Adapted from notes by
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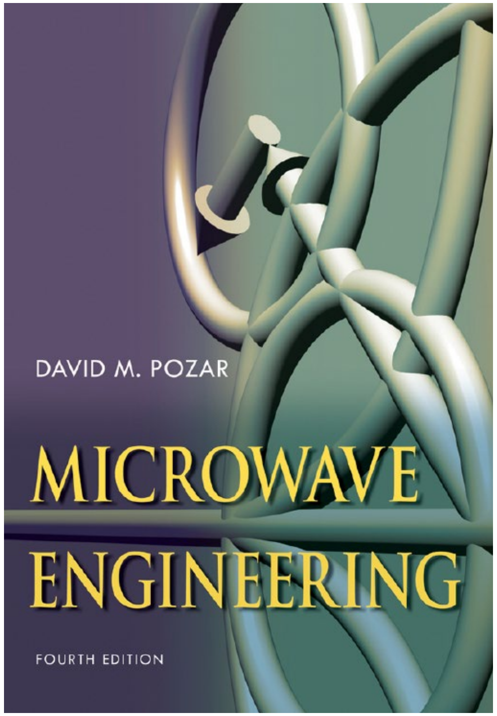
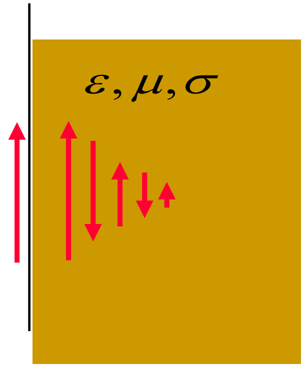


Notes 7
Waveguiding Structures
Part 2: Attenuation



Attenuation on Waveguiding Structures

For most practical waveguides and transmission lines the loss associated with dielectric loss and conductor loss is relatively small.

To account for these losses we will make this approximation:

$$\alpha \approx \alpha_c + \alpha_d$$

Attenuation constant due to conductor loss (ignore dielectric loss) → α_c

Attenuation constant due to dielectric loss (ignore conductor loss) → α_d

Attenuation due to Dielectric Loss: α_d

Lossy dielectric $\Rightarrow$ complex permittivity

$\Rightarrow$ complex wavenumber k

$$k = \omega\sqrt{\mu\varepsilon_c} = k_0\sqrt{\mu_r\varepsilon_{rc}} = k_0\sqrt{\mu_r\varepsilon_r(1 - j\tan\delta_d)}$$

$$k = k' - jk''$$

Note: $k' = \mathrm{Re}\{k\}$

$$\begin{aligned}\varepsilon_c &= \varepsilon - j\frac{\sigma}{\omega} \\ &= \varepsilon' - j\varepsilon'' - j\frac{\sigma}{\omega} \\ &= \varepsilon_c' - j\varepsilon_c'' \\ &= \varepsilon_c'\left(1 - j\frac{\varepsilon_c''}{\varepsilon_c'}\right) \\ &= \varepsilon_c'(1 - j\tan\delta_d) \\ &= \varepsilon_0\varepsilon_r(1 - j\tan\delta\ )\end{aligned}$$

Note : ε_{rc}' is denoted as ε_r.

(e.g., $\varepsilon_r = 2.1$ for Teflon).

Dielectric Attenuation for TEM Mode

TEM mode: $k_z = k$

where

$$k = k_0 \sqrt{\mu_r \varepsilon_r} \sqrt{(1 - j \tan \delta_d)}$$

Assume a small dielectric loss in medium: $\tan \delta << 1$

Use $\sqrt{1 - z} \approx 1 - z/2$ for $|z| << 1$

$$\Rightarrow \; k \approx k_0 \sqrt{\mu_r \varepsilon_r} \left(1 - j\left(\tan \delta_d\right)/2\right)$$

$$k' \approx k_0 \sqrt{\mu_r \varepsilon_r}$$

$$k'' \approx \frac{1}{2} k_0 \sqrt{\mu_r \varepsilon_r} \tan \delta_d$$

Summary of Dielectric Attenuation

TEM mode

$$k_z = \beta - j\alpha_d = k$$

$$\beta = \mathrm{Re}(k) = k'$$

$$\alpha_d = -\mathrm{Im}(k) = k''$$

$$\beta \approx k_0 \sqrt{\mu_r \varepsilon_r}$$

$$\alpha_d \approx \frac{1}{2} k_0 \sqrt{\mu_r \varepsilon_r} \tan \delta_d$$

$$k = k' - jk'' = k_0 \sqrt{\mu_r \varepsilon_r} \sqrt{1 - j \tan \delta_d}$$

Dielectric Attenuation for Waveguide Mode

An exact general expression for the dielectric attenuation:

$$k_z = \beta - j\alpha_d = \sqrt{k^2 - k_c^2}$$

$$k = k' - jk'' = k_0 \sqrt{\mu_r \varepsilon_r} \sqrt{1 - j \tan \delta_d}$$

Remember: The value k_c is always real, regardless of whether the waveguide filling material is lossy or not.

Note: The radical sign denotes the *principal square root*:

$$-\pi < arg\left(z\right) < \pi$$

Dielectric Attenuation for Waveguide Mode (cont.)

Approximation for the wavenumber of a waveguide mode :

$$k_z = \beta - j\alpha_d = \sqrt{k^2 - k_c^2}$$
$$= \sqrt{k_0^2 \mu_r \varepsilon_r (1 - j\tan\delta_d) - k_c^2}$$
$$= \sqrt{\left(k_0^2 \mu_r \varepsilon_r - k_c^2\right) - jk_0^2 \mu_r \varepsilon_r \tan\delta_d}$$

Assume a small dielectric loss:

$$k_0^2 \mu_r \varepsilon_r \tan\delta_d << k_0^2 \mu_r \varepsilon_r - k_c^2$$

Then use:

$$\sqrt{a - z} = \sqrt{a}\sqrt{1 - (z/a)} \approx \sqrt{a}\left[1 - \frac{1}{2}\left(\frac{z}{a}\right)\right] = \sqrt{a} - \frac{1}{2}\left(\frac{z}{\sqrt{a}}\right) \qquad \text{for} \quad |z| << a$$

Approximate Dielectric Attenuation (cont.)

Hence, we have:

$$k_z \approx \sqrt{\left(k_0^2 \mu_r \varepsilon_r - k_c^2\right)} - \frac{1}{2}\left(\frac{j k_0^2 \mu_r \varepsilon_r \tan\delta_d}{\sqrt{\left(k_0^2 \mu_r \varepsilon_r - k_c^2\right)}}\right)$$

We assume here that we are above cutoff.

This gives us:

$$\beta \approx \sqrt{\left(k_0^2 \mu_r \varepsilon_r - k_c^2\right)}$$

$$\alpha_d \approx \frac{k_0^2 \mu_r \varepsilon_r \tan\delta_d}{2\beta}$$

Summary of Dielectric Attenuation

Waveguide mode (TM$_z$ or TE$_z$)

$$k_z = \beta - j\alpha_d = \sqrt{k^2 - k_c^2}$$

$$\beta = \operatorname{Re}\sqrt{k^2 - k_c^2}$$

$$\alpha_d = -\operatorname{Im}\sqrt{k^2 - k_c^2}$$

$$\beta \approx \sqrt{k_0^2 \mu_r \varepsilon_r - k_c^2}$$

$$\alpha_d \approx \frac{k_0^2 \mu_r \varepsilon_r \tan\delta_d}{2\beta}$$

$$k = k' - jk'' = k_0 \sqrt{\mu_r \varepsilon_r}\sqrt{1 - j\tan\delta_d}$$

Attenuation due to Conductor Loss

Assuming a small amount of conductor loss:

We can assume that the fields of the lossy guide are approximately the same as those for lossless guide, except with a small amount of attenuation.

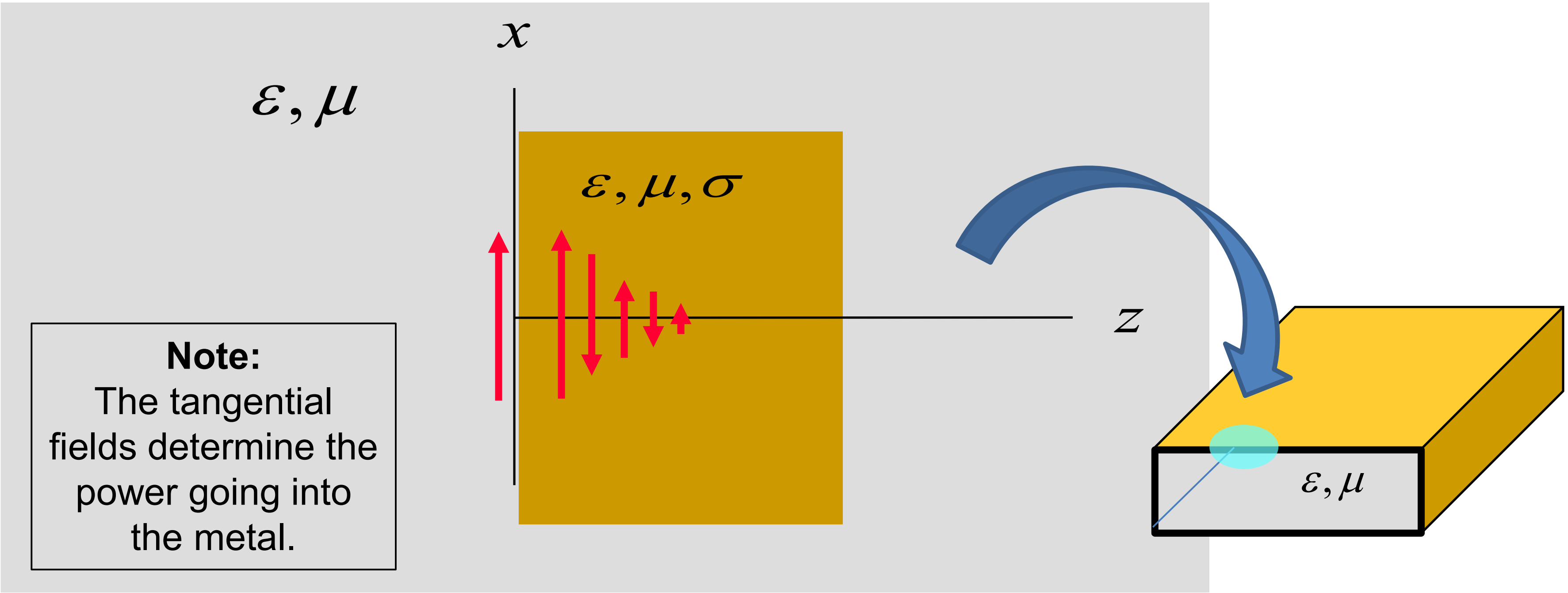
$\Rightarrow$ We can use a "perturbation" method to determine α_c .

Notes:

- Dielectric loss does not change the shape of the fields at all in a waveguide or transmission line, since the boundary conditions remain the same (PEC).
- Conductor loss does disturb the fields slightly.

Surface Resistance

This is a very important concept for calculating loss at a metal surface.



Plane wave in a good conductor

Note: In this figure, z is the direction normal to the metal surface, not the axis of the waveguide. Also, the electric field is assumed to be in the x direction for simplicity.

Surface Resistance (cont.)

Assume $\left|\dfrac{\sigma}{\omega\varepsilon}\right| \gg 1$ (for the metal)

The we have (for the metal):

$$k = \omega\sqrt{\mu}\left(\varepsilon - j\frac{\sigma}{\omega}\right)^{1/2} \approx \omega\sqrt{\mu}\left(-j\frac{\sigma}{\omega}\right)^{1/2} = \omega\sqrt{\frac{\mu\sigma}{\omega}}\left(\frac{1-j}{\sqrt{2}}\right) = \sqrt{\frac{\omega\mu\sigma}{2}}(1-j)$$

Hence

$$k = k' - jk'' \approx \sqrt{\frac{\omega\mu\sigma}{2}}\left(1-j\right)$$

Therefore

$$k' \approx k'' \approx \sqrt{\frac{\omega\mu\sigma}{2}}$$

Surface Resistance (cont.)

$$k' \approx k'' \approx \sqrt{\frac{\omega\mu\sigma}{2}}$$

Denote $\delta \equiv d_p = \dfrac{1}{k''}$ "skin depth" = "depth of penetration"

Note : $e^{-1} \approx 0.37$ (For $z = \delta$, fields are down to 37% of their values at the surface.)

Then we have

$$\delta = d_p = \sqrt{\frac{2}{\omega\mu\sigma}}$$

$$k' \approx k'' \approx \frac{1}{\delta}$$

At 3 GHz, the skin depth for copper is about 1.2 microns.

Surface Resistance (cont.)

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$$

Example: copper (pure)

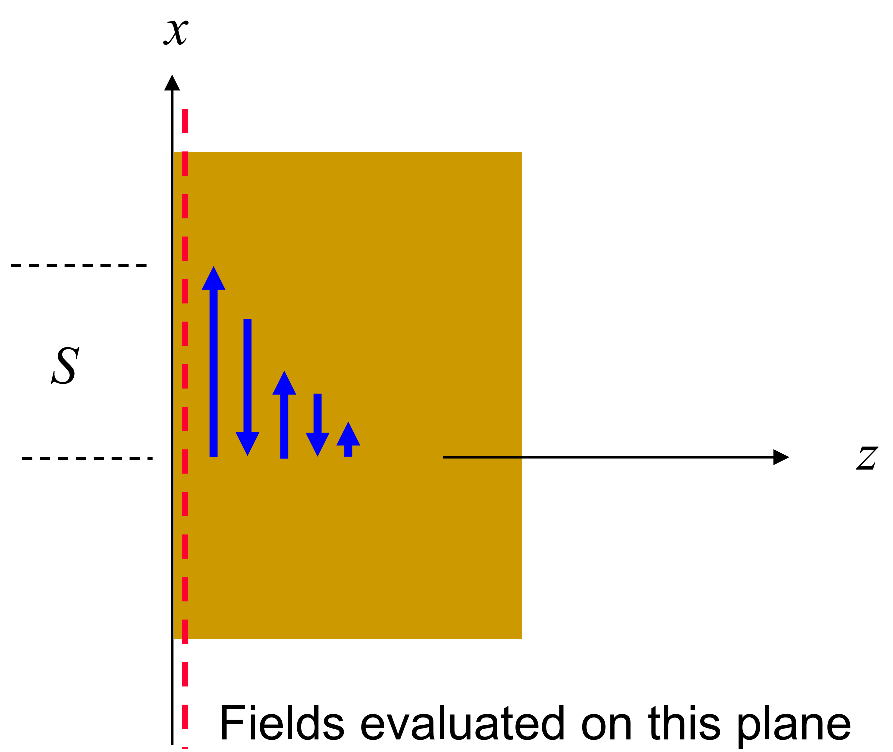
$$\mu = \mu_0 = 4\pi \times 10^{-7}\ [\text{H/m}]$$

$$\sigma = 5.8 \times 10^{7}\ [\text{S/m}]$$

Note:
A value of 3.0×10^7 [S/m] is often assumed for "practical" copper.

Frequency	δ
1 [Hz]	6.6 [cm]
10 [Hz]	2.1 [cm]
100 [Hz]	6.6 [mm]
1 [kHz]	2.1 [mm]
10 [kHz]	0.66 [mm]
100 [kHz]	0.21 [mm]
1 [MHz]	66 [μm]
10 [MHz]	20.1 [μm]
100 [MHz]	6.6 [μm]
1 [GHz]	2.1 [μm]
10 [GHz]	0.66 [μm]
100 [GHz]	0.21 [μm]

Surface Resistance (cont.)



$$\left\langle \mathscr{P}_d \right\rangle = \text{time-average power dissipated} / \text{m}^2 \text{ on } S$$

$$\left\langle \mathscr{P}_d \right\rangle = \frac{1}{2}\text{Re}\left(\underline{E} \times \underline{H}^*\right) \cdot \underline{\hat{z}} = \frac{1}{2}\text{Re}\left(E_x H_y^{\;*}\right)_{z=0}$$

Surface Resistance (cont.)

Inside conductor: $E_x = \eta H_y$

where

$$\eta = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon - j\frac{\sigma}{\omega}}} \approx \sqrt{\frac{\mu}{-j\frac{\sigma}{\omega}}} = \sqrt{j}\sqrt{\frac{\omega\mu}{\sigma}}$$

$$= \frac{1+j}{\sqrt{2}}\sqrt{\frac{\omega\mu}{\sigma}}$$

$$= (1+j)\sqrt{\frac{\omega\mu}{2\sigma}}$$

$$= (1+j)R_s$$

$$= Z_s$$

"Surface resistance (Ω)"

$$R_s \equiv \sqrt{\frac{\omega\mu}{2\sigma}}$$

"Surface impedance (Ω)"

$$Z_s = (1+j)R_s$$

Note:

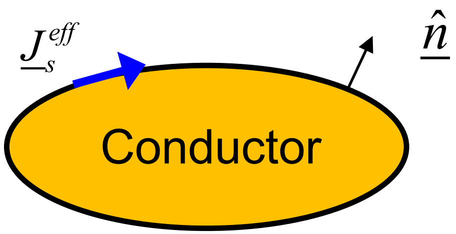
$$\left.\frac{E_x}{H_y}\right|_{z=0,\ \text{air}} = \left.\frac{E_x}{H_y}\right|_{\text{plane wave in metal}}$$

Note: To be more general:

$$\underline{E}_t = Z_s\left(\underline{\hat{n}} \times \underline{H}_t\right)$$

$\underline{\hat{n}}$ = outward normal

Surface Resistance (cont.)



$$\underline{E}_t = Z_s \left(\underline{\hat{n}} \times \underline{H}_t \right)$$

$\underline{E}_t$ = tangential electric field at surface

$\underline{H}_t$ = tangential magnetic field at surface

$\underline{\hat{n}}$ = outward unit normal to conductor surface

"Effective surface current"

$$\underline{J}_s^{eff} \approx \left(\underline{\hat{n}} \times \underline{H}_t \right)$$

Hence we have

$$\underline{E}_t \approx Z_s \, \underline{J}_s^{eff}$$

For the "effective" surface current density we imagine the actual volume current density to be collapsed into a planar surface current.

The surface impedance gives us the ratio of the tangential electric field at the surface to the effective surface current flowing on the object.

Surface Resistance (cont.)

Summary for a Good Conductor

$$\underline{E}_t = Z_s \left(\underline{\hat{n}} \times \underline{H}_t \right)$$ (fields at the surface)

$$\underline{E}_t \approx Z_s \, \underline{J}_s^{eff}$$ (effective surface current)

$$Z_s = (1 + j)\, R_s$$

$$R_s = \sqrt{\frac{\omega\mu}{2\sigma}} = \frac{1}{\sigma\delta}$$

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$$

Surface Resistance (cont.)

$$R_s = \sqrt{\frac{\omega\mu}{2\sigma}} = \frac{1}{\sigma\delta}$$

Example: copper (pure)

$$\mu = \mu_0 = 4\pi \times 10^{-7}\ [\text{H/m}]$$

$$\sigma = 5.8 \times 10^{7}\ [\text{S/m}]$$

Note:
A value of 3.0×10^{7} [S/m] is often assumed for "practical" copper.

Frequency	R_s
1 [Hz]	2.61×10^{-7} [Ω]
10 [Hz]	8.25×10^{-7} [Ω]
100 [Hz]	2.61×10^{-6} [Ω]
1 [kHz]	8.25×10^{-6} [Ω]
10 [kHz]	2.61×10^{-5} [Ω]
100 [kHz]	8.25×10^{-5} [Ω]
1 [MHz]	2.61×10^{-4} [Ω]
10 [MHz]	8.25×10^{-4} [Ω]
100 [MHz]	0.00261 [Ω]
1 [GHz]	0.00825 [Ω]
10 [GHz]	0.0261 [Ω]
100 [GHz]	0.0825 [Ω]

Surface Resistance (cont.)

Returning to the power calculation, we have:

$$\langle \mathscr{P}_d \rangle = \frac{1}{2}\mathrm{Re}\left(E_x H_y^*\right)_{z=0} = \frac{1}{2}\mathrm{Re}\left(\left(Z_s H_{y0}\right) H_{y0}^*\right) = \frac{1}{2} R_s \left|H_{y0}\right|^2$$

In general, $\langle \mathscr{P}_d \rangle = \frac{1}{2} R_s \left|\underline{H}_{t0}\right|^2$

For a good conductor, $\underline{J}_s^{eff} \approx \underline{\hat{n}} \times \underline{H}_{t0}$

Hence $$\langle \mathscr{P}_d \rangle = \frac{1}{2} R_s \left|\underline{J}_s^{eff}\right|^2$$

This gives us the power dissipated per square meter of conductor surface, if we know the effective surface current density flowing on the surface.

PEC limit: $\underline{J}_s^{eff} \to \underline{J}_s^{PEC}$

Perturbation method : Assume that $\underline{J}_s^{eff} \approx \underline{J}_s^{PEC}$

Perturbation Method for α_c

Power flow along the guide: $P(z) = P_0 \, e^{-2\alpha z}$

Power P_0 @ $z = 0$ is calculated from the lossless case.

Power loss (dissipated) per unit length: $P_l = -\dfrac{dP(z)}{dz}$ $\quad \left(\Delta P_{\text{abs}} = -\Delta P_{\text{flow}}(z)\right)$

$$\Rightarrow \; P_l(z) = 2\alpha P_0 e^{-2\alpha z} = 2\alpha P(z)$$

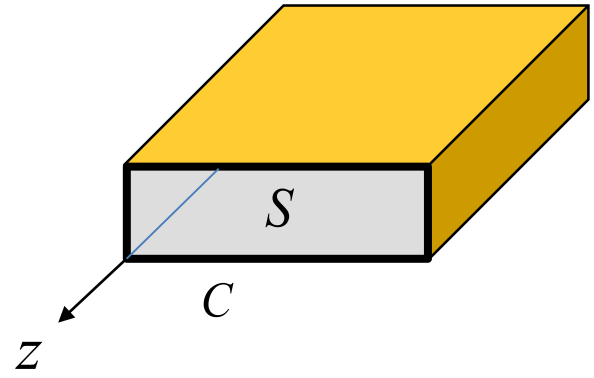
$$\Rightarrow \quad \alpha = \frac{P_l(z)}{2P(z)} = \frac{P_l(0)}{2P_0}$$

Note:
$\alpha = \alpha_c$ for conductor loss

Perturbation Method: Waveguide Mode

$$\alpha_c = \frac{P_l(0)}{2P_0}$$

There is a single conducting boundary.



$$P_0 = \mathrm{Re}\left\{ \iint_S \left(\frac{1}{2} \underline{E} \times \underline{H}^* \right) \cdot \hat{z}\, dS \bigg|_{z=0} \right\}$$

$$P_l(0) = \frac{R_s}{2} \int_C \left| \underline{J}_s \right|^2 \bigg|_{z=0} d\ell$$

For these calculations, we neglect loss when we determine the fields and currents.

$$\underline{J}_s = \underline{\hat{n}} \times \underline{H}$$

On PEC conductor

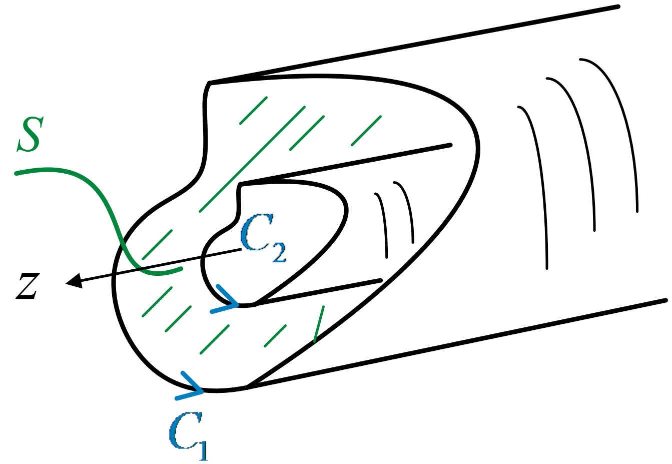
Surface resistance of metal conductors:

$$R_s = \sqrt{\frac{\omega\mu}{2\sigma}}$$

Perturbation Method: TEM Mode

$$\alpha_c = \frac{P_l(0)}{2P_0}$$

There are two conducting boundaries.



$$P_0 = \mathrm{Re}\left\{ \iint_S \left(\frac{1}{2} \underline{E} \times \underline{H}^* \right) \cdot \hat{z}\, dS \Big|_{z=0} \right\}$$

$$= \frac{1}{2} Z_0 |I|^2$$

For these calculations, we neglect loss when we determine the fields and currents.

$$\left(Z_0 = Z_0^{lossless} \right)$$

$$P_l(0) = \frac{1}{2} \int_{C_1} R_{s1} \left| \underline{J}_s \right|^2 \Big|_{z=0} d\ell \;+\; \frac{1}{2} \int_{C_2} R_{s1} \left| \underline{J}_s \right|^2 \Big|_{z=0} d\ell$$

$$\underline{J}_s = \underline{\hat{n}} \times \underline{H}$$

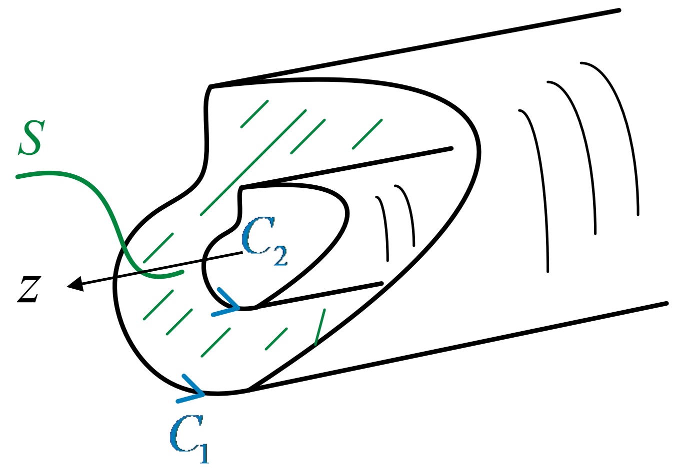
On PEC conductor

Surface resistance of metal conductors:

$$R_s = \sqrt{\frac{\omega\mu}{2\sigma}} \quad \left(\sigma = \sigma_1 \text{ or } \sigma_2 \right)$$

Wheeler Incremental Inductance Rule

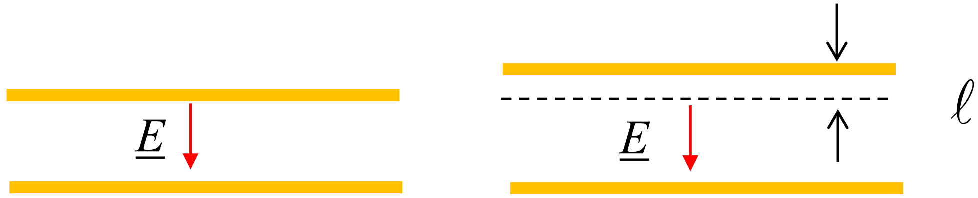
The Wheeler incremental inductance rule gives an alternative method for calculating the conductor attenuation on a transmission line (TEM mode): It is useful when you have a formula for Z_0.



$$\alpha_c = \left(\frac{R_s}{2Z_0\eta}\right)\frac{\partial Z_0}{\partial \ell}$$

The formula is applied for each conductor and the conductor attenuation from each of the two conductors is then added.

In this formula, ℓ (for a given conductor) is the distance by which the conducting boundary is receded away from the field region.



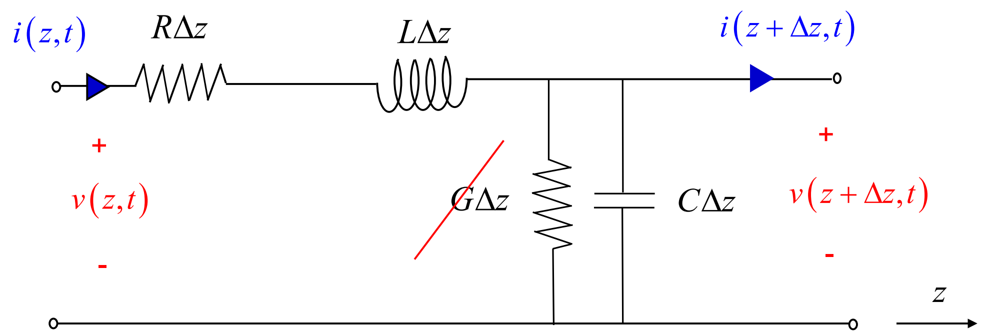
The top plate of a PPW line is shown being receded.

H. Wheeler, "Formulas for the skin-effect," Proc. IRE, vol. 30, pp. 412-424, 1942.

Calculation of R for TEM Mode

From α_c we can calculate R (the resistance per unit length of the transmission line):

$$\alpha_c = \frac{P_l(0)}{2P_0}$$



$$P_0 = \frac{1}{2} Z_0 \left| I_0 \right|^2$$

$$P_l(0) = \frac{1}{2} R \left| I_0 \right|^2$$

Assume conductor loss only.

$$\Rightarrow \quad \alpha_c = \frac{R}{2Z_0}$$

$$\Rightarrow \quad R = 2Z_0\alpha_c$$

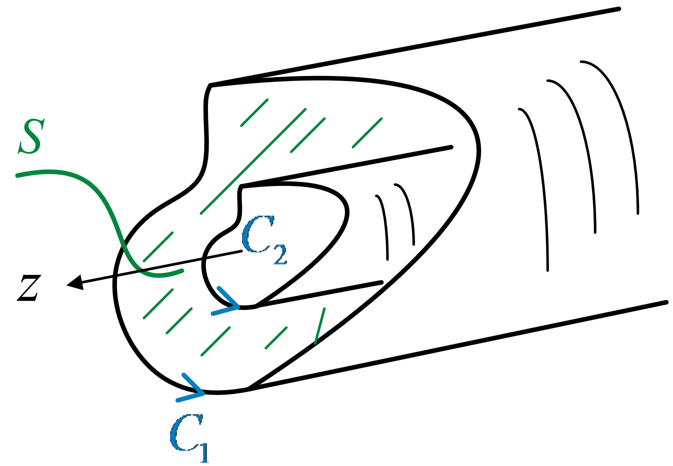
Summary of Attenuation Formulas

Transmission Line (TEM Mode)

Method #1

$$\alpha = \operatorname{Re}\sqrt{(R + j\omega L)(G + j\omega C)}$$

Note: Set $R = 0$ for α_d. Set $G = 0$ for α_c.



Method #2

$$k = k' - jk'' = k_0\sqrt{\mu_r \varepsilon_r}\sqrt{1 - j\tan\delta_d}$$

$$\alpha = \alpha_d + \alpha_c$$

$$\alpha_d = k'' \approx \frac{1}{2} k_0 \sqrt{\mu_r \varepsilon_r}\tan\delta_d$$

$$\alpha_c = \frac{R}{2Z_0} = \frac{P_l(0)}{2P_0}$$

$$P_0 = \frac{1}{2} Z_0 \left|I_0\right|^2$$

$$P_l(0) = \frac{R_{s1}}{2}\int_{C_1} \left|\underline{J}_s\right|^2\Big|_{z=0} d\ell + \frac{R_{s2}}{2}\int_{C_2} \left|\underline{J}_s\right|^2\Big|_{z=0} d\ell$$

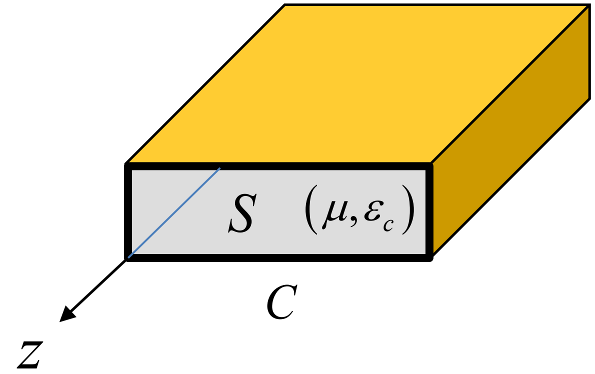
Summary of Attenuation Formulas (cont.)

Waveguide (TM_z or TE_z Mode)

$$\alpha = \alpha_d + \alpha_c$$

$$\alpha_d = -\mathrm{Im}\sqrt{k^2 - k_c^2}$$

$$\alpha_c = \frac{P_l(0)}{2P_0}$$



$$k = k' - jk'' = k_0\sqrt{\mu_r \varepsilon_r}\sqrt{1 - j\tan\delta_d}$$

$$P_0 = \mathrm{Re}\left\{ \iint_S \left(\frac{1}{2} \underline{E} \times \underline{H}^* \right) \cdot \hat{z}\, dS \bigg|_{z=0} \right\}$$

$$P_l(0) = \frac{R_s}{2} \int_C \left| \underline{J}_s \right|^2 \bigg|_{z=0} d\ell$$

Comparison of Attenuation

Approximate attenuation in dB per meter

Waveguides are getting smaller →

Frequency	RG59 Coax		WR975 WG	WR159 WG	WR90 WG	WR42 WG	WR19 WG	WR10 WG
1 [MHz]	0.01		NA	NA	NA	NA	NA	NA
10 [MHz]	0.03		NA	NA	NA	NA	NA	NA
100 [MHz]	0.11		NA	NA	NA	NA	NA	NA
1 [GHz]	0.4		0.004	NA	NA	NA	NA	NA
5 [GHz]	1.0		OM	0.04	NA	NA	NA	NA
10 [GHz]	1.5		OM	OM	0.11	NA	NA	NA
20 [GHz]	2.3		OM	OM	OM	0.37	NA	NA
50 [GHz]	OM		OM	OM	OM	OM	1.0	NA
100 [GHz]	OM		OM	OM	OM	OM	OM	3.0

OM = overmoded

NA = below cutoff

Typical single-mode fiber optic cable: 0.3 dB/km
Typical multimode fiber optic cable: 3 dB/km

Comparison of Waveguide with Wireless System (Two Antennas)

Waveguide: $P(z) = P(0)e^{-2\alpha z}$ (attenuating wave)

Antenna: $P(r) = \dfrac{A}{r^2}$ (spreading spherical wave)

- For small distances, the waveguide delivers more power (no spreading).
- For large distances, the antenna (wireless) system will deliver more power.

Wireless System (Two Antennas)

Here we examine a wireless system in more detail.

Two antennas (transmit and receive):

$$P_r = \frac{P_t}{4\pi r^2} G_t A_{er}$$

$P_t =$ power transmitted

$P_r =$ power received

Matched receive antenna: $$A_{er} = G_r \left(\frac{\lambda_0^2}{4\pi} \right)$$ (from antenna theory)

$A_{er} =$ effective area of receive antenna

Hence, we have:

$$P_r = P_t \frac{G_t G_r}{4\pi r^2} \frac{\lambda_0^2}{4\pi}$$

Wireless System (Two Antennas) (cont.)

$$\frac{P_r}{P_t} = G_t G_r \left(\frac{\lambda_0}{4\pi r} \right)^2$$

Friis transmission formula

Total dB of attenuation:

$$\text{dB} = -10\log_{10}\left(\frac{P_r}{P_t} \right)$$

Hence, we have:

$$\text{dB} = -10\log_{10}\left(G_t G_r \right) - 20\log_{10}\left(\frac{\lambda_0}{4\pi} \right) + 20\log_{10}\left(r \right)$$

↑

The dB attenuation increases slowly with distance

dB Attenuation:

Comparison of Waveguiding system with Wireless System

Waveguiding system: $$\text{dB} = 8.686(\alpha z)$$

Wireless system: $$\text{dB} = -10\log_{10}(G_t G_r) - 20\log_{10}\left(\frac{\lambda_0}{4\pi}\right) + 20\log_{10}(r)$$

Examples:

(a) Two Half - Wavelength Dipole Antennas: $G_t = G_r = 1.64$

(b) Dish antenna + Dipole antenna: $G_t = A_{et}\left(\frac{4\pi}{\lambda_0^2}\right), \quad A_{et} \approx \pi (D/2)^2 \qquad G_r = 1.64$

transmit receive

$D =$ diameter of dish (choose 34 meters)

(large dish in NASA Deep Space Network)

dB Attenuation:

Comparison of Waveguiding System with Wireless System

1 GHz	RG59	Single Mode	Two Dipoles	34m Dish+Dipole
Distance	Coax	Fiber	Wireless	Wireless
1 m	0.4	0.0003	28.2	-
10 m	4	0.003	48.2	-
100 m	40	0.03	68.2	-
1 km	400	0.3	88.2	39.3
10 km	4000	3	108.2	59.3
100 km	-	30	128.2	79.3
1000 km	-	300	148.2	99.3
10,000 km	-	3000	168.2	119.3
100,000 km	-	-	188.2	139.3
1,000,000 km	-	-	208.2	159.3
10,000,000 km	-	-	228.2	179.3
100,000,000 km	-	-	248.2	199.3