Adapted from notes by
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ECE 5317-6351
Microwave Engineering

Fall 2019

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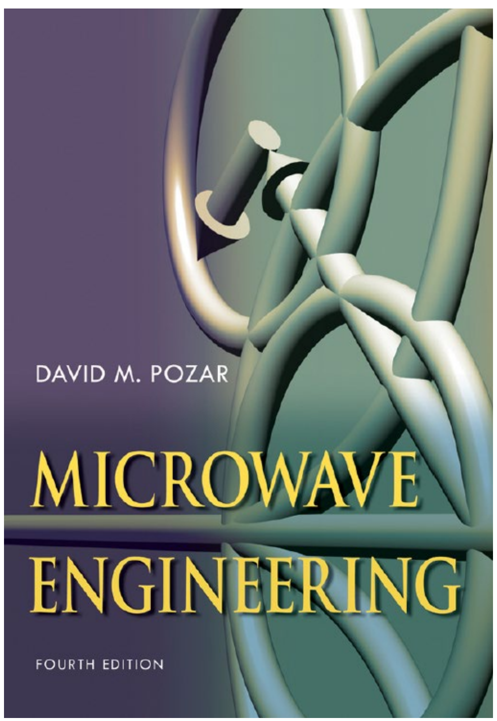
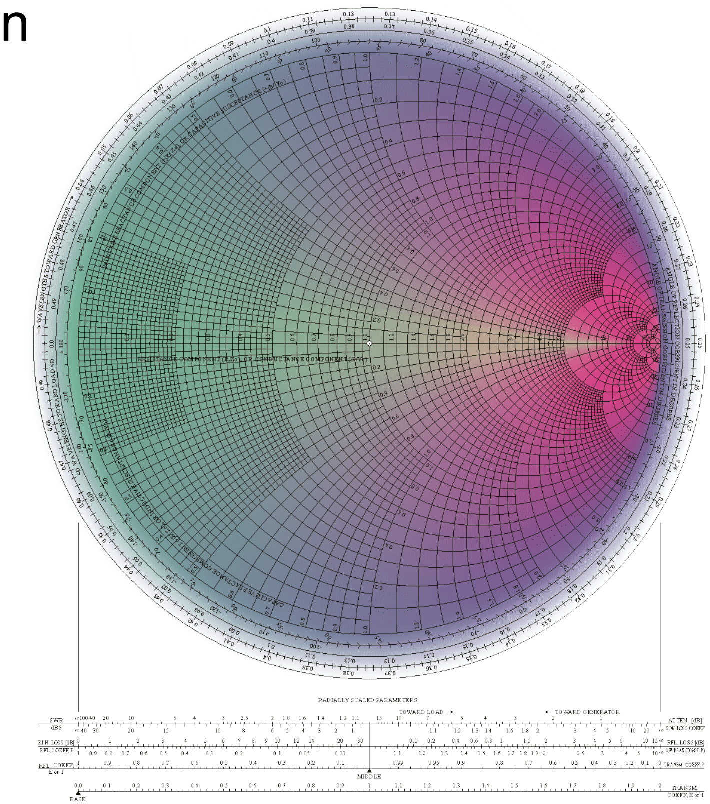
Dept. of ECE

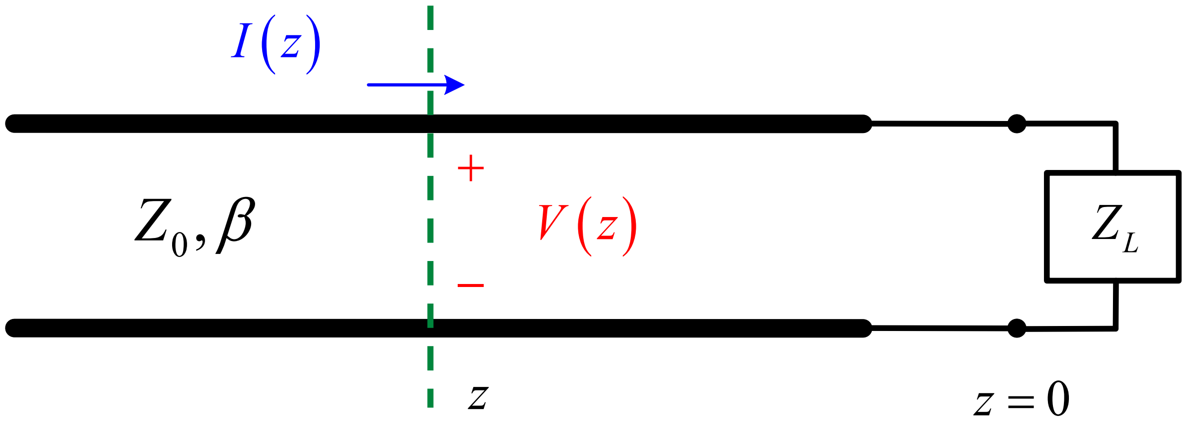


Notes 5

Smith Charts



Generalized Reflection Coefficient



$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Recall:

$$V(z) = V_0^+ e^{-\gamma z}\left(1 + \Gamma_L e^{+2\gamma z}\right) = V_0^+ e^{-\gamma z}\left(1 + \Gamma(z)\right)$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-\gamma z}\left(1 - \Gamma_L e^{+2\gamma z}\right) = \frac{V_0^+}{Z_0} e^{-\gamma z}\left(1 - \Gamma(z)\right)$$

$$Z(z) = \frac{V(z)}{I(z)} = Z_0\left(\frac{1 + \Gamma_L e^{+2\gamma z}}{1 - \Gamma_L e^{+2\gamma z}}\right) = Z_0\left(\frac{1 + \Gamma(z)}{1 - \Gamma(z)}\right)$$

Generalized reflection Coefficient: $\Gamma(z) = \Gamma_L\, e^{+2\gamma z}$

Generalized Reflection Coefficient (cont.)

Different forms for $\Gamma(z)$

$$\Gamma(z) = \Gamma_L e^{+2\gamma z}$$
$$= |\Gamma_L| e^{j\phi_L} e^{+2\gamma z}$$
$$= \Gamma_R(z) + j\Gamma_I(z)$$

Lossless transmission line ($\alpha = 0$)

$$\Gamma(z) = |\Gamma_L| e^{j(\phi_L + 2\beta z)}$$

Magnitude property of $\Gamma(z)$

$$\mathrm{Re}\{Z_L\} \geq 0 \quad \Rightarrow \quad |\Gamma_L| \leq 1$$

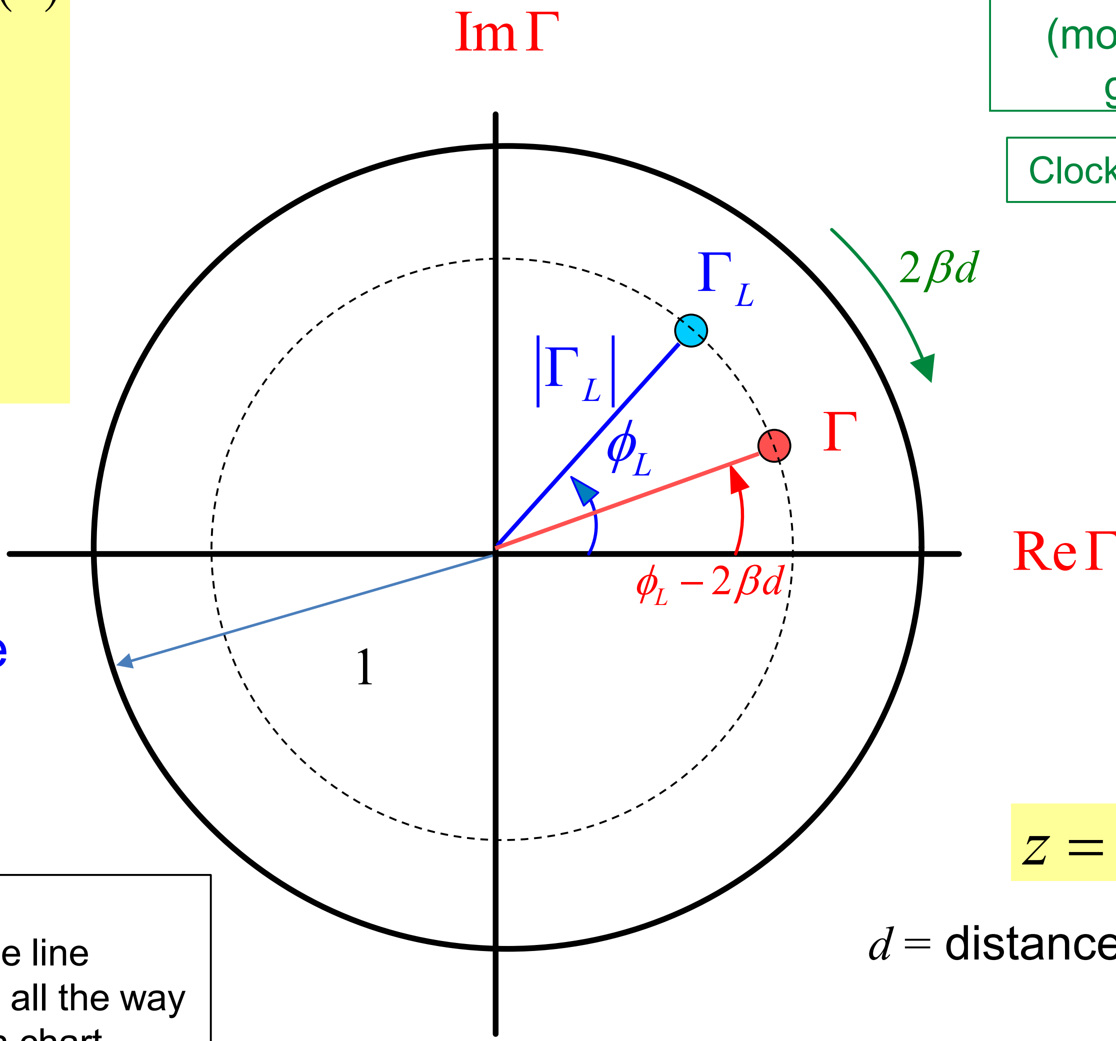
Proof:

$$\Gamma_L = \frac{(R_L + jX_L) - Z_0}{(R_L + jX_L) + Z_0}$$
$$= \frac{(R_L - Z_0) + jX_L}{(R_L + Z_0) + jX_L}$$

$$\Rightarrow |\Gamma_L|^2 = \frac{(R_L - Z_0)^2 + X_L^2}{(R_L + Z_0)^2 + X_L^2} \leq 1$$

Complex Γ Plane

$$
\begin{aligned}
\Gamma &= \Gamma(z) \\
&= \Gamma_R(z) + j\Gamma_I(z) \\
&= \Gamma_L\, e^{j(+2\beta z)} \\
&= |\Gamma_L|\, e^{j(\phi_L + 2\beta z)} \\
&= |\Gamma_L|\, e^{j(\phi_L - 2\beta d)} \\
&= \Gamma_L\, e^{-j(2\beta d)}
\end{aligned}
$$



Increasing d (moving towards generator)

Clockwise movement!

Lossless line

$z = -d$

d = distance from load

Note:
Going $\lambda/2$ on the line corresponds to going all the way around the Smith chart.

Z Chart

Start with

$$Z(z) = Z_0 \left(\frac{1+\Gamma(z)}{1-\Gamma(z)} \right)$$

The Z chart is the "usual" Smith chart.

Define $$Z_n(z) \equiv \frac{Z(z)}{Z_0} = \left(\frac{1+\Gamma(z)}{1-\Gamma(z)} \right)$$

$$Z_n = R_n + jX_n$$

Note:
The z dependence is being suppressed here.

Hence we have:

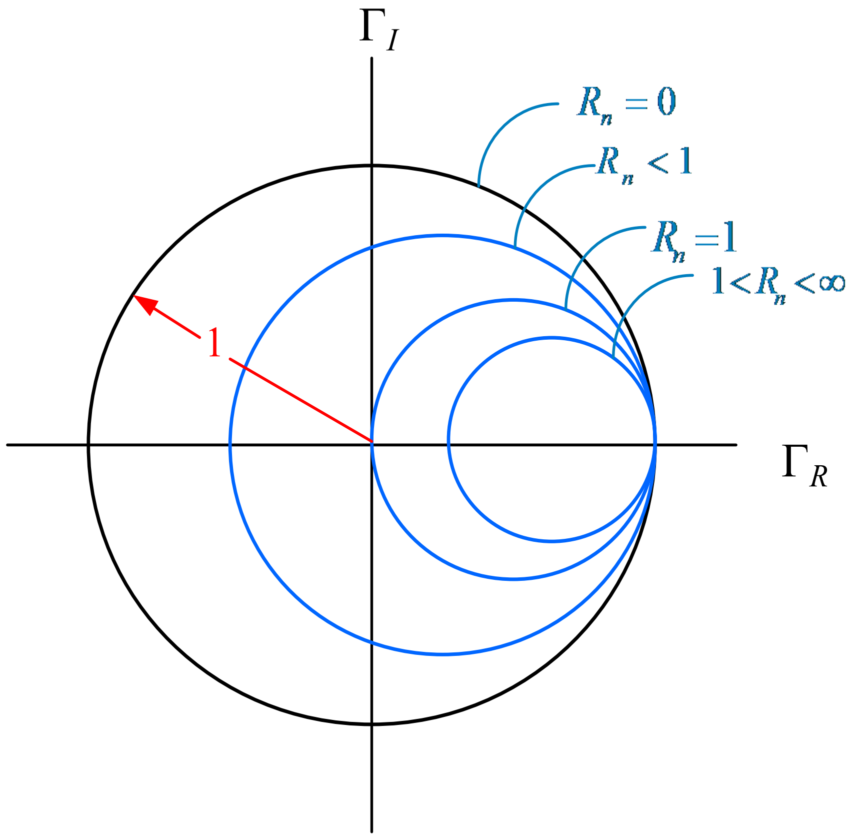
$$R_n + jX_n = \left(\frac{1+(\Gamma_R + j\Gamma_I)}{1-(\Gamma_R + j\Gamma_I)} \right)$$

Next, multiply both sides by the RHS denominator term and equate real and imaginary parts. Then solve the resulting equations for Γ_R and Γ_I in terms of R_n or X_n. This gives two equations.

Z Chart (cont.)

1) Equation #1:

$$\left(\Gamma_R - \frac{R_n}{1+R_n}\right)^2 + \Gamma_I^2 = \left(\frac{1}{1+R_n}\right)^2$$



Equation for a circle in the Γ plane

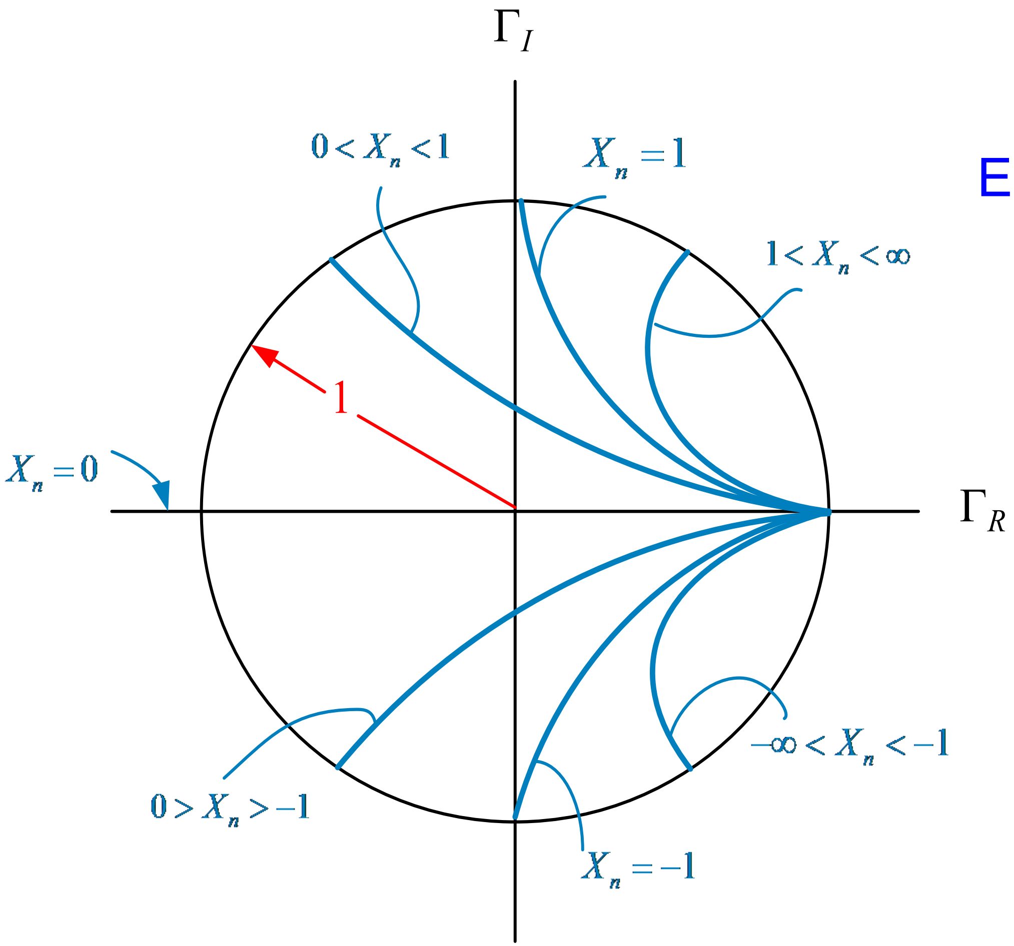
$$\text{Center} = \left(\frac{R_n}{1+R_n}, 0\right)$$

$$\text{Radius} = \frac{1}{1+R_n}$$

Z Chart (cont.)

2) Equation #2:

$$(\Gamma_R - 1)^2 + \left(\Gamma_I - \frac{1}{X_n}\right)^2 = \left(\frac{1}{X_n}\right)^2$$



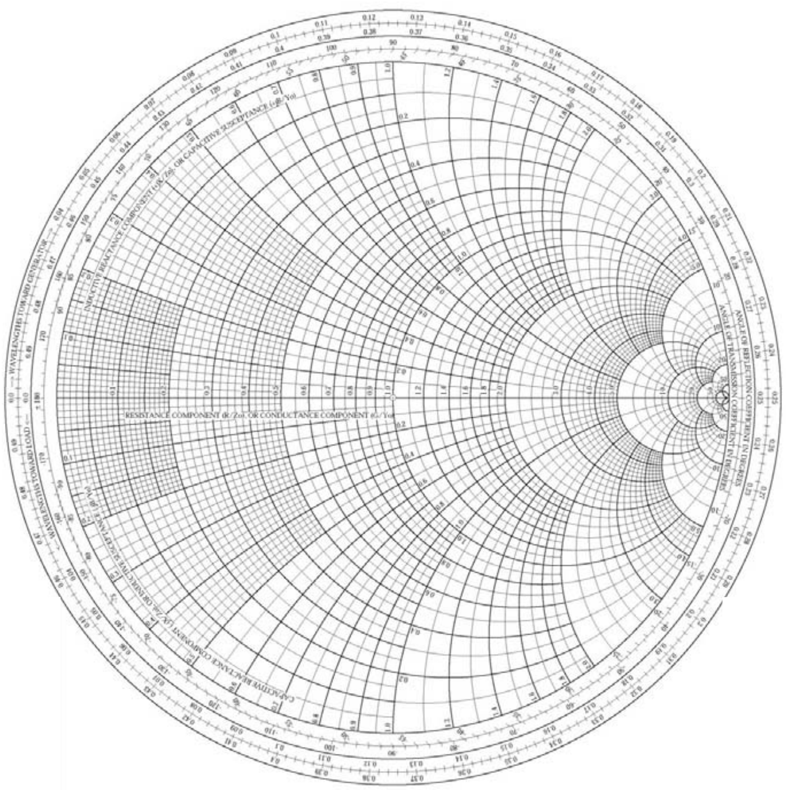
Equation for a circle in the Γ plane

$$\text{Center} = \left(1, \frac{1}{X_n}\right)$$

$$\text{Radius} = \frac{1}{|X_n|}$$

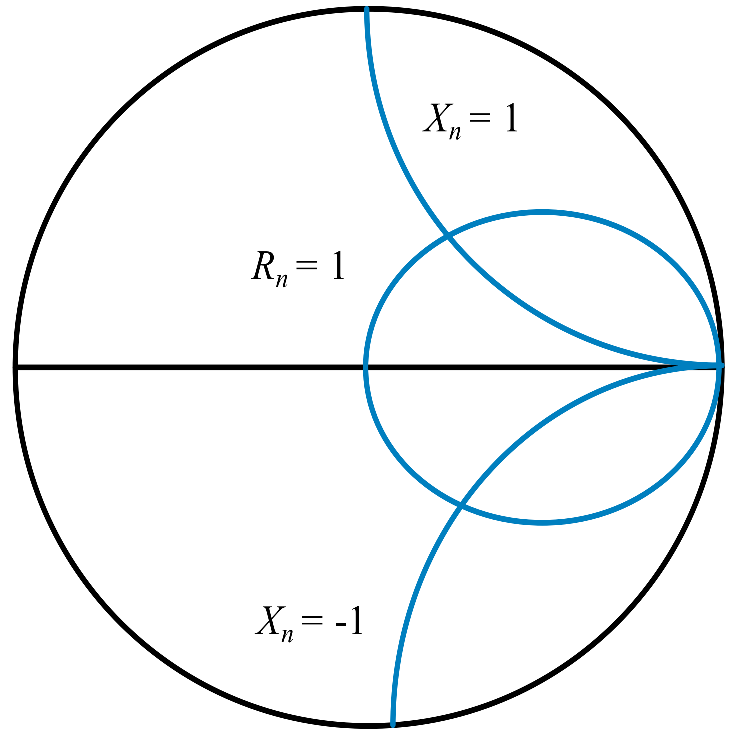
Z Chart (cont.)

Smith Chart
(Z-Chart)



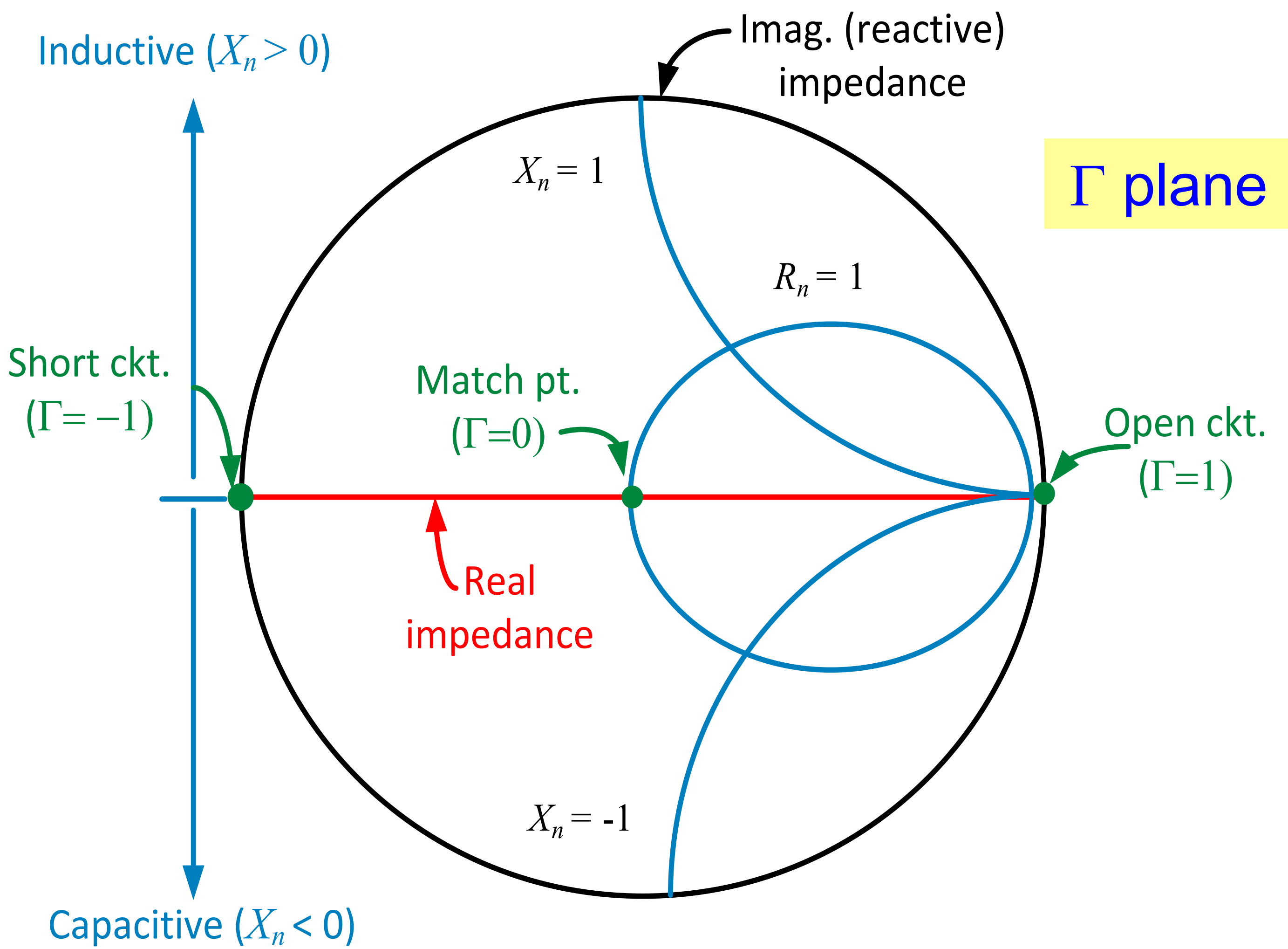
Γ plane

Short-hand version



Γ plane

Z Chart (cont.)



Inductive ($X_n > 0$)
Capacitive ($X_n < 0$)
Imag. (reactive) impedance
Γ plane
$X_n = 1$
$R_n = 1$
$X_n = -1$
Short ckt. ($\Gamma = -1$)
Match pt. (Γ=0)
Open ckt. (Γ=1)
Real impedance

Admittance Calculations with the Z Chart

Note: $$Y(z)=\frac{1}{Z(z)}=\frac{1}{Z_0}\left(\frac{1-\Gamma(z)}{1+\Gamma(z)}\right)$$

$$=Y_0\left(\frac{1+(-\Gamma(z))}{1-(-\Gamma(z))}\right) \qquad (Y_0\equiv 1/Z_0)$$

$$\Rightarrow Y_n(z)\equiv\frac{Y(z)}{Y_0}=\left(\frac{1+(-\Gamma(z))}{1-(-\Gamma(z))}\right)=G_n(z)+jB_n(z)$$

Define:

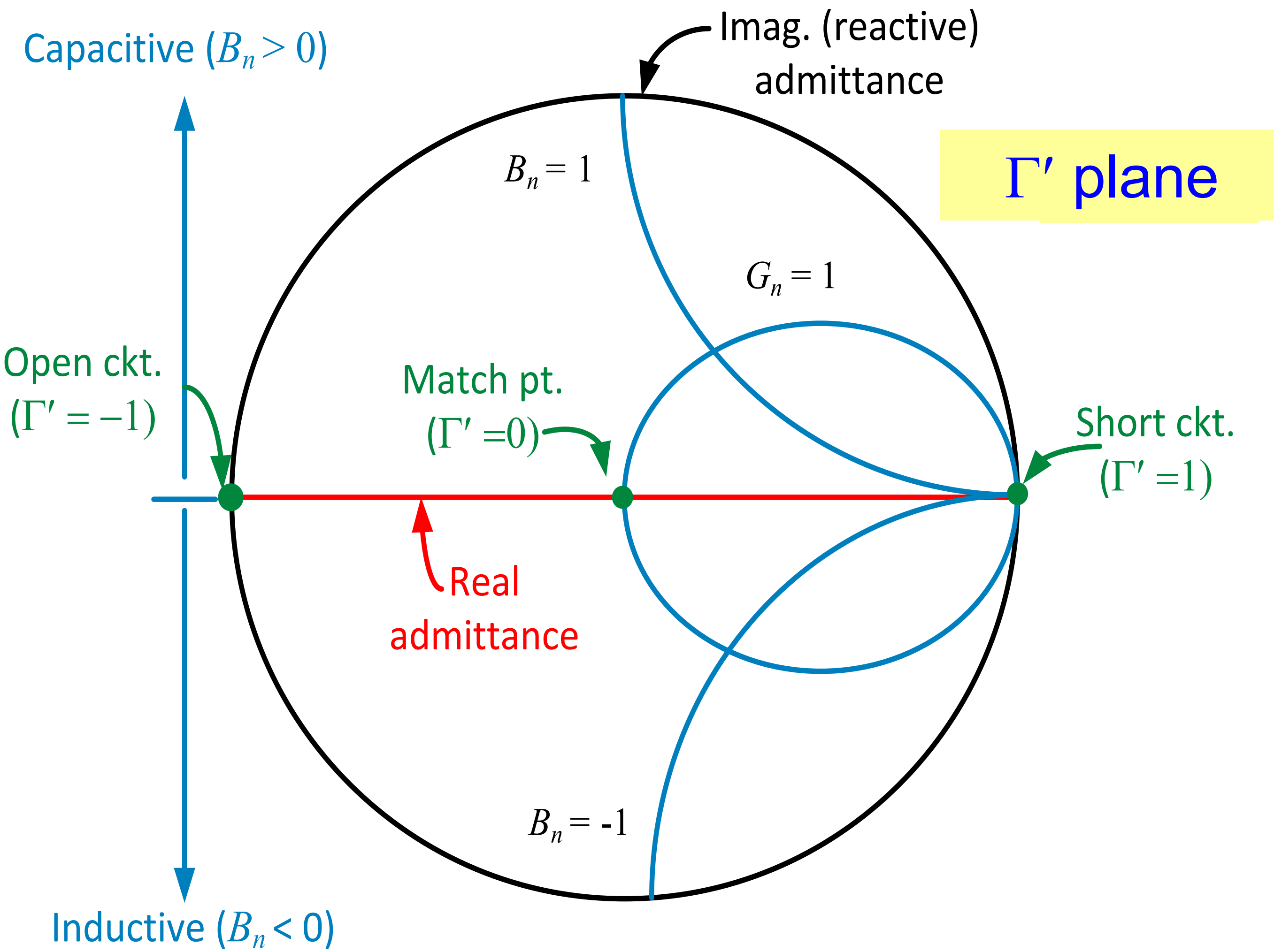
$$\Gamma'(z)\equiv-\Gamma(z)$$

$$Y_n(z)=\left(\frac{1+\Gamma'}{1-\Gamma'}\right)$$

Conclusion:
The same Smith chart can be used as an admittance calculator.

Same mathematical form as for Z_n: $$Z_n(z)=\left(\frac{1+\Gamma}{1-\Gamma}\right)$$

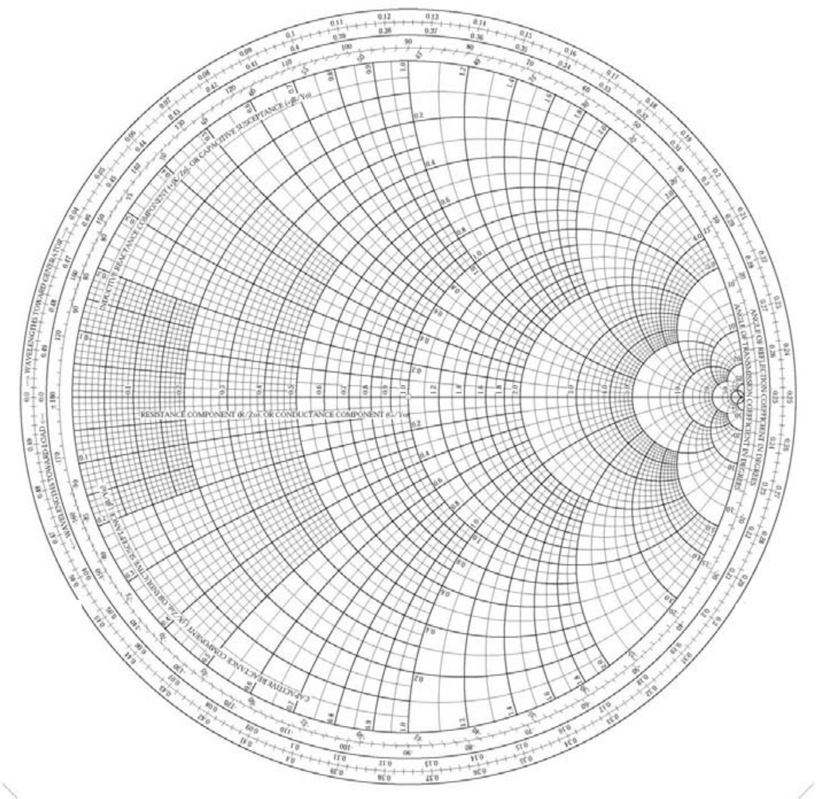
Admittance Calculations with the Z Chart (cont.)



Impedance or Admittance Calculations with the Z Chart

Normalized impedance <u>or</u> admittance coordinates

The Smith chart can be used for either impedance or admittance calculations, as long as we are consistent.



The complex plane is either the Γ plane <u>or</u> the Γ' plane.

Y Chart

As an alternative way to do admittance calculations, we can continue to use the original Γ plane, and add admittance curves to the chart.

$$Y_n(z) = \left(\frac{1 + \left(-\Gamma(z)\right)}{1 - \left(-\Gamma(z)\right)} \right) = G_n(z) + jB_n(z)$$

Compare with previous Smith chart derivation, which started with this equation:

$$Z_n(z) = \left(\frac{1 + \left(\Gamma(z)\right)}{1 - \left(\Gamma(z)\right)} \right) = R_n(z) + jX_n(z)$$

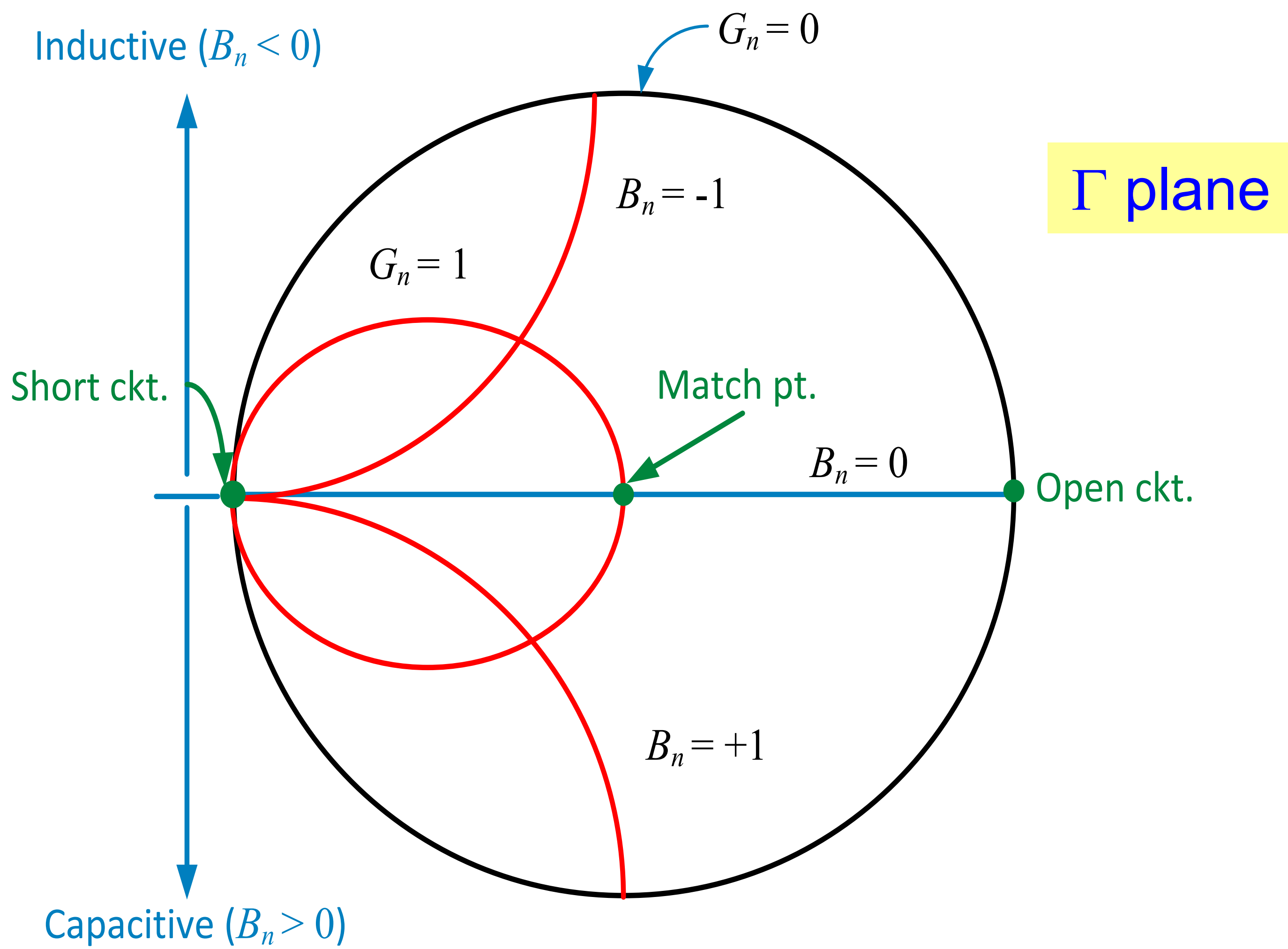
$$\left.\begin{array}{l} R_n(z) \to G_n(z) \\ X_n(z) \to B_n(z) \end{array}\right\} \Rightarrow \quad \Gamma(z) \to -\Gamma(z)$$ (rotation of 180°)

Examples:

$R_n = 1$ circle, rotated 180°, becomes $G_n = 1$ circle.
$X_n = 1$ circle, rotated 180°, becomes $B_n = 1$ circle.

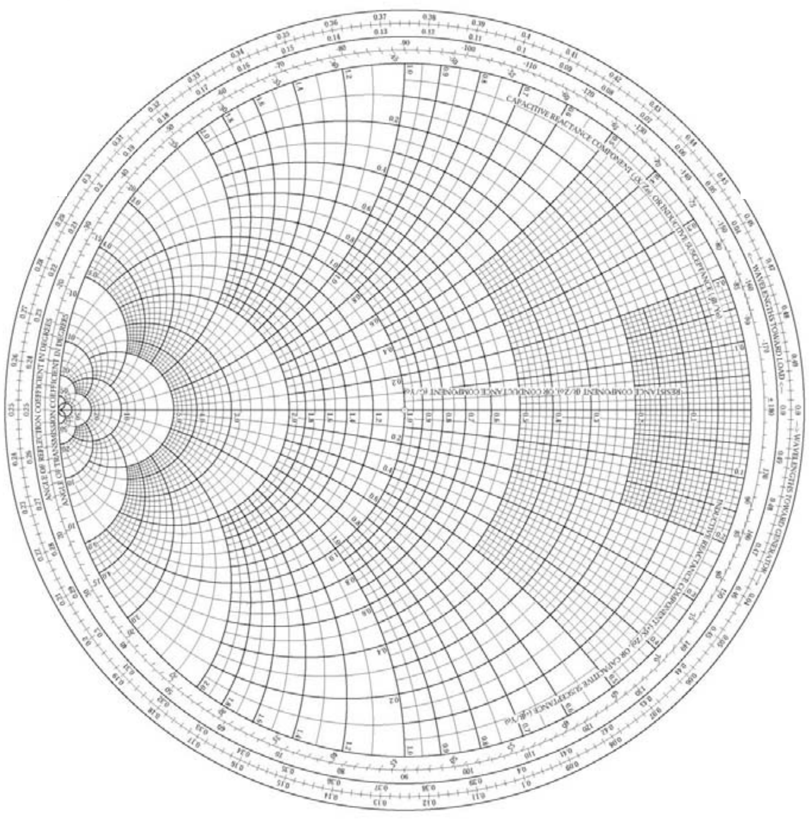
Side note: A 180° rotation on a Smith chart makes a normalized impedance become its reciprocal.

Y Chart (cont.)

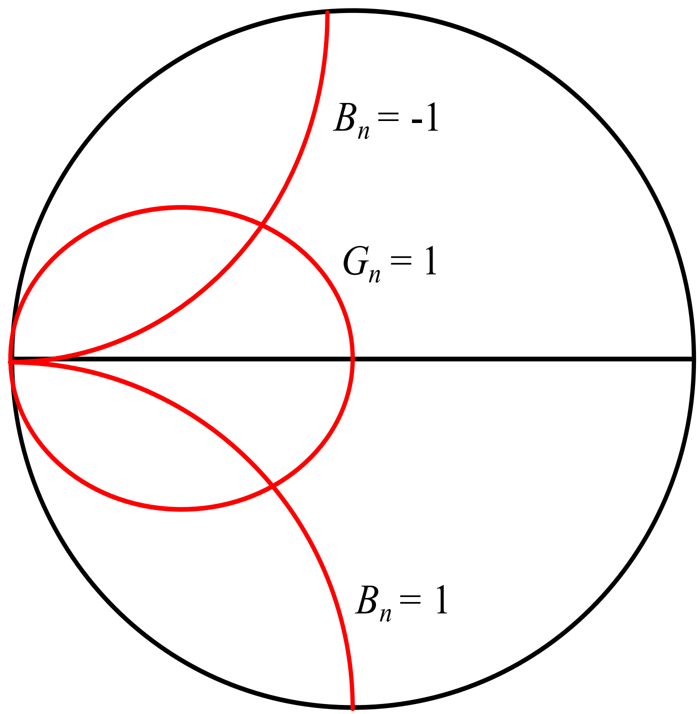


The Y chart is the "mirror image" of the usual Smith chart.

Y Chart (cont.)



Short-hand version

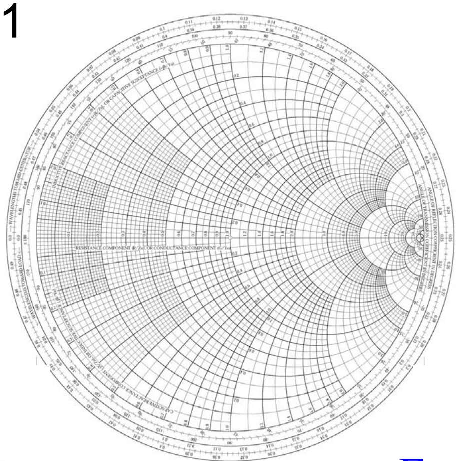


Γ plane

All Four Possibilities for Smith Charts

Z chart, used for impedance

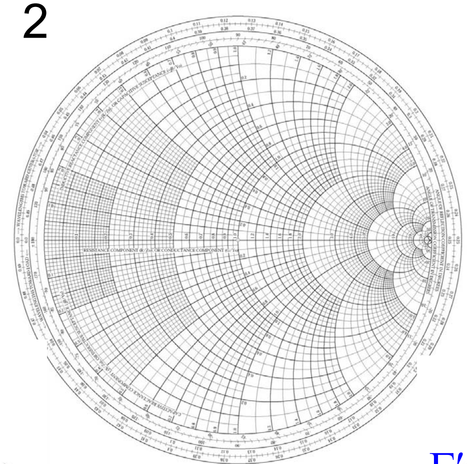
1



Γ plane

Z chart, used for admittance

2



Γ' plane

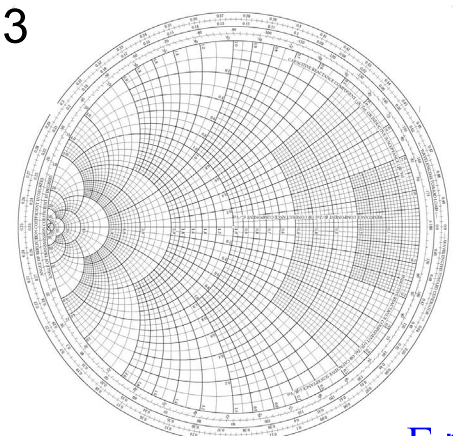
The first two are the most common.

The third is sometimes convenient.

The fourth is almost never used.

Y chart, used for admittance

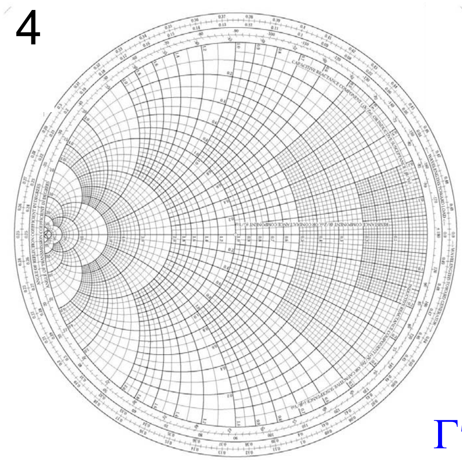
3



Γ plane

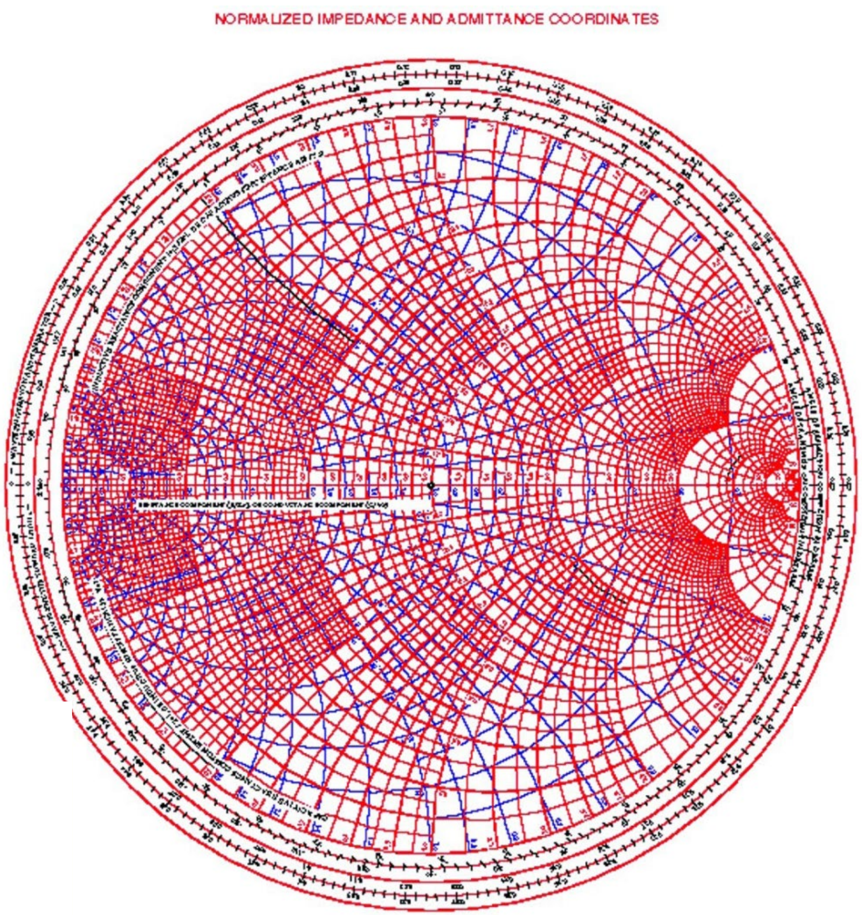
Y chart, used for impedance

4

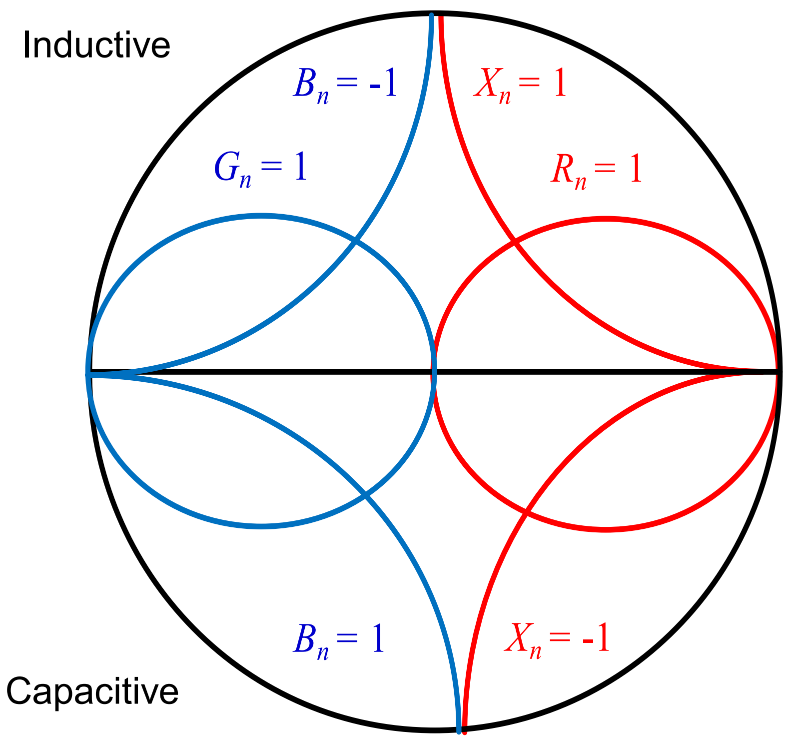


Γ' plane

ZY Chart



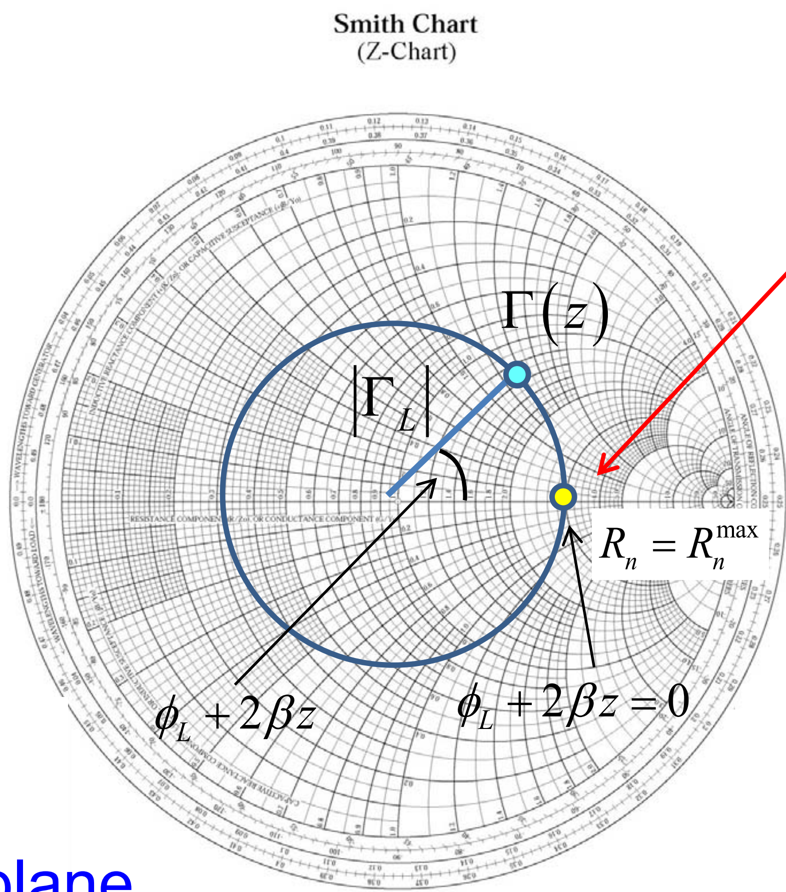
Short-hand version



Γ plane

This is convenient for doing matching problems that involve <u>both</u> series and shunt elements (done later).

Standing Wave Ratio



Γ plane

The SWR is given by the value of R_n on the positive real axis of the Smith chart (R_n^{max}).

Proof:

$$\text{SWR} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

$$Z_n = \frac{1+\Gamma(z)}{1-\Gamma(z)} = \frac{1+\Gamma_L e^{+2j\beta z}}{1-\Gamma_L e^{+2j\beta z}} = \frac{1+|\Gamma_L| e^{j\phi_L} e^{+2j\beta z}}{1-|\Gamma_L| e^{j\phi_L} e^{+2j\beta z}}$$

$$\phi_L + 2\beta z = 0 \;\Rightarrow\; R_n^{\max} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

Electronic Smith Chart

At this link:

http://www.sss-mag.com/topten5.html

Download the following zip file:

smith_v191.zip

Extract the following files:

smith.exe　　mith.hlp　　smith.pdf

This is the application file

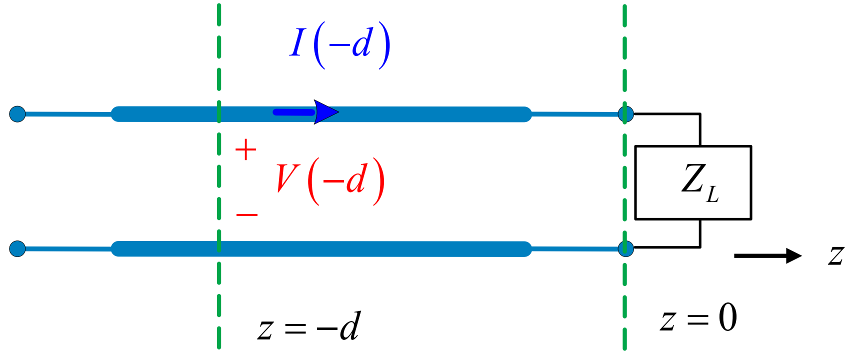
Example 1

$Z_0 = 50\ \Omega$

$Z_L = 100 + j50\ \Omega$

Find $Z(\text{-}d)$ at $d/\lambda_g = 1/4,\ \ 3/8,\ \ 1/2$

Use the *Z* chart.



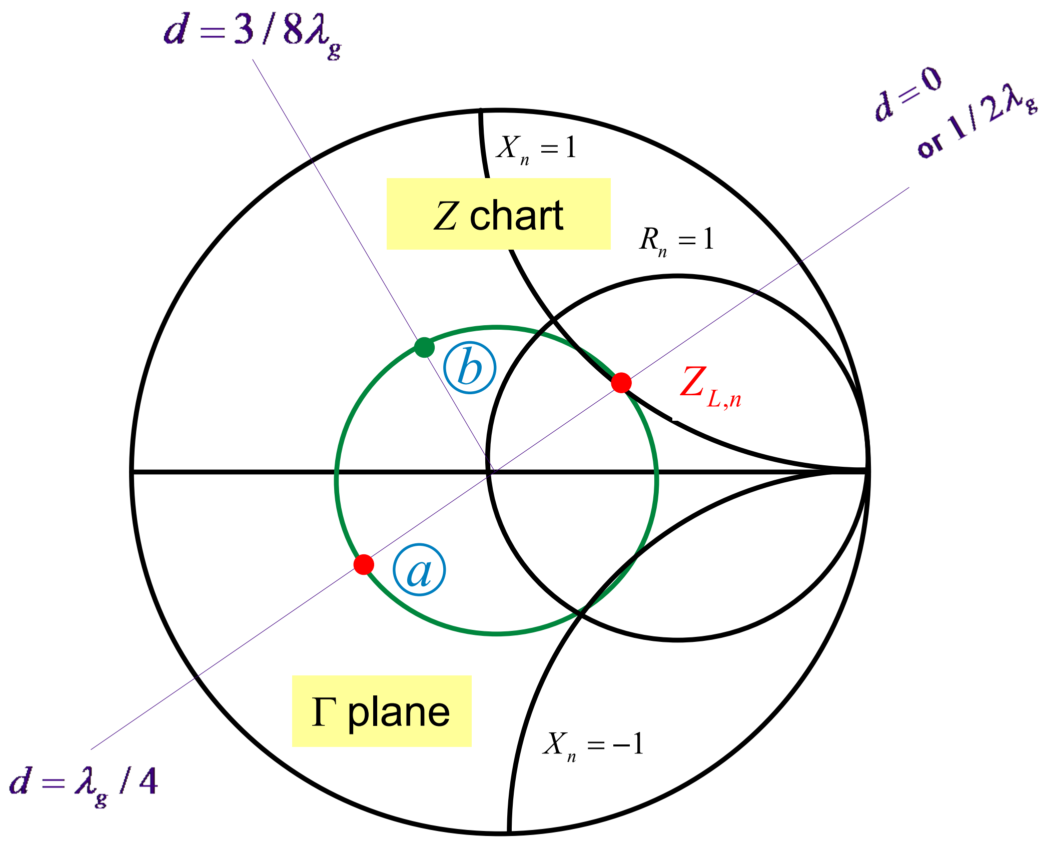
$$Z_{L,n} = \frac{Z_L}{Z_0} = 2 + j1$$

ⓐ

$d = \lambda_g / 4$

$Z_n \approx 0.4 - j0.2$

$\Rightarrow Z(-\lambda_g/4) \approx 20 - j10\,\Omega$



Example 1 (cont.)

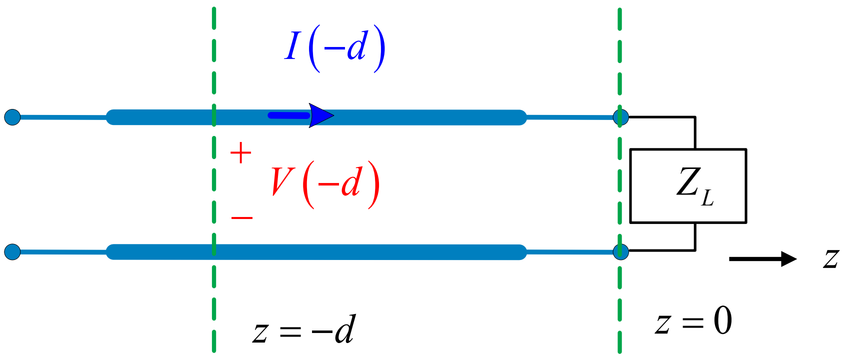
ⓑ

$d = 3/8\lambda_g$

$Z_n \approx 0.5 + j0.5$

$\Rightarrow Z(-3/8\lambda_g) \approx 25 + j25\Omega$

Note : $3/8 + 0.212 - 0.5 = 0.087$

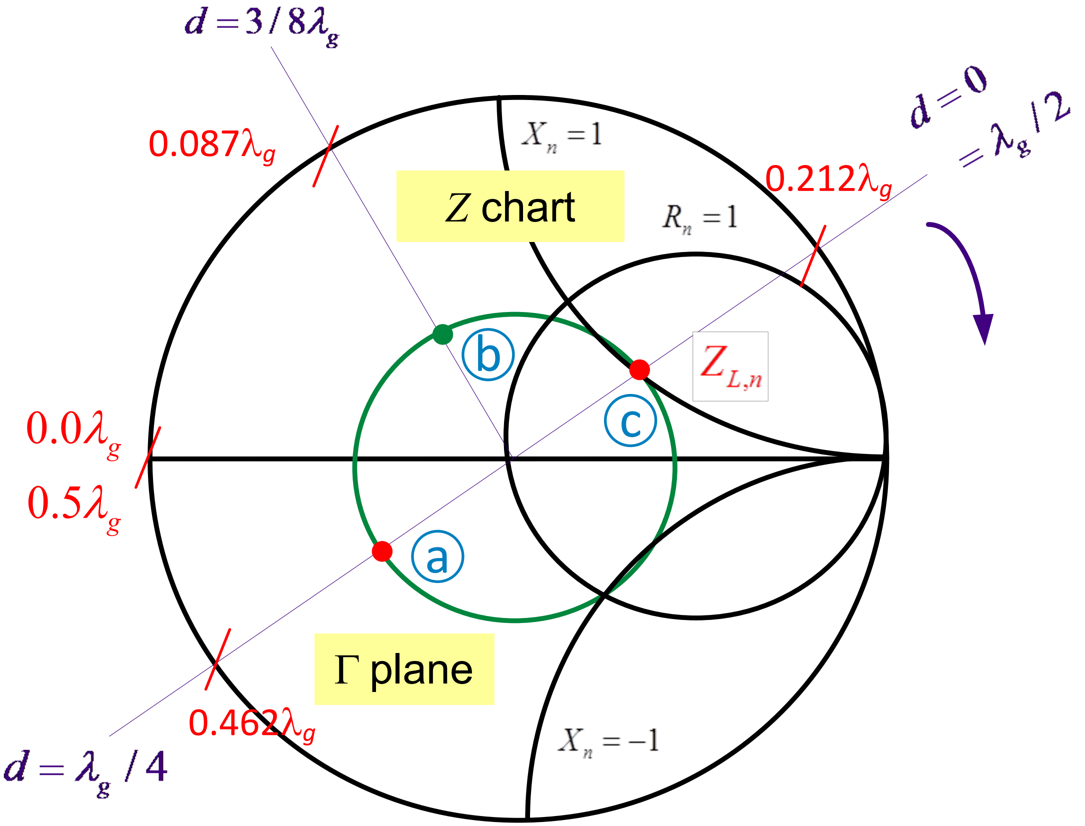


ⓒ

$d = \lambda_g/2$

$Z_n \approx 2 + j1$

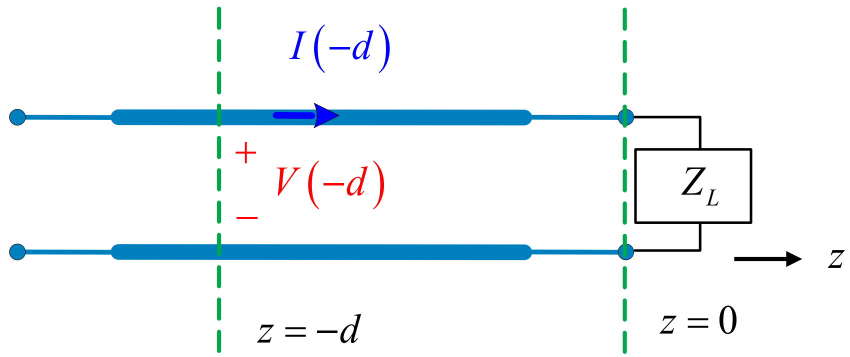
$\Rightarrow Z(-\lambda_g/2) = 100 + j50\Omega$



Example 2

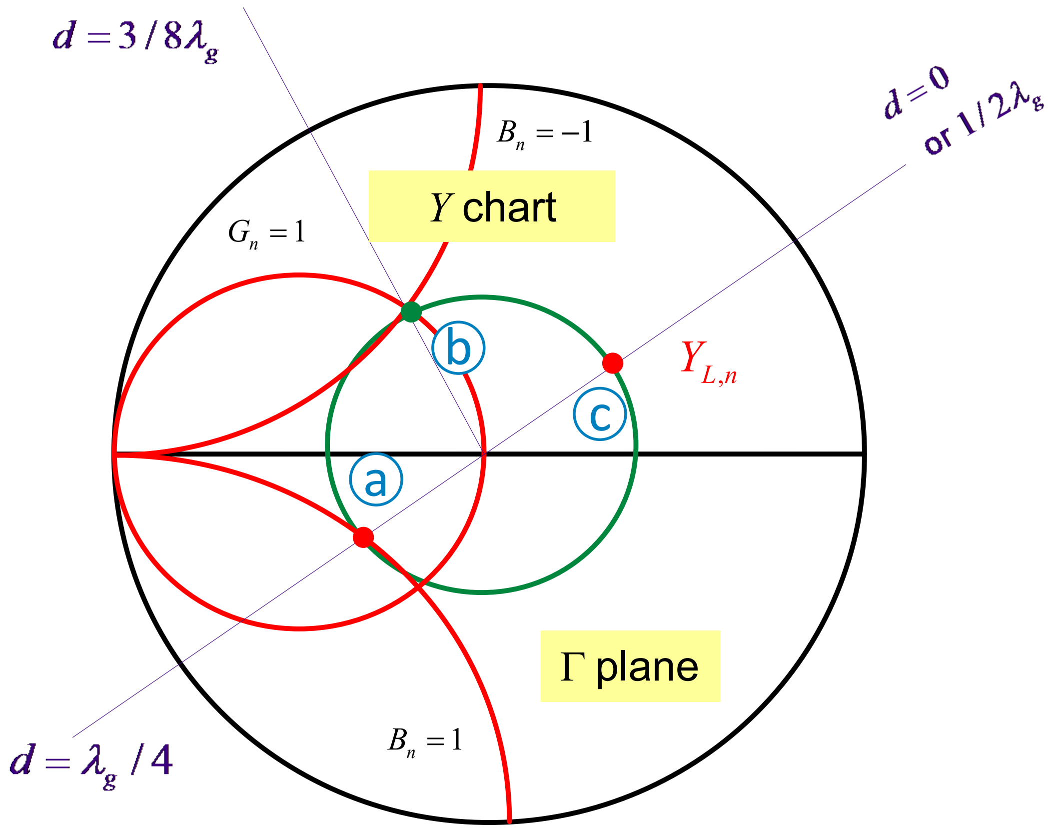
$$Z_0 = 50\ \Omega\ \left(Y_0 = 20\,\text{mS}\right)$$
$$Y_L = 8\,\text{mS} - j4\,\text{mS}$$

Find $Y(\text{-}d)$ at $d/\lambda_g = 1/4,\ 3/8,\ 1/2$



Use the Y chart.

$$Y_{L,n} = \frac{Y_L}{Y_0} = 0.4 - j0.2$$



ⓐ

$$d = \lambda_g / 4$$
$$Y_n \approx 2 + j1$$
$$\Rightarrow Y\left(-\lambda_g/4\right) \approx 40\,\text{mS} + j20\,\text{mS}$$

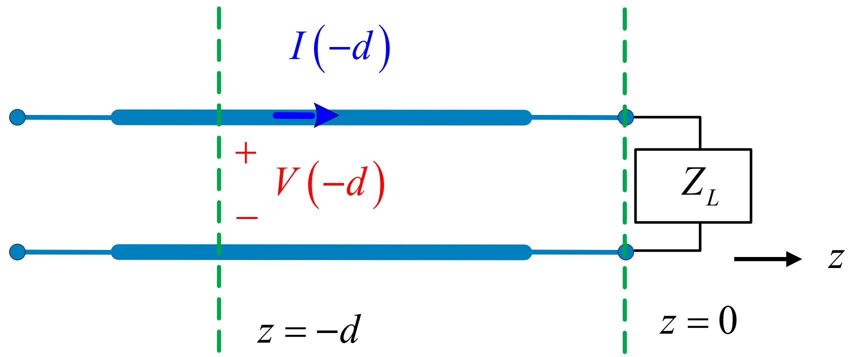
Example 2 (cont.)

ⓑ

$d = 3/8\lambda_g$

$Y_n \approx 1 - j1$

$\Rightarrow Y\left(-3/8\lambda_g\right) \approx 20\,\text{mS} - j20\,\text{mS}$

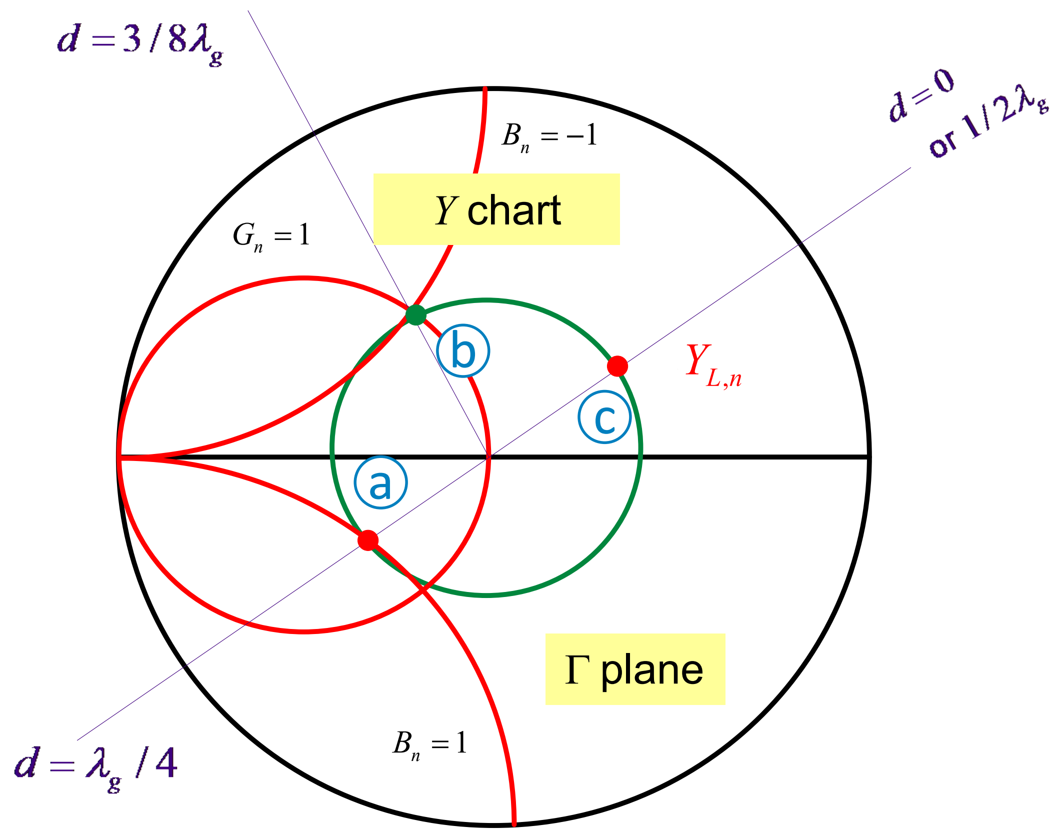


ⓒ

$d = \lambda_g/2$

$Y_n \approx 0.4 - j0.2$

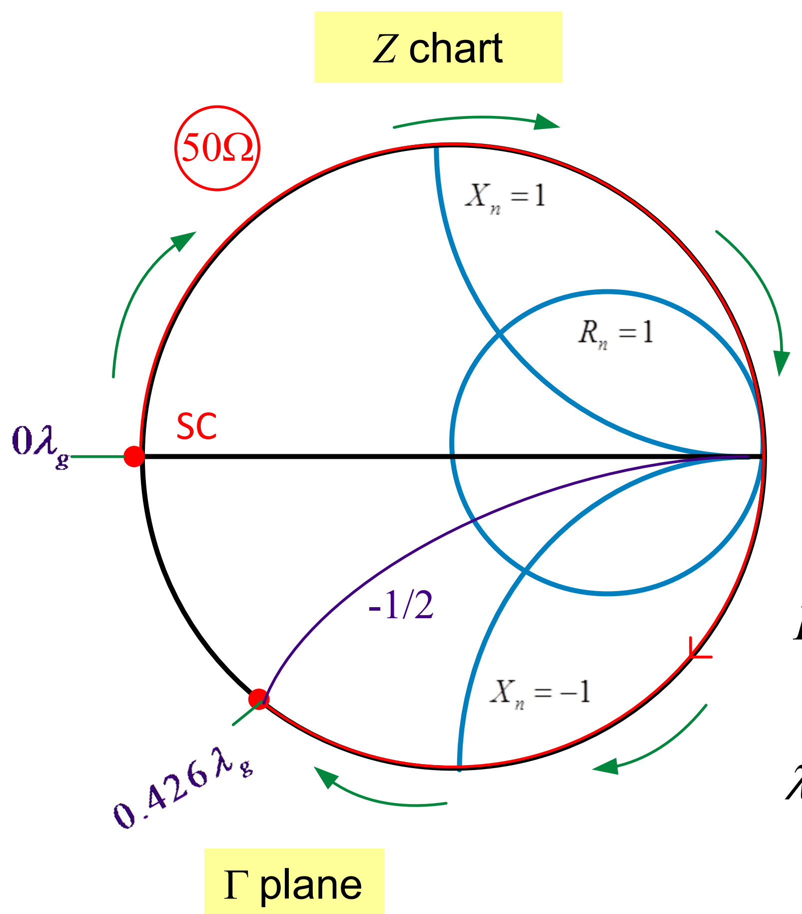
$\Rightarrow Y\left(-\lambda_g/2\right) = 8\,\text{mS} - j4\,\text{mS}$



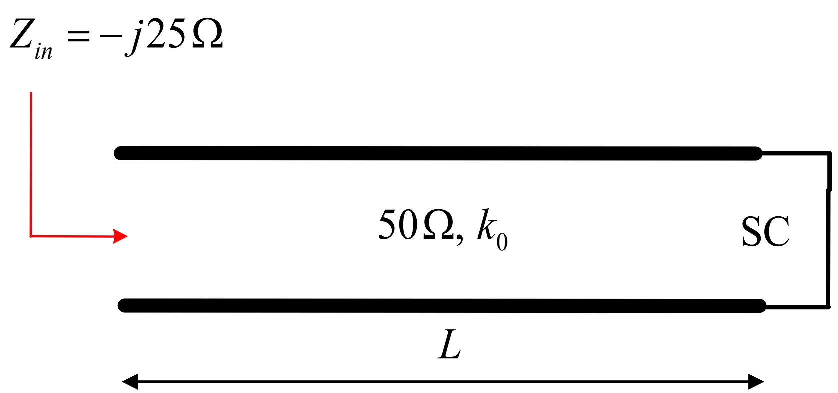
Example 3

Use a short-circuited section of air-filled TEM, 50 Ω transmission line ($\beta = k_0$, $\lambda_g = \lambda_d = \lambda_0$) to create an impedance of Z_{in} = -j25 Ω at f = 10 GHz.

Use the Z chart.



$$Z_{in,n} = -j\frac{25}{50} = -j(0.5)$$



$$L = 0.426\lambda_g - 0\lambda_g = 0.426\lambda_g = 0.426\lambda_0$$

$$\lambda_0 = \frac{c}{f} = \frac{2\pi}{k_0} = \frac{2\pi}{\omega\sqrt{\mu_0\varepsilon_0}}$$

$\lambda_0 = 3.0$ cm

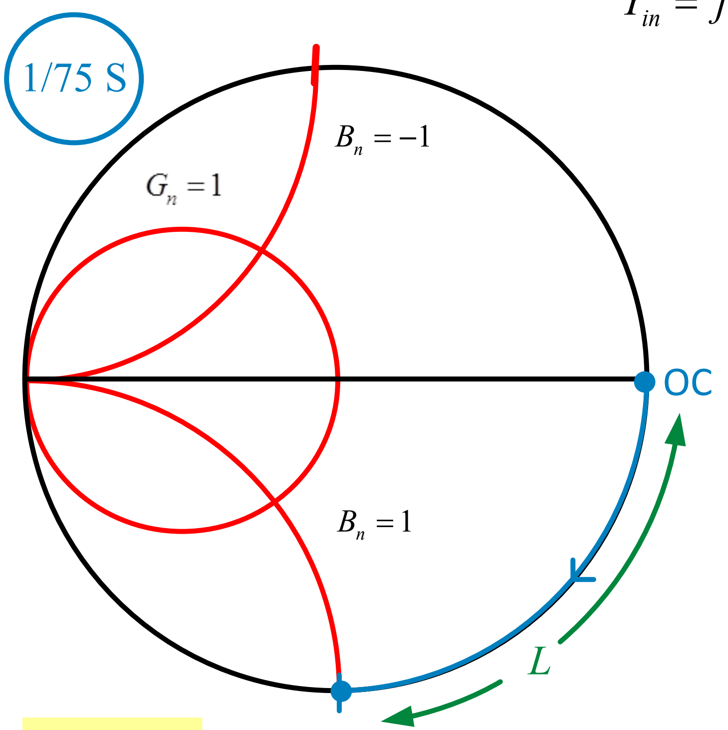
⇨ $L = 1.28\,\text{cm}$

Example 4

Use an open-circuited section of 75 Ω ($Y_0 = 1/75$ S) air-filled transmission line at $f = 10$ GHz to create an admittance of $Y_{in} = j1/75$ S:

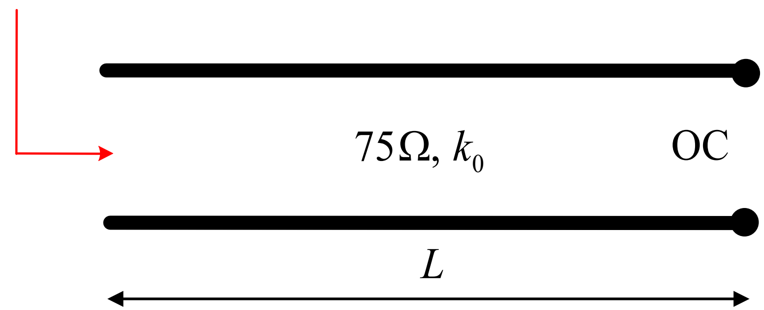
Use the Y chart.

Y chart



Γ plane

$$Y_{in} = j(1/75)\text{S} \Rightarrow Y_{in,n} = j(1)$$



$$L = 0.125\lambda_g = 0.125\lambda_0$$

$\lambda_0 = 3.0$ cm

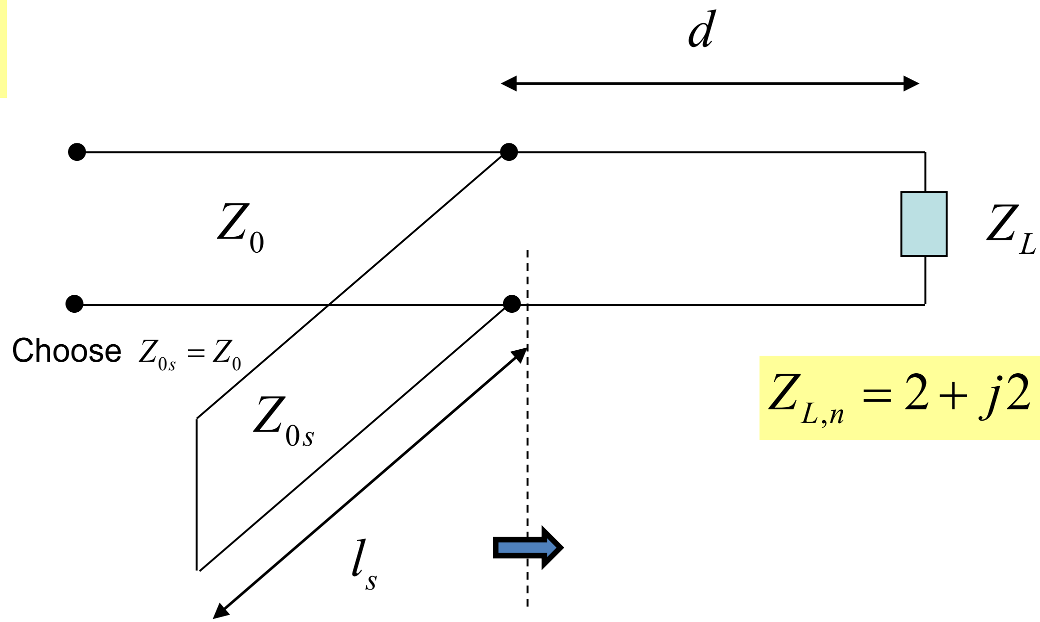
$$\Rightarrow L = 0.375\,\text{cm}$$

Example 5

$$Z_0 = 50\,\Omega$$

$$Z_L = 100 + j100\ \Omega$$

Single-stub matching



$$Z_{L,n} = 2 + j2$$

We want $G_{in} = Y_0 \; (G_{in,n} = 1)$

In this example we will use the "usual" Smith chart (Z chart), but as an admittance calculator.

$$Y_{L,n} = \frac{1}{2 + j2} = 0.25 - j(.25)$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Z_{Ln} - 1}{Z_{Ln} + 1} \;\Rightarrow\; \Gamma_L = 0.62\, e^{j\pi/6} = 0.62\angle 30^\circ$$

Example 5 (cont.)

$0.041\lambda + 0.178\lambda = 0.219\lambda_g$

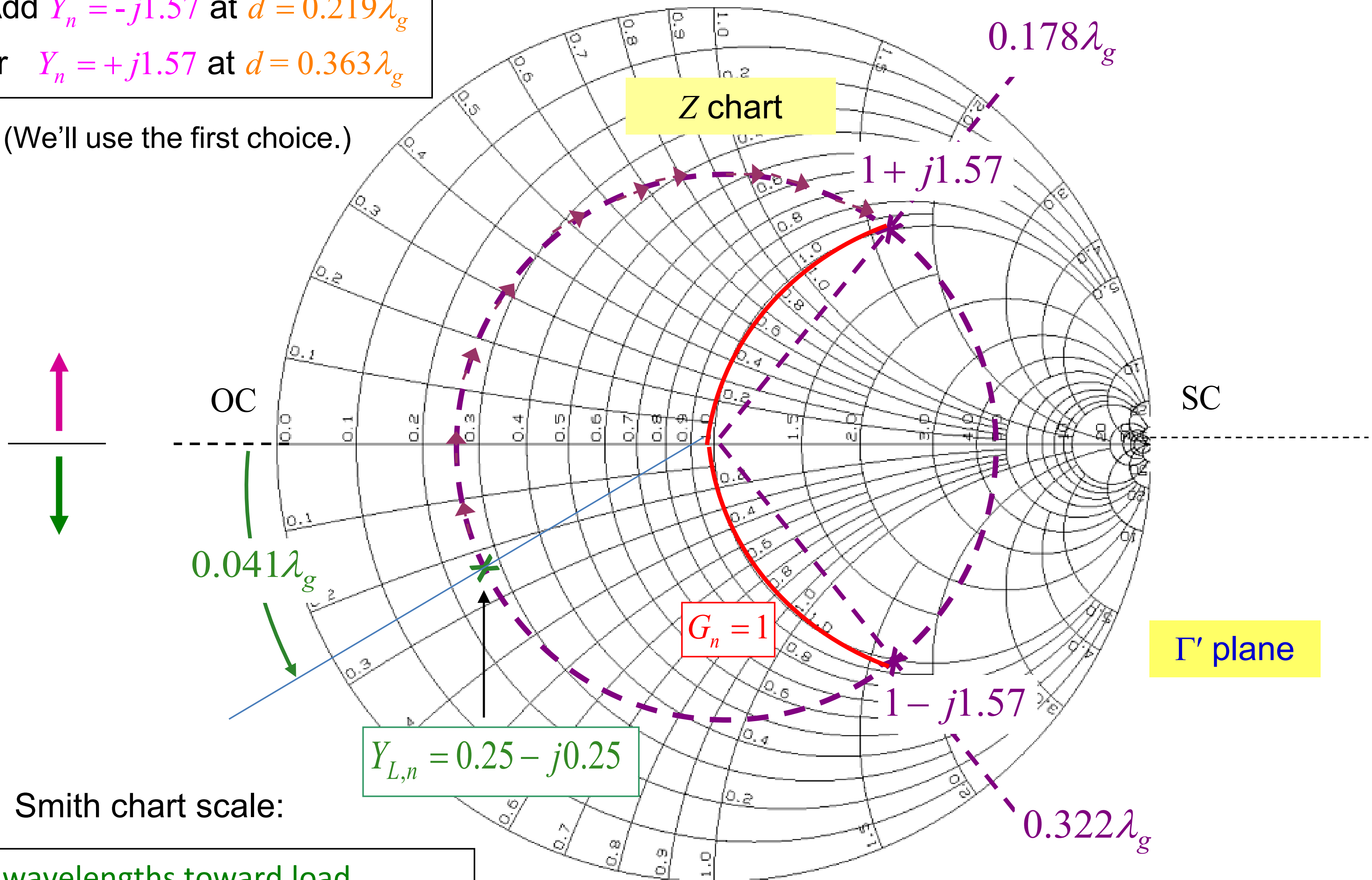
$0.041\lambda + 0.322\lambda = 0.363\lambda_g$

Solution:

Add $Y_n = -j1.57$ at $d = 0.219\lambda_g$

or $Y_n = +j1.57$ at $d = 0.363\lambda_g$

(We'll use the first choice.)



Smith chart scale:

wavelengths toward load
wavelengths toward generator

Example 5 (cont.)

From the Smith chart:

$$l_s = 0.09\lambda_g$$

Analytically:

$$Z_{in}^{sc} = jZ_0 \tan\left(\beta l_s\right)$$

$$Y_{in}^{sc} = -jY_0 \cot\left(\beta l_s\right)$$

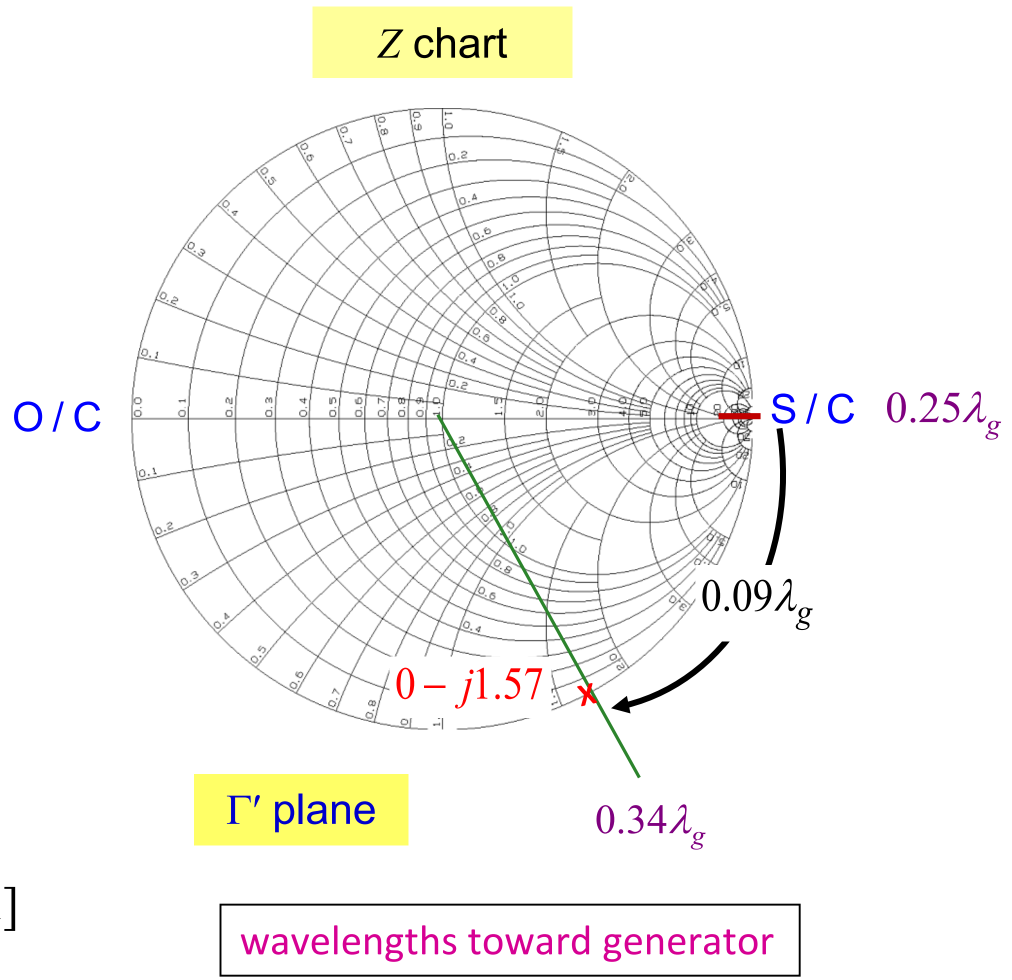
$$B_{s,n} = -\cot\left(\beta l_s\right)$$

$$-1.57 = -\cot \beta l_s$$

$$\cot \beta l_s = 1.57; \;\; \tan \beta l_s = \frac{1}{1.57} = 0.637$$

$$\beta l_s = \frac{2\pi}{\lambda_g} l_s = \tan^{-1}\left(0.637\right) = 0.567 \text{ [rad]}$$

$$l_s = 0.0903\lambda_g$$

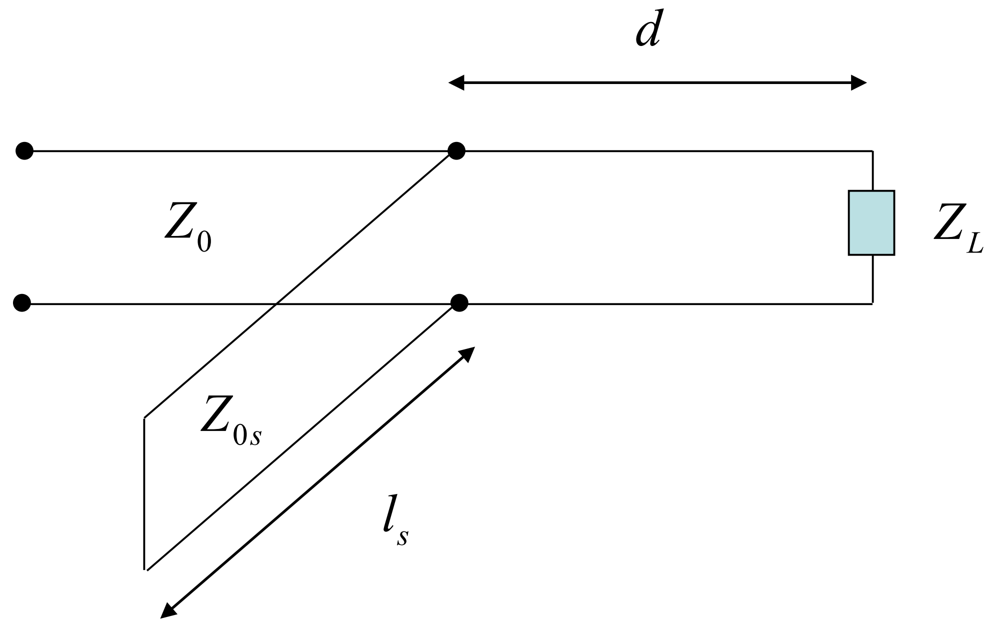


Example 5 (cont.)

$$Z_0 = 50\ \Omega$$

$$Z_L = 100 + j100\ \Omega$$

Single-stub matching



Summary:

$$d = 0.219\lambda_g$$

$$l_s = 0.09\lambda_g$$