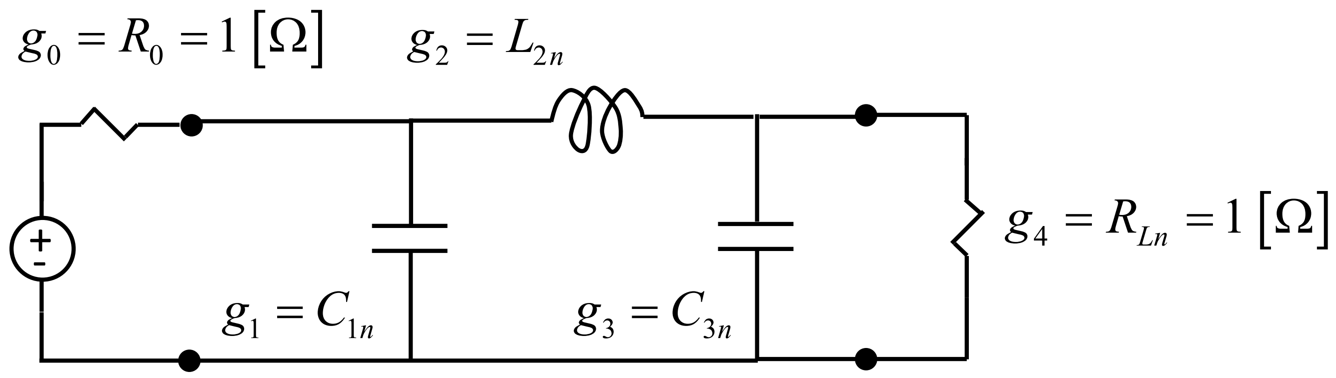
From attenuation plot: $N > 2$

Choose: $N = 3$

Choose "a" design

Example (cont.)

Proposed filter layout:



Recall : $g_{N+1} = \begin{cases} \text{normalized load resistance if } g_N \text{ is a shunt capacitance} \\ \text{normalized load conductance if } g_N \text{ is a series indutance} \end{cases}$

From Table:

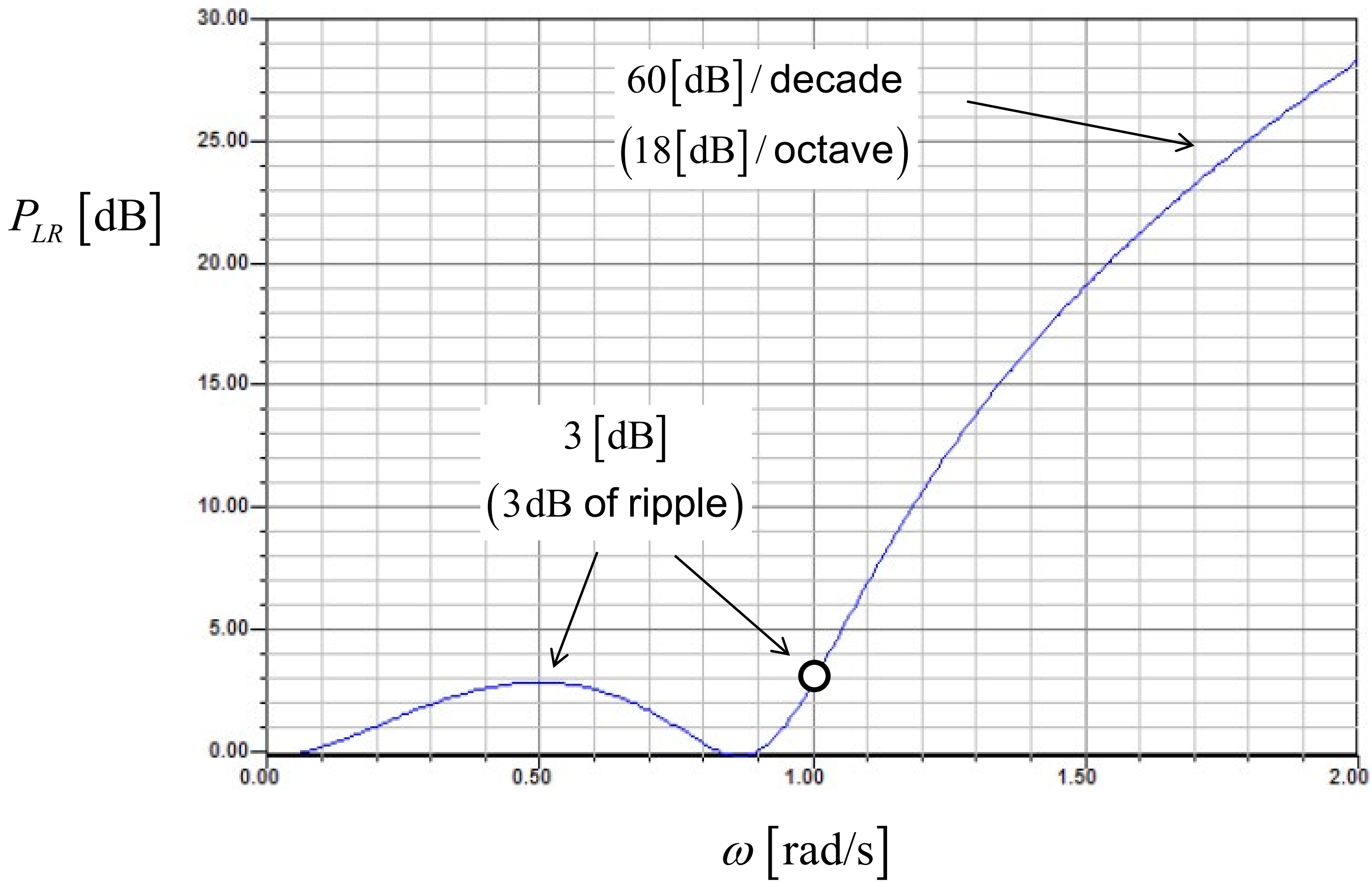
$$g_1 = 3.3487$$
$$g_2 = 0.7117$$
$$g_3 = 3.3487$$

Hence:

$$C_{1n} = 3.3487\ [\mathrm{F}]$$
$$L_{2n} = 0.7117\ [\mathrm{H}]$$
$$C_{3n} = 3.3487\ [\mathrm{F}]$$

Example (cont.)

Results from Ansys Designer



Denormalization

Impedance scaling:

- This accounts for arbitrary R_s and R_L
- Scale all impedances by R_0.

$$R_s' = (1)\,R_0$$
$$R_L' = (R_{Ln})\,R_0$$
$$L' = (L_n)\,R_0$$
$$C' = (C_n)\,/\,R_0$$

Example : $R_0 = 50\ [\Omega]$

The prime denotes that there is no longer impedance scaling, but a normalized frequency is still being used ($\omega_c = 1$).

Denormalization (cont.)

Frequency scaling:

- This allows us to shift from $\omega_c = 1$ to arbitrary ω_c
- Replace ω with ω / ω_c (and require same impedances)

ω in prototype $\qquad$ ω in final filter

$$j\omega L' \to j\left(\frac{\omega}{\omega_c}\right)L' = j\omega\left(\frac{L'}{\omega_c}\right) = j\omega L$$

$$j\omega C' \to j\left(\frac{\omega}{\omega_c}\right)C' = j\omega\left(\frac{C'}{\omega_c}\right) = j\omega C$$

Hence:

$$R_s = R_0$$
$$R_L = R_0$$
$$L = (L') / \omega_c$$
$$C = (C') / \omega_c$$

Denormalization (cont.)

Impedance and frequency scaling:

- This scales the impedance and shifts from $\omega_c = 1$ to arbitrary ω_c .

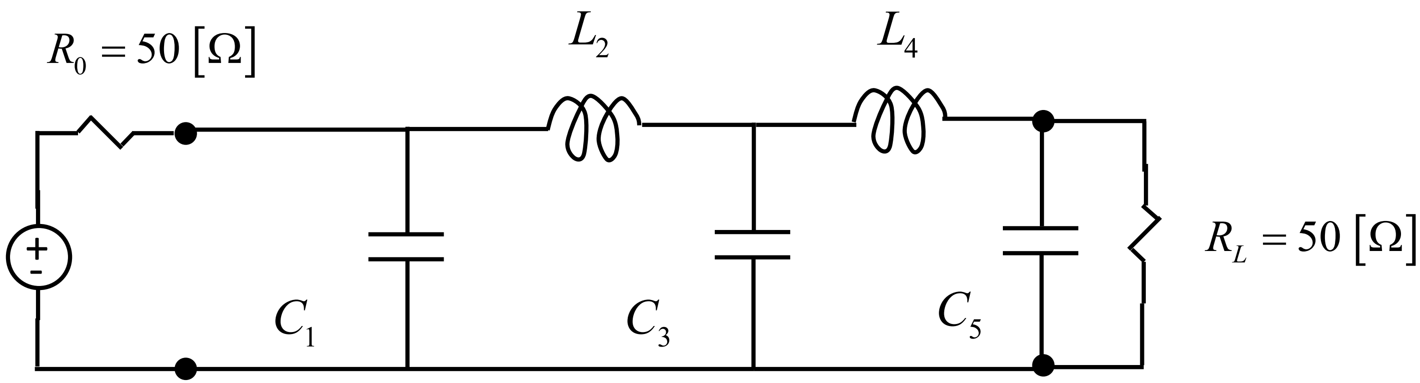
$$
\begin{aligned}
R_s &= R_0 \\
R_L &= R_{Ln} R_0 \\
L &= \left(L_n R_0 \right) / \omega_c \\
C &= \left(C_n / R_0 \right) / \omega_c
\end{aligned}
$$

This takes us from the normalized "prototype" low-pass filter
to the final low-pass filter.

Example

Design a low-pass Butterworth filter for a matched 50 Ω load with $f_c = 1.0 \text{ GHz}$ and an attenuation greater than 15 dB when $\omega / \omega_c > 1.5$.

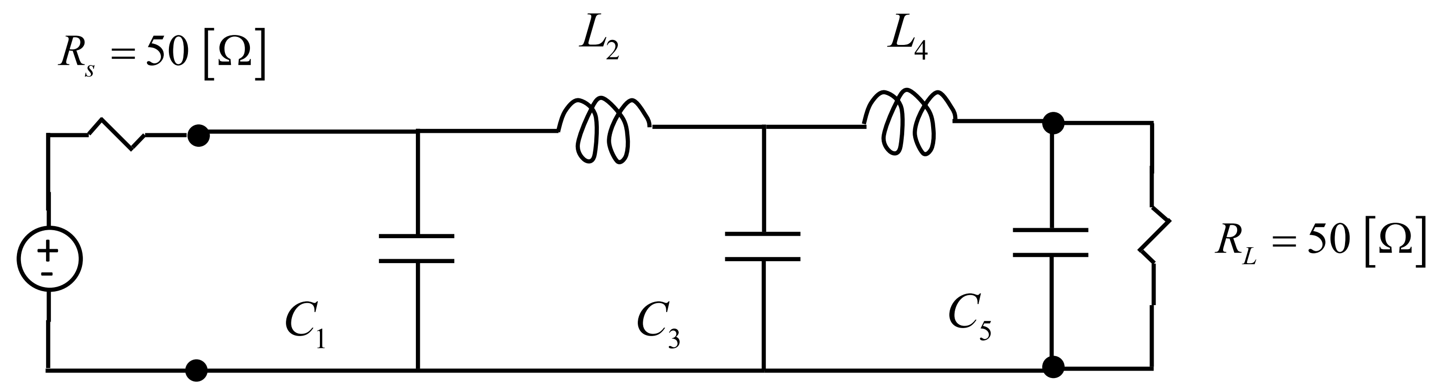
Choose type “a” design (arbitrary choice)



Recall the normalized design:

$$g_1 = 0.618 \qquad C_{1n} = 0.618\ [\mathrm{F}]$$
$$g_2 = 1.618 \qquad L_{2n} = 1.618\ [\mathrm{H}]$$
$$g_3 = 2.000 \qquad C_{3n} = 2.000\ [\mathrm{F}]$$
$$g_4 = 1.618 \qquad L_{4n} = 1.618\ [\mathrm{H}]$$
$$g_5 = 0.618 \qquad C_{5n} = 0.618\ [\mathrm{F}]$$

Example (cont.)



De-normalization:

$$R_s = R_0$$
$$R_L = R_{Ln} R_0$$
$$L = \left(L_n R_0\right) / \omega_c$$
$$C = \left(C_n / R_0\right) / \omega_c$$

$\Rightarrow$

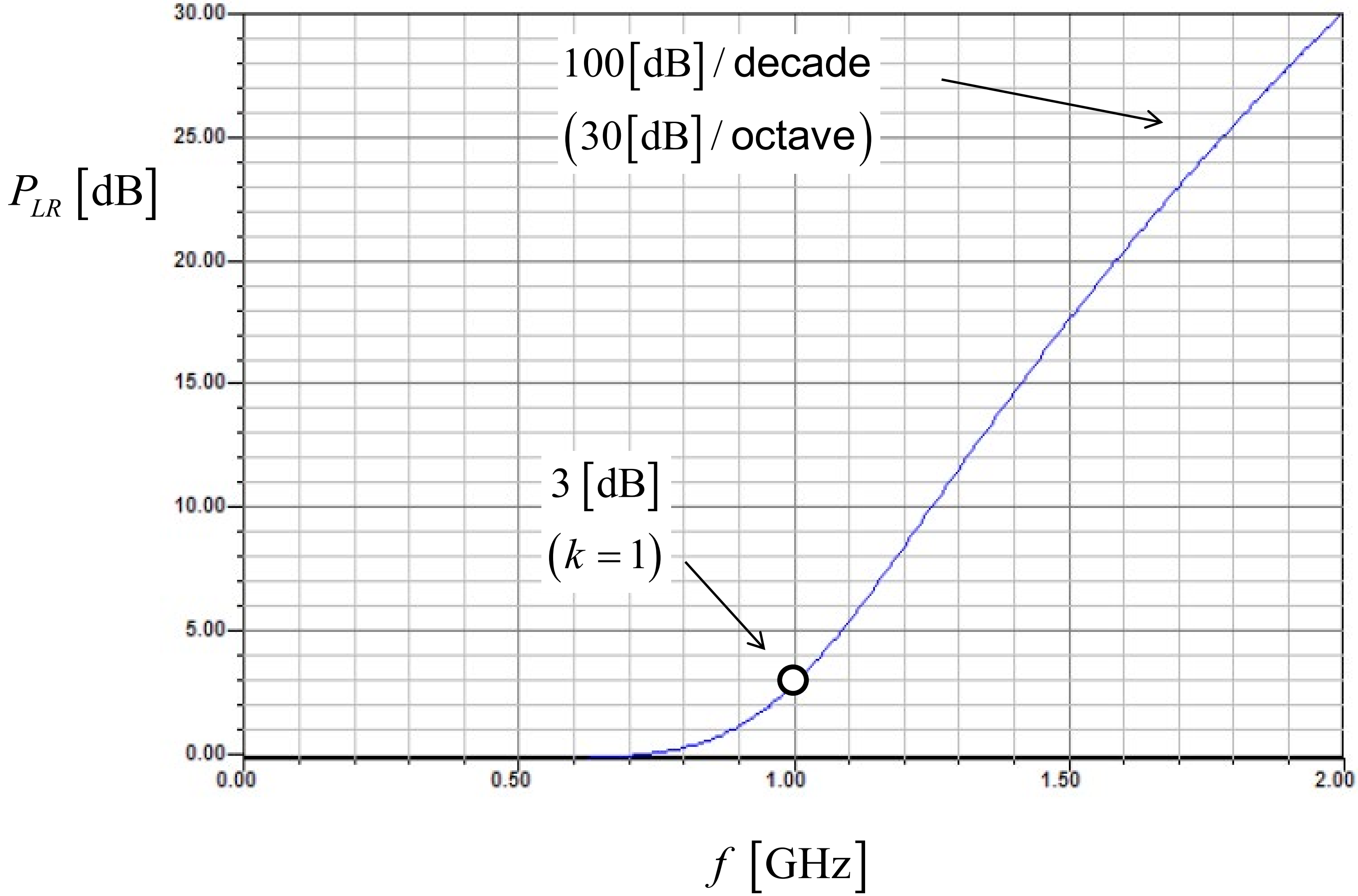
$$R_s = 50$$
$$R_L = R_{Ln} 50$$
$$L = \left(L_n 50\right) / \left(2\pi 10^9\right)$$
$$C = \left(C_n / 50\right) / \left(2\pi 10^9\right)$$

$\Rightarrow$

$$C_1 = 1.967\ [\mathrm{pF}]$$
$$L_2 = 12.88\ [\mathrm{nH}]$$
$$C_3 = 6.366\ [\mathrm{pF}]$$
$$L_4 = 12.88\ [\mathrm{nH}]$$
$$C_5 = 1.967\ [\mathrm{pF}]$$

Example (cont.)

Results (from Ansys Designer)



Frequency Transformation

Normalized low-pass $\rightarrow$ High-pass

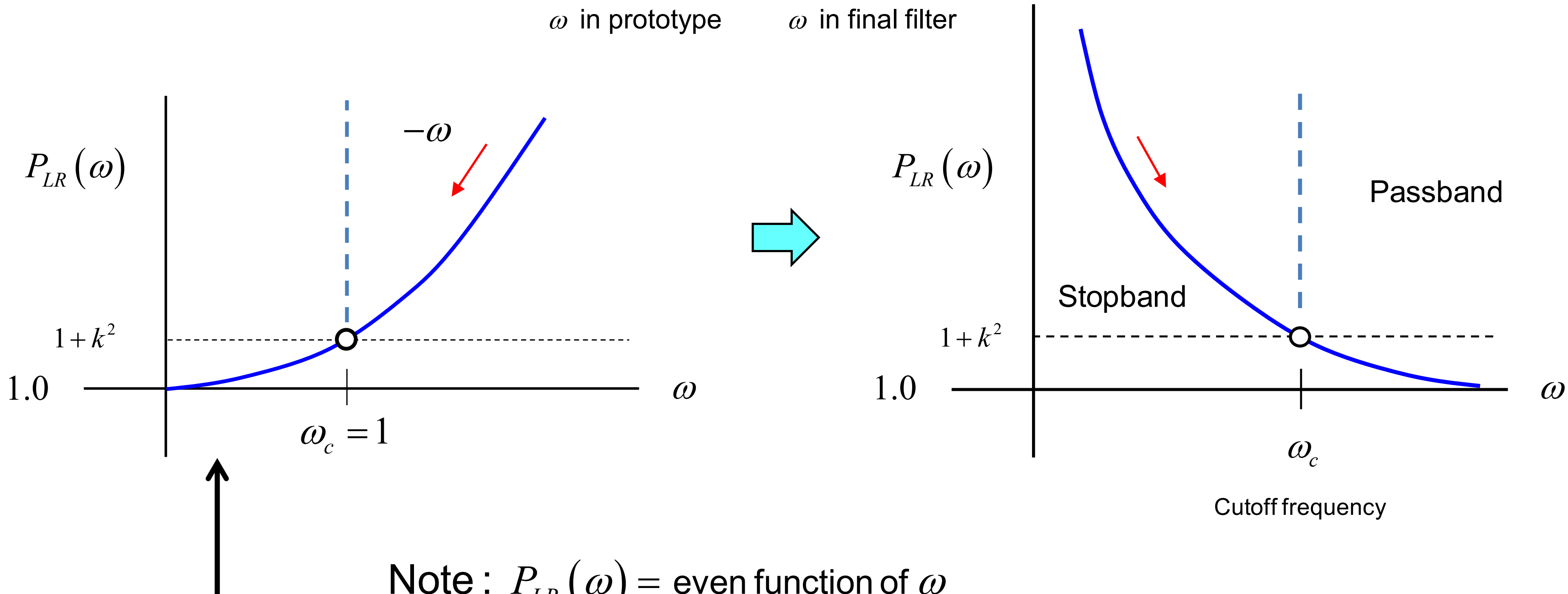
Replace: $\omega \rightarrow -\dfrac{\omega_c}{\omega}$

ω in prototype $\qquad$ ω in final filter

$\omega = -\infty \rightarrow \omega = 0$

$\omega = -1 \rightarrow \omega = \omega_c$

$\omega = 0 \rightarrow \omega = \infty$



Note : $P_{LR}(\omega) =$ even function of ω

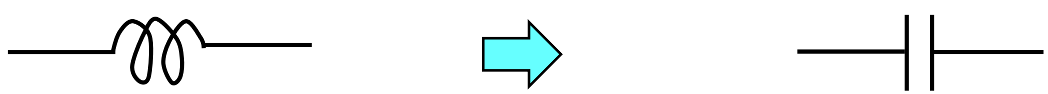
(Negative values of ω in the normalized prototype (red color) have been converted to positive values.)

Frequency Transformation (cont.)

What happens to the circuit elements in the prototype?

$$\omega \rightarrow -\frac{\omega_c}{\omega}$$

$$j\omega L_n \rightarrow j\left(-\frac{\omega_c}{\omega}\right)L_n = \frac{1}{j\omega\left(1/\left(\omega_c L_n\right)\right)} = \frac{1}{j\omega C}$$



L_n $\quad$ C

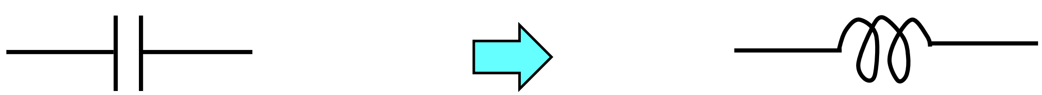
$$C = \frac{1}{\omega_c L_n}$$

We also need to <u>divide</u> C by a factor of R_0 to account for impedance scaling.

Frequency Transformation (cont.)

$$\omega \rightarrow -\frac{\omega_c}{\omega}$$

$$j\omega C_n \rightarrow j\left(-\frac{\omega_c}{\omega}\right)C_n = \frac{1}{j\omega\left(1/\left(\omega_c C_n\right)\right)} = \frac{1}{j\omega L}$$



C_n $\qquad$ L

$$L = \frac{1}{\omega_c C_n}$$

Also, we need to <u>multiply</u> L by a factor of R_0 to account for impedance scaling.

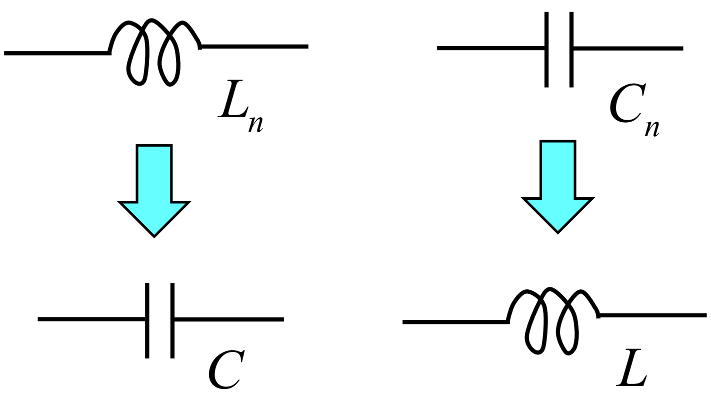
Frequency Transformation (cont.)

Summary

Normalized low-pass $\rightarrow$ High-pass

$$\omega \rightarrow -\frac{\omega_c}{\omega}$$

Normalized low-pass



Final high-pass

$$R_s = R_0$$

$$R_L = R_{Ln} R_0$$

$$C = \frac{1}{\omega_c L_n R_0}$$

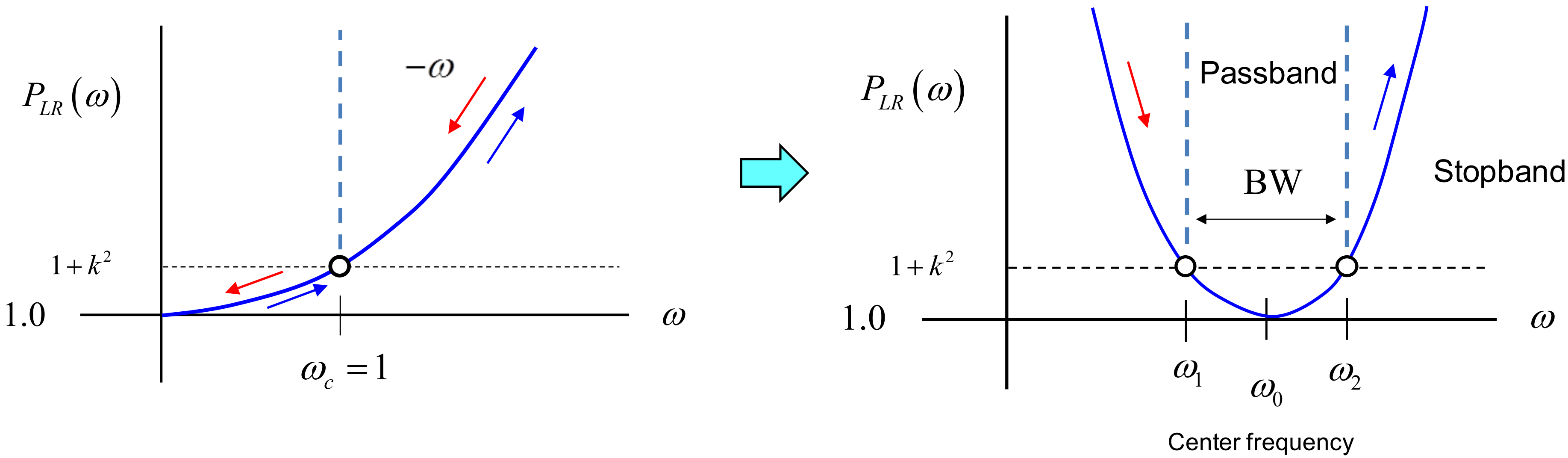
$$L = \frac{R_0}{\omega_c C_n}$$

Frequency Transformation (cont.)

Normalized low pass $\rightarrow$ Bandpass

Replace: $\omega \rightarrow \dfrac{1}{\Delta}\left(\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega}\right)$ $\quad \Delta = \dfrac{\omega_2 - \omega_1}{\omega_0}$ $\quad \omega_0 = \sqrt{\omega_1 \omega_2}$

Relative bandwidth



Note : $P_{LR}(\omega) =$ even function of ω

(Negative values of ω in the normalized prototype (red color) have been converted to positive values.)

Frequency Transformation (cont.)

Verification of mapping

Denormalized : $\omega = \omega_1$

Normalized :

$$\omega = \frac{1}{\Delta}\left(\frac{\omega_1}{\omega_0} - \frac{\omega_0}{\omega_1}\right)$$

$$= \frac{1}{\Delta}\left(\frac{\omega_1^2 - \omega_0^2}{\omega_0\omega_1}\right)$$

$$= \frac{\omega_1^2 - \omega_0^2}{(\omega_2 - \omega_1)\omega_1}$$

$$= \frac{\omega_1^2 - \omega_0^2}{\omega_1\omega_2 - \omega_1^2}$$

$$= \frac{\omega_1^2 - \omega_0^2}{\omega_0^2 - \omega_1^2}$$

$$= -1$$

Denormalized : $\omega = \omega_2$

Normalized :

$$\omega = \frac{1}{\Delta}\left(\frac{\omega_2}{\omega_0} - \frac{\omega_0}{\omega_2}\right)$$

$$= \frac{1}{\Delta}\left(\frac{\omega_2^2 - \omega_0^2}{\omega_0\omega_2}\right)$$

$$= \frac{\omega_2^2 - \omega_0^2}{(\omega_2 - \omega_1)\omega_2}$$

$$= \frac{\omega_2^2 - \omega_0^2}{\omega_2^2 - \omega_1\omega_2}$$

$$= \frac{\omega_2^2 - \omega_0^2}{\omega_2^2 - \omega_0^2}$$

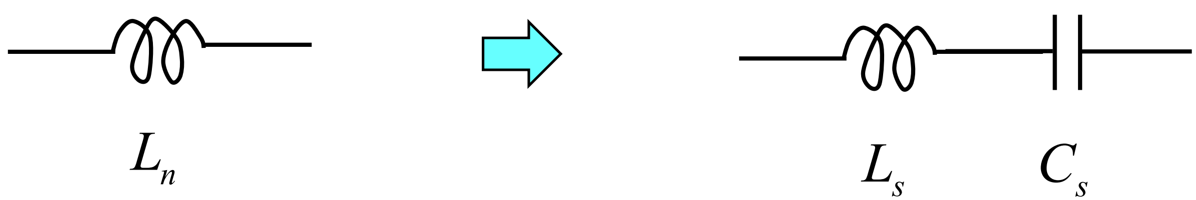
$$= 1$$

Frequency Transformation (cont.)

Transformation of elements

$$\omega \rightarrow \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$

$$j\omega L_n \rightarrow j\frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)L_n = j\omega\left(\frac{L_n}{\omega_0 \Delta}\right) + \frac{1}{j\omega\left(\Delta / \left(L_n \omega_0\right)\right)} = j\omega L_s + \frac{1}{j\omega C_s}$$



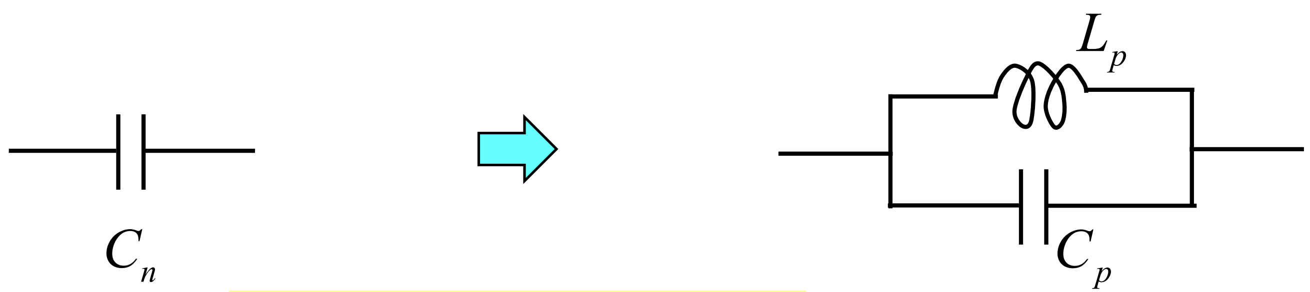
$$L_s = \frac{L_n}{\omega_0 \Delta} \quad , \quad C_s = \frac{\Delta}{L_n \omega_0}$$

Also, we need to add factors of R_0 to account for impedance scaling (multiply L_s with R_0, divide C_s by R_0).

Frequency Transformation (cont.)

$$\omega \to \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$

$$j\omega C_n \to j\frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)C_n = j\omega\left(\frac{C_n}{\omega_0\Delta}\right) + \frac{1}{j\omega\left(\Delta / \left(C_n\omega_0\right)\right)} = j\omega C_p + \frac{1}{j\omega L_p}$$



$$C_p = \frac{C_n}{\omega_0\Delta} \quad , \quad L_p = \frac{\Delta}{C_n\omega_0}$$

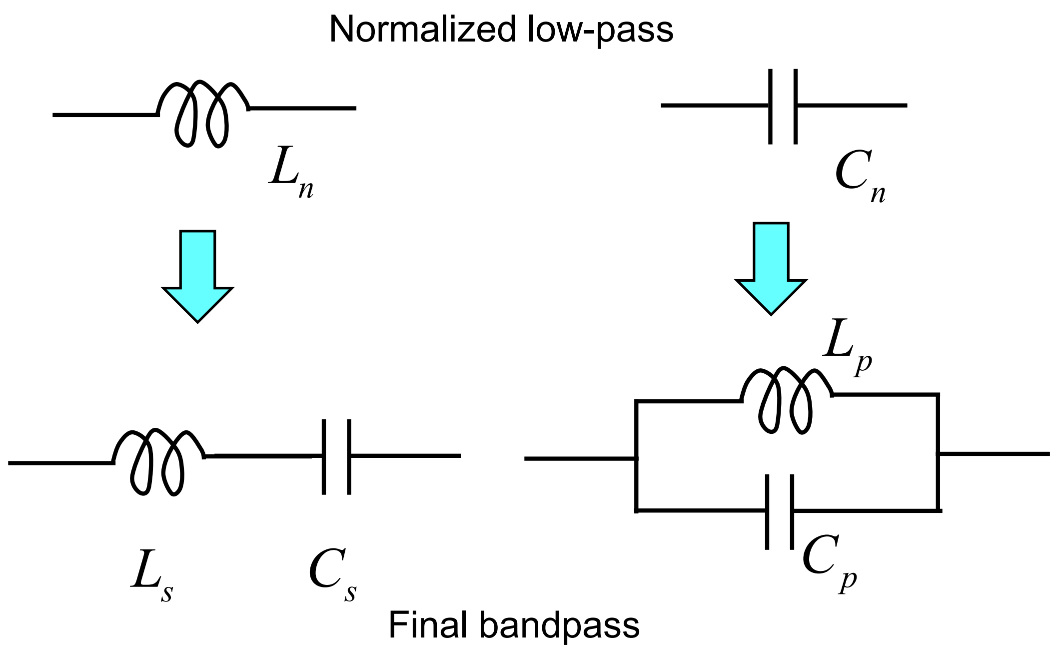
Also, we need to add factors of R_0 to account for impedance scaling (multiply L_p with R_0, divide C_p by R_0).

Frequency Transformation (cont.)

Summary

Normalized low-pass → Bandpass

$$\omega \rightarrow \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$



$$R_s = R_0$$

$$R_L = R_{Ln} R_0$$

$$L_s = \frac{L_n R_0}{\omega_0 \Delta}$$

$$C_s = \frac{\Delta}{L_n \omega_0 R_0}$$

$$L_p = \frac{\Delta R_0}{C_n \omega_0}$$

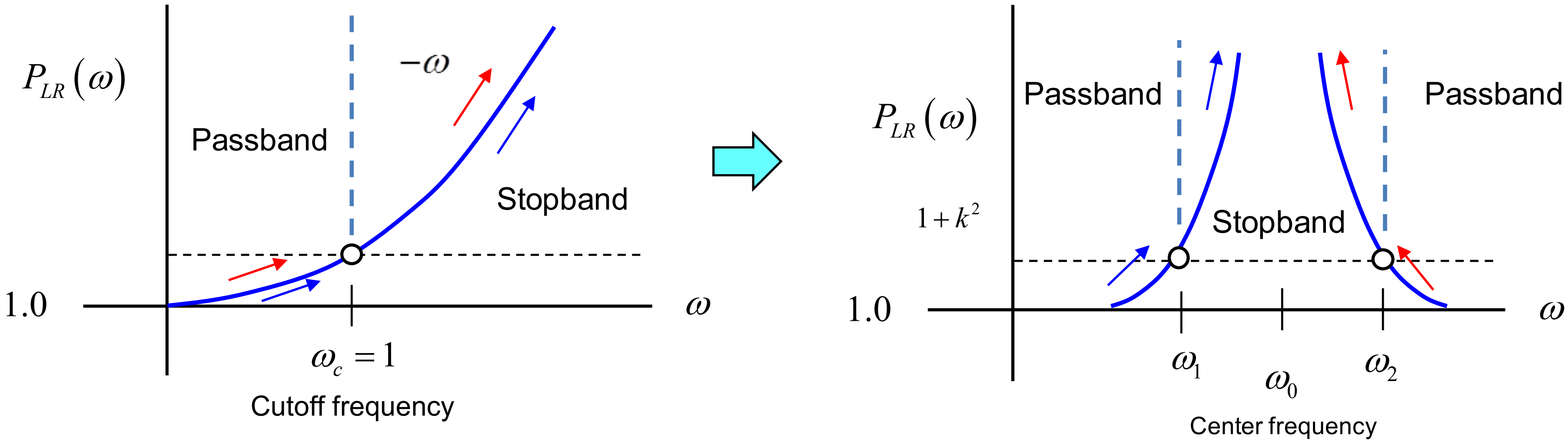
$$C_p = \frac{C_n}{\omega_0 \Delta R_0}$$

Frequency Transformation (cont.)

Normalized low pass $\rightarrow$ Bandstop

Replace: $\omega \rightarrow -\Delta\left(\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega}\right)^{-1}$ $\quad \Delta = \dfrac{\omega_2 - \omega_1}{\omega_0}$ $\quad \omega_0 = \sqrt{\omega_1 \omega_2}$

Relative bandwidth



Note : $P_{LR}(\omega)$ = even function of ω

(Negative values of ω in the normalized prototype (red color) have been converted to positive values.)

Frequency Transformation (cont.)

Verification of mapping

Denormalized : $\omega = \omega_1$

Normalized :

$$\omega = -\Delta\left(\frac{\omega_1}{\omega_0} - \frac{\omega_0}{\omega_1}\right)^{-1}$$

$$= -\Delta\left(\frac{\omega_1^2 - \omega_0^2}{\omega_0\omega_1}\right)^{-1}$$

$$= -\frac{\omega_2 - \omega_1}{\omega_0}\frac{\omega_0\omega_1}{\omega_1^2 - \omega_0^2}$$

$$= -\frac{\omega_1\omega_2 - \omega_1^2}{\omega_1^2 - \omega_0^2}$$

$$= \frac{\omega_1^2 - \omega_0^2}{\omega_1^2 - \omega_0^2}$$

$$= 1$$

Denormalized : $\omega = \omega_2$

Normalized :

$$\omega = -\Delta\left(\frac{\omega_2}{\omega_0} - \frac{\omega_0}{\omega_2}\right)^{-1}$$

$$= -\Delta\left(\frac{\omega_2^2 - \omega_0^2}{\omega_0\omega_2}\right)^{-1}$$

$$= -\frac{\omega_2 - \omega_1}{\omega_0}\frac{\omega_0\omega_2}{\omega_2^2 - \omega_0^2}$$

$$= -\frac{\omega_2^2 - \omega_1\omega_2}{\omega_2^2 - \omega_0^2}$$

$$= -\frac{\omega_2^2 - \omega_0^2}{\omega_2^2 - \omega_0^2}$$

$$= -1$$

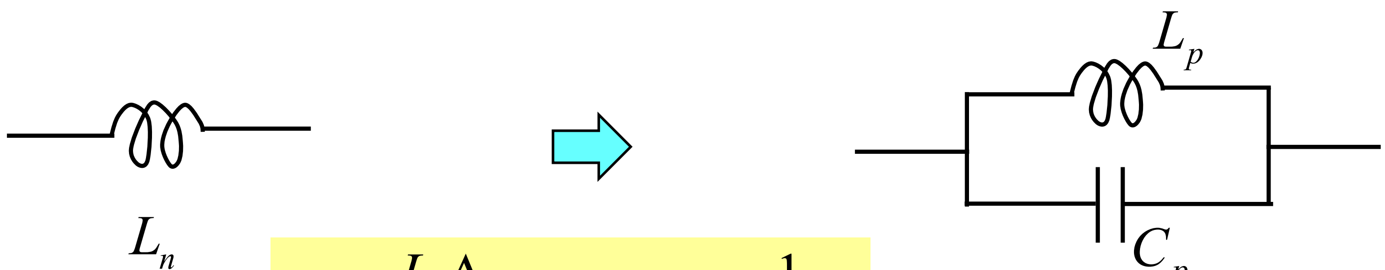
Frequency Transformation (cont.)

Transformation of elements

$$\omega \to -\Delta\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)^{-1}$$

$$j\omega L_n \to j\left(-\Delta\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)^{-1}\right)L_n \quad , \quad \left(j\omega L_n\right)^{-1} \to \frac{j}{\Delta}\left(\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)\right)\frac{1}{L_n}$$

so $$\left(j\omega L_n\right)^{-1} \to j\omega\left(\frac{1}{\omega_0 L_n \Delta}\right)+\frac{1}{j\omega\left(L_n\Delta/\omega_0\right)}$$



$$L_p = \frac{L_n\Delta}{\omega_0} \quad , \quad C_p = \frac{1}{L_n\omega_0\Delta}$$

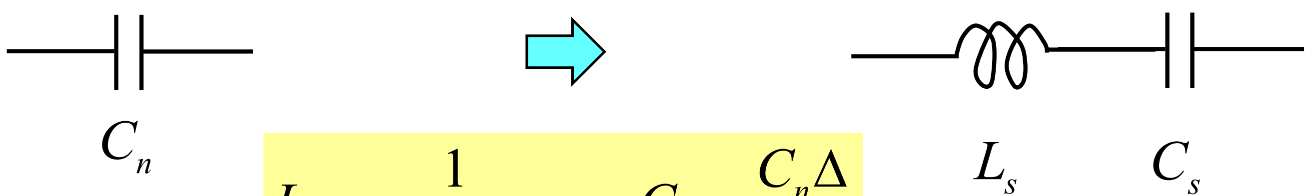
Also, we need to add factors of R_0 to account for impedance scaling (multiply L_p with R_0, divide C_p by R_0).

Frequency Transformation (cont.)

$$\omega \rightarrow -\Delta\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^{-1}$$

$$j\omega C_n \rightarrow j\left(-\Delta\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^{-1}\right)C_n \quad , \quad \left(j\omega C_n\right)^{-1} \rightarrow \frac{j}{\Delta}\left(\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right)\frac{1}{C_n}$$

so $$\left(j\omega C_n\right)^{-1} \rightarrow j\omega\left(\frac{1}{\omega_0 C_n \Delta}\right) + \frac{1}{j\omega\left(C_n\Delta/\omega_0\right)}$$



$$L_s = \frac{1}{\omega_0 C_n \Delta} \quad , \quad C_s = \frac{C_n \Delta}{\omega_0}$$

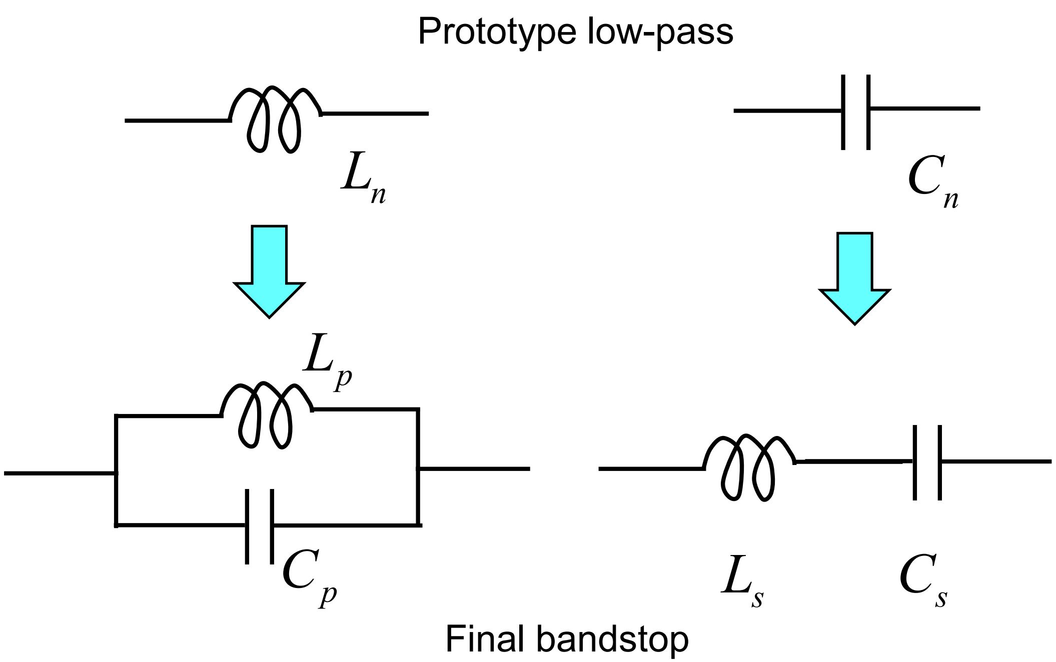
Also, we need to add factors of R_0 to account for impedance scaling (multiply L_s with R_0, divide C_s by R_0).

Frequency Transformation (cont.)

Summary

Normalized low-pass → Bandstop

$$\omega \to -\Delta\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)^{-1}$$



$$R_s = R_0$$

$$R_L = R_{Ln} R_0$$

$$L_p = \frac{L_n R_0 \Delta}{\omega_0}$$

$$C_p = \frac{1}{L_n \omega_0 R_0 \Delta}$$

$$L_s = \frac{R_0}{\omega_0 C_n \Delta}$$

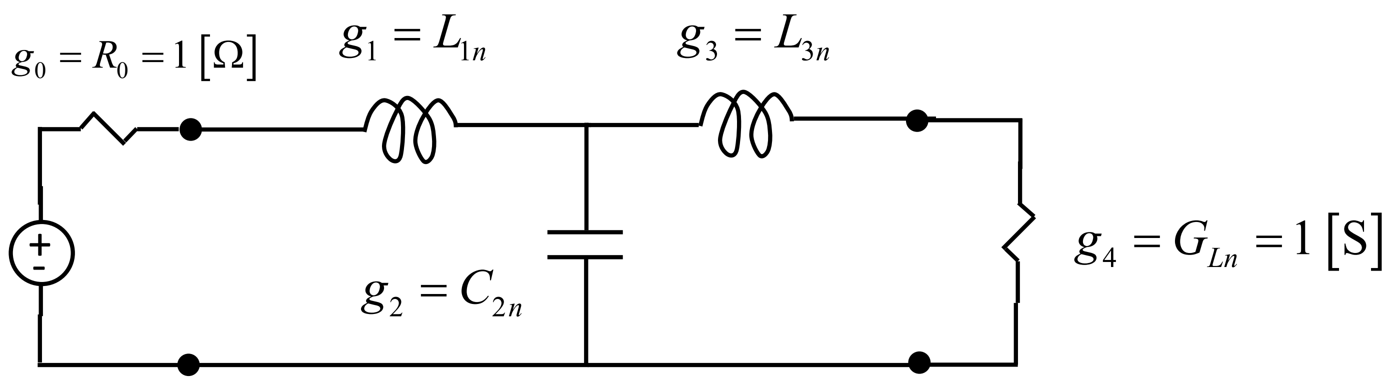
$$C_s = \frac{C_n \Delta}{\omega_0 R_0}$$

Example

Design an $N = 3$ Chebyshev bandpass filter for a matched 50 Ω load with 0.5 dB of ripple in the passband, a 10% bandwidth, and a center frequency of 1.0 GHz.

Choose type "b" low-pass prototype:

$$N = 3 = \text{odd}$$
$$\Rightarrow g_4 = 1$$



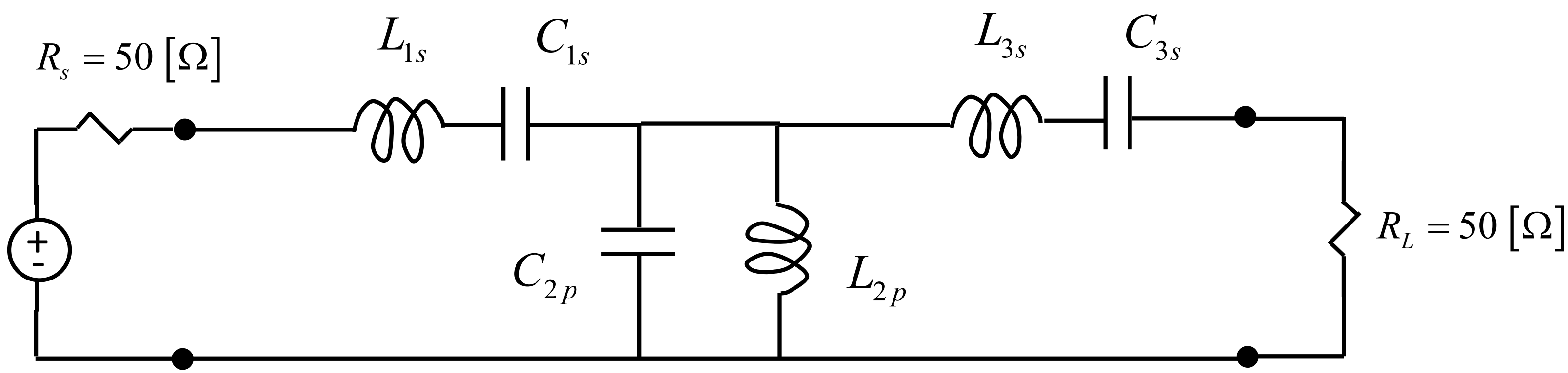
From table:

$$g_1 = L_{1n} = 1.5963$$
$$g_2 = C_{2n} = 1.0967$$
$$g_3 = L_{3n} = 1.5963$$

$$g_{N+1} = \begin{cases} \text{normalized load resistance if } g_N \text{ is a shunt capacitance} \\ \text{normalized load conductance if } g_N \text{ is a series inductance} \end{cases}$$

Example (cont.)

Transform to bandpass:



From table:

$$g_1 = L_{1n} = 1.5963$$
$$g_2 = C_{2n} = 1.0967$$
$$g_3 = L_{3n} = 1.5963$$

For $k = 1, 3$:

$$L_s = \frac{L_n R_0}{\omega_0 \Delta}$$
$$C_s = \frac{\Delta}{L_n \omega_0 R_0}$$

For $k = 2$:

$$L_p = \frac{R_0 \Delta}{C_n \omega_0}$$
$$C_p = \frac{C_n}{\omega_0 R_0 \Delta}$$

Bandpass:

$$R_s = R_0$$
$$R_L = R_{Ln} R_0$$
$$L_s = \frac{L_n R_0}{\omega_0 \Delta}$$
$$C_s = \frac{\Delta}{L_n \omega_0 R_0}$$
$$L_p = \frac{\Delta R_0}{C_n \omega_0}$$
$$C_p = \frac{C_n}{\omega_0 \Delta R_0}$$

Example (cont.)

Hence we have:

$$L_{1s} = \frac{L_{n1}R_0}{\omega_0 \Delta}$$

$$C_{1s} = \frac{\Delta}{L_{n1}\omega_0 R_0}$$

$$L_{2p} = \frac{R_0 \Delta}{C_{n2}\omega_0}$$

$$C_{2p} = \frac{C_{n2}}{\omega_0 R_0 \Delta}$$

$$L_{3s} = \frac{L_{n3}R_0}{\omega_0 \Delta}$$

$$C_{3s} = \frac{\Delta}{L_{n3}\omega_0 R_0}$$

$$L_{1s} = \frac{(1.5963)(50)}{\left(2\pi 10^9\right)(0.1)}$$

$$C_{1s} = \frac{(0.1)}{(1.5963)\left(2\pi 10^9\right)(50)}$$

$$L_{2p} = \frac{(50)(0.1)}{(1.0967)\left(2\pi 10^9\right)}$$

$$C_{2p} = \frac{(1.0967)}{\left(2\pi 10^9\right)(50)(0.1)}$$

$$L_{3s} = \frac{(1.5963)(50)}{\left(2\pi 10^9\right)(0.1)}$$

$$C_{3s} = \frac{(0.1)}{(1.5963)\left(2\pi 10^9\right)(50)}$$

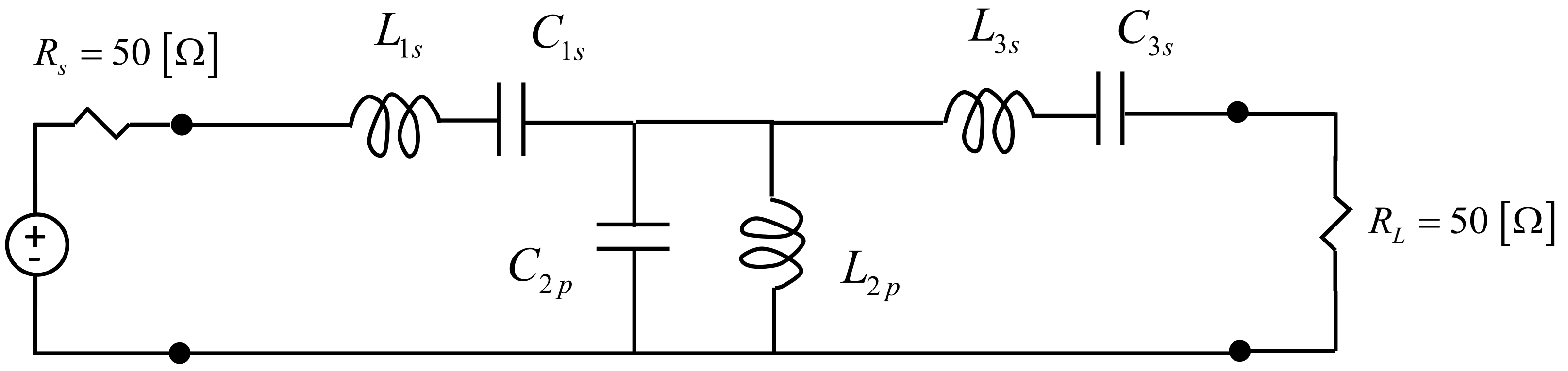
Example (cont.)

This gives us:

$$L_{1s} = 127\ [\mathrm{nH}]$$
$$C_{1s} = 0.199\ [\mathrm{nF}]$$

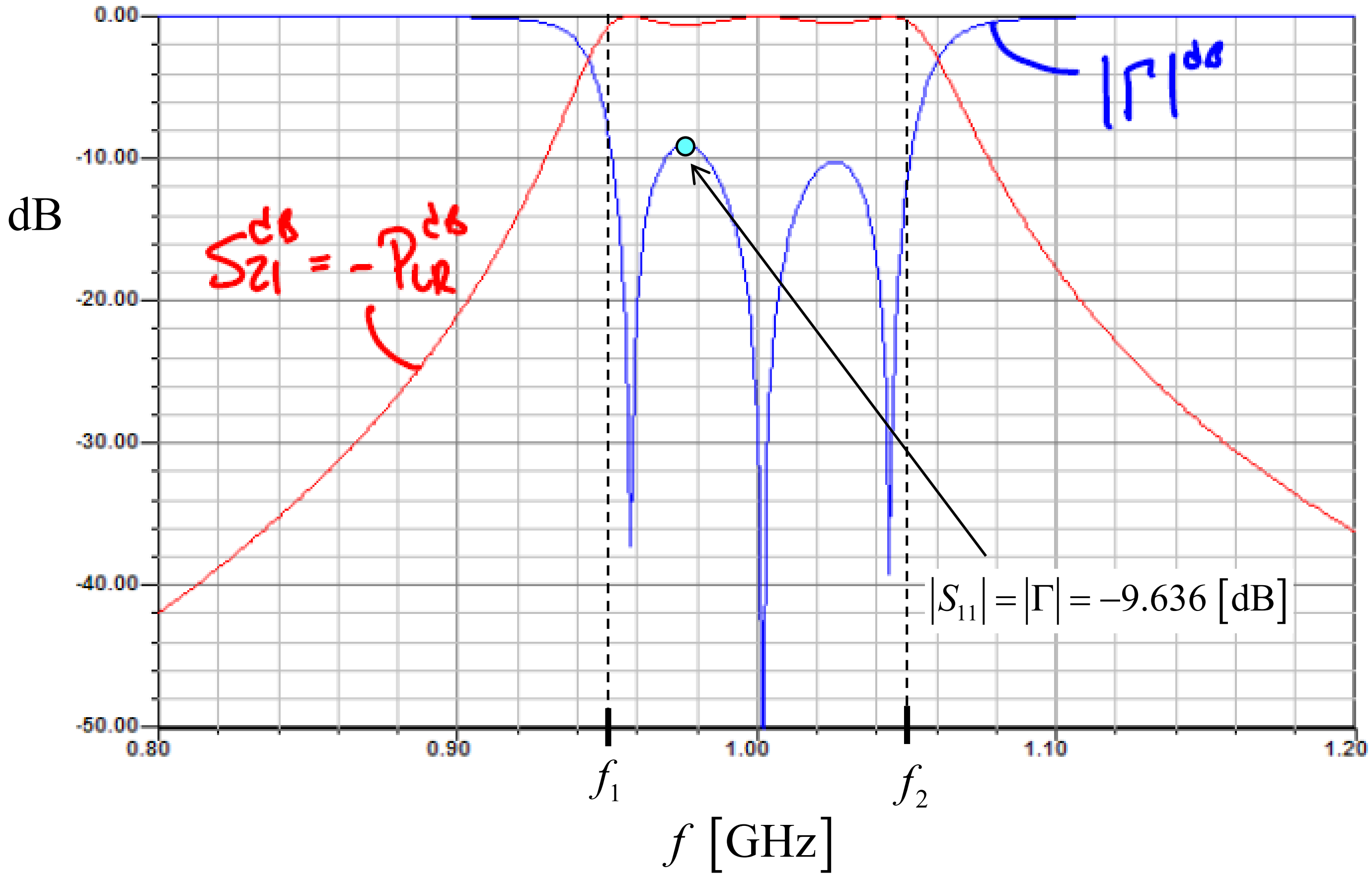
$$L_{2p} = 0.726\ [\mathrm{nH}]$$
$$C_{2p} = 34.9\ [\mathrm{pF}]$$

$$L_{3s} = 127\ [\mathrm{nH}]$$
$$C_{3s} = 0.199\ [\mathrm{nF}]$$



Example (cont.)

Results from Ansys Designer



$$\omega_0 = \sqrt{\omega_1 \omega_2}$$
$$\frac{\omega_2 - \omega_1}{\omega_0} = 0.1$$

$\Downarrow$

$$f_0 = \sqrt{f_1 f_2}$$
$$\frac{f_2 - f_1}{f_0} = 0.1$$

$\Downarrow$

$$f_2 / f_0 = 1.051$$
$$f_1 / f_0 = 0.951$$

Ripple $= 0.5\,\text{dB} \Rightarrow P_{LR} = 0.5\,\text{dB} \Rightarrow P_{LR} = 1.122 \Rightarrow |S_{21}|^2 = 1/1.122 = 0.8913$

$$\Rightarrow |S_{11}|^2 = 1 - |S_{21}|^2 = 0.1087 \Rightarrow |S_{11}| = -9.636\ \text{dB}$$