Tunable semiconductor lasers

Thesis qualifying exam presentation by

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Outline

1. Introduction and motivation
2. Semiconductor laser physics
3. Tunable laser fundamentals
4. Technologies for tunable lasers
5. Summary and Conclusion

Introduction: Laser History

Milestones:

- 1917 Origin of laser can be traced back to Einstein's treatment of stimulated emission and Planck’s description of the quantum.
- 1951 Development of the maser by C.H. Townes.
- 1958 Laser was proposed by C.H. Townes and A.L. Schawlow
- **1960** T.H. Maiman at Hughes Laboratories reports the first laser: the pulsed ruby laser.
- 1961 The first continuous wave laser was reported (the helium neon laser).
- 1962 First semiconductor laser

Introduction: Laser types and applications

Common Daily Applications

- Compact disk
- Laser printer
- Optical disc drives
- Optical computer
- Bar code scanner
- Holograms against forgery
- Fiber optic communications
- Free space communications
- Laser shows
- Holograms
- Kinetic sculptures

Scientific Applications

- Basic Scientific Research
- Spectroscopy
- Nuclear Fusion
- Cooling Atoms
- Short Pulses

Medical Applications

- Surgery:
  - •Eyes
  - •General
  - •Dentistry
  - •Dermatology
- Diagnostic fluorescence
- Soft lasers

Military Applications

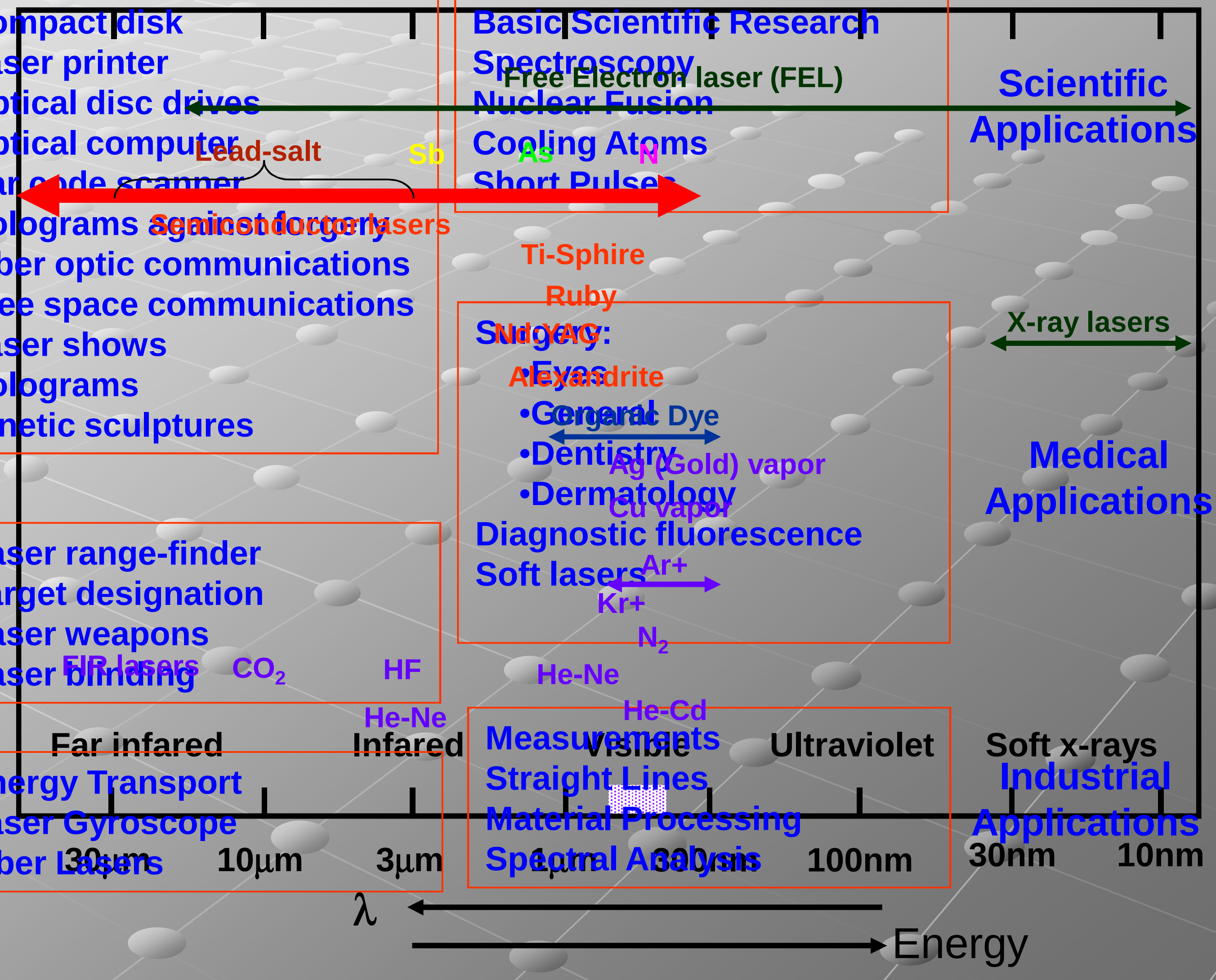
- Laser range-finder
- Target designation
- Laser weapons
- Laser blinding

Industrial Applications

- Measurements
- Straight Lines
- Material Processing
- Spectral Analysis

Special Applications

- Energy Transport
- Laser Gyroscope
- Fiber Lasers



Introduction: Semiconductor Laser

What made the semiconductor lasers the most popular light sources ?

- Small physical size
- Electrical pumping
- High efficiency in converting electric power to light
- High speed direct modulation (high-data-rate optical communication systems)
- Possibility of monolithic integration with electronic and optical components to form OEICs (optoelectronic integrated circuits)
- Optical fiber compatibility
- Mass production using the mature semiconductor-based manufacturing technology.

Motivations

Application interests in tunable mid-IR semiconductor lasers:

- **Spectroscopy**
  - Single frequency mode, tunable
- **Environmental sensing and pollution monitoring**
  - Lidar
  - Requires Ruggedness, Correct Wavelength
- **Industrial Process Monitoring**
  - Requirements similar to Environmental Monitoring
- **Medical Diagnostics**
  - Breath analysis; Non-invasive Glucose monitoring, Cancer Detection, etc.
- **Military and law enforcement**
- **Optical communication**

A key requirement:

Broad, continuous wavelength tunability

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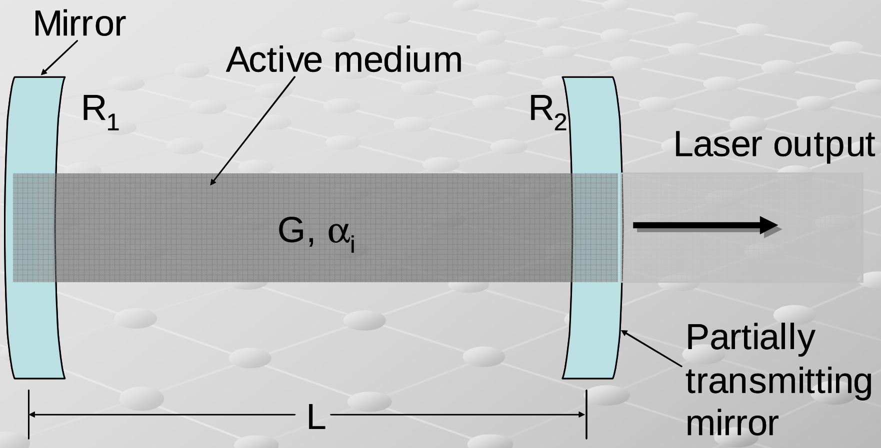
Laser physics



Oscillation condition is reached when

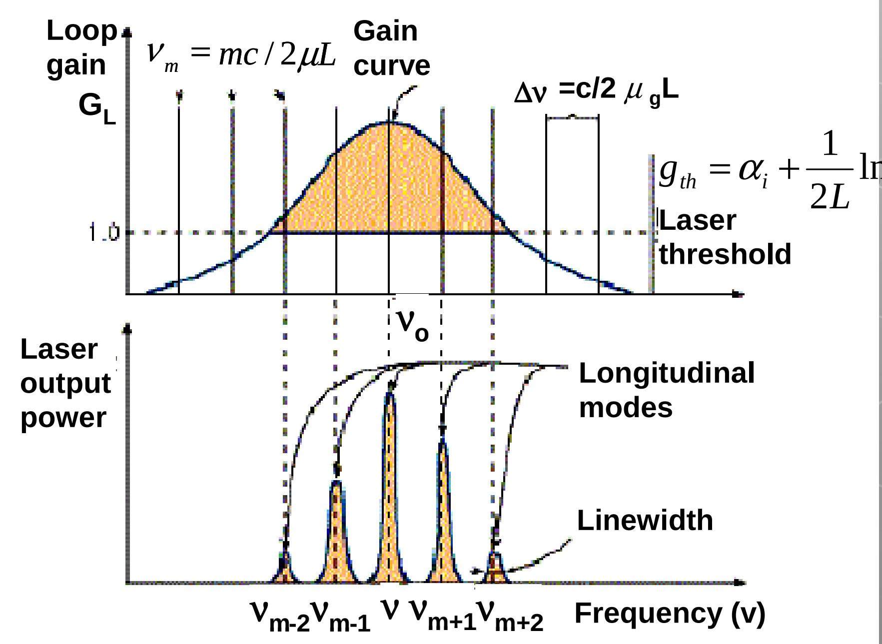
$$(R_1R_2)^{1/2}e^{2i\tilde{\beta}L}=1$$

$$\tilde{\beta}=\mu k_0-i\alpha/2$$

Real part: Threshold condition

$$\begin{cases}(R_1R_2)^{1/2}e^{(g-\alpha_i)L}=1\\ 2\mu k_0L=2m\pi, \qquad (m=1,2,3,..)\end{cases}$$

Imaginary part: wavelength condition

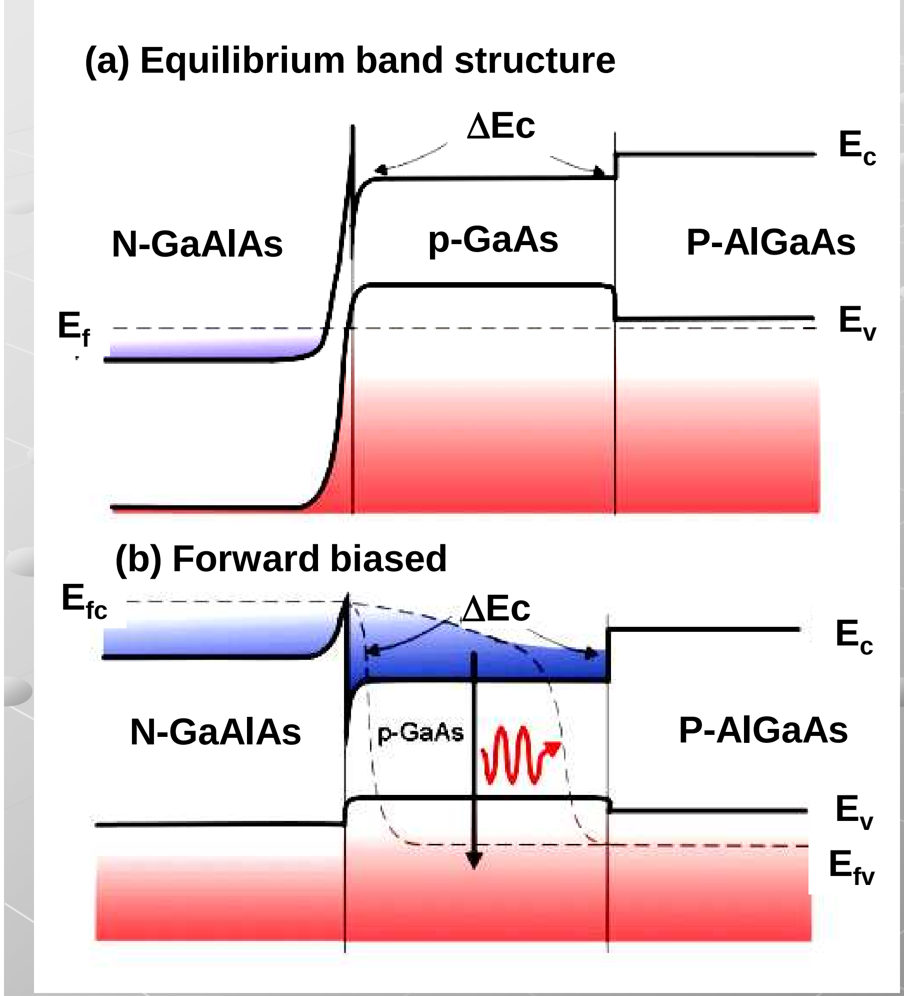


$$g_{th}=\alpha_i+\frac{1}{2L}\ln\left|\frac{1}{R_1R_2}\right|$$

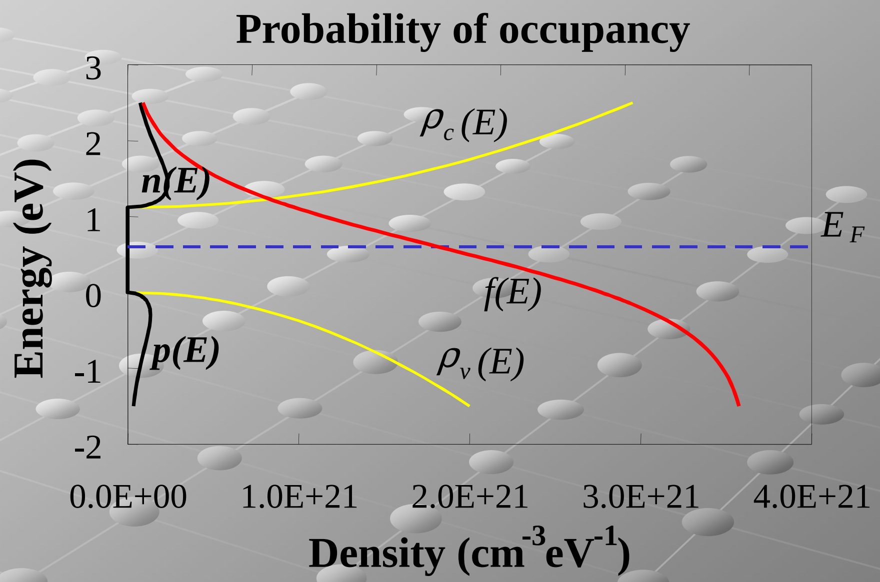
Semiconductor laser physics

- Threshold: carrier density, cavity loss
- Wavelength: bandgap physics, or intraband energy level (QC)
- Tunability: gain bandwidth:
  - for interband D.H. and quantum well laser: Fermi Distribution
  - for quantum cascade laser: energy level linewidth
- Power and efficiency
- Operating temperature

Semiconductor laser band diagram



Energy band diagrams for a double hetero-structure laser (a) unbiased, (b) forward biased



Carrier concentration in each band:

$$n = \int \frac{\rho_c(E_c)}{\exp[(E_c - E_{FC})/KT]+1} dE_c$$

$$p = \int \frac{\rho_v(E_v)}{\exp[(E_v - E_{FV})/KT]+1} dE_v$$

Threshold current density

Carrier density rate equation: $\frac{\partial n}{\partial t} = D(\nabla^2 n) + \frac{J}{qd} - R(n)$

At steady state, $\frac{\partial n}{\partial t} = 0$

So: $R(n) = \frac{J}{qd}$

$$R(n) = A_{nr} n + Bn^2 + Cn^3 + R_{st} N_{ph}$$

A_{nr} nonradiative process

Bn^2 spontaneous radiative rate

Cn^3 nonradiative Auger recombination

R_{st} stimulated recombination that leads to emission of light and it is proportional to the photon density N_{ph}.

Below or near threshold: $R(n) = \frac{n}{\tau_e(n)}$

So, the threshold current density is $J_{th} = \frac{qdn_{th}}{\tau_e(n_{th})}$

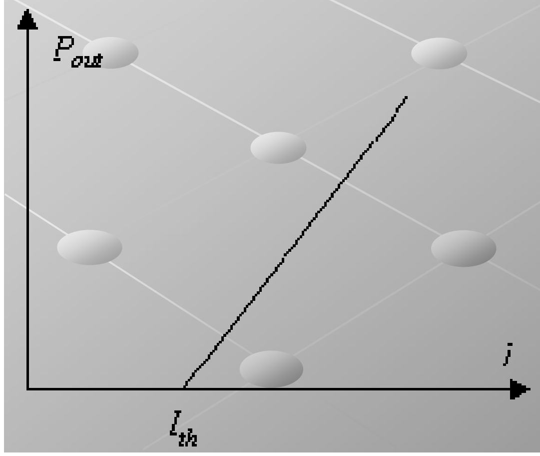
Power and efficiency of semiconductor lasers

If the current is above threshold, this leads to stimulated emission

$$J = J_{th} + qdv_g g_{th} N_{ph}$$

Optical output power P$_{out}$ vs. injection current is determined by

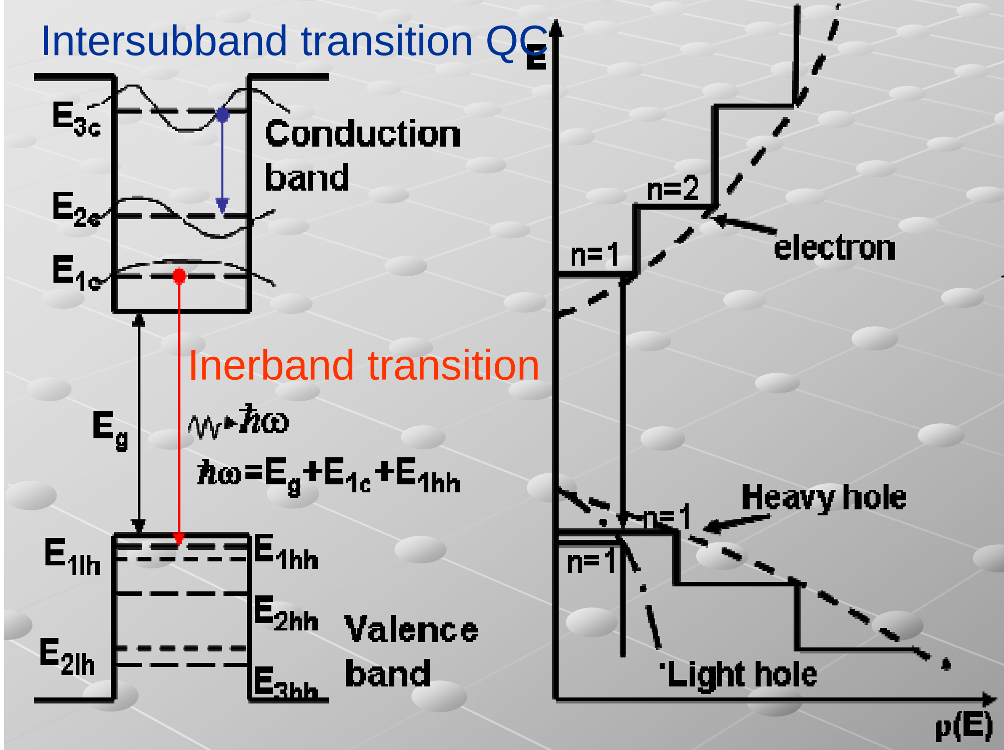
$$P_{out} = v_g \alpha_m V N_{ph} h\nu = \frac{h\nu}{q} \eta_i \frac{\alpha_m}{\alpha_m + \alpha_i} (I - I_{th})$$



External quantum efficiency is defined as:

$$\eta_e = \frac{dP_{out}/dI}{\hbar\omega/q} = \frac{\alpha_m}{\alpha_m + \alpha_i}\eta_i = \eta_i \frac{\ln(1/R)}{\alpha_i L + \ln(1/R)}$$

Quantum Well lasers



Energy eigenvalues for a particle confined in the quantum well are:

$$E(n,k_x,k_y) = E_n + \frac{\hbar^2}{2m_n^*}(k_x^2+k_y^2)$$

Density of states:

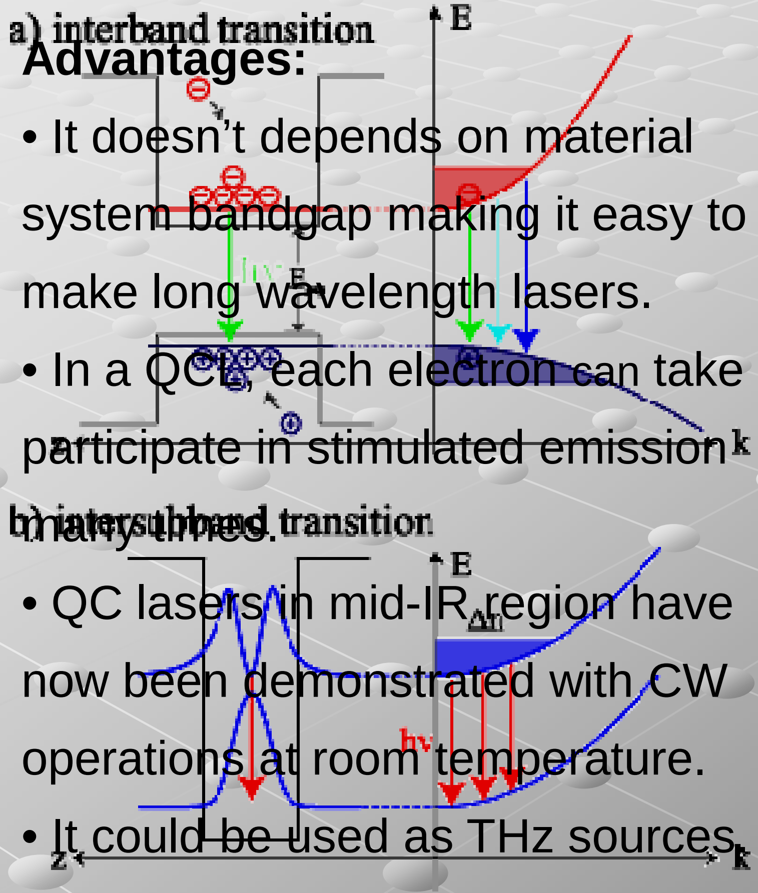
$$\boxed{>>>\rho_{ci} = \frac{m_{ci}}{\pi\hbar^2 L_z}}$$

Quantum wells are important in semiconductor lasers because they allow some degree of freedom in the design of the emitted wavelength through adjustment of the energy levels within the well by careful consideration of the well width.

Compare with Heterostructure:

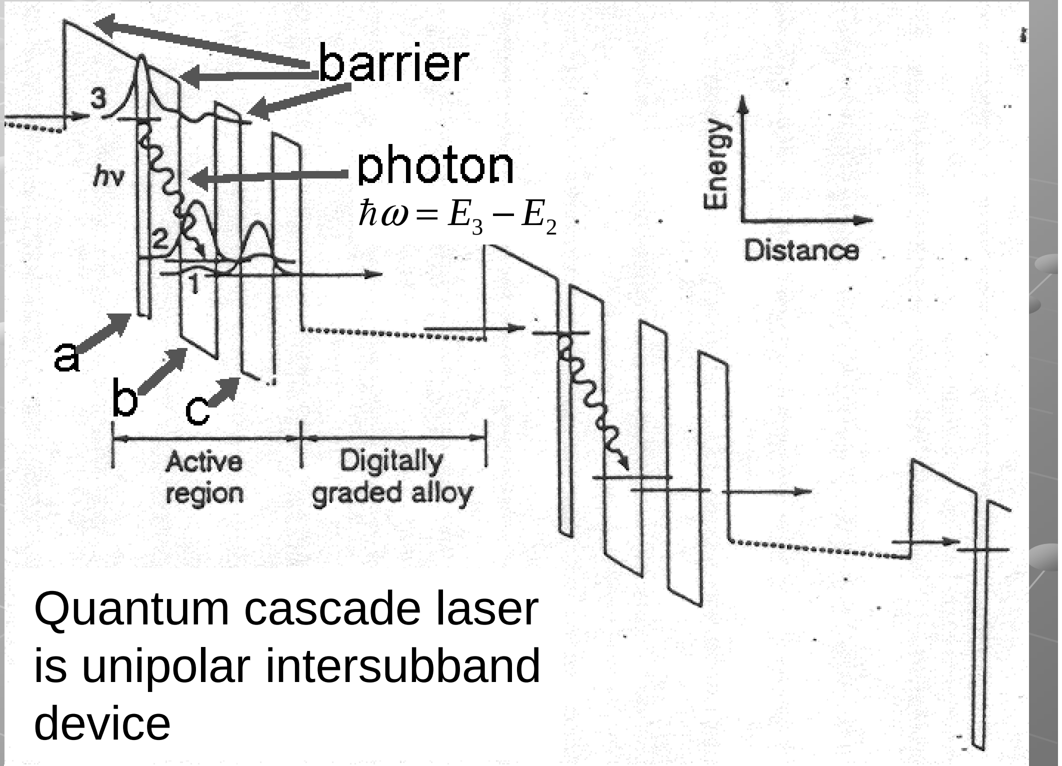
$$\rho_c(E) = 4\pi\left(\frac{2m_c}{h^2}\right)^{3/2} E^{1/2}$$

Quantum cascade lasers



Advantages:

- It doesn't depends on material system bandgap making it easy to make long wavelength lasers.
- In a QCL, each electron can take participate in stimulated emission many times.
- QC lasers in mid-IR region have now been demonstrated with CW operations at room temperature.
- It could be used as THz sources.



Quantum cascade laser is unipolar intersubband device

ΔE_{32} defines ħω for lasing

$\tau_{21} << \tau_{32}$ necessary for population inversion

The QC laser relies on only one type of carrier, making electronic transition between conduction band states arising from size quantization.

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4. **Technologies of tunable lasers**
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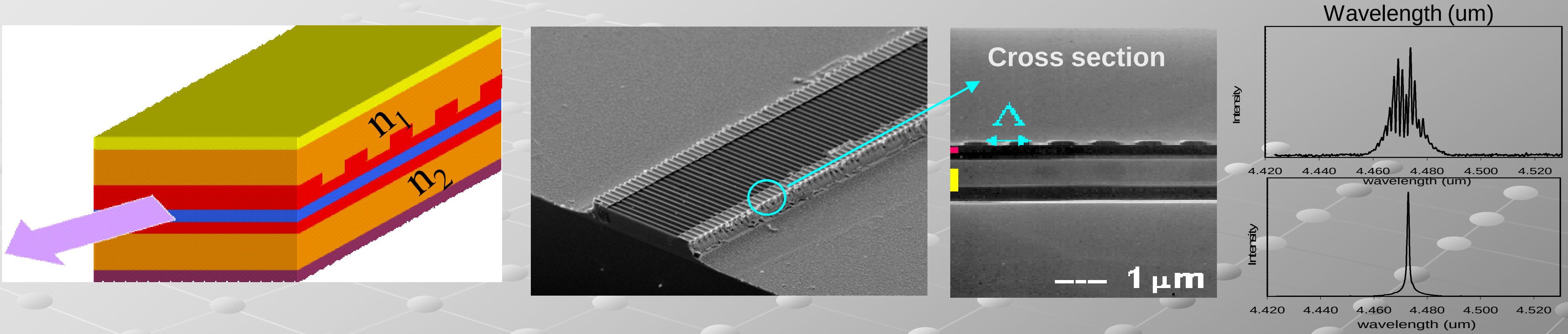
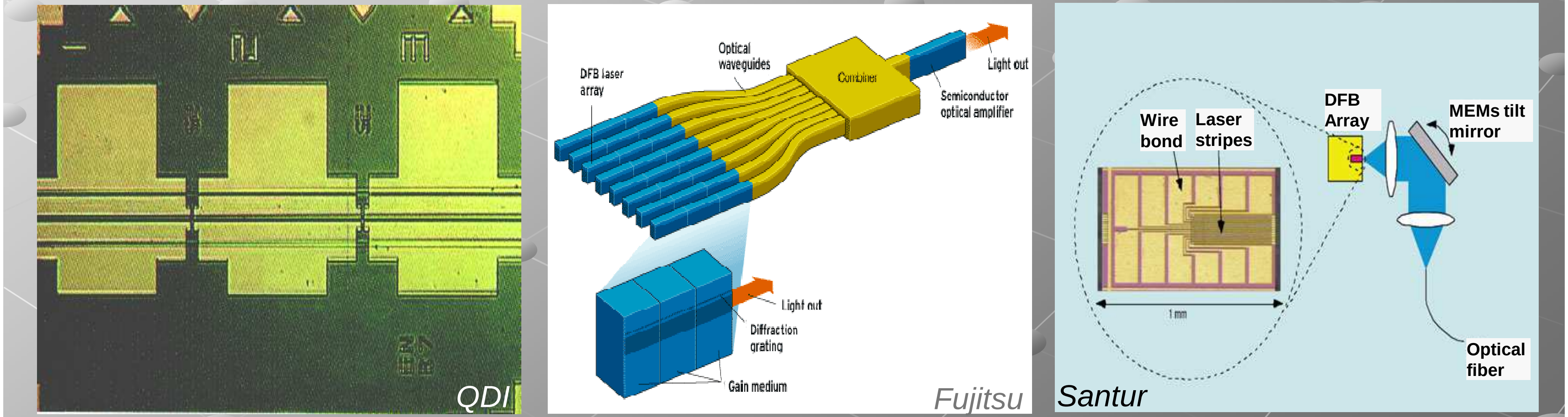
Existing Tunable Laser technologies

- Distributed Feedback Bragg Grating (DFB)
- Sampled grating Distributed Bragg Reflectors (DBR)
- MEM-VCSEL
- Grating coupled external cavity (ECLD)

Existing Tunable Laser Structures (DFB)

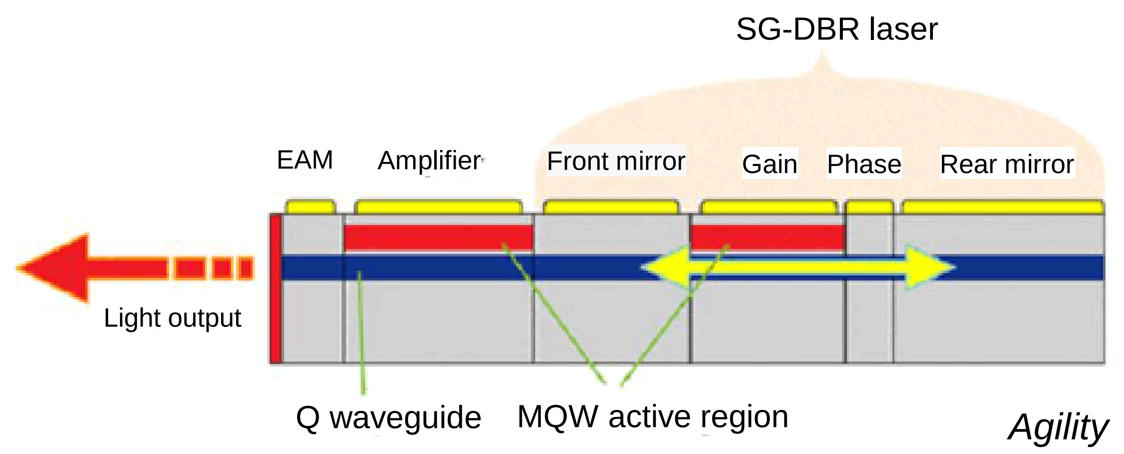


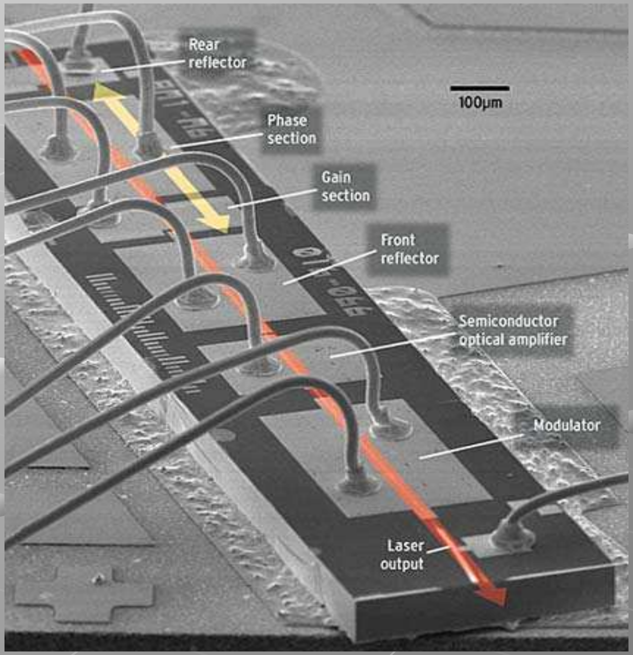
DFB laser structure. The grating is etched onto one of the cladding layers. Grating period is determined by Λ=mλ/2. λ is the wavelength inside the medium.

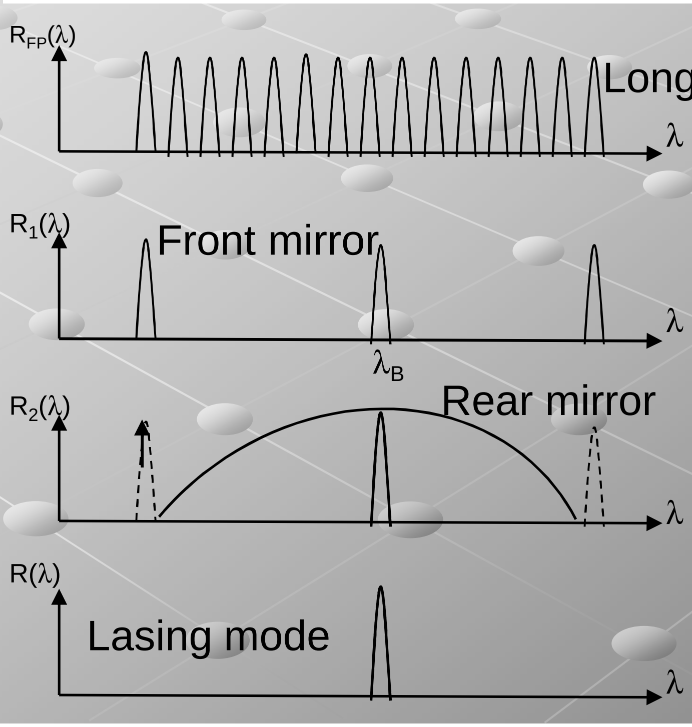


Tunable DFB laser arrays (a) Serial (b) Parallel (c) Mem mirror

Existing Tunable Laser Structures (DBR)

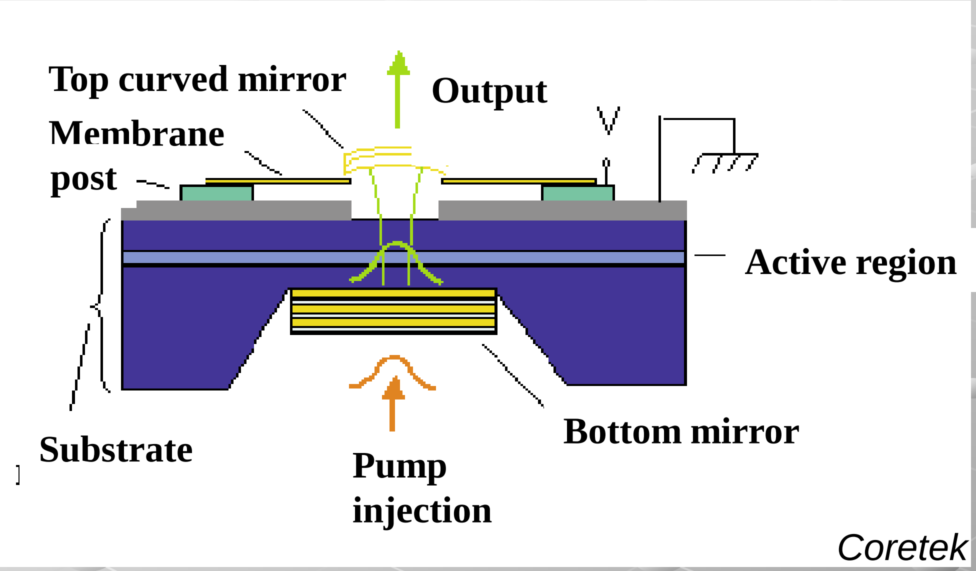




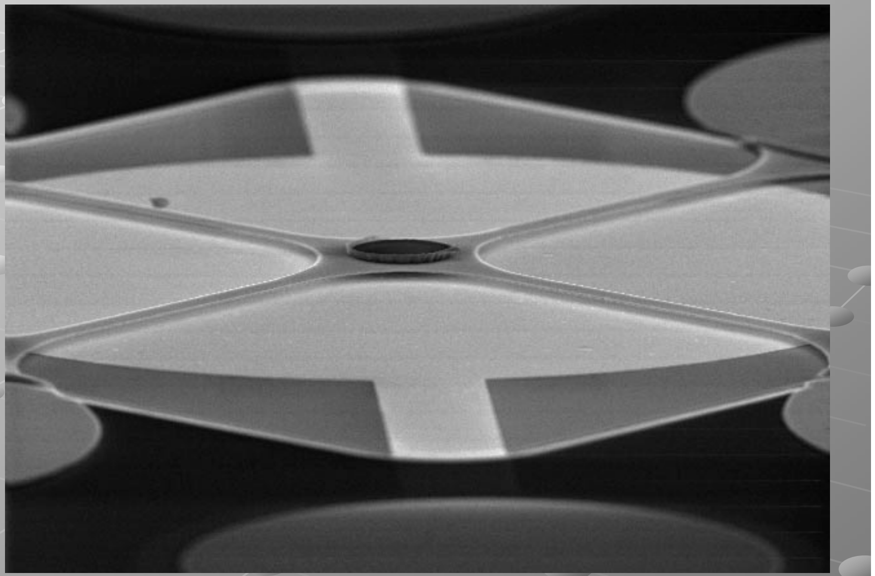


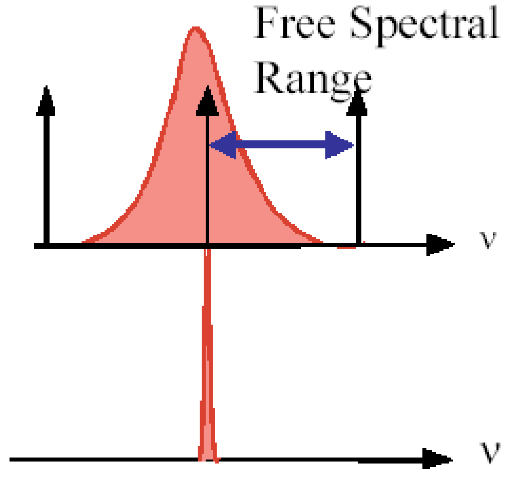
- ***Fabry-Perot cavity:***
  - Modes with FP spacing
- ***Sampled Bragg Reflectors*:**
  - Spectra with wider spacing and broader profile
  - Filters out 1 Bragg mode
  - Increasing I_{dbr} shifts filter profile to shorter λ
- ***Phase Section:*** Fine tuning

Existing Tunable Laser Structures (Mem-VCSEL)



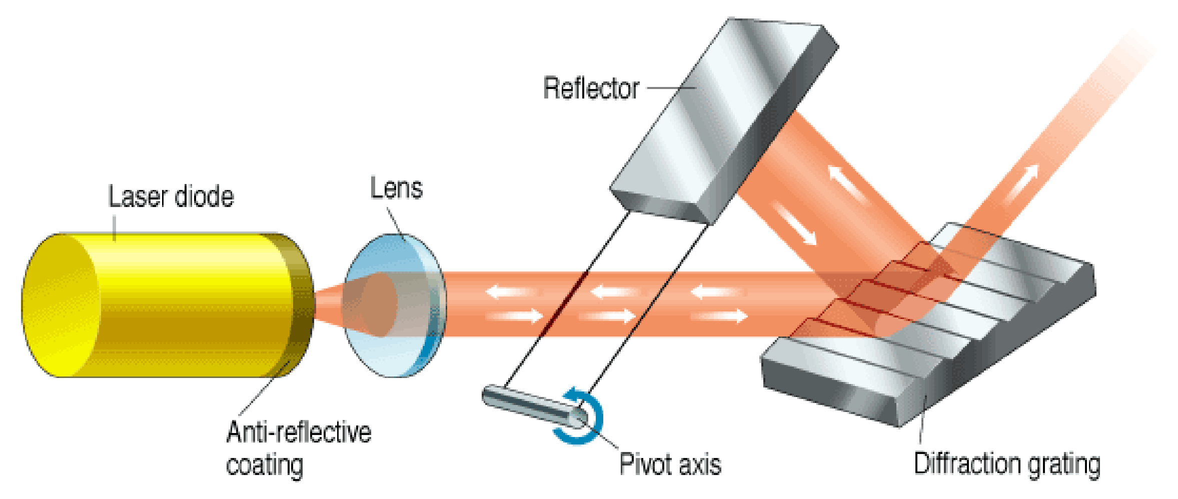
Coretek



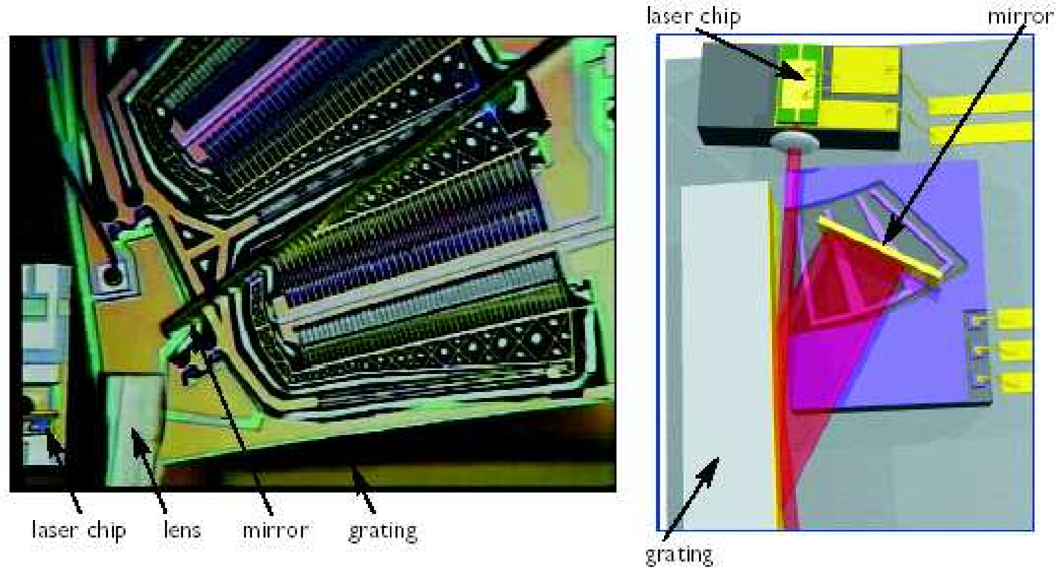


Schematic of micromechanically tunable VCSEL from Coretek / Nortel Networks.The upper membrane is moved electrostically to modify the cavity length and tune the lasing wavelength of the vertical cavity laser.

Existing Tunable Laser Structures (ECLD)



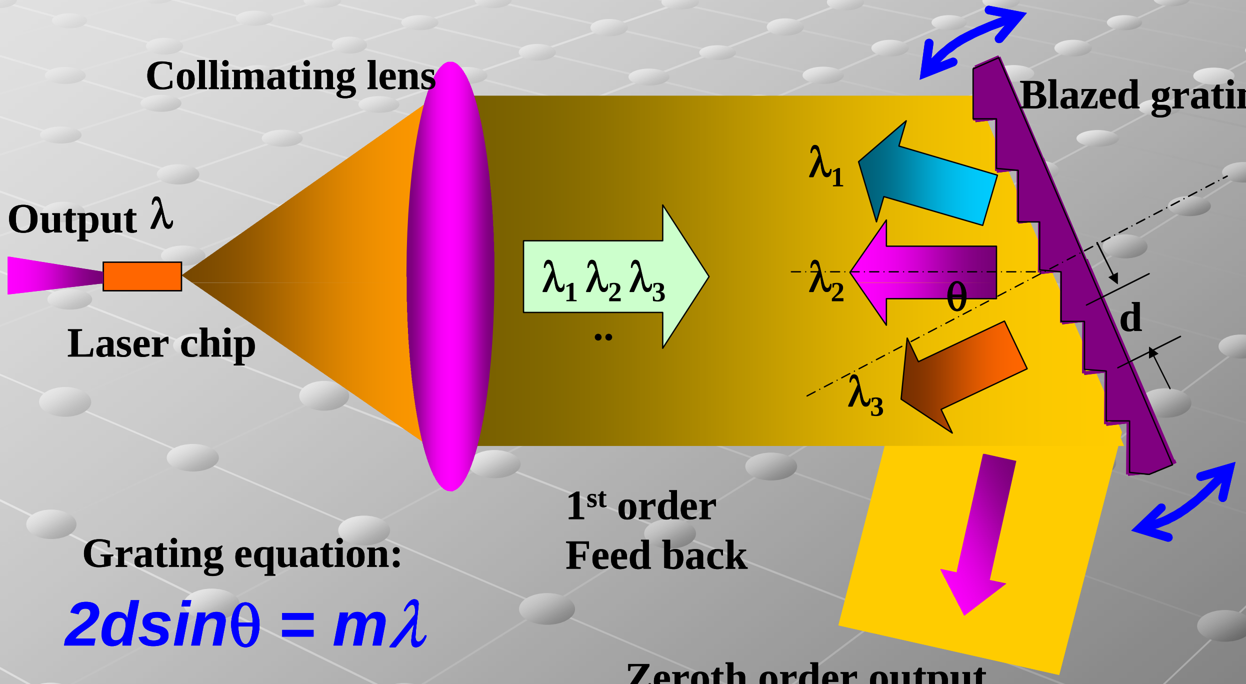
Schematic representations of the Littman Cavities.



Photograph and schematic of Iolon's MEMs based external cavity laser configuration.

Grating coupled external cavity tunable laser fundamentals

Example



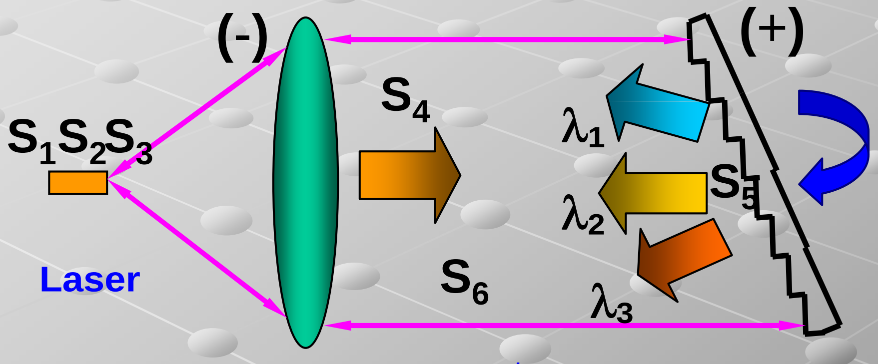
Grating equation:

$$2d\sin\theta = m\lambda$$

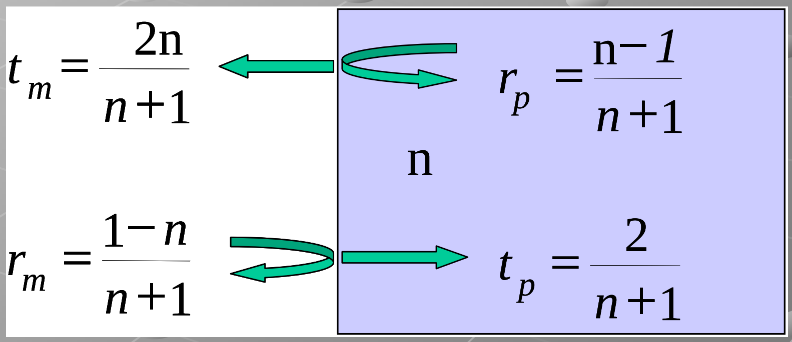
Grating coupled external cavity tunable laser structure.

External cavity laser modeling

$$S1=\begin{pmatrix} t_{1p}-\dfrac{r_{1m}r_{1p}}{t_{1m}} & \dfrac{r_{1p}}{t_{1m}} \\ -\dfrac{r_{1m}}{t_{1m}} & \dfrac{1}{t_{1m}} \end{pmatrix} \qquad S2=\begin{pmatrix} e^{ik_{sem}l} & 0 \\ 0 & e^{-ik_{sem}l} \end{pmatrix} \qquad S3=\begin{pmatrix} t_{2p}-\dfrac{r_{2m}r_{2p}}{t_{2m}} & \dfrac{r_{2p}}{t_{2m}} \\ -\dfrac{r_{2m}}{t_{2m}} & \dfrac{1}{t_{2m}} \end{pmatrix}$$



1st order Feed back

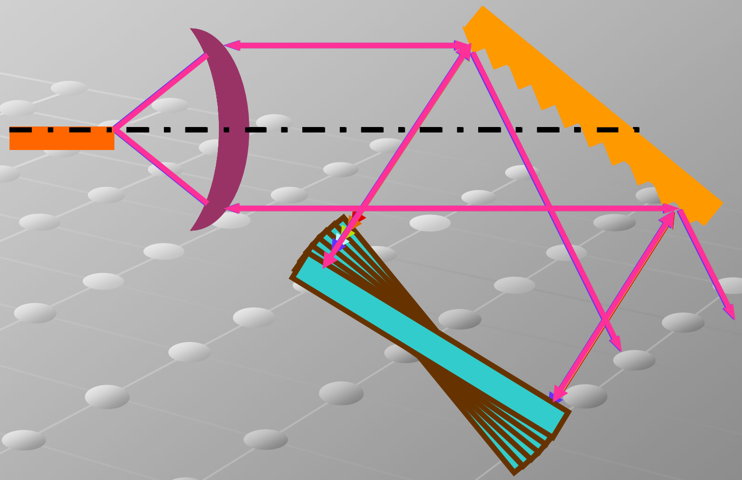


$$t_m=\frac{2n}{n+1} \qquad r_p=\frac{n-1}{n+1}$$

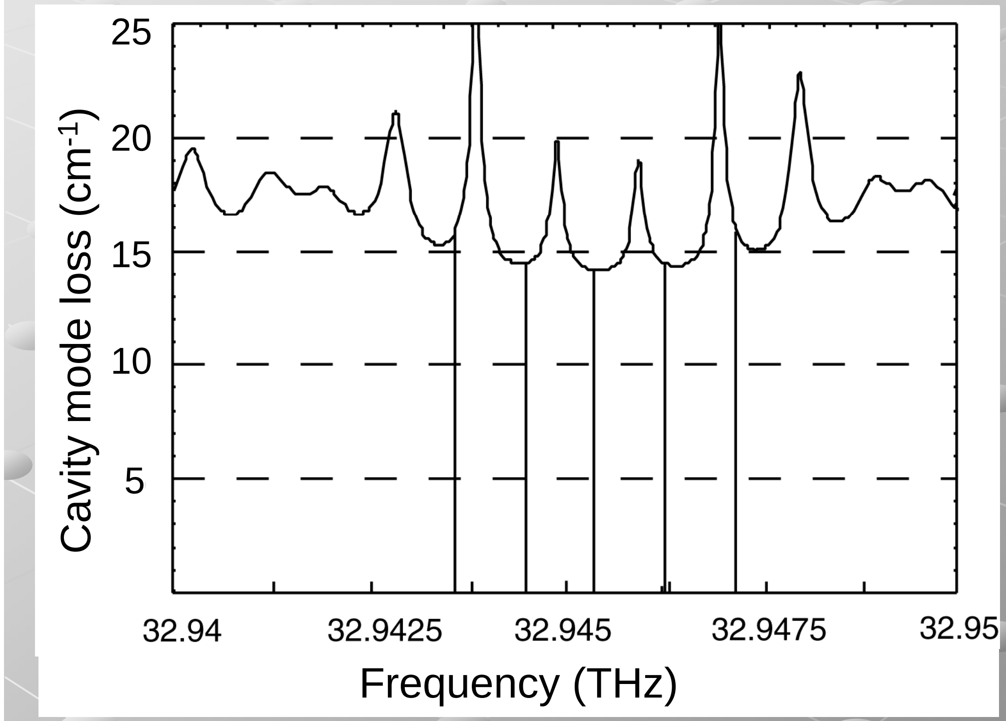
$$r_m=\frac{1-n}{n+1} \qquad t_p=\frac{2}{n+1}$$

$$S4=\begin{pmatrix} e^{ik_{air}L} & 0 \\ 0 & e^{-ik_{air}L} \end{pmatrix} \qquad S5=\begin{pmatrix} t_{Gp}-\dfrac{r_{Gm}r_{Gp}}{t_{Gm}} & \dfrac{r_{Gp}}{t_{Gm}} \\ -\dfrac{r_{Gm}}{t_{Gm}} & \dfrac{1}{t_{Gm}} \end{pmatrix} \qquad S6=\begin{pmatrix} closs & 0 \\ 0 & \dfrac{1}{closs} \end{pmatrix}$$

Grating coupled external cavity tunable Laser



An example: Cavity mode loss curve with longitudinal modes.



Laser oscillation condition

$$e^{(g_{th}-\alpha_i)l}\, e^{2il\beta_{sem}} = \frac{1}{r_{efft}\ r_{1p}}$$

$$r_{eff} = |r_{eff}|\, e^{i\phi}$$

$$\begin{cases} g_{th} = \alpha + \frac{1}{l}\ln\left|\frac{1}{r_{eff}\ r_{1p}}\right| \\ 2l\beta_{sem} + \phi = 2m\pi \end{cases}$$

Current tunable laser development

Laser type	Power	Tuning range
DBR	30 mW*	> 40 nm
DFB	40 mW*	< 5 nm per λ
VCSEL	<2 mW*	~ 40 nm
External cavity	30 mW*	>100 nm

DBR

Advantages:

- Wide tuning range
- Fast switching speed
- Good side mode suppression
- Moderate output power
- Low power consumption
- Integrated functions (Modulators, optical amplifiers)

Disadvantages:

- Yield
- Wavelength stability
- Relatively broad linewidth
- Complex software
- Power

Companies in brackets no longer exist, but honorable

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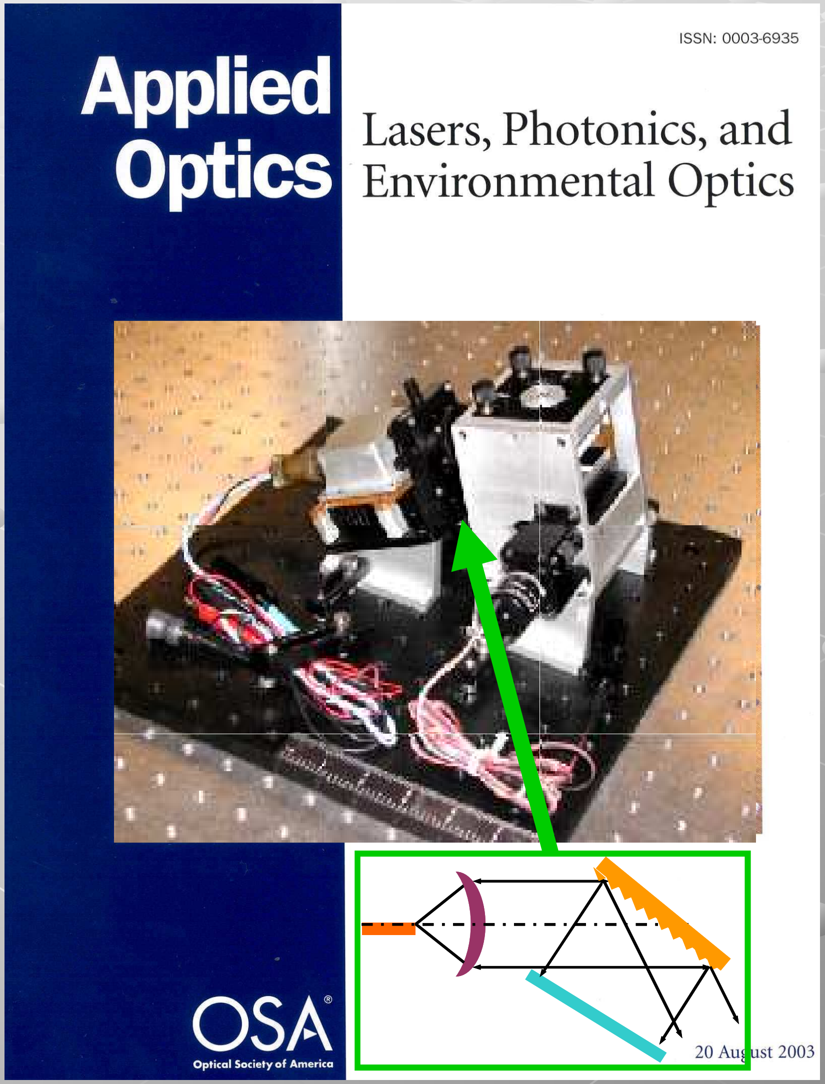
1. Introduction and motivation
2. Semiconductor laser physics
3. Tunable laser fundamentals
4. Technologies of tunable lasers
5. **Proposed research and summary**

Key points of the Proposed Research

- **Theoretical calculations and modeling in order to get single mode and continuous long tuning range**
- **Design and construct the E.C. laser testing apparatus and programs**
- **Perform testing to qualify lasers for more in depth testing**

- **Laser spectroscopy of some real samples**
- **Design compact and robust EC laser source**

Recent performance of tunable Mid-IR lasers



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Lasers, Photonics, and Environmental Optics

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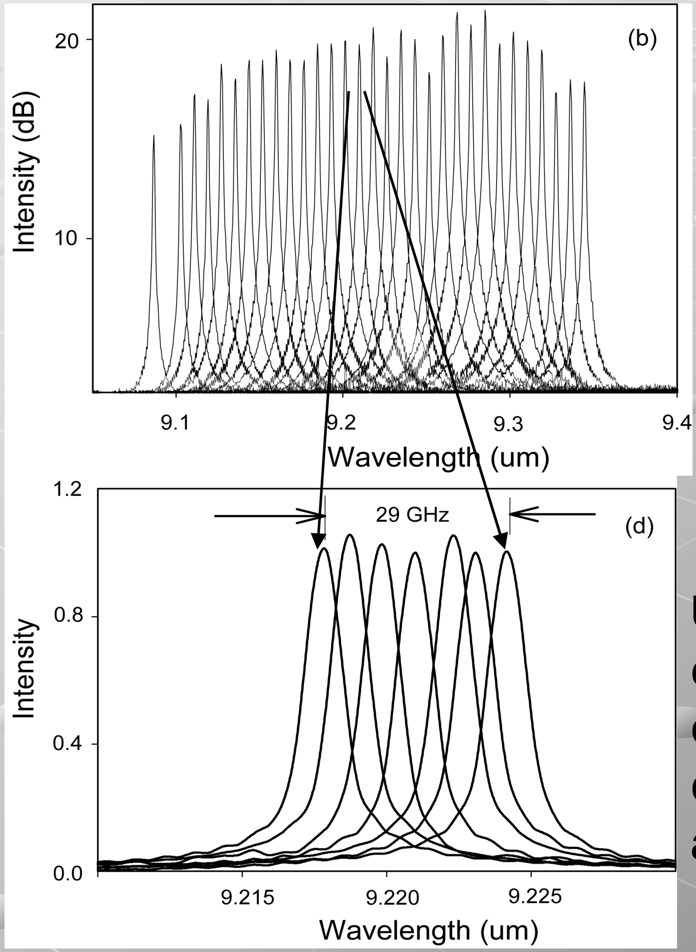
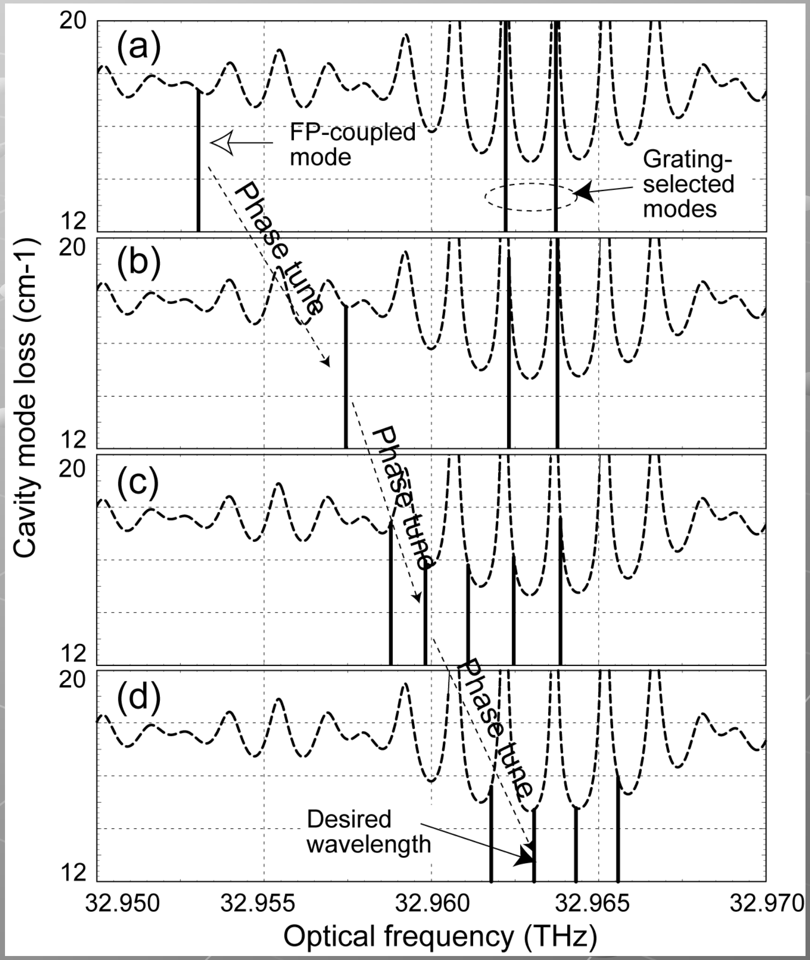
- For basic research, breadboard systems
- Study of basic properties: thermo-optic behavior, gain property, power, efficiency, noise model developed
- Overcome challenges: λ-scaling issue, facet coating problem, non-optimum optics
- Single-mode or nearly so (pulse operation), 7-12-dB side mode suppression ratio
- Linewidth dominated by thermo-optics (~500 MHz; theoretical model projects near transform-limited for < 20-ns pulse)
- Wavelength bands: 4.6, 5.2, 7.1, 9 μm
- Tuning range ~ 2-2.5% of center wavelength, limited by the gain band
- Power ~1 - 40 mW peak (up to 5% d.c.)
- Turn-key, high stability

Thermal fine phase control of tunable laser



Use the coupled-cavity effect to advantage

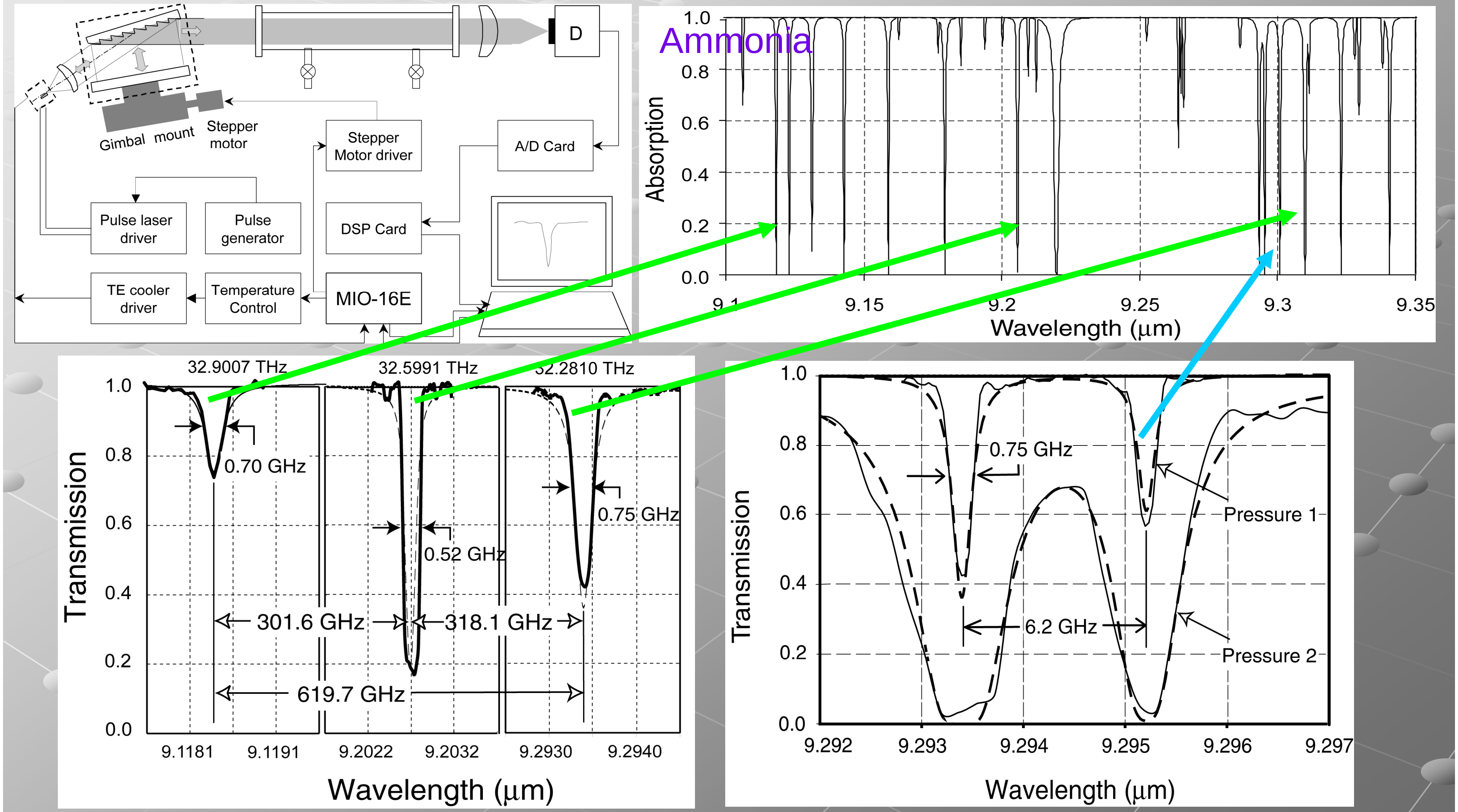


- Substantial thermal-induced phase tuning and grating tuning provide continuous and broad tuning even without AR coating

Peng et al., Appl. Optics (8/20/03)

Tuning performance of tunable Mid-IR lasers

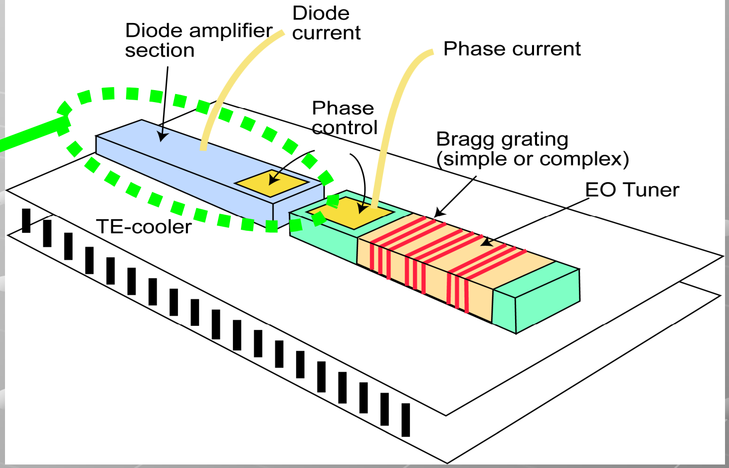
Peng et al., Appl. Optics (8/20/03)

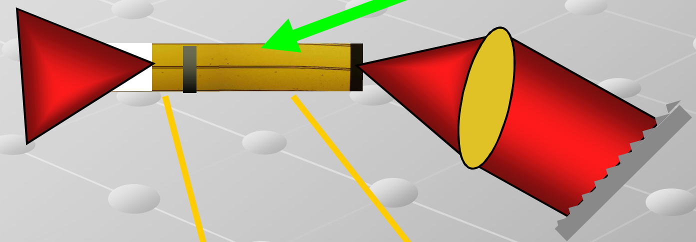


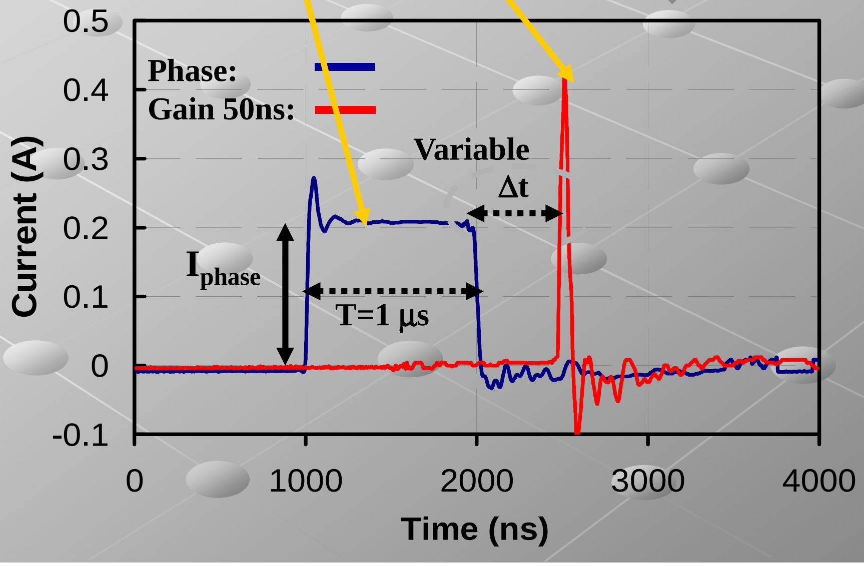
Approach for miniature tunable module

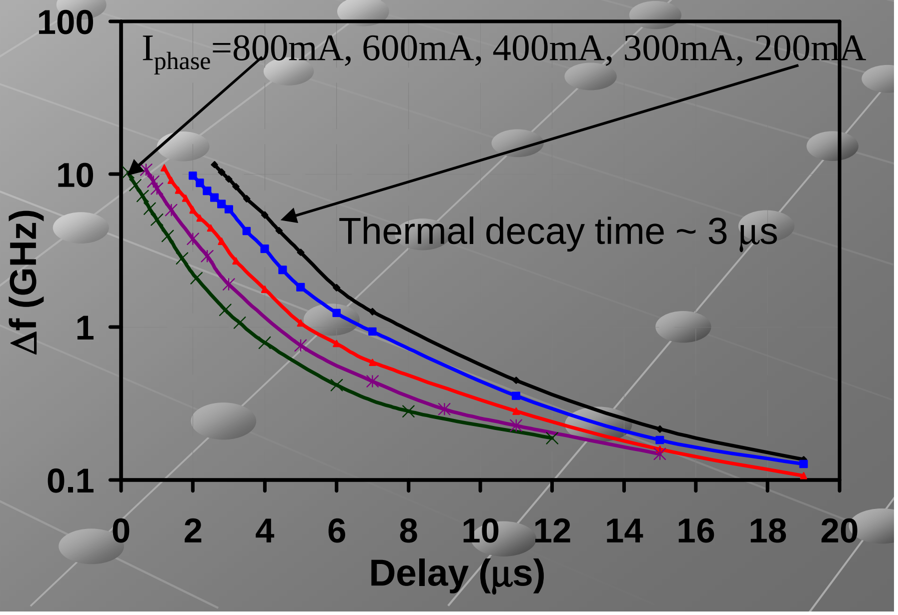
Phase tuning is needed: 2-segment laser

- Use only thermo-optic effect
- No need of specialized phase-segment in monolithic device
- A key test of the miniature module design

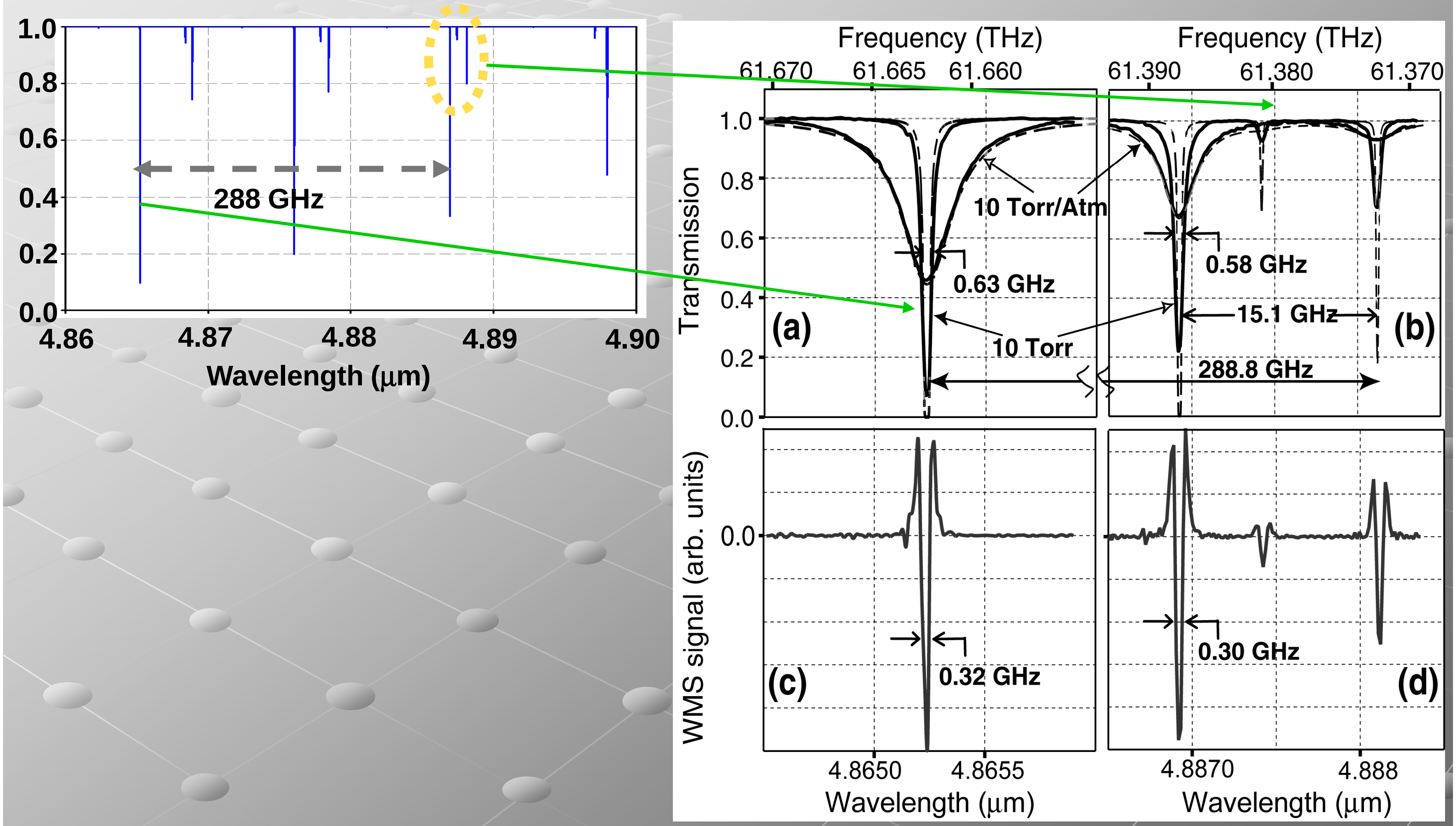








CO gas absorption spectroscopy



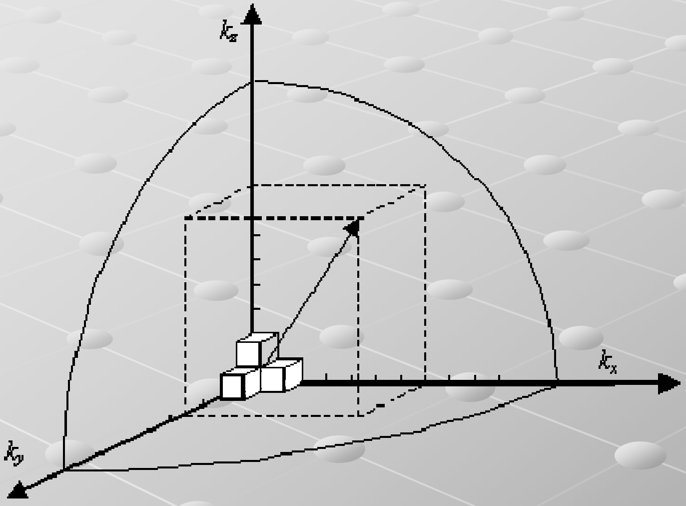
Summary and Conclusion

- Various types of semiconductor lasers were investigated and studied.
- Most common used tunable laser structures were introduced and evaluated. One example of tuning principle was explained.
- Dissertation research topics will be novel mid-IR tunable semiconductor laser concept and demonstration.
- Research strategy and recent results were presented.

Thank you !

Additional readings

Semiconductor Laser Physics: Density of states calculation



$$N = 2\times\frac{1}{8}\times(\frac{L}{\pi})^3\times\frac{4}{3}\times\pi\times k^3$$

$$\frac{dN}{dE} = \frac{dN}{dk}\frac{dk}{dE} = (\frac{L}{\pi})^3\times\pi\times k^2\frac{dk}{dE}$$

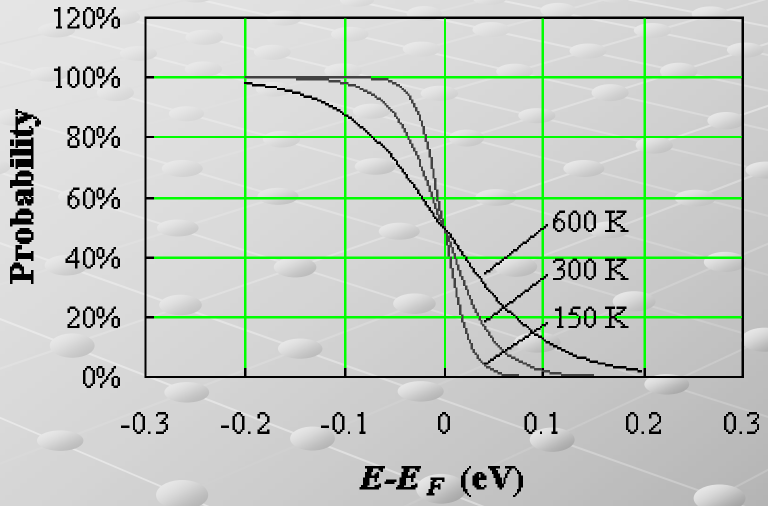
Where $E(k) = \frac{\hbar^2 k^2}{2m^*}$

$$\rho(E) = \frac{1}{L^3}\frac{dN}{dE} = 2\frac{4\pi k^2}{(2\pi)^3}\frac{dk}{dE} = \frac{k^2}{\pi^2}\frac{dk}{dE}$$

$$\rho_c = (2)\frac{4\pi k^2}{(2\pi)^3}\frac{dk}{dE} = \frac{1}{\pi^2}(\frac{2m_c}{\hbar^2})^{3/2}E^{1/2}$$

$$\rho_v = \frac{1}{\pi^2}(\frac{2m_v}{\hbar^2})^{3/2}E^{1/2}$$

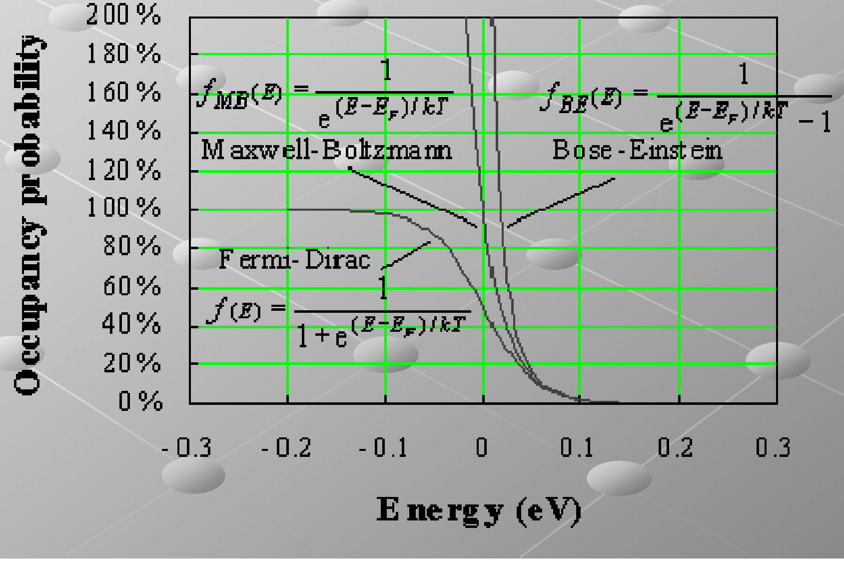
Semiconductor Laser Physics: Occupation factors for both bands



Occupation factor for each band by Fermi-Dirac distribution function

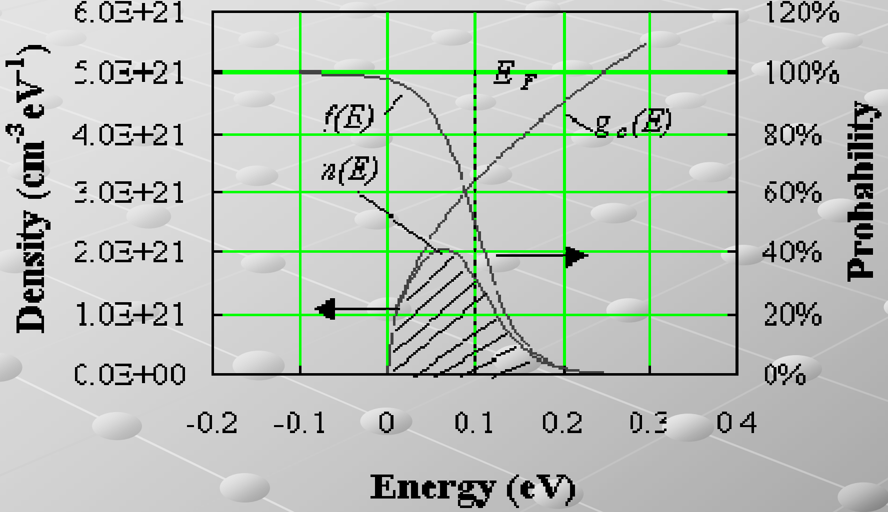
$$f_c = \frac{1}{1+\exp(\frac{E_c - E_{FC}}{KT})}$$

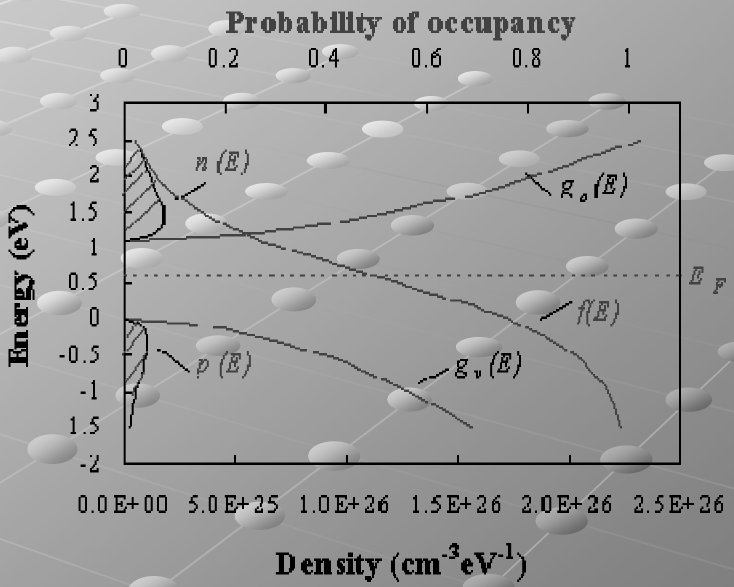
$$f_v = \frac{1}{1+\exp(\frac{E_v - E_{FV}}{KT})}$$



Probability of occupancy versus energy of the Fermi-Dirac, the Bose-Einstein and the Maxwell-Boltzmann distribution

Semiconductor Laser Physics:
Carrier concentration calculation





Electron concentration *n* and hole concentration *p* are given by

$$n = \int \frac{\rho_c(E_c)}{\exp[(E_c - F_c)/KT]+1}\, dE_c$$

$$p = \int \frac{\rho_v(E_v)}{\exp[(E_v - F_v)/KT]+1}\, dE_v$$

Semiconductor Laser Physics: Fermi energy and occupation factor calculation

$$n = \int \frac{\rho_c(E_c)}{\exp[(E_c - F_c)/KT] + 1} dE_c, \quad p = \int \frac{\rho_v(E_v)}{\exp[(E_v - F_v)/KT] + 1} dE_v$$

Fermi energy, occupation probability can be approximately derived from above equations, which is often referred as Boltzmann approximation.

For conduction band:

$$E_{FC} = KT \ln(\frac{n}{N_C})$$

$$f_c(E) \cong \frac{n}{N_C} \exp(\frac{-E}{KT})$$

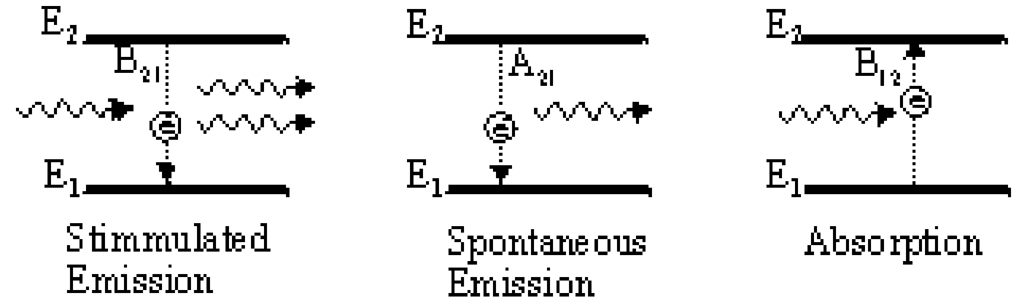
For valence band:

$$E_{FV} = KT \ln(\frac{p}{N_V})$$

$$f_v(E) \cong \frac{p}{N_V} \exp(\frac{-E}{KT})$$

where $N_C = 2(2\pi m_c KT / h^2)^{3/2}$

Semiconductor Laser Physics:
Stimulated emission rate and gain calculation



Three different processes during the interaction of light with matter

Net stimulated emission rate per unit volume per unit energy interval at photon energy *hν*:

$$R_{stim}(h\nu) = P(h\nu)\int_{-\infty}^{+\infty} B_{21}(E_c, h\nu)V\rho_c(E_c)\rho_v(E_c - h\nu)(f_c - f_v)\left(1+\frac{\rho_v}{\rho_c}\right)^{-1} dE_c$$

Gain per unit length can be derived as

$$g(h\nu) = R_{stim}\frac{\mu_g}{c}\Gamma = \left(\frac{\Gamma\mu_g}{c}\right)\int_{-\infty}^{+\infty} B(E_c)V\rho_c(E_c)\rho_v(E_c - h\nu)(f_c - f_v)\left(1+\frac{\rho_v}{\rho_c}\right)^{-1} dE_c$$

Bernard and Duraffourg condition for stimulated emission

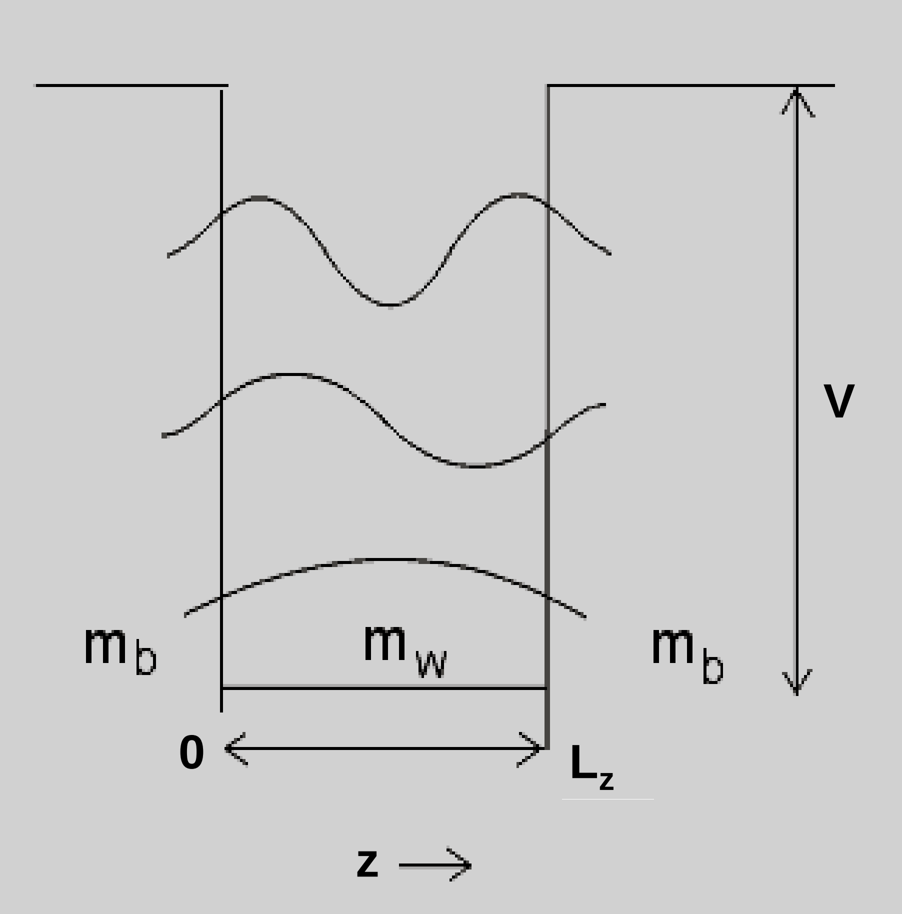
$$\exp\left(\frac{E_c - h\nu - E_{FV}}{kT}\right) > \exp\left(\frac{E_c - E_{FC}}{kT}\right)$$

Or

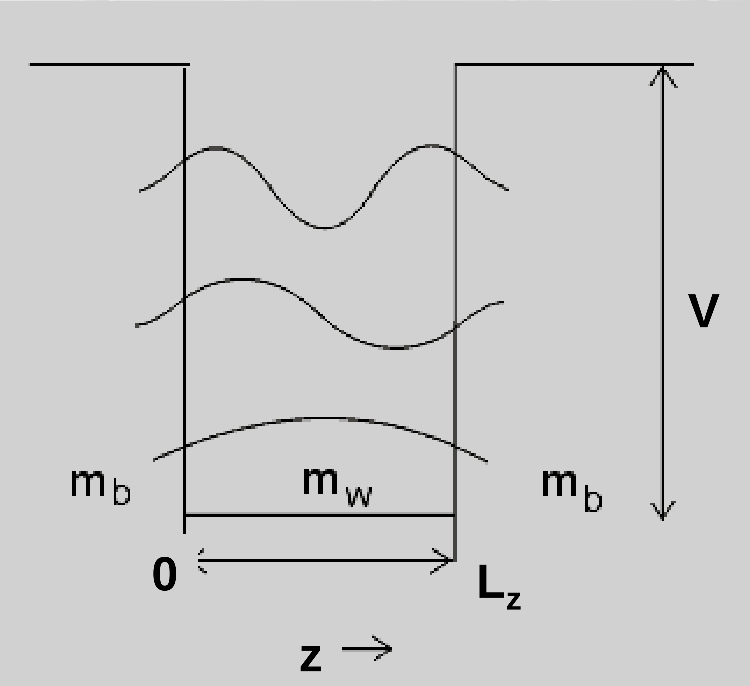
$$E_{FC} - E_{FV} > h\nu$$

Quantum well lasers

Another innovation, now nearly ubiquitous, was the use of a **Quantum Well (QW)** active region where the gain region thickness is reduced until the electronic states are quantised in one dimension. The quantisation allows the density of states to be engineered, and the tiny active volume reduces greatly the injection current required to achieve transparency.



Quantum well lasers:
Energy levels calculations by Schrodinger equation



Along the x, y direction, the energy levels form a continuum of states given by

$$E = \frac{\hbar^2}{2m}(k_x^2 + k_y^2)$$

The energy levels in z direction can be solved by Schrodinger equation for one dimensional potential well:

$$E\psi = -\frac{\hbar^2}{2m}\frac{d^2\psi}{dz^2}$$

inside the well ($0<z<L_z$)

$$E\psi = -\frac{\hbar^2}{2m}\frac{d^2\psi}{dz^2} + V\psi$$

outside the well ($z>L_z$, $z<0$)

Quantum well lasers:
Solution in finite quantum well

Boundary conditions are that ψ and dψ/dz are continuum at z=0 and z=L_z, The solution is:

$$\psi = \begin{cases} A\exp(k_1 z) & z \le 0 \\ B\sin(k_2 z + \delta) & 0 \le z \le L_z \\ C\exp(-k_1 z) & z \ge L_z \end{cases}$$

where

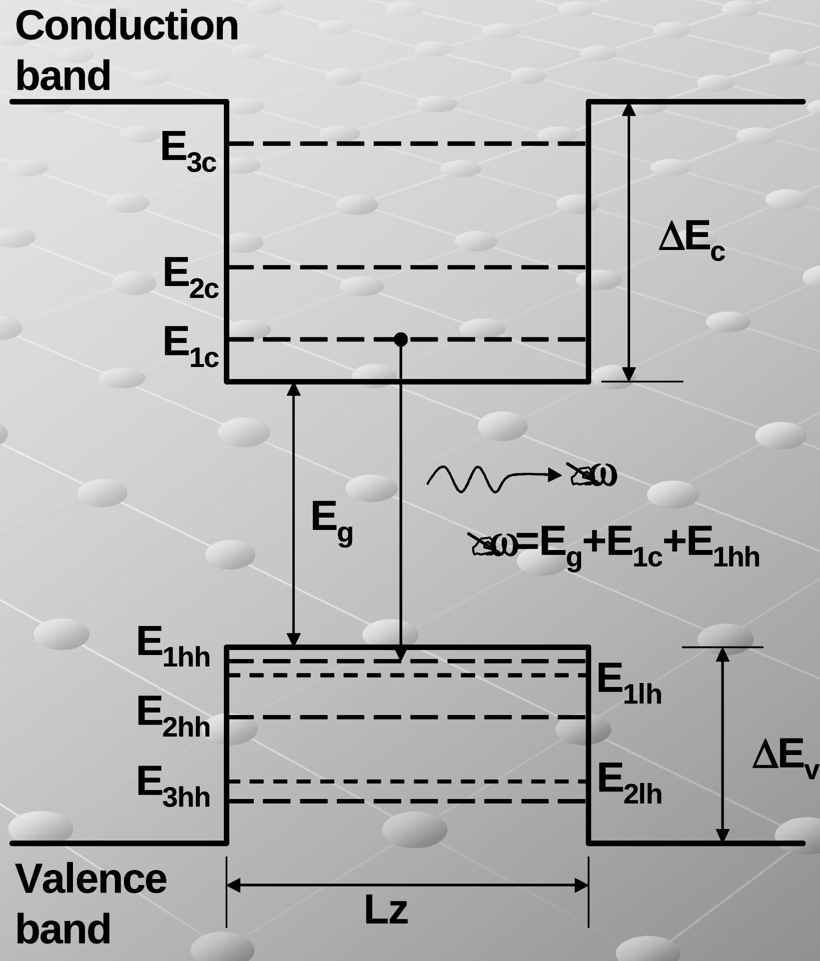
$$k_1 = [\frac{2m(V-E)}{\hbar^2}]^{1/2}$$

$$k_2 = (\frac{2mE}{\hbar^2})^{1/2}$$

The eigenvalue equation is:

$$\tan(k_2 L_z) = k_1 / k_2$$

Quantum well lasers:
Energy quantization in quantum well



Energy eigenvalues for a particle confined in the quantum well are:

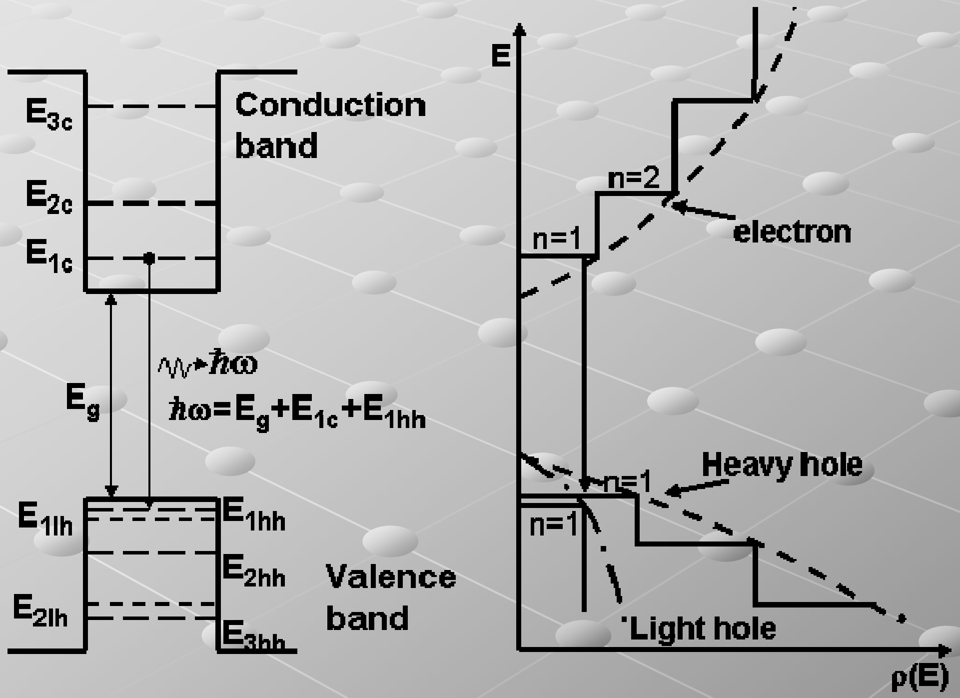
$$E(n,k_x,k_y) = E_n + \frac{\hbar^2}{2m_n^*}(k_x^2 + k_y^2)$$

E_n is the nth confined particle energy level for carrier motion normal to the well and m* is the effective mass for this level.

The confined energy levels E_n are denoted by E_{1c}, E_{2c}, E_{3c}, for electrons, … heavy holes and light holes. They can be calculated by the solution above.

Quantum well lasers:
Density of sates calculation

Density of states: The number of electron states per unit area in the x-y plane for the i_{th} subband, within an energy interval dE is given by:



$$D_i(E)dE = 2\frac{d^2k}{(2\pi)^2}, \quad E = \frac{\hbar^2k^2}{2m_{ci}}$$

$$>>> D_i = \frac{m_{ci}}{\pi\hbar^2}$$

Density of states per unit volume:

$$>>> \rho_{ci} = \frac{D_i}{L_z} = \frac{m_{ci}}{\pi\hbar^2 L_z}$$

Compare with Heterostructure: $\rho_c(E) = 4\pi(\frac{2m_c}{h^2})^{3/2}E^{1/2}$

Quantum well lasers

Carrier concentration calculation

Calculated number of electrons in the conduction band

$$n = \sum_i \int_{E_{io}}^{\infty} \rho_{ci} f_c(E_i) dE_i \qquad \text{where} \qquad f_c(E_i) = \frac{1}{1+\exp[(E_i - E_{fc})/k_B T]}$$

It is easy to obtain

$$n = k_B T \sum_i \rho_{ci} \ln[1+\exp(\frac{E_{fc} - E_{io}}{k_B T})] = N_c \exp(\frac{E_{fc}}{k_B T})$$

where $N_c = \rho_{ci} k_B T$

Similar solutions hold for holes in the valence band.

Electrical properties of semiconductor lasers
Carrier density derivation from photoluminance

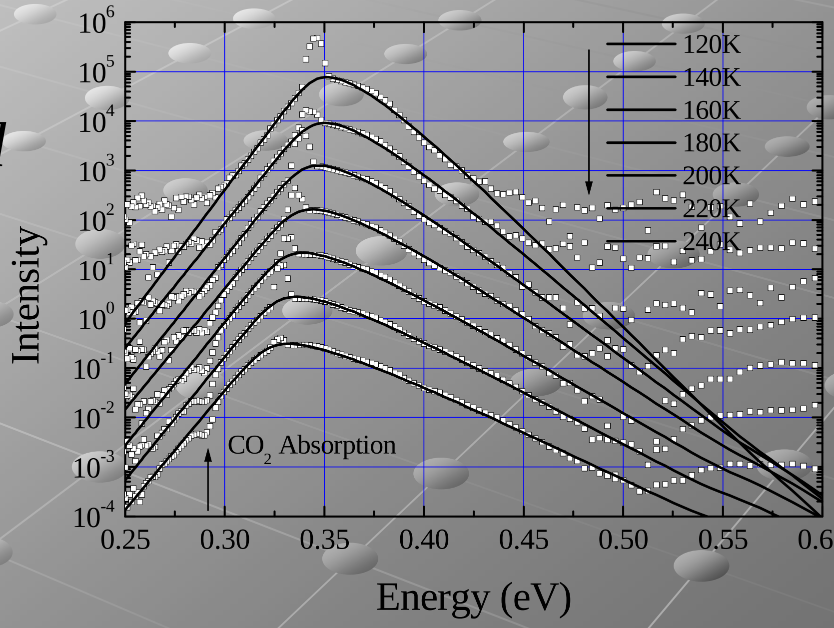
The light spectrum taken from facet is the amplified spontaneous emission,

$$L_F(\hbar\omega) = \hbar\omega \int_{z=0}^{L} wdR^{spon}(\hbar\omega)e^{G_n(\hbar\omega)z}dz$$

$$= \hbar\omega R^{spon}(\hbar\omega)[\frac{e^{G_n(\hbar\omega)L} - 1}{G_n(\hbar\omega)}]wd$$

where $G_n(\hbar\omega) = \Gamma g - \alpha_i$ is net gain.

By fitting the gain curve of the device, the carrier density can be derived at different current. The threshold carrier density will increase with increasing temperature.



Threshold photoluminescence spectra at different temperatures. Experimental data are shown in square symbols. Solid lines show the calculations.

Temperature dependence of electrical properties of semiconductor lasers

- **Temperature dependence of threshold current** $I_{th} = I_0 \exp(T / T_0)$

  T_0 is characteristic temperature. A lower T_0 value implies that the threshold current increases more rapidly with increasing temperature.

- **Temperature dependence of carrier lifetime** $J_{th} = qdn_{th} / \tau_{th}$

  It decreases with increasing temperature.

- **Temperature dependence of optical gain**

  It increases with increasing temperature.

- **Temperature dependence of differential quantum efficiency**

  It decreases with increasing temperature.

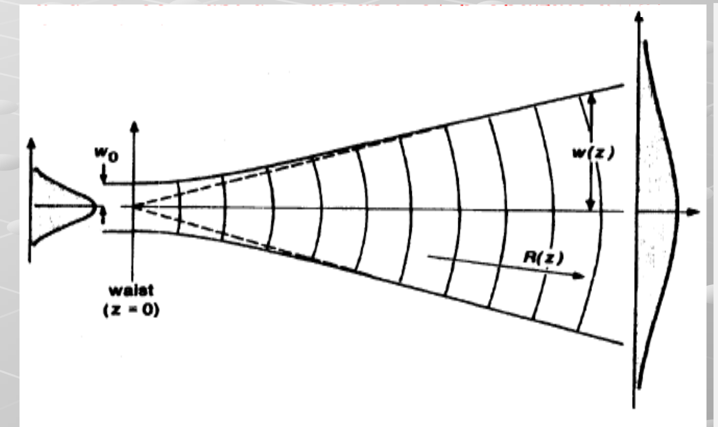
- **Leakage current**

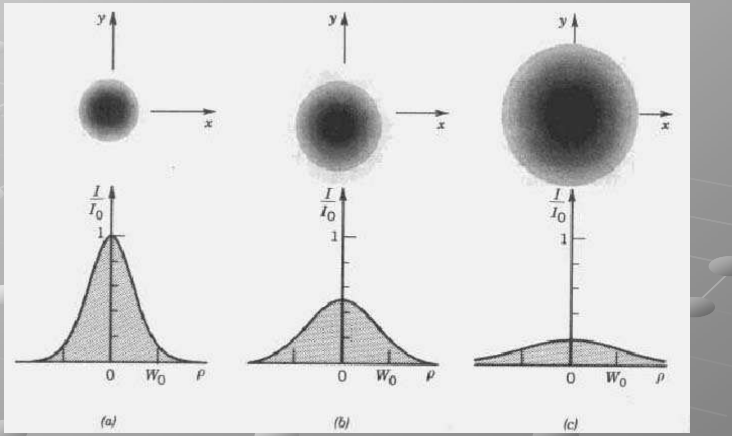
  Caused by diffusion and drift of electrons and holes from the edges of the active region to the cladding layers.

  It is higher at higher temperature
  It increases when the barrier height decreases.
  It increases with the decreases in cladding layer doping.

Optical properties of semiconductor lasers: Gaussian beam profile

The expression for a real laser beam's electric field is given by:





$$E(x,y,z) \propto \frac{\exp[-ikz - i\psi(z)]}{w(z)} \exp[-\frac{x^2+y^2}{w^2(z)} - ik\frac{x^2+y^2}{2R(z)}]$$

Expression for spot size, radius of curvature, and phase shift:

$$w(z) = w_0\sqrt{1+(z/z_R)^2}$$

$$R(z) = z + z_R^2/z$$

$$\psi(z) = \arctan(z/z_R)$$

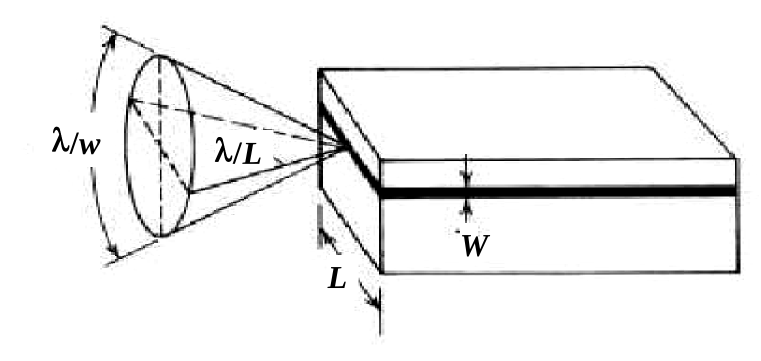
Where Z_R is the Rayleigh Range, it is given by

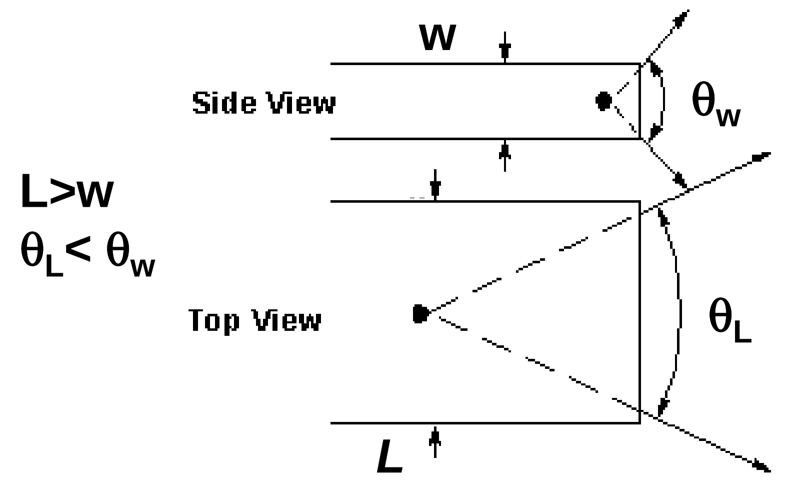
$$z_R = \pi w_0^2/\lambda$$

The beam divergence half angle is given by

$$\theta_{1/e} = \frac{w(z)}{z} = \frac{w_0 z}{z_R z} = \frac{w_0}{z_R} = \frac{\lambda}{\pi w_0}$$

Optical properties of semiconductor lasers: Astigmatism





An edge emitting laser has astigmatism, the divergent angles are:

$$\theta_w = \frac{\lambda}{\pi w}$$

$$\theta_L = \frac{\lambda}{\pi L}$$

Semiconductor Laser gain dynamics

Consider two coupled aspects. We therefore consider two simple “rate equations” – first order differential equations that are coupled to one another.

$$\begin{cases} \dfrac{dN}{dt} = \boxed{\dfrac{\eta_i I}{eV_{gain}}} - \boxed{\dfrac{N}{\tau}} - \boxed{v_g g N_p} \\ \dfrac{dN_p}{dt} = \boxed{\Gamma v_g g N_p} - \boxed{\dfrac{N_p}{\tau_p}} \end{cases}$$

Number of carriers added per unit volume per unit time

Number of undesired carrier recombination per unit volume per unit time

Number of stimulated carrier recombination per unit volume per unit time

Number of photons added per unit cavity volume per unit time

Number of photons lost from the cavity per unit cavity volume per unit time

At steady state, the carrier and photon density are stable, so

$$0 = \frac{\eta_i I_0}{eV_{gain}} - \frac{N_0}{\tau} - v_g g N_{p0}$$

$$0 = \Gamma v_g g_0 N_{po} - \frac{N_{po}}{\tau_p}$$

Semiconductor Laser gain dynamics

Suppose there is small variation at steady condition.

$$\begin{cases} \dfrac{d\delta N}{dt} = \dfrac{\eta_i \delta I}{eV_{gain}} - \dfrac{\delta N}{\tau} - \dfrac{\delta N_p}{\Gamma \tau_p} - v_g a N_{po} \delta N \\ \dfrac{d\delta N_p}{dt} = \Gamma v_g a N_{po} \delta N \end{cases}$$

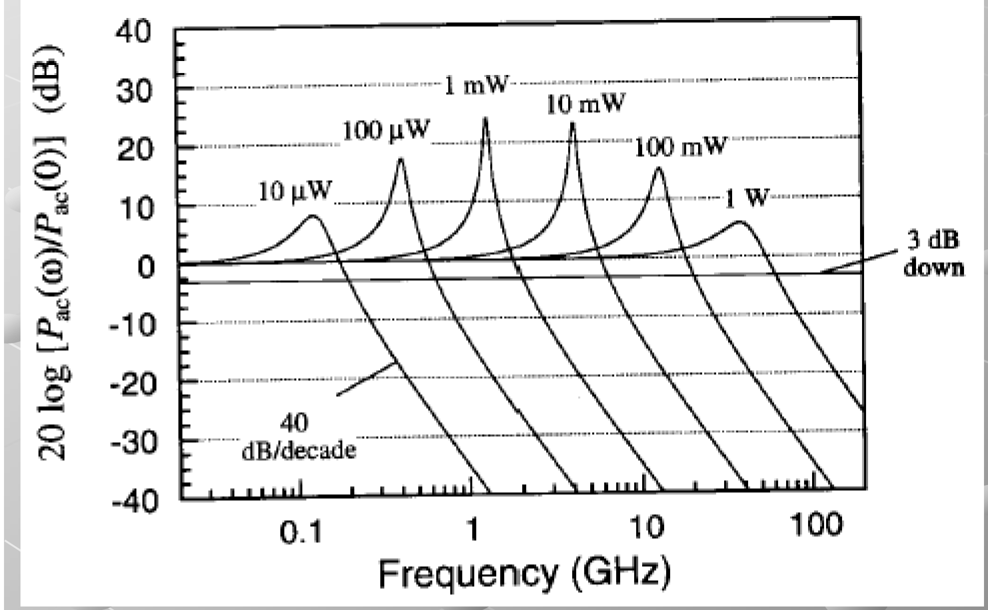
Solving them:

$$\frac{d^2 \delta N}{dt^2} + \left(\frac{1}{\tau} + v_g a N_{po}\right)\frac{d\delta N}{dt} + \left(\frac{v_g a N_{po}}{\tau_p}\right)\delta N = \frac{\eta_i}{eV_{gain}}\frac{d\delta I}{dt}$$

Relaxation oscillation frequency: $\omega_R = \sqrt{\dfrac{v_g a N_{po}}{\tau_p}}$

Semiconductor Laser gain dynamics

General form of the frequency response $$\frac{\delta P(\omega)}{\delta P(0)} = \frac{1}{1-\frac{\omega^2}{\omega_R^2}+i\frac{\omega}{\omega_R}\left(\frac{1}{\omega_R\tau}+\omega_R\tau_p\right)}$$



Frequency response of the output power modulation of a laser diode as the frequency of electrical drive is increased.

Note that

1. **there is an intrinsic limit to the modulation speed**
2. **the modulation speed tend to rise with the square root of the differential gain,**
3. **the modulation speed tends to rise as the square root of the laser output power.**

Noise properties of semiconductor lasers

Noise arises from the spontaneous emission process and carrier-generation-recombination process, can be modeled by adding Langevin noise source.

Modified rate equations with added noise

$$\dot{P} = (G-\gamma)P + R_{sp} + F_p(t)$$

$$\dot{N} = I/q - \gamma_e N - GP + F_N(t)$$

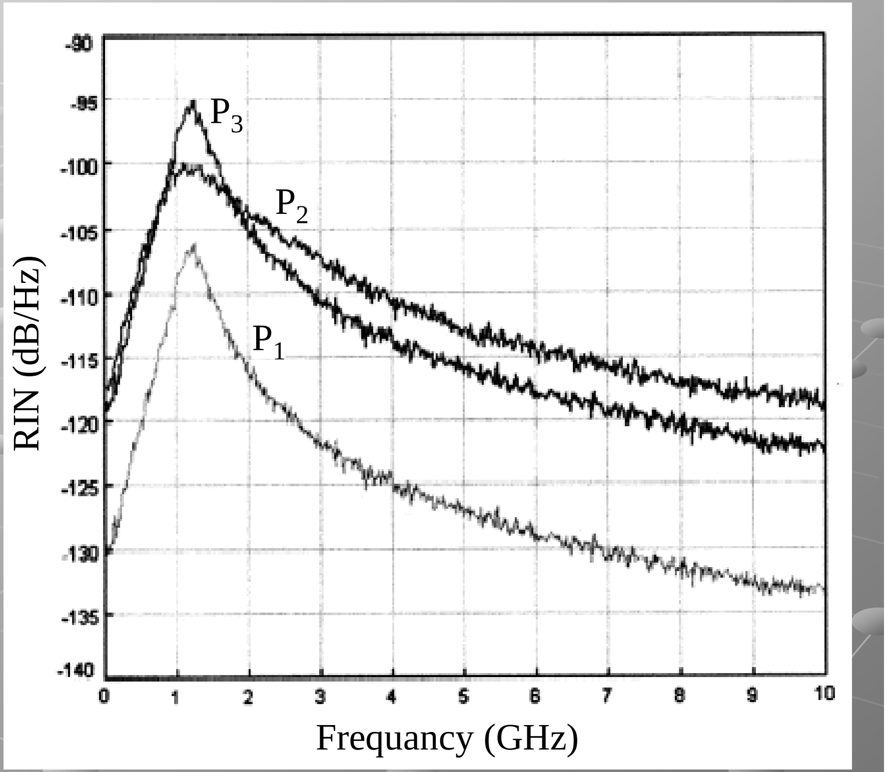
$$\dot{\phi} = -(\omega_0 - \omega_{th}) + \frac{1}{2}\beta_c(G-\gamma) + F_\phi(t)$$

F_p and F_ϕ are from spontaneous emission, F_N has its origin in the discrete nature of carrier generation and recombination processes (shot noise)

Electrical and optical properties of semiconductor lasers

Intensity noise, RIN

RIN decreases with increasing power (P_3>P_2>P_1). It shows maximum at the relaxation-oscillation peak.



Phase noise

Phase fluctuation comes from spontaneous emission, change in optical gain and refractive index induced by carrier population, linewidth enhancement factor (huge for semiconductor laser).

$$\delta\nu = \frac{R_{sp}}{4\pi P}(1+\beta_c^2)$$