ECE 6341

Spring 2016

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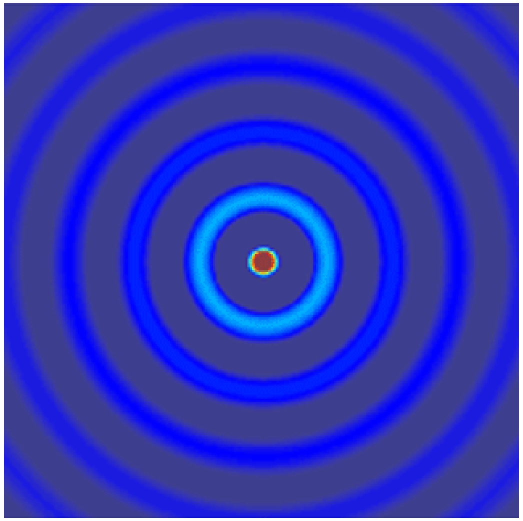
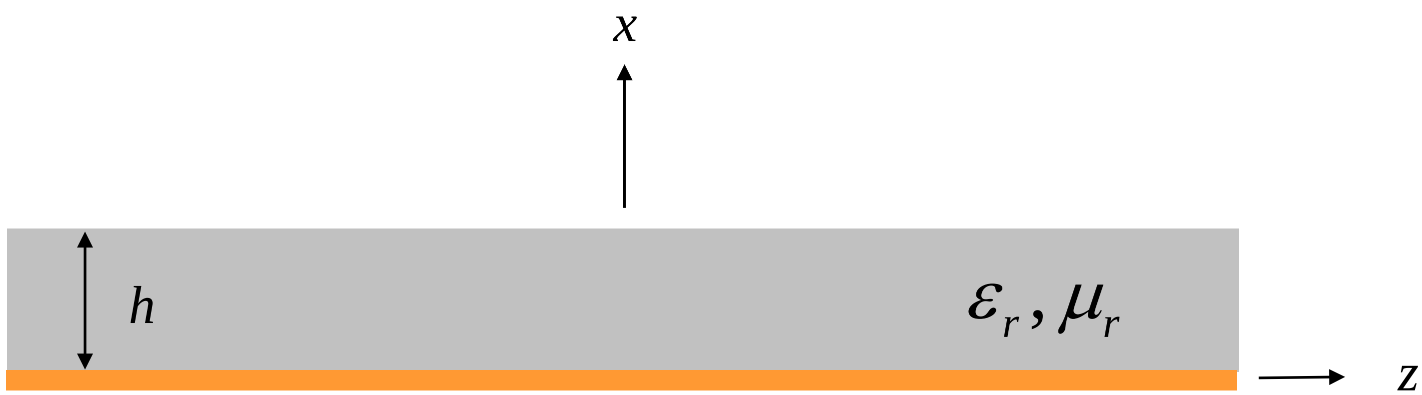
ECE Dept.

Notes 4

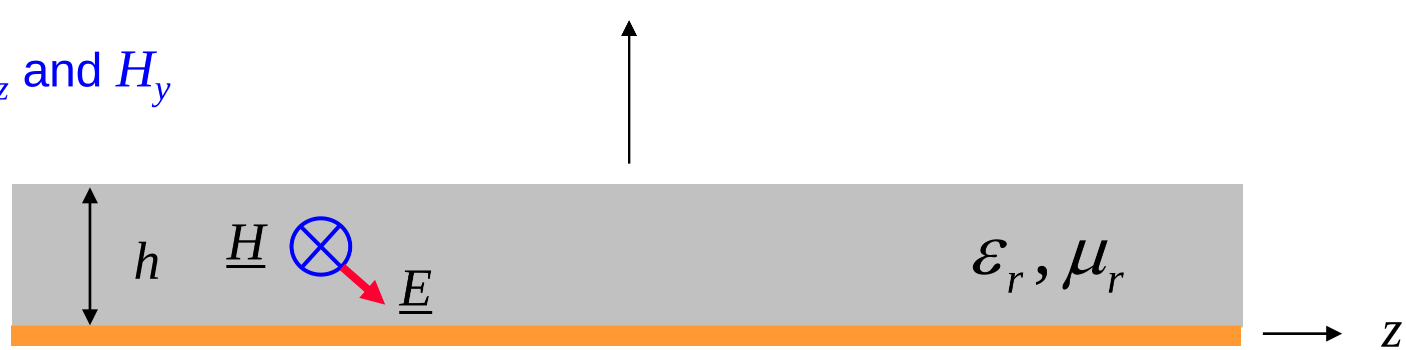


Transverse Equivalent Network (TEN)



Assume TM_x surface wave:

E_x, E_z and H_y



$$\underline{E}_t = \underline{\hat{z}}\, E_z \qquad \underline{H}_t = \underline{\hat{y}}\, H_y$$

TEN (cont.)

$$V(x) \leftrightarrow \underline{E}_t$$

$$I(x) \leftrightarrow \underline{H}_t$$

Note: t denotes transverse to x.

In particular,

$$\underline{E}_t(x,z) = \underline{\hat{z}}\, V(x)\, e^{-jk_z z}$$

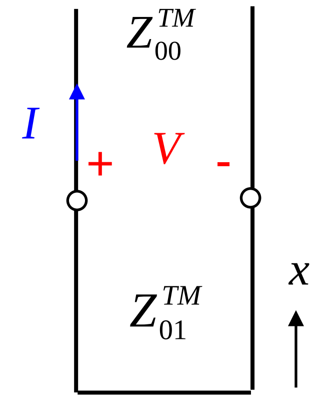
$$\underline{H}_t(x,z) = -\underline{\hat{y}}\, I(x)\, e^{-jk_z z}$$

We will show that V and I behave like voltage and current on a TL.

$$\nabla^2 \underline{E}_t + k^2 \underline{E}_t = \underline{0} \quad \Rightarrow \quad \nabla^2 E_z + k^2 E_z = 0$$

so

$$\frac{d^2 V}{dx^2} + (k^2 - k_z^2)V = 0$$



TEN (cont.)

$$\frac{d^2V}{dx^2} + k_x^2 V = 0 \qquad k_x^2 \equiv k^2 - k_z^2$$

$$V(x) = Ae^{-jk_x x} + Be^{+jk_x x}$$

(same conclusion for $I(x)$)

Next, consider a wave traveling upward in the + x direction:

$$E_z(x,z) = V(x)\, e^{-jk_z z}$$
$$H_y(x,z) = -I(x)\, e^{-jk_z z}$$

$$\Rightarrow \quad \frac{V^+(x)}{I^+(x)} = \frac{E_z^+}{-H_y^+}$$

TEN (cont.)

From the TE_x-TM_x table:

$$E_z^+ = \frac{1}{j\omega\mu\varepsilon}\frac{\partial^2 A_x^+}{\partial x \partial z} \qquad H_y^+ = \frac{1}{\mu}\frac{\partial A_x^+}{\partial z}$$

Hence

$$\frac{V^+(x)}{I^+(x)} = -\frac{1}{j\omega\varepsilon}\frac{\dfrac{\partial^2 A_x^+}{\partial x \partial z}}{\dfrac{\partial A_x^+}{\partial z}} = -\frac{1}{j\omega\varepsilon}\frac{(-jk_x)\dfrac{\partial A_x^+}{\partial z}}{\dfrac{\partial A_x^+}{\partial z}} = \frac{k_x}{\omega\varepsilon}$$

Similarly,

$$\frac{V^-(x)}{I^-(x)} = -\frac{k_x}{\omega\varepsilon}$$

We define:

$$Z_0^{TM_x} = \frac{k_x}{\omega\varepsilon}$$

TEN (cont.)

Hence:

$$Z_{00}^{TM_x} = \frac{k_{x0}}{\omega\varepsilon_0}$$

$$Z_{01}^{TM_x} = \frac{k_{x1}}{\omega\varepsilon_1}$$

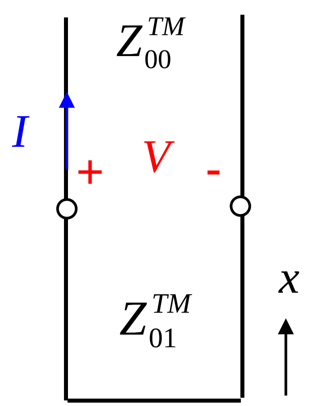
Similarly,

$$Z_{00}^{TE_x} = \frac{\omega\mu_0}{k_{x0}}$$

$$Z_{01}^{TE_x} = \frac{\omega\mu_1}{k_{x1}}$$

Note:
TM_x and TE_x waves do not couple at the boundaries, since they each have their own separate TEN circuits.

TEN:



$$Z_{00}^{TM} = \frac{k_{x0}}{\omega\varepsilon_0}$$

$$Z_{01}^{TM} = \frac{k_{x1}}{\omega\varepsilon_1}$$

$$k_{x0} = \left(k_0^{\,2} - k_z^2\right)^{1/2}$$

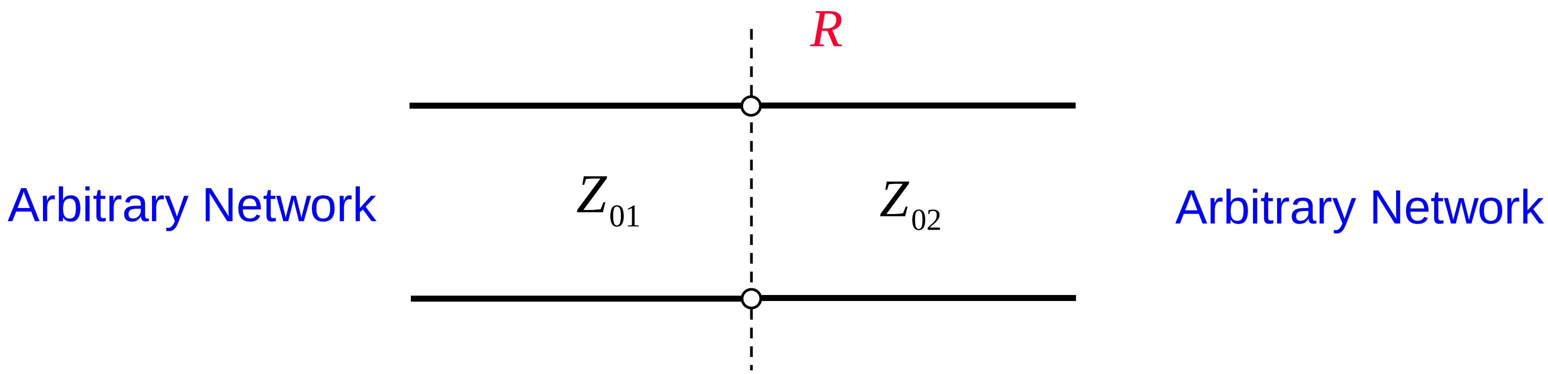
$$k_{x1} = \left(k_1^{\,2} - k_z^2\right)^{1/2}$$

Transverse Resonance Equation (TRE)

The TRE is useful for various purposes:

- Finding the unknown wavenumber of a waveguiding structure, given the frequency
- Finding the cutoff frequency of a waveguiding structure
- Finding the resonance frequency of a resonator structure

Transverse Resonance Equation (TRE)



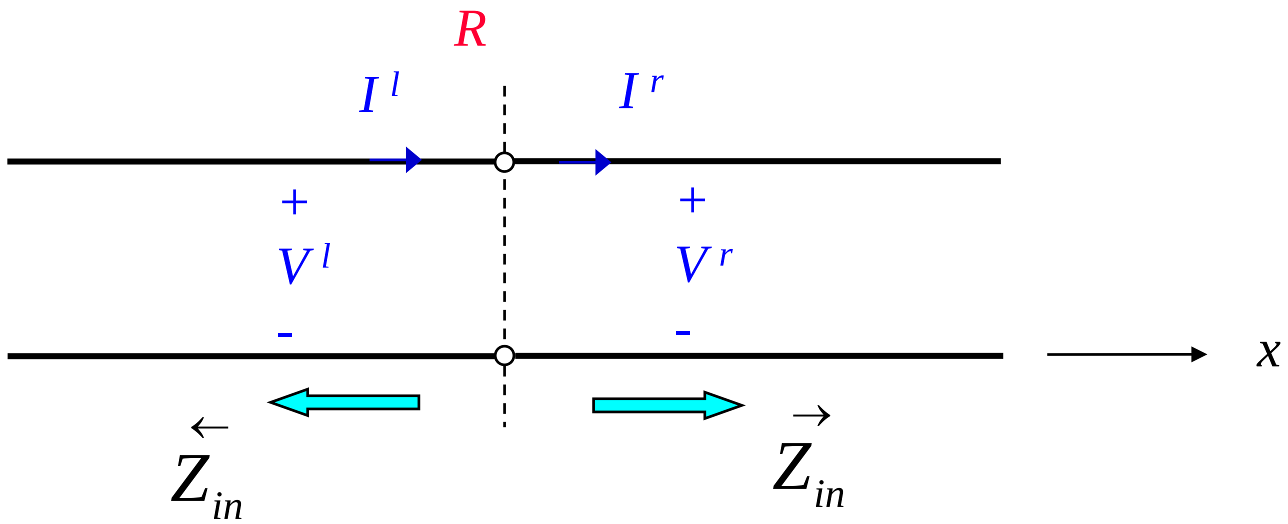
R = reference plane within a network (no sources, but nontrivial fields)

Assume: no sources, but nontrivial fields

(a "resonator problem")

TRE (cont.)

Examine the behavior at the reference plane:



Define impedances:

$$\overrightarrow{Z}_{in} = \frac{V^r}{I^r}$$

$$\overleftarrow{Z}_{in} = \frac{V^l}{-I^l}$$

Boundary conditions:

$$V^r = V^l$$

$$I^r = I^l$$

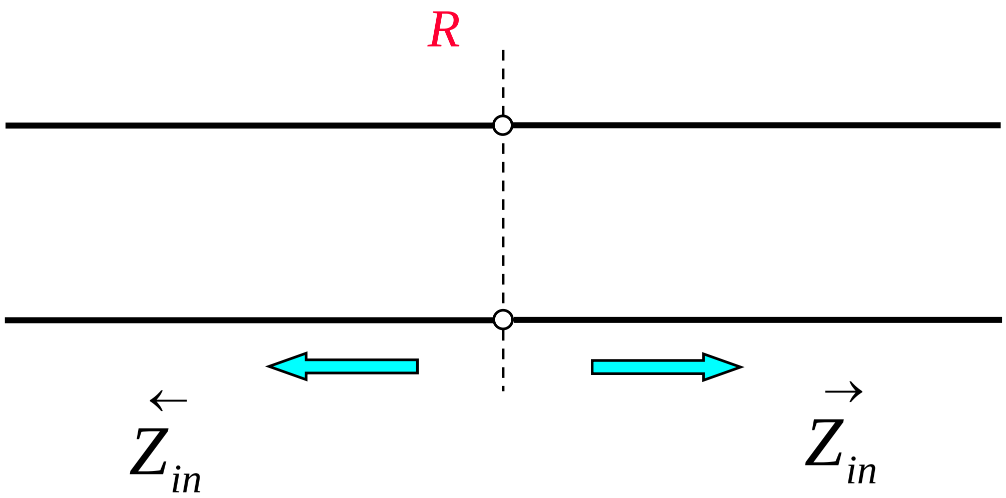
Hence:

$$\overleftarrow{Z}_{in} = -\overrightarrow{Z}_{in}$$

Note: We can take the reciprocal of both sides to get the same equation for admittance.

TRE (cont.)

Summary

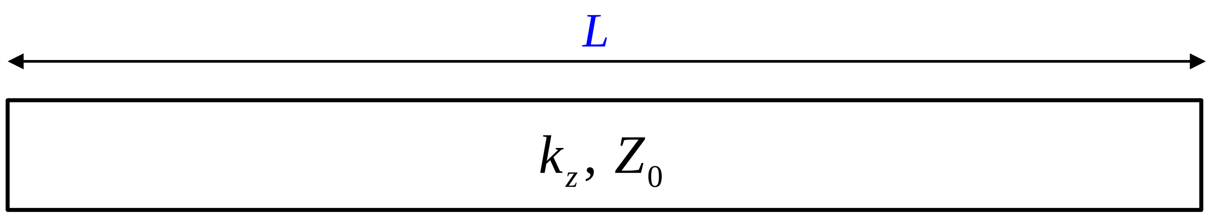


$$\overleftarrow{Z}_{in} = -\overrightarrow{Z}_{in}$$

$$\overleftarrow{Y}_{in} = -\overrightarrow{Y}_{in}$$

TRE (cont.)

Example:



Short-circuited transmission-line resonator

$k_z = k_0 \sqrt{\varepsilon_r}$ (TEM mode)

Put reference place at left end (short circuit):

$$\overleftarrow{Z}_{in} = 0$$

$$\overrightarrow{Z}_{in} = jZ_0 \tan\left(k_z L\right)$$

TRE (cont.)

TRE: $$0 = -jZ_0 \tan\left(k_z L\right)$$

$$\Rightarrow \quad \tan\left(k_z L\right) = 0$$

$$\Rightarrow \quad k_z L = m\pi,\ m = 1, 2 \ldots$$

$$\Rightarrow \quad k_0 \sqrt{\varepsilon_r} L = m\pi$$

$$\Rightarrow \quad k_0 = \frac{m\pi}{\sqrt{\varepsilon_r} L}$$

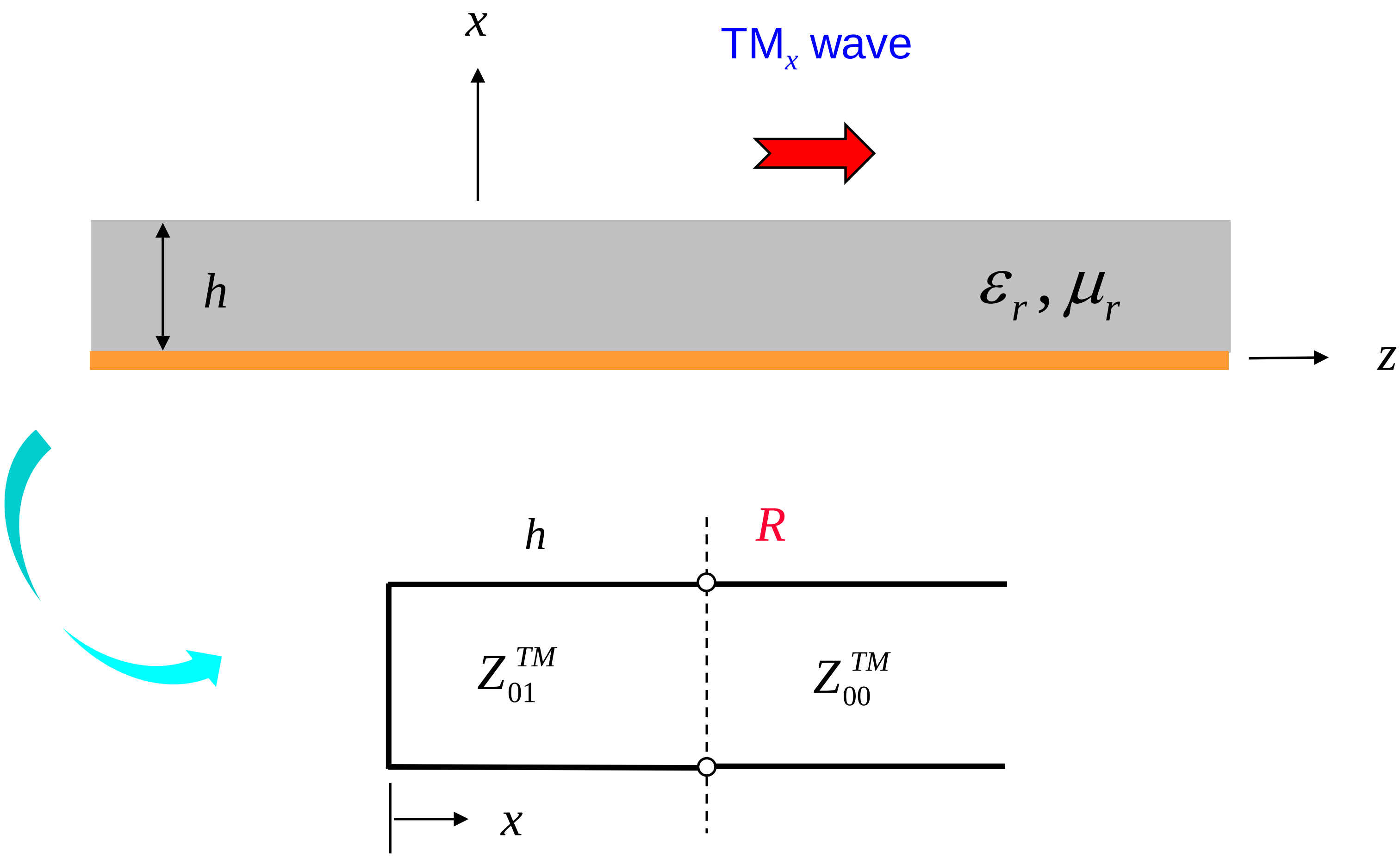
$$\Rightarrow \quad 2\pi f \sqrt{\mu_0 \varepsilon_0} = \frac{m\pi}{\sqrt{\varepsilon_r} L}$$

Hence

$$f = \frac{mc}{2\sqrt{\varepsilon_r} L}$$

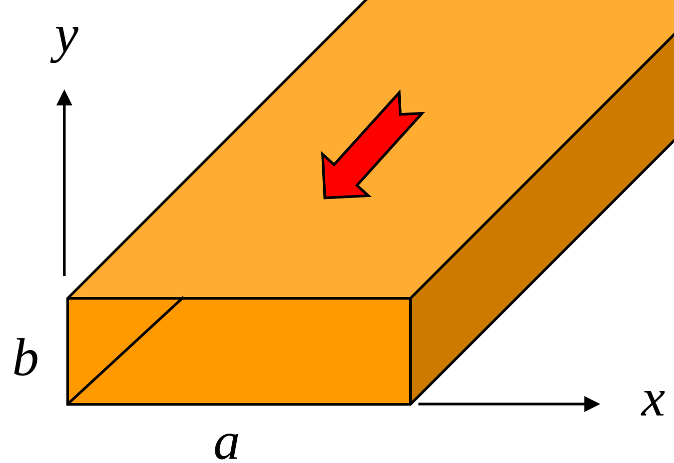
TRE (cont.)

When we apply the TRE to a waveguiding problem, we think of the "resonator" as being formed from the TEN that models the waveguiding structure in the *transverse direction*.



TRE (cont.)

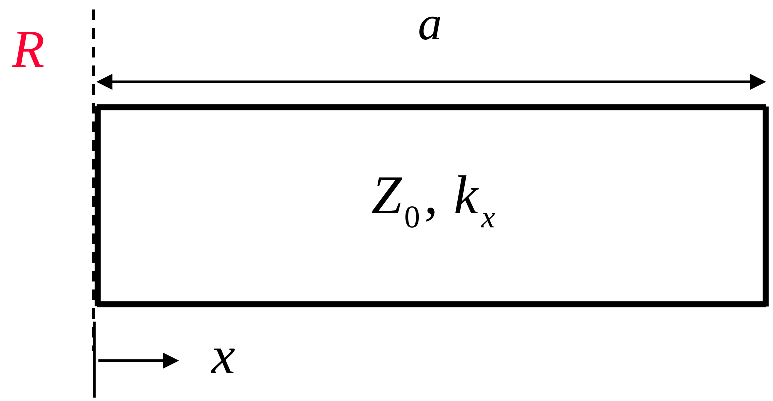
Example:



We focus on the x variation of the fields.

Rectangular waveguide

Note:
A TE_z or TM_z mode is a linear combination of TM_x and TE_x modes.



$$Z_0 = Z_0^{TM_x} \text{ or } Z_0^{TE_x}$$

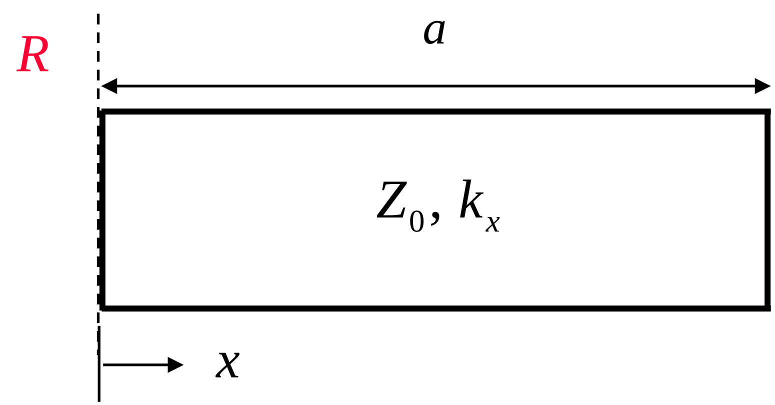
TRE (cont.)

$$\overleftarrow{Z}_{in} = 0$$

$$\overrightarrow{Z}_{in} = jZ_0 \tan\left(k_x a\right)$$

TRE: $$0 = -jZ_0 \tan\left(k_x a\right)$$

$$\Rightarrow \quad k_x a = m\pi, \; m = 0, 1, 2\ldots$$



$$Z_0 = Z_0^{TM_x} \text{ or } Z_0^{TE_x}$$

TRE (cont.)

Therefore,

$$k_x = \frac{m\pi}{a},\ m = 0,1,2\ldots$$

Similarly,

$$k_y = \frac{n\pi}{b},\ n = 0,1,2\ldots$$

Also,

$$k_z = \sqrt{k^2 - k_x^2 - k_y^2}$$

Hence, we have,

$$k_z = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$