

**ECE 6340**  
**Fall 2016**

**Homework 6**

Assignment: Please do Probs. 1-7 and 10 from the set below. You are welcome to do other problems for your own benefit.

- 1) Calculate the skin depth and the surface impedance for aluminum at a frequency of 2.45 [GHz] (this is the frequency of a microwave oven). The conductivity of aluminum is taken as  $2.0 \times 10^7$  [S/m]. Aluminum is nonmagnetic ( $\mu = \mu_0$ ).
- 2) A 75 [ $\Omega$ ] coaxial line used for TV has an outer radius of  $b = 0.25$  [cm] and an inner radius of  $a = 0.039$  [cm]. The coax is filled with Teflon ( $\epsilon_r = 2.2$ ) that has a loss tangent of 0.001. The conductors are made of copper, which is nonmagnetic. Assume that the conductivity of copper is  $3.0 \times 10^7$  [S/m]. Make a table that shows the conductive attenuation  $\alpha_c$ , the dielectric attenuation  $\alpha_d$ , and the total attenuation  $\alpha$  in [dB/m] for various frequencies, including 1 KHz, 10 KHz, 100 KHz, 1 MHz, 10 MHz, 100 MHz, 1 GHz, 10 GHz, and 100 GHz. Use the formulas derived in class for the conductor and dielectric attenuations, and take the total attenuation to be the sum of the conductor and dielectric attenuations. That is, use “method 1”.
- 3) Repeat the previous problem, calculating the attenuation by using the transmission-line formula for  $\gamma$ . That is, find ( $R, L, G, C$ ) at each frequency, and then use this in the formula for  $\gamma$  in order to then find  $\alpha$  at each frequency. (Assume that  $L = L_0$  is based on external inductance only). That is, use “method 2”. Compare with the results obtained in the previous problem.
- 4) A twin-lead transmission line used for television has a characteristic impedance of 300 [ $\Omega$ ]. The separation between the centers of the two wires is 0.6 [cm]. The wires are made of copper, having a conductivity of  $3.0 \times 10^7$  [S/m]. The wires may be assumed to be in air. Find the attenuation in dB/m for a VHF frequency of 100 [MHz]. Compare with the result for the 75 [ $\Omega$ ] coax, using your results from Prob. 2. Note: You should find the radius of the wires by using the known characteristic impedance of the line. Use the exact characteristic impedance, given by

$$Z_0 = \frac{\eta_0}{\pi} \cosh^{-1} \left( \frac{h}{2a} \right).$$

- 5) A coaxial line has a fixed outer radius  $b$ . What should the ratio  $x = a / b$  be in order to minimize the conductor attenuation constant  $\alpha_c$ ? (give a final numerical value). What is the characteristic impedance for such a line, in term of  $\epsilon_r$ ? Assume  $\epsilon_r = 2.2$  (Teflon) and

calculate a numerical value for the resulting  $Z_0$ . (Note: You can directly search numerically for the value of  $x$  that minimizes the attenuation. Or, you can set the derivative of the attenuation with respect to  $x$  equal to zero and solve the resulting equation numerically.)

- 6) A  $75 \text{ } [\Omega]$  coaxial line used for TV has an outer radius of  $b = 0.25 \text{ [cm]}$  and an inner radius of  $a = 0.039 \text{ [cm]}$ . The coax is filled with Teflon ( $\epsilon_r = 2.2$ ) that has a loss tangent of 0.001. The conductors are made of copper, which is nonmagnetic. Assume that the conductivity of copper is  $3.0 \times 10^7 \text{ [S/m]}$ . At a frequency of 100 MHz, calculate the parameters  $R, L, G, C$  (where  $L = L_0$  is based on external inductance only). Next, calculate  $\Delta L$  from the skin inductance formula. Finally, compare the normalized phase and attenuation constants ( $\beta / k_0$  and  $\alpha / k_0$ ) that you obtain by including and excluding the skin inductance (internal inductance) in the transmission-line ( $R L G C$ ) model.
- 7) Consider a round solid wire of radius  $a$  running along the  $z$  axis. Show that the DC internal inductance per unit length in the  $z$  direction is

$$L_{int} = \frac{\mu_0 \mu_r}{8\pi},$$

where  $\mu_r$  is the relative permeability of the wire material (which would be unity for copper or aluminum). Do this by assuming a DC current  $I$  flowing along the wire in the  $z$  direction, and then using Ampere's law to calculate the magnetic field inside the wire. From this, calculate the stored magnetic energy inside the wire, given by

$$U_H = \frac{1}{2} \int_V \mu_0 \mu_r |\underline{H}|^2 dV,$$

where  $V$  is the region inside the wire, assuming a one meter length in the  $z$  direction. Then use the energy formula for an inductor,

$$U_H = \frac{1}{2} L_{int} |I|^2.$$

- 8) Consider a conducting tube of inner radius  $b$  and outer radius  $c$  (modeling the outer shield of a coax) running along the  $z$  axis. Show that the DC internal inductance per unit length in the  $z$  direction is

$$L_{int} = \frac{\mu_0 \mu_r}{2\pi} \left[ \frac{c^4 \ln\left(\frac{c}{b}\right)}{(c^2 - b^2)^2} + \frac{b^2 - 3c^2}{4(c^2 - b^2)} \right],$$

where  $\mu_r$  is the relative permeability of the wire material (which would be unity for copper or aluminum). Do this by assuming a DC current  $I$  flowing along the tube in the  $z$  direction, and then using Ampere's law to calculate the magnetic field inside the tube. From this, calculate the stored magnetic energy inside the tube, given by

$$U_H = \frac{1}{2} \int_V \mu_0 \mu_r |\underline{H}|^2 dV ,$$

where  $V$  is the region inside the tube, assuming a one meter length in the  $z$  direction. Then use the energy formula for an inductor,

$$U_H = \frac{1}{2} L_{int} |I|^2 .$$

- 9) Derive a formula for the attenuation constant for the  $TE_{10}$  mode in an air-filled rectangular waveguide, assuming that the waveguide metal has a surface resistance  $R_s$ . The dimensions of the waveguide are  $a \times b$ , with  $a$  being the long dimension. Evaluate all integrals to obtain a closed-form result, which should be

$$\alpha_c = \frac{R_s}{b\eta_0 \sqrt{1 - (f_c/f)^2}} \left[ 1 + 2 \frac{b}{a} \left( \frac{f_c}{f} \right)^2 \right] .$$

- 10) The  $TE_{10}$  mode propagates in an X-band air-filled rectangular waveguide, having  $a = 2.2225$  [cm] and  $b = 1.0319$  [cm]. The waveguide walls are made of copper, having a conductivity of  $3.0 \times 10^7$  [S/m]. Using the answer to the previous problem, compare (as explained below) the total attenuation for this waveguide with that of a 50 [ $\Omega$ ] coaxial cable. The coax has an outer radius of 0.5 [cm] and an inner radius of 0.145 [cm], and is filled with Teflon, having  $\epsilon_r = 2.2$  and a loss tangent of 0.001. The conductivity of the copper conductors is  $3.0 \times 10^7$  [S/m].

Make the comparison by plotting the total attenuation in dB/m for the waveguide and the coax versus frequency. Plot from the cutoff frequency of the  $TE_{10}$  mode up to twice the cutoff frequency of the  $TE_{10}$  mode (which is the frequency at which the next mode, the  $TE_{20}$  mode, starts to propagate). For the vertical scale, plot up to a maximum of 2 [dB/m]. (Note: The waveguide has only conductive loss because it is air filled, while the coaxial cable has both conductive loss and dielectric loss.) For the coax, calculate the total attenuation as the sum of the conductive and dielectric attenuations (i.e., use “method 1”).