Adapted from notes by
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Microwave Engineering

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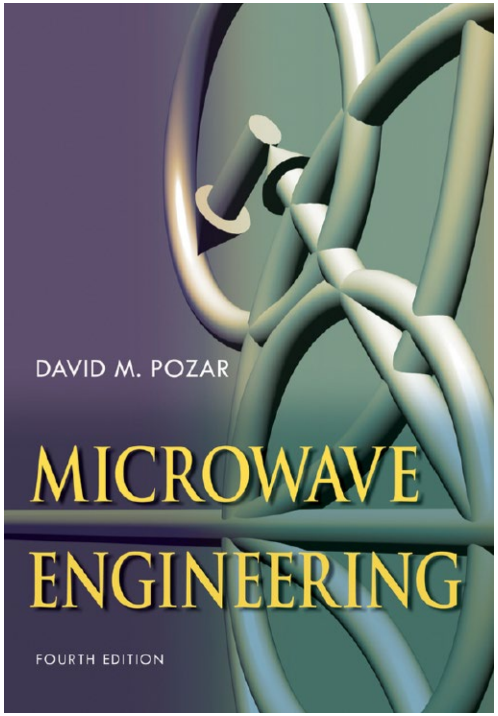
Dept. of ECE



Notes 24

Filter Design Part 3: Transmission Line Filters

Filter Design Using Transmission Lines

In this set of notes we examine filter design using transmission Lines

Recipe:

- We start with a lumped-element design.
- We can use low and high impedance lines to approximate lumped elements: works for series L and shunt C elements (what you need in a low-pass design).
- We can also apply Richard's transformation to change any lumped element design into one with transmission lines.
- We can also use the Kuroda identities to change series TL elements into parallel ones (more convenient for microstrip implementation).

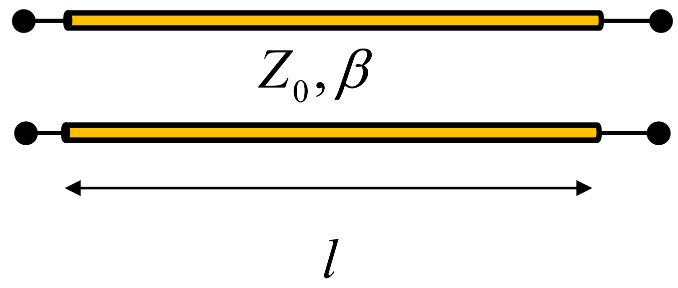
Approximate Realization of Lumped Elements

Approximate method:

- Narrow and wide sections of microstrip line can be used to approximately realize series lumped L and shunt C elements.
- Although approximate, this is a simple technique.
- This works well for low-pass filters (where we need series L and shunt C elements).

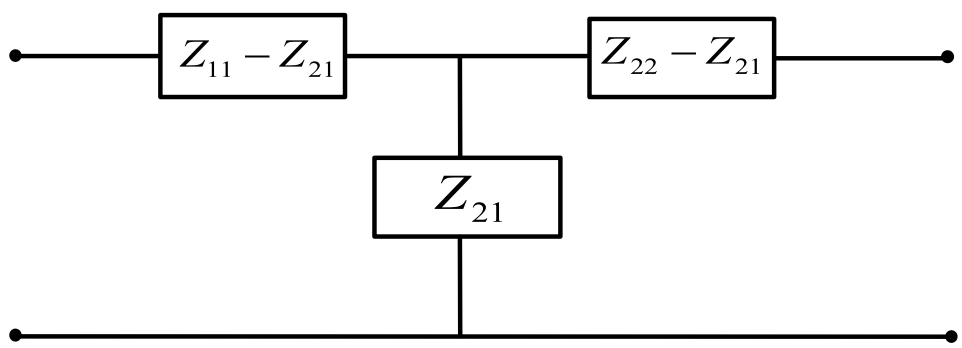
Approximate Realization of Lumped Elements (cont.)

Consider a section of transmission line (e.g., microstrip line):



$$Z_{11} = Z_{22} = -jZ_0 \cot\left(\beta l\right) \qquad Z_{21} = Z_{12} = -jZ_0 / \sin\left(\beta l\right)$$

Model:



Approximate Realization of Lumped Elements (cont.)

Element values in the model:

$$Z_{11} - Z_{21} = jZ_0 \tan\left(\beta l / 2\right)$$

$$Z_{21} = -jZ_0 / \sin\left(\beta l\right)$$

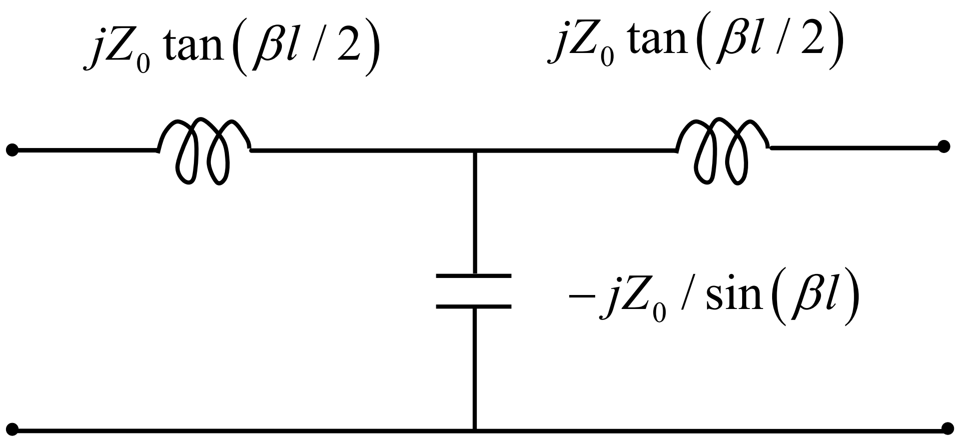
Details of calculation:

$$Z_{11} - Z_{21} = -jZ_0\left(\cot\left(\beta l\right) - 1/\sin\left(\beta l\right)\right)$$

$$= -jZ_0\left(\frac{\cos\left(\beta l\right) - 1}{\sin\left(\beta l\right)}\right)$$

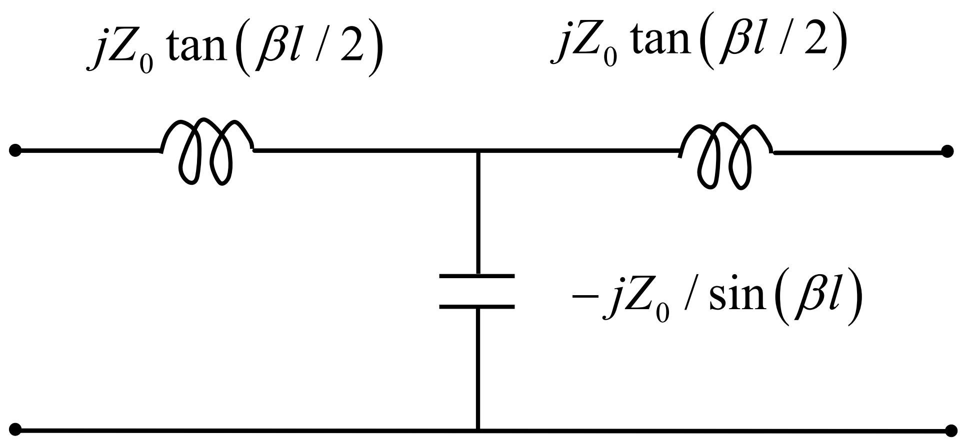
$$= jZ_0 \tan\left(\beta l / 2\right)$$

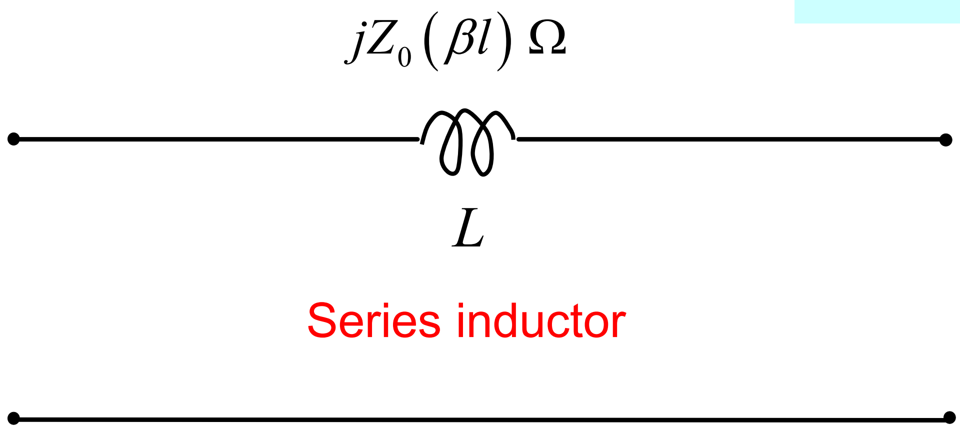
Note : $\tan\left(x/2\right) = \dfrac{1-\cos x}{\sin x}$



Approximate Realization of Lumped Elements (cont.)

Assume: $Z_0 >> 1,\ \beta l << 1$ (narrow and short microstrip line)



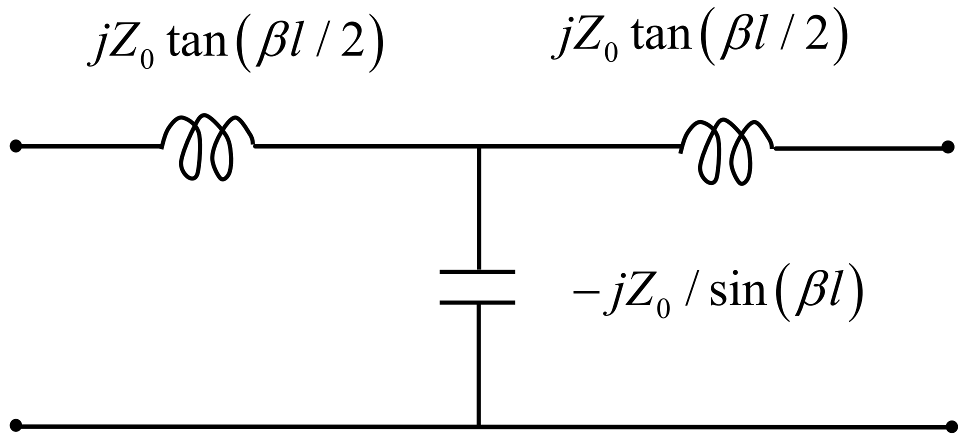


$$\Rightarrow\ \omega L = Z_0 (\beta l)$$

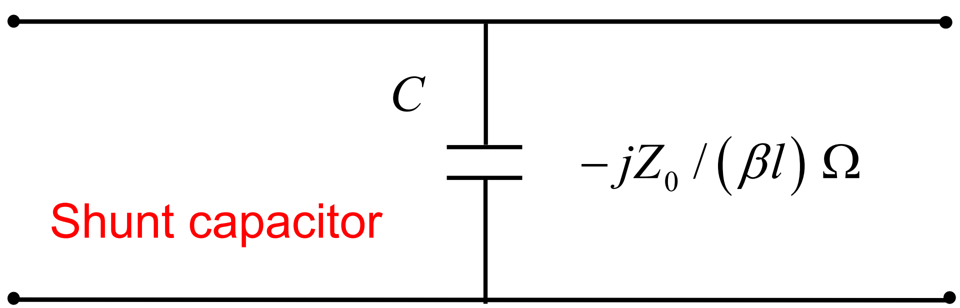
Series inductor

Approximate Realization of Lumped Elements (cont.)

Assume: $Z_0 << 1,\ \beta l << 1$ (wide and short microstrip line)



$$\Rightarrow \frac{1}{\omega C} = Z_0 / (\beta l)$$



Approximate Realization of Lumped Elements (cont.)

From the previous derivation:

$$\omega L = Z_0^{high}\left(\beta l\right)$$

$$\frac{1}{\omega C} = Z_0^{low} / \left(\beta l\right)$$

Solve for the lengths of the lines that are needed

Choose: $\omega = \omega_c$

Series inductors: $\omega_c L = Z_{high}\left(\beta l\right)_{\omega_c}$

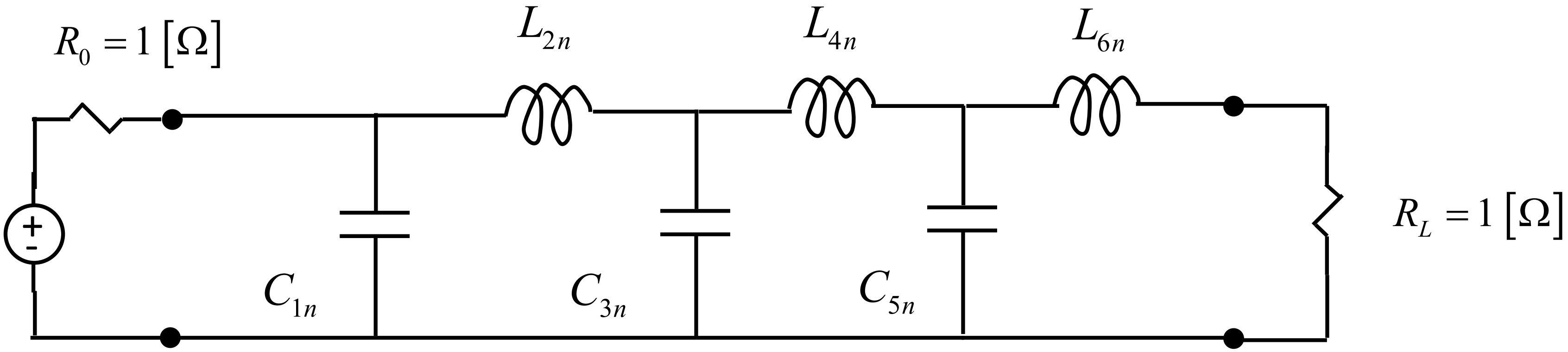
Shunt capacitors: $\dfrac{1}{\omega_c C} = Z_{low} / \left(\beta l\right)_{\omega_c}$

Narrow lines (series inductors): $\left(\beta l\right)_{\omega_c} = \dfrac{\omega_c L}{Z_{high}}$

Wide lines (shunt capacitors): $\left(\beta l\right)_{\omega_c} = \omega_c C Z_{low}$

Example

Design an $N = 6$ Butterworth stepped-impedance low-pass filter for a matched 50 Ω load with a cutoff frequency of 2.5 GHz. Choose $Z_{low} = 20\ \Omega$, $Z_{high} = 120\ \Omega$.



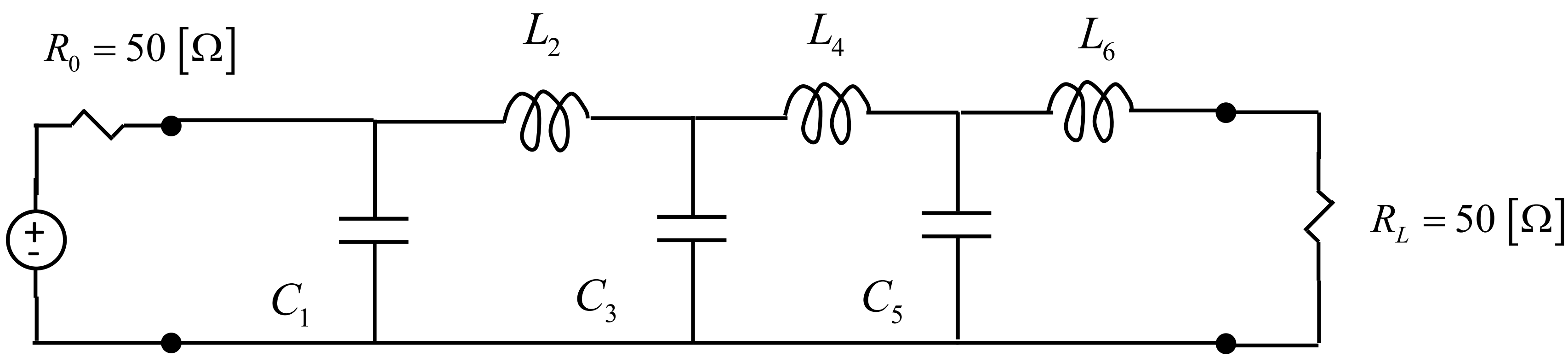
Choose type "*a*"

From table:

$$g_1 = C_{1n} = 0.517$$
$$g_2 = L_{2n} = 1.414$$
$$g_3 = C_{3n} = 1.932$$
$$g_4 = L_{4n} = 1.932$$
$$g_5 = C_{5n} = 1.414$$
$$g_6 = L_{6n} = 0.517$$

Example (cont.)

De-normalized filter:



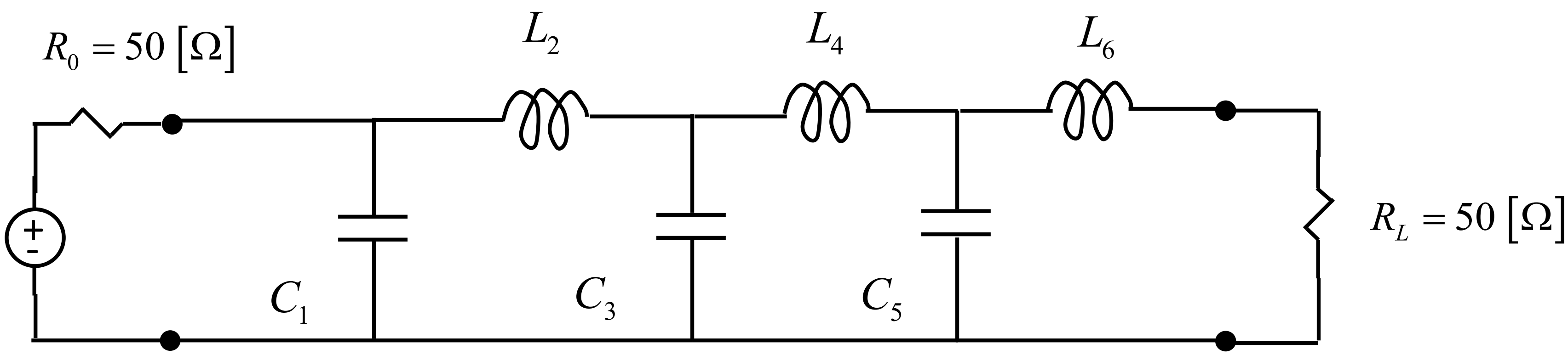
$C_{1n} = 0.517\ [\mathrm{F}]$
$L_{2n} = 1.414\ [\mathrm{H}]$
$C_{3n} = 1.932\ [\mathrm{F}]$
$L_{4n} = 1.932\ [\mathrm{H}]$
$C_{5n} = 1.414\ [\mathrm{F}]$
$L_{6n} = 0.517\ [\mathrm{H}]$

$$L = \left(L_n R_0\right) / \omega_c$$
$$C = \left(C_n / R_0\right) / \omega_c$$

$\Rightarrow$

$C_1 = 0.658\ [\mathrm{pF}]$
$L_2 = 4.5\ [\mathrm{nH}]$
$C_3 = 2.46\ [\mathrm{pF}]$
$L_4 = 6.15\ [\mathrm{nH}]$
$C_5 = 1.80\ [\mathrm{pF}]$
$L_6 = 1.65\ [\mathrm{nH}]$

Example (cont.)



Choice:

$$Z_{low} = 20\,\Omega$$
$$Z_{high} = 120\,\Omega$$

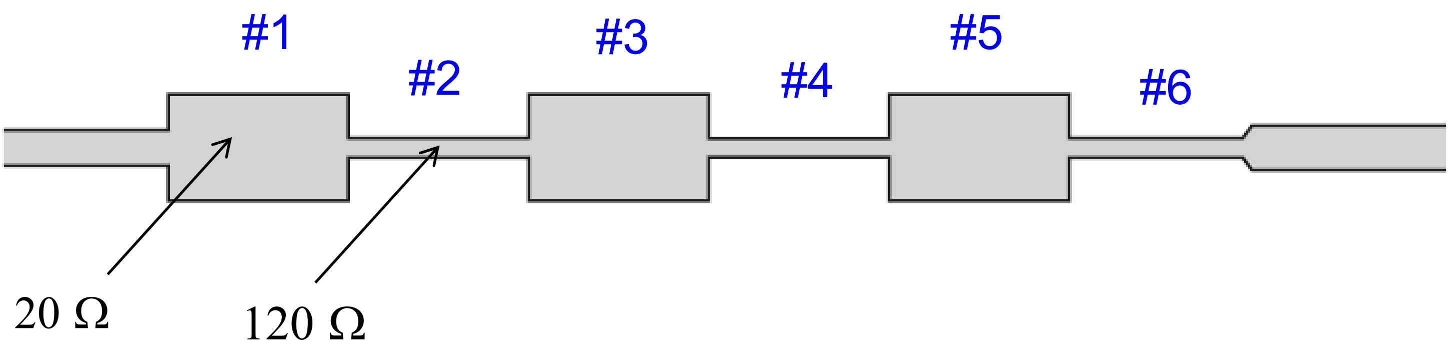
$$C_1 = 0.658\ [\text{pF}]$$
$$L_2 = 4.5\ [\text{nH}]$$
$$C_3 = 2.46\ [\text{pF}]$$
$$L_4 = 6.15\ [\text{nH}]$$
$$C_5 = 1.80\ [\text{pF}]$$
$$L_6 = 1.65\ [\text{nH}]$$

$$(\beta l)_{\omega_c} = \frac{\omega_c L}{Z_{high}} \quad (\#2,\ \#4,\ \#6)$$
$$(\beta l)_{\omega_c} = \omega_c C Z_{low} \quad (\#1,\ \#3,\ \#5)$$

$$(\beta l)_1 = 0.207\ [\text{rad}]\ \ (11.8^{\circ})$$
$$(\beta l)_2 = 0.589\ [\text{rad}]\ \ (11.8^{\circ})$$
$$(\beta l)_3 = 0.773\ [\text{rad}]\ \ (44.3^{\circ})$$
$$(\beta l)_4 = 0.805\ [\text{rad}]\ \ (46.1^{\circ})$$
$$(\beta l)_5 = 0.565\ [\text{rad}]\ \ (32.4^{\circ})$$
$$(\beta l)_6 = 0.215\ [\text{rad}]\ \ (12.3^{\circ})$$

Example (cont.)

Microstrip Realization



(not drawn to scale)

Figure 8.40 from Pozar

Note:
TX line can first be used to find the line widths (from the chosen Z_0 values), and then used to find the line lengths (from the calculated electrical lengths).

Example (cont.)

Microstrip Realization

$$h = 1.58\,[\text{mm}],\ \varepsilon_r = 4.2,\ \tan\delta = 0.02$$

Section	$Z_i = Z_\ell$ or $Z_h\,(\Omega)$	$\beta\ell_i$ (deg)	W_i (mm)	ℓ_i (mm)
1	20	11.8	11.3	2.05
2	120	33.8	0.428	6.63
3	20	44.3	11.3	7.69
4	120	46.1	0.428	9.04
5	20	32.4	11.3	5.63
6	120	12.3	0.428	2.41

From p. 425 of Pozar

Example (cont.)

Results

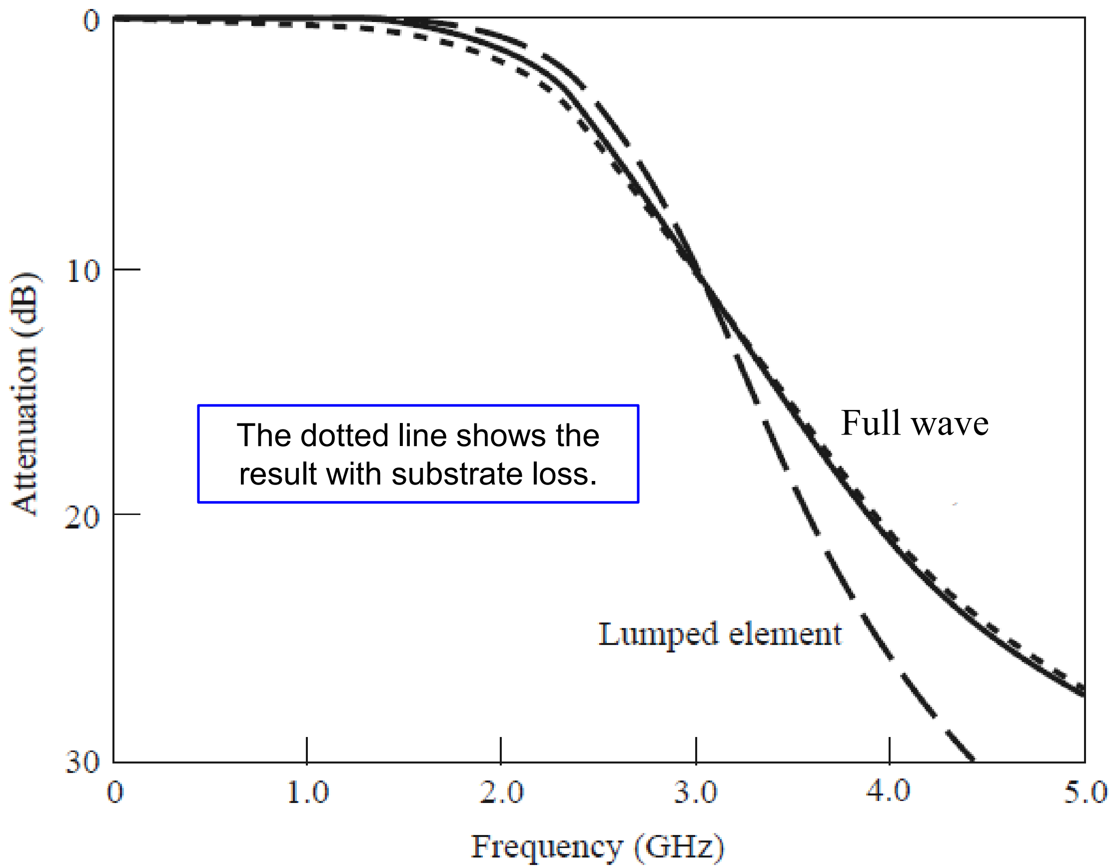
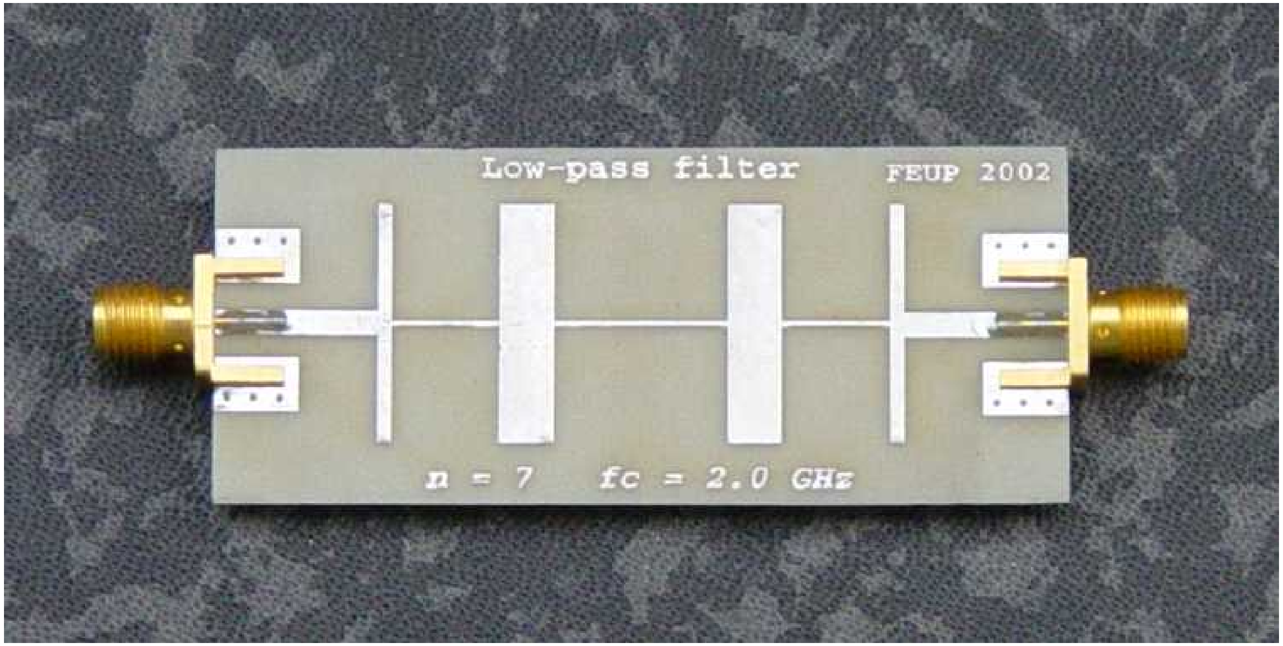


Figure 8.41 from Pozar

Example (cont.)

Figure of an Actual Low-Pass Filter



Richard's Transformation

Richard's transformation

Main idea:
Short-circuited and open-circuited transmission lines are chosen to mimic the performance of the lumped L and C elements, respectively.

Richard's transformation:

$$\Omega(\omega) \equiv \omega_c \tan(\beta l) = \omega_c \tan\left(\omega\sqrt{\mu\varepsilon}\, l\right)$$

$\Omega =$ radian frequency in lumped-element design

$\omega =$ radian frequency in transmission line design

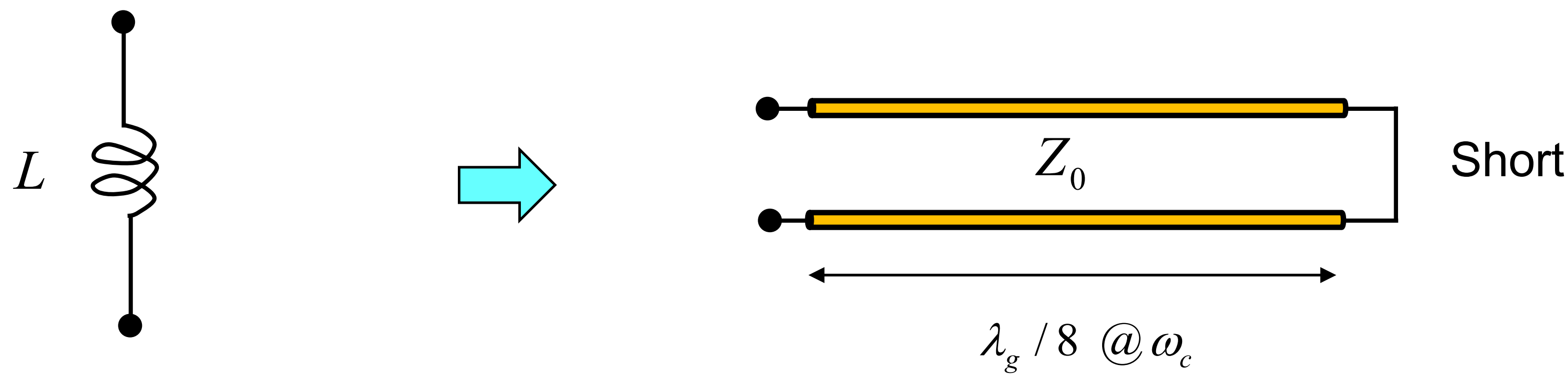
Require: $\Omega(\omega_c) = \omega_c$ ⇨ $\tan\left(\omega_c\sqrt{\mu\varepsilon}\, l\right) = 1$ $\left(l = \lambda_g / 8 \;\; @\, \omega_c\right)$

(The lumped-element and transmission-line circuits will have the same cutoff frequency.)

Richard's Transformation (cont.)

At any frequency ω, TLs can behave the same as lumped elements at frequency Ω.

Equivalent TL model for a lumped inductor:



Require: $j\Omega L = jZ_0 \tan(\beta l)$ (same impedance property)

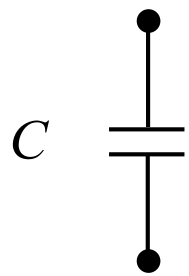
$$\Rightarrow \quad j\omega_c \tan(\beta l) L = jZ_0 \tan(\beta l)$$

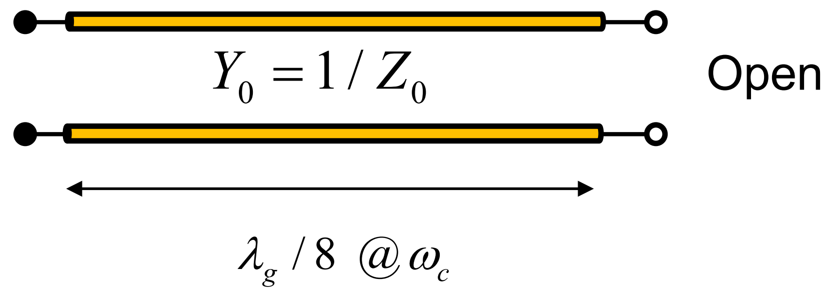
$$\Rightarrow \quad Z_0 = \omega_c L$$

The inductor then has the same impedance at <u>any</u> frequency Ω as does the short-circuited transmission line at corresponding frequency ω.

Richard's Transformation (cont.)

Equivalent TL model for a lumped capacitor:





Require: $j\Omega C = jY_0 \tan(\beta l)$ (same admittance property)

$$\Rightarrow \quad j\omega_c \tan(\beta l) C = jY_0 \tan(\beta l)$$

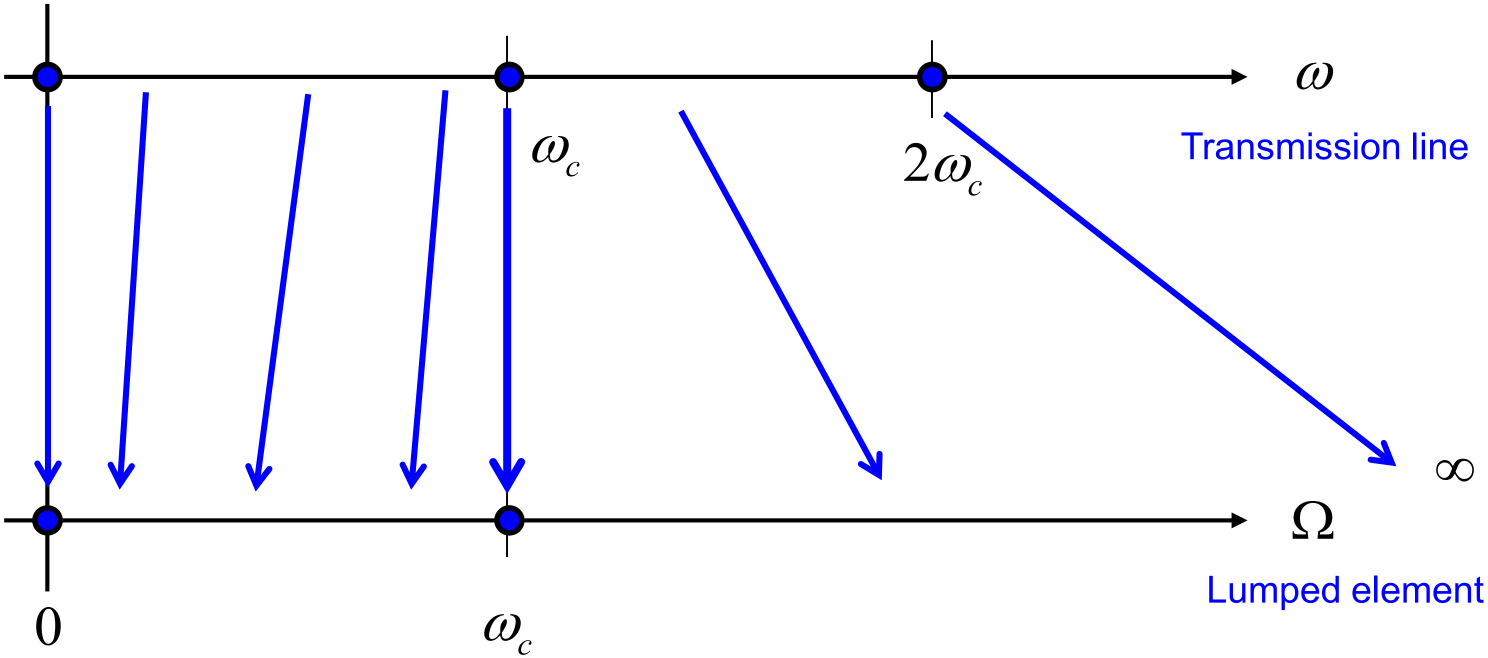
$$\Rightarrow \quad Y_0 = \omega_c C$$

The capacitor then has the same admittance at <u>any</u> frequency Ω as does the open-circuited transmission line at corresponding frequency ω.

Richard's Transformation (cont.)

Illustration of Mapping

$$\Omega(\omega) \equiv \omega_c \tan(\beta l) = \omega_c \tan\left(\omega\sqrt{\mu\varepsilon} l\right)$$



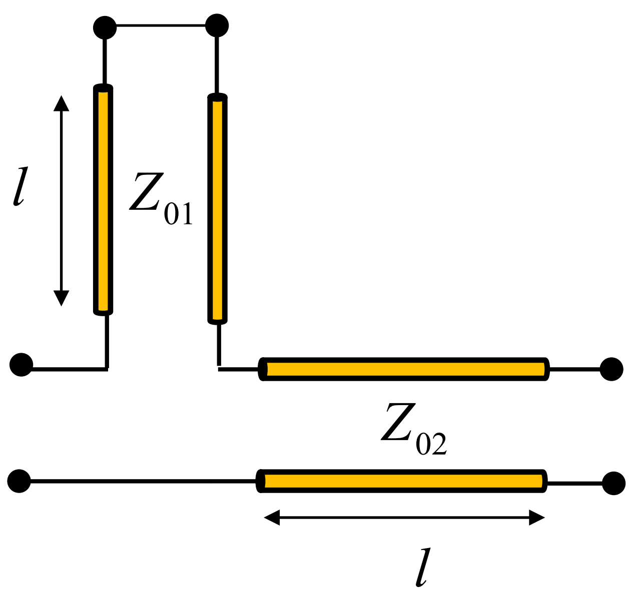
$$@\,\omega_c: \quad \beta l = \frac{\pi}{4} \Rightarrow \tan(\beta l) = 1$$

$$@\,2\omega_c: \quad \beta l = \frac{\pi}{2} \Rightarrow \tan(\beta l) = \infty$$

Kuroda's Identities

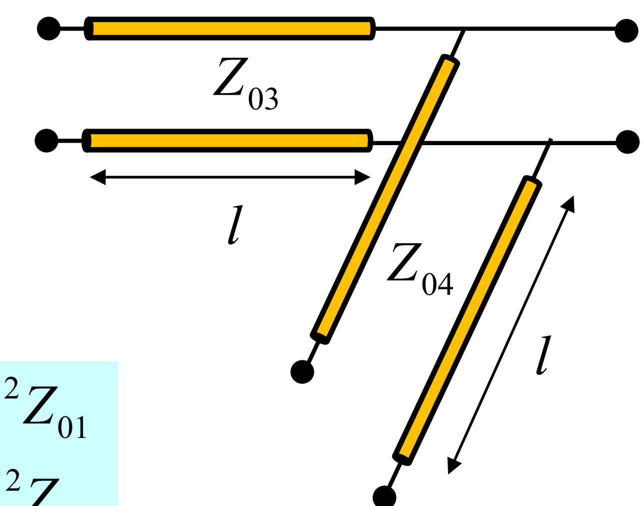
Kuroda's Identity #2 is useful for transforming a series shorted TL into a parallel open-circuited TL.

Note: The TLs all have the same electrical length.



Line with series shorted stub

Line with parallel open stub



$$Z_{03} = n^2 Z_{01}$$

$$Z_{04} = n^2 Z_{02}$$

where

$$n^2 = 1 + \frac{Z_{02}}{Z_{01}}$$

Please see the Pozar book for a derivation.

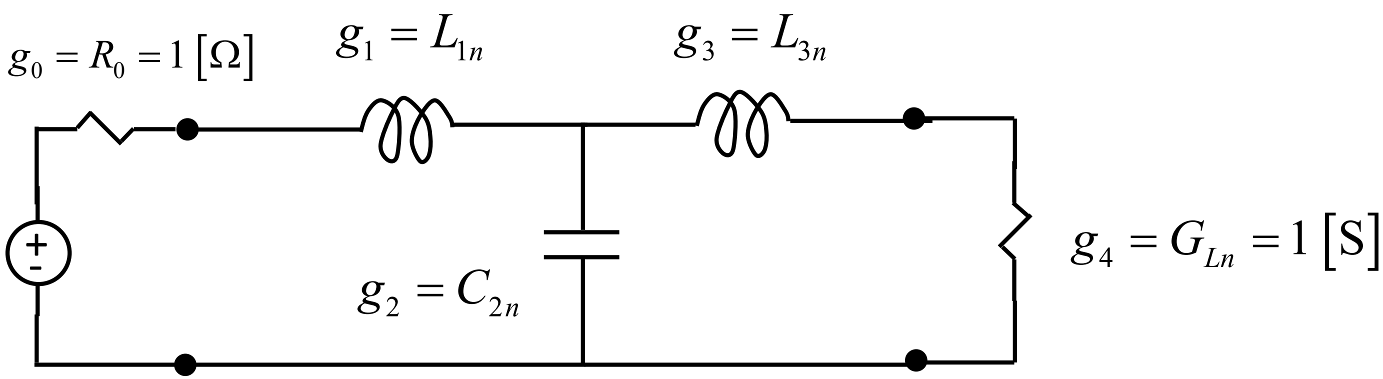
Example

Design an $N = 3$ Chebyshev low-pass filter for a matched 50 Ω load with 3.0 dB of ripple in the passband and a cutoff frequency of 4.0 GHz.

Choose type "b" low-pass prototype:

$N = 3 = \text{odd}$

$\Rightarrow g_4 = 1$



From table:

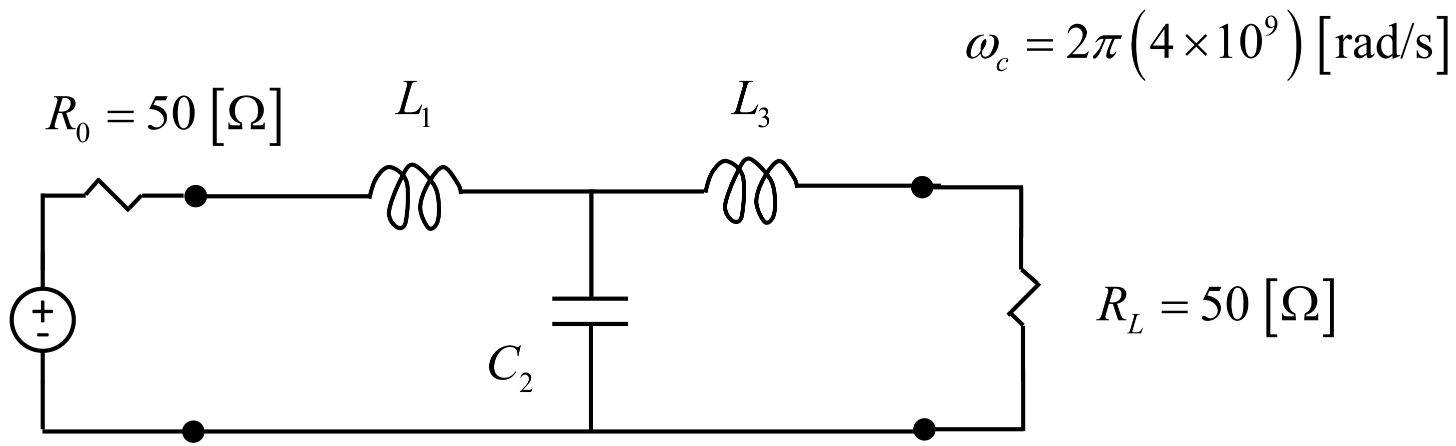
$$g_1 = L_{1n} = 3.3487$$

$$g_2 = C_{2n} = 0.7117$$

$$g_3 = L_{3n} = 3.3487$$

Example (cont.)

Final lumped-element design:



From table:

$L_{1n} = 3.3487$ [H]

$C_{2n} = 0.7117$ [F]

$L_{3n} = 3.3487$ [H]

Denormalization:

$L = \left(L_n R_0\right)/\omega_c$

$C = \left(C_n / R_0\right)/\omega_c$

Lumped elements:

$L_1 = 6.66$ [nH]

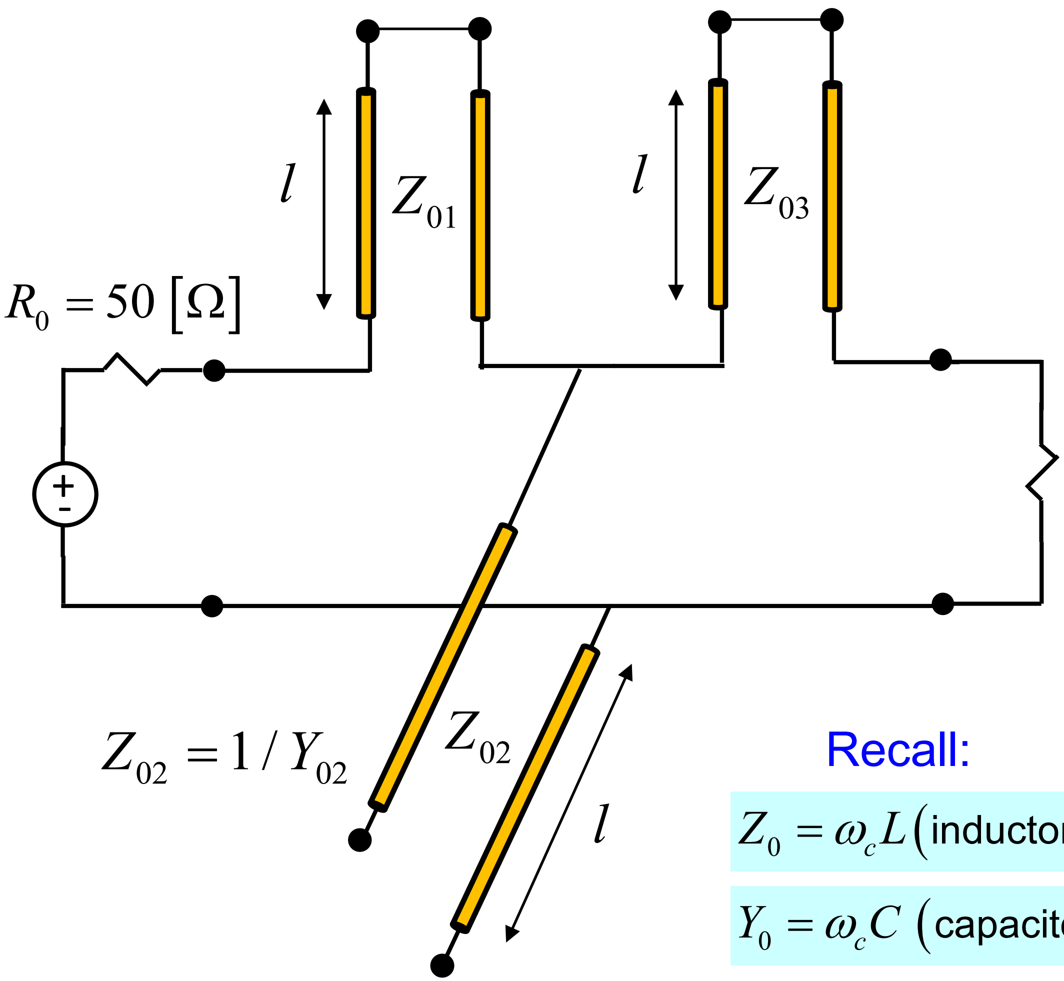
$C_2 = 0.566$ [pF]

$L_3 = 6.66$ [nH]

Example (cont.)

Convert to TLs (Richard's transformation):

$$Z_{01} = \omega_c L_1 = 167.4\ [\Omega]$$
$$Z_{02} = 1/(\omega_c C_2) = 70.3\ [\Omega]$$
$$Z_{03} = \omega_c L_3 = 167.4\ [\Omega]$$



$$\omega_c = 2\pi(4\times 10^9)\ [\text{rad/s}]$$

Recall:

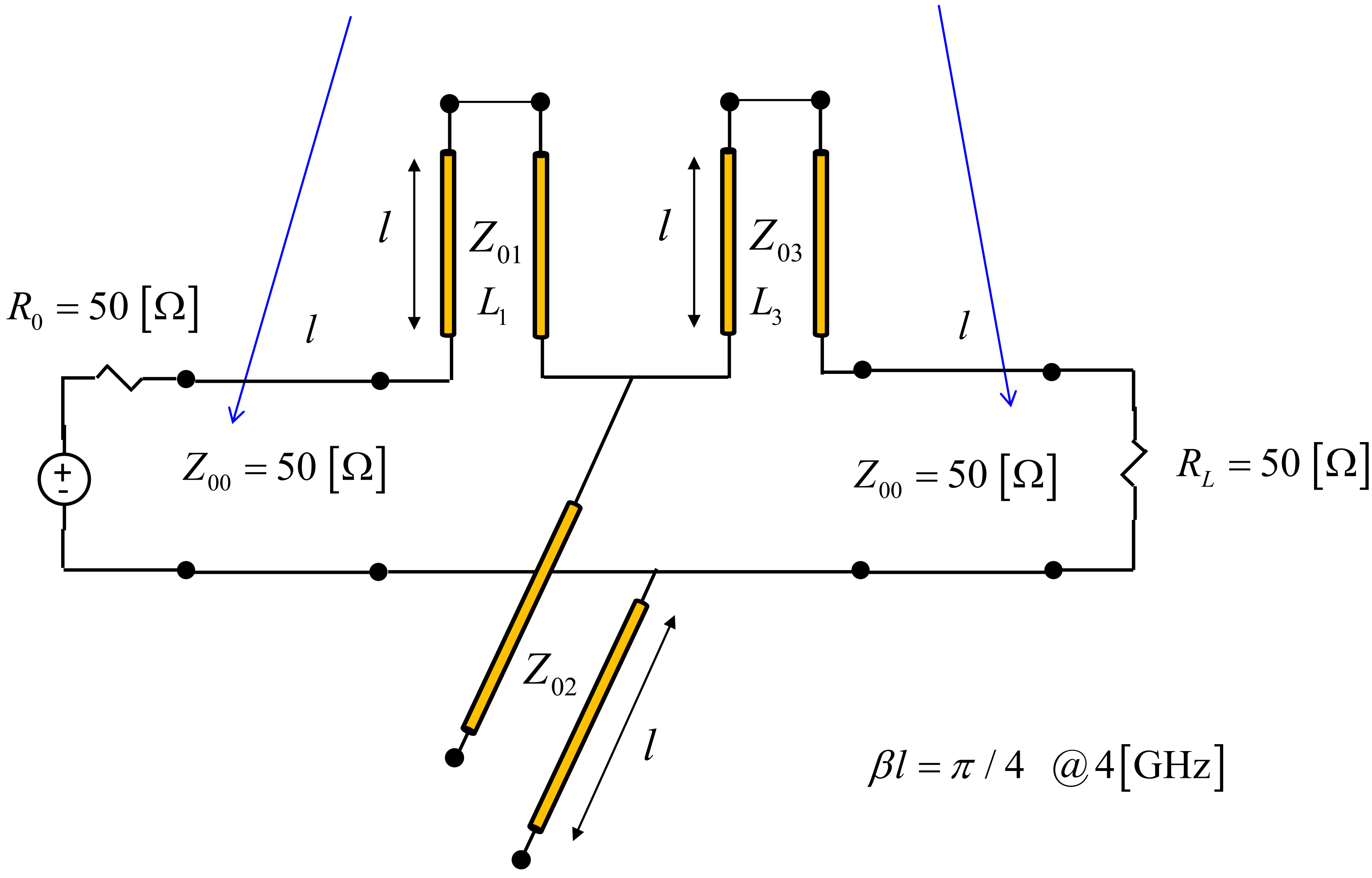
$Z_0 = \omega_c L$ (inductor)

$Y_0 = \omega_c C$ (capacitor)

$\beta l = \pi/4 \quad @4[\text{GHz}]$

Example (cont.)

Add extra 50 Ω transmission lines.



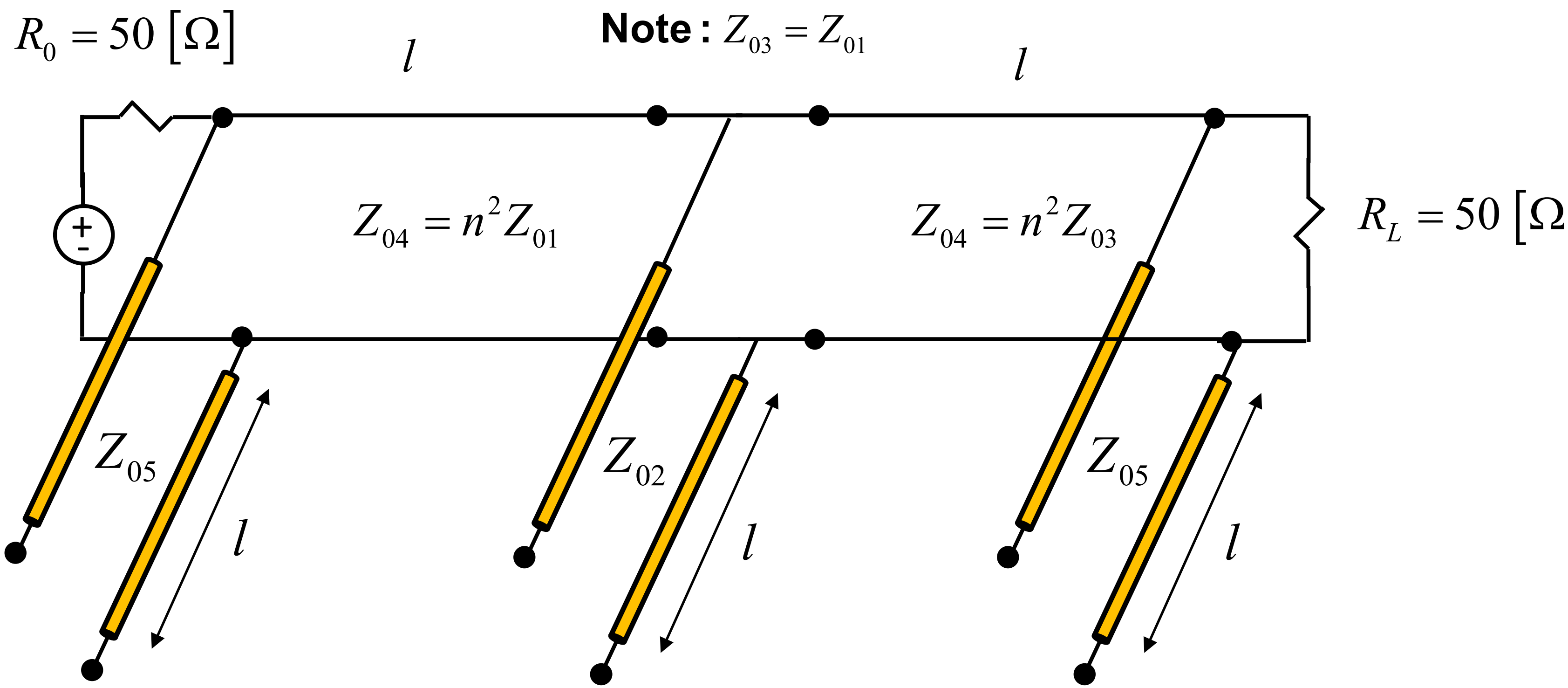
$\beta l = \pi / 4 \quad @\, 4\,[\text{GHz}]$

These extra lines do not affect the filter performance.

Example (cont.)

Apply the Kuroda identity #2:

$\beta l = \pi / 4 \quad @ 4[\text{GHz}]$



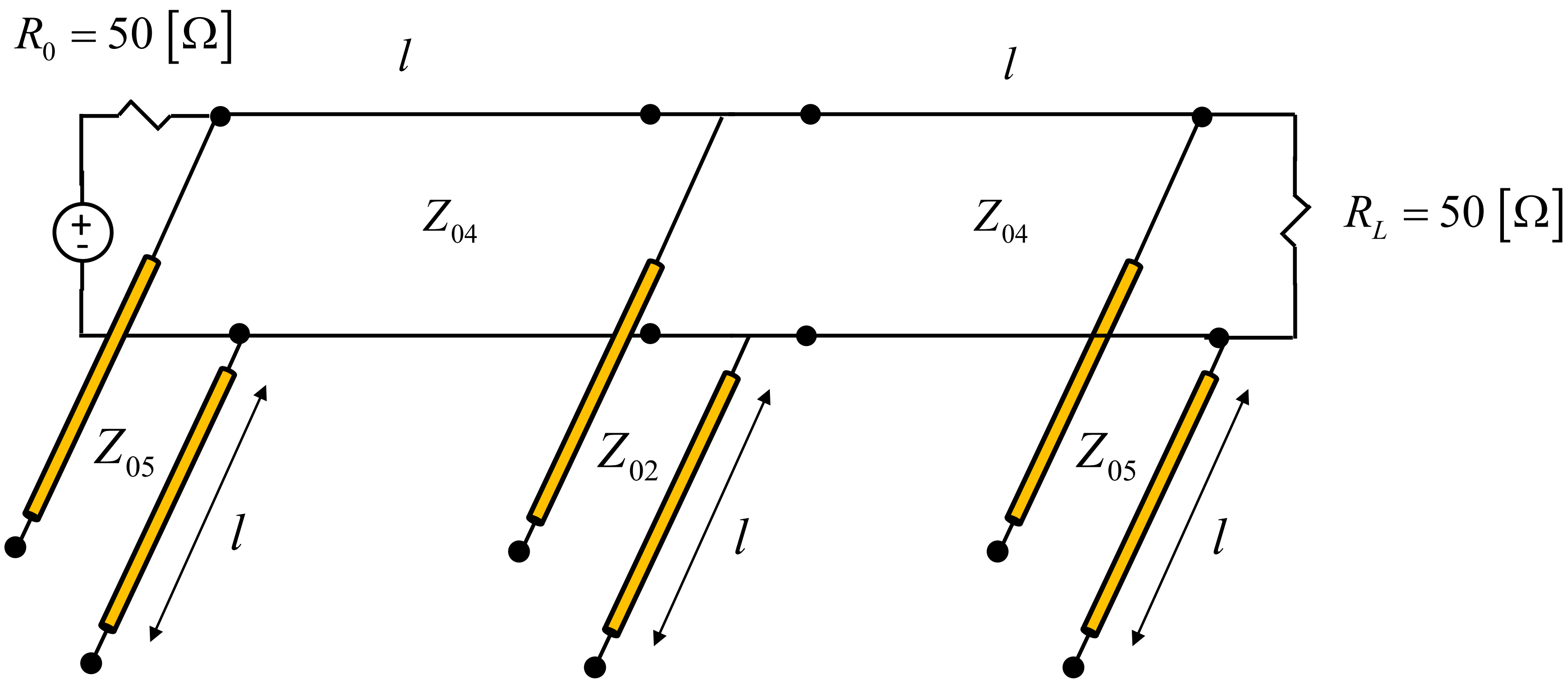
$$n^2 = 1 + \frac{Z_{00}}{Z_{03}} = 1.299$$

$$Z_{04} = n^2 Z_{01}$$
$$Z_{05} = n^2 Z_{00}$$

Example (cont.)

Final Design

$\beta l = \pi / 4 \quad @ 4[\text{GHz}]$



$Z_{02} = 70.3\ [\Omega]$

$Z_{04} = 217.5\ [\Omega]$

$Z_{05} = 64.9\ [\Omega]$

Example (cont.)

Microstrip Realization

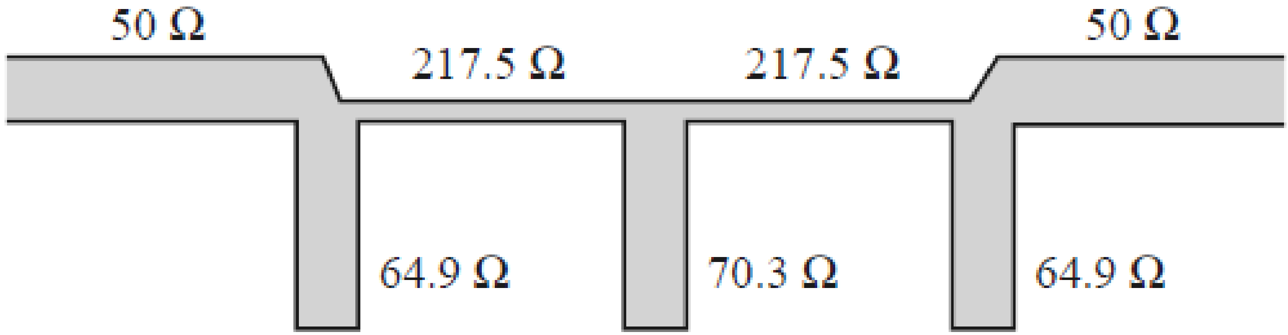
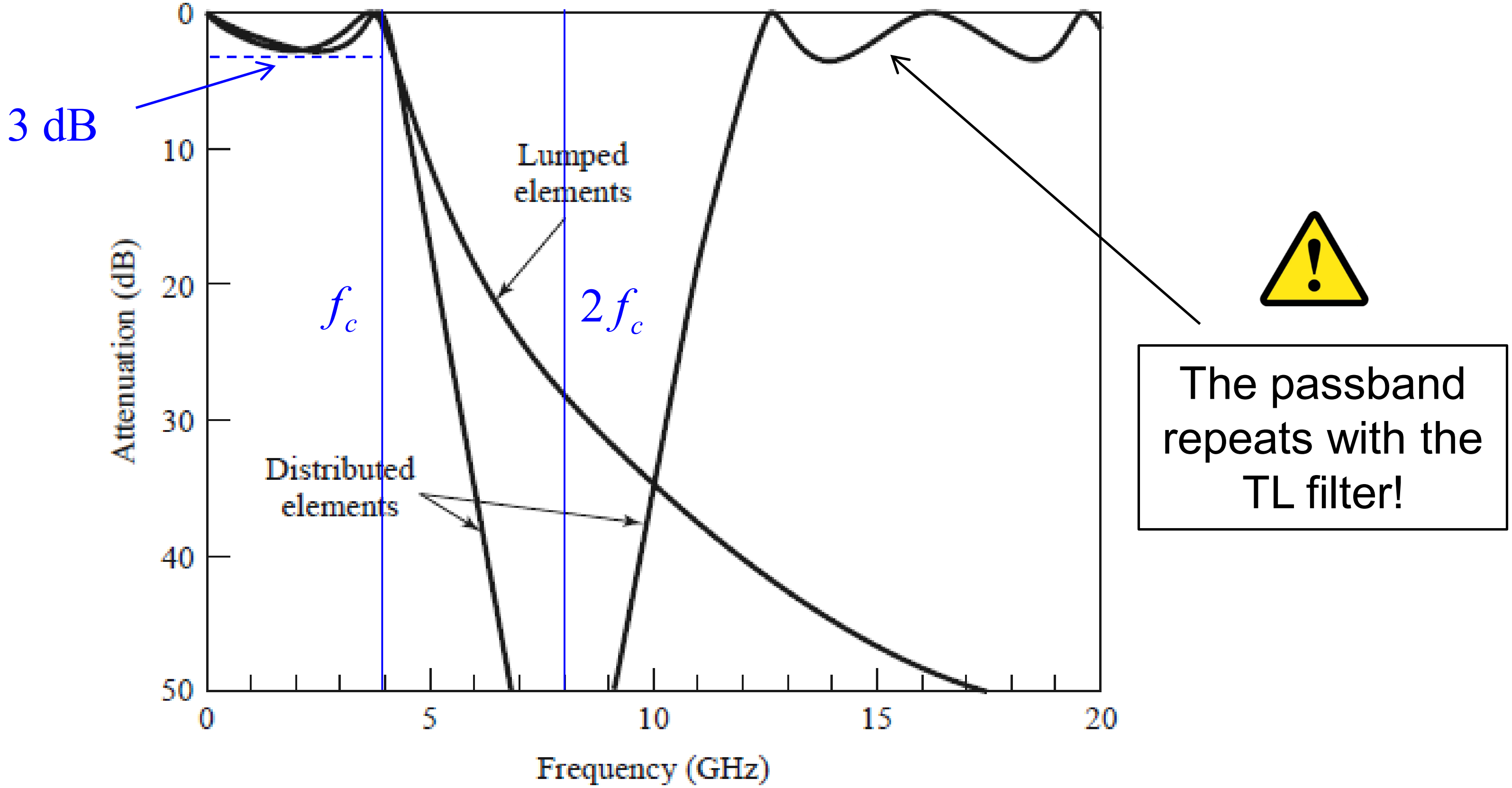


Figure 8.36 from Pozar

Example (cont.)

Results



$$\mathrm{IL} = \text{attenuation}_{\mathrm{dB}} = -\left|S_{21}\right|_{\mathrm{dB}}$$

Figure 8.37 from Pozar