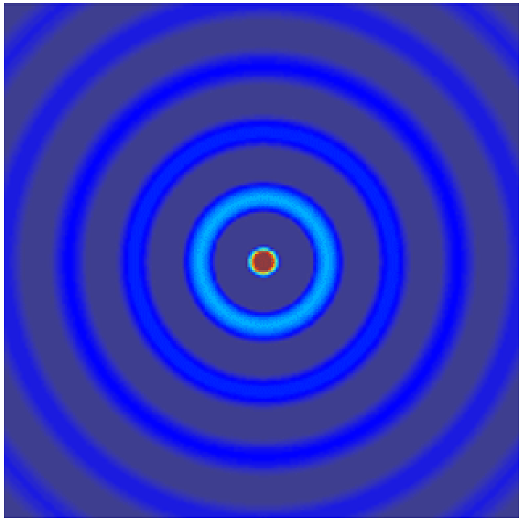
ECE 6341

Spring 2016

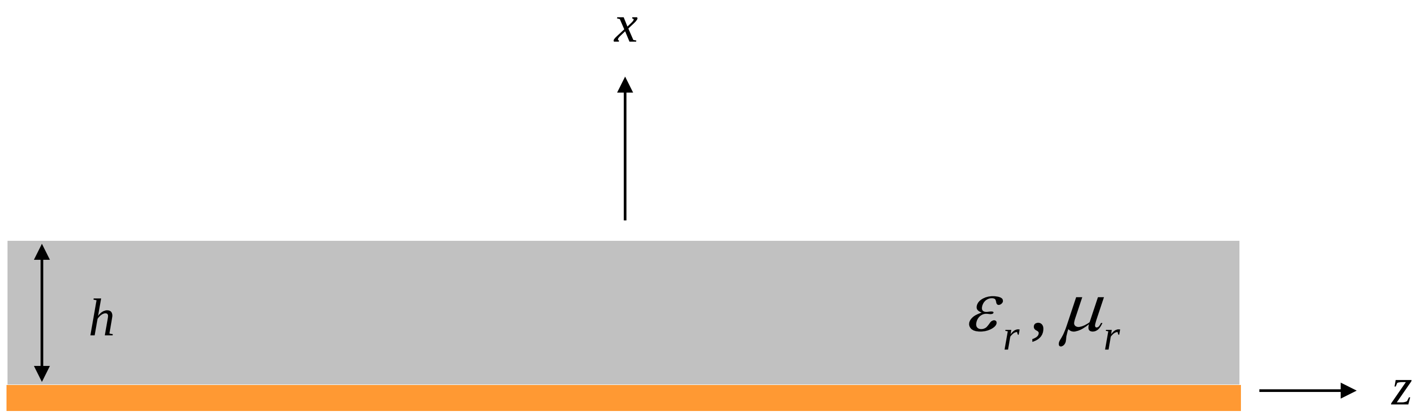
Prof. David R. Jackson
ECE Dept.



Notes 5

TM_x Surface-Wave Solution

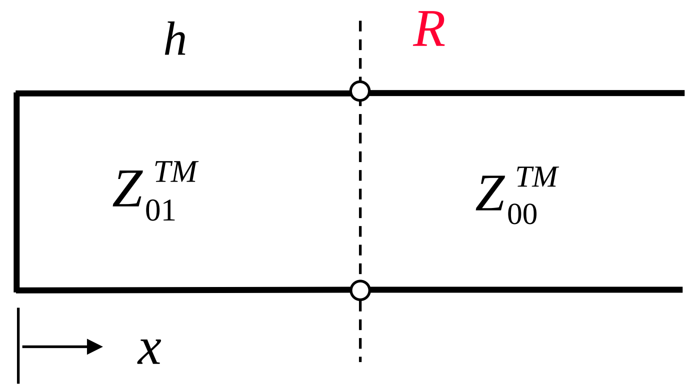
Problem under consideration:



A TM_x surface wave is propagating in the z direction (no y variation).

TM_x Surface-Wave Solution

TEN:



The reference plane is put at the top of the substrate.

$$Z_{01}^{TM} = \frac{k_{x1}}{\omega \varepsilon_1} \qquad Z_{00}^{TM} = \frac{k_{x0}}{\omega \varepsilon_0}$$

$$k_{x1} = (k_1^2 - k_z^2)^{1/2} \qquad k_{x0} = (k_0^2 - k_z^2)^{1/2} = -j\sqrt{k_z^2 - k_0^2}$$

$$\overleftarrow{Z}_{in} = jZ_{01}^{TM} \tan(k_{x1}h) \qquad \overrightarrow{Z}_{in} = Z_{00}^{TM}$$

TM$_x$ Surface-Wave Solution (cont.)

TRE: $\overleftarrow{Z}_{in} = -\overrightarrow{Z}_{in}$

so

$$jZ_{01}^{TM}\tan(k_{x1}h) = -Z_{00}^{TM}$$

Hence

$$j\frac{k_{x1}}{\omega\varepsilon_1}\tan(k_{x1}h) = -\frac{k_{x0}}{\omega\varepsilon_0}$$

or

$$\varepsilon_r = -j\left(\frac{k_{x1}}{k_{x0}}\right)\tan(k_{x1}h)$$

Note:

$k_{x1}\tan(k_{x1}h)$ is always real

(regardless of whether k_{x1} is real or imaginary)

Note: Assuming a real k_z, a solution will only exist if k_{x0} is imaginary.

$$\Rightarrow k_z > k_0$$

TM_x Surface-Wave Solution (cont.)

Let $$k_{x0} = -j\alpha_{x0}\,, \qquad \alpha_{x0} = \sqrt{k_z^2 - k_0^2}$$

Then we have $$\varepsilon_r = \left(\frac{k_{x1}}{\alpha_{x0}}\right)\tan(k_{x1}h)$$

or

$$\varepsilon_r = \frac{(k_1^2 - k_z^2)^{1/2}}{\sqrt{k_z^2 - k_0^2}}\tan\left((k_1^2 - k_z^2)^{1/2} h\right)$$

Note: This must be solved numerically.

Properties of Surface-Wave Solution

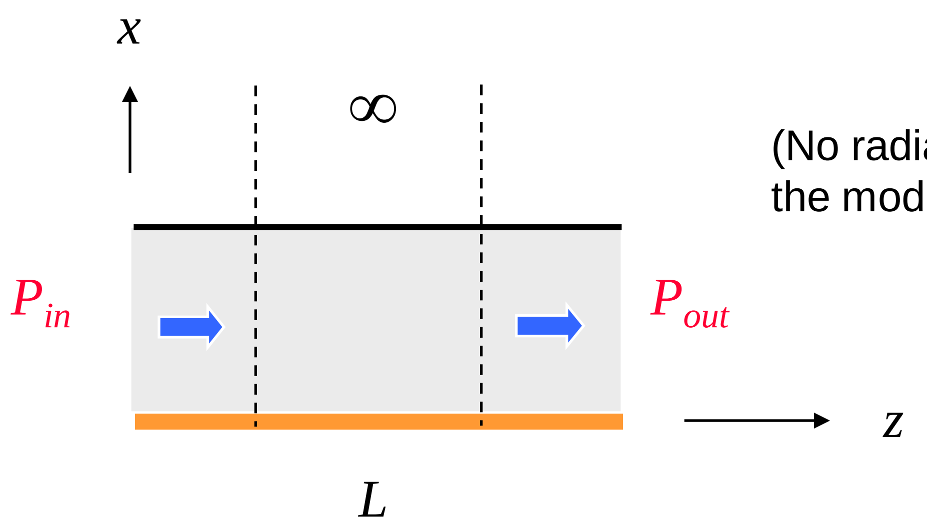
Assumptions:

- A lossless structure
- A proper surface-wave solution (the fields decay at $x = \infty$)

Properties of SW Solution (cont.)

Property 1) k_z is real

Otherwise, conservative of energy is violated:



$$P_{out} = P_{in}$$

(No radiation can escape since the mode decays at infinity.)

Assume

$$k_z = \beta - j\alpha \qquad P_{out} = P_{in}\, e^{-2\alpha L}$$

$$\Rightarrow \quad \alpha = 0$$

Properties of SW Solution (cont.)

Property 2) $k_z \geq k_0$

Otherwise ε_r = imaginary (no solution possible)

$$\varepsilon_r = \left(\frac{k_{x1}}{\alpha_{x0}}\right)\tan(k_{x1}h)$$

$$\alpha_{x0} = \sqrt{k_z^2 - k_0^2}$$

Recall: $k_{x1}\tan(k_{x1}h)$ is always real

Properties of SW Solution (cont.)

Property 3) $k_z < k_1$

Otherwise, $$k_{x1} = (k_1^2 - k_z^2)^{1/2} = -j\alpha_{x1} = -j\sqrt{k_z^2 - k_1^2}$$

In this case,

$$\varepsilon_r = \left(\frac{k_{x1}}{\alpha_{x0}}\right)\tan(k_{x1}h) = \frac{-j\alpha_{x1}}{\alpha_{x0}}\tan\left[(-j\alpha_{x1})h\right]$$

$$= -j\left(\frac{\alpha_{x1}}{\alpha_{x0}}\right)(-j)\tanh(\alpha_{x1}h)$$

$$= -\left(\frac{\alpha_{x1}}{\alpha_{x0}}\right)\tanh(\alpha_{x1}h)$$

$=$ negative real number

Properties of SW Solution (cont.)

Hence, we have for a lossless layer:

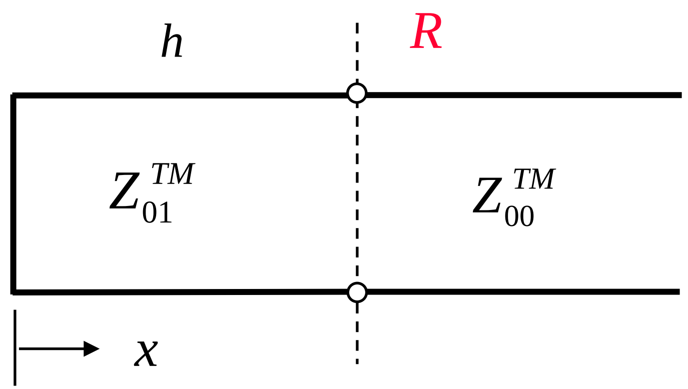
$$k_z = \text{real}$$

$$k_0 < k_z < k_1$$

Generalization to an arbitrary number of layers:

$$k_1 \rightarrow k_{max}$$

TE_x Solution for Slab



$$Z_{00}^{TE} = \frac{\omega\mu_0}{k_{x0}}$$

$$Z_{01}^{TE} = \frac{\omega\mu_1}{k_{x1}}$$

TRE: $$\overleftarrow{Z}_{in} = -\overrightarrow{Z}_{in}$$

$$j\left(\frac{\omega\mu_1}{k_{x1}}\right)\tan(k_{x1}h) = -\frac{\omega\mu_0}{k_{x0}}$$

or

$$\frac{1}{\mu_r} = -j\left(\frac{k_{x0}}{k_{x1}}\right)\tan(k_{x1}h)$$

TE$_x$ Solution for Slab (cont.)

Using $\quad k_{x0} = -j\alpha_{x0} \qquad \alpha_{x0} = \sqrt{k_z^2 - k_0^2}$

we have

$$\frac{1}{\mu_r} = -j\left(\frac{-j\alpha_{x0}}{k_{x1}}\right)\tan(k_{x1}h)$$

or

$$\frac{1}{\mu_r} = -\left(\frac{\alpha_{x0}}{k_{x1}}\right)\tan(k_{x1}h)$$

or

$$\frac{1}{\mu_r} = -\frac{\sqrt{k_z^2 - k_0^2}}{(k_1^2 - k_z^2)^{1/2}}\tan\left((k_1^2 - k_z^2)^{1/2}h\right)$$

Graphical Solution for SW Modes

Consider TM_x: $$\alpha_{x0}\varepsilon_r = k_{x1}\tan(k_{x1}h)$$

or

$$\alpha_{x0}h = \frac{1}{\varepsilon_r}(k_{x1}h)\tan(k_{x1}h)$$

Let

$$u \equiv k_{x1}h$$

$$v \equiv \alpha_{x0}h$$

Then

$$v = \frac{1}{\varepsilon_r}u\tan u$$

Graphical Solution (cont.)

We can develop another equation by relating u and v:

$$u = h\sqrt{k_1^2 - k_z^2}$$

$$v = h\sqrt{k_z^2 - k_0^2}$$

Hence

$$u^2 = h^2 (k_1^2 - k_z^2)$$

$$v^2 = h^2 (k_z^2 - k_0^2)$$

} add

Graphical Solution (cont.)

$$u^2 + v^2 = h^2(k_1^2 - k_0^2)$$
$$= (k_0 h)^2 (n_1^2 - 1)$$

Define:

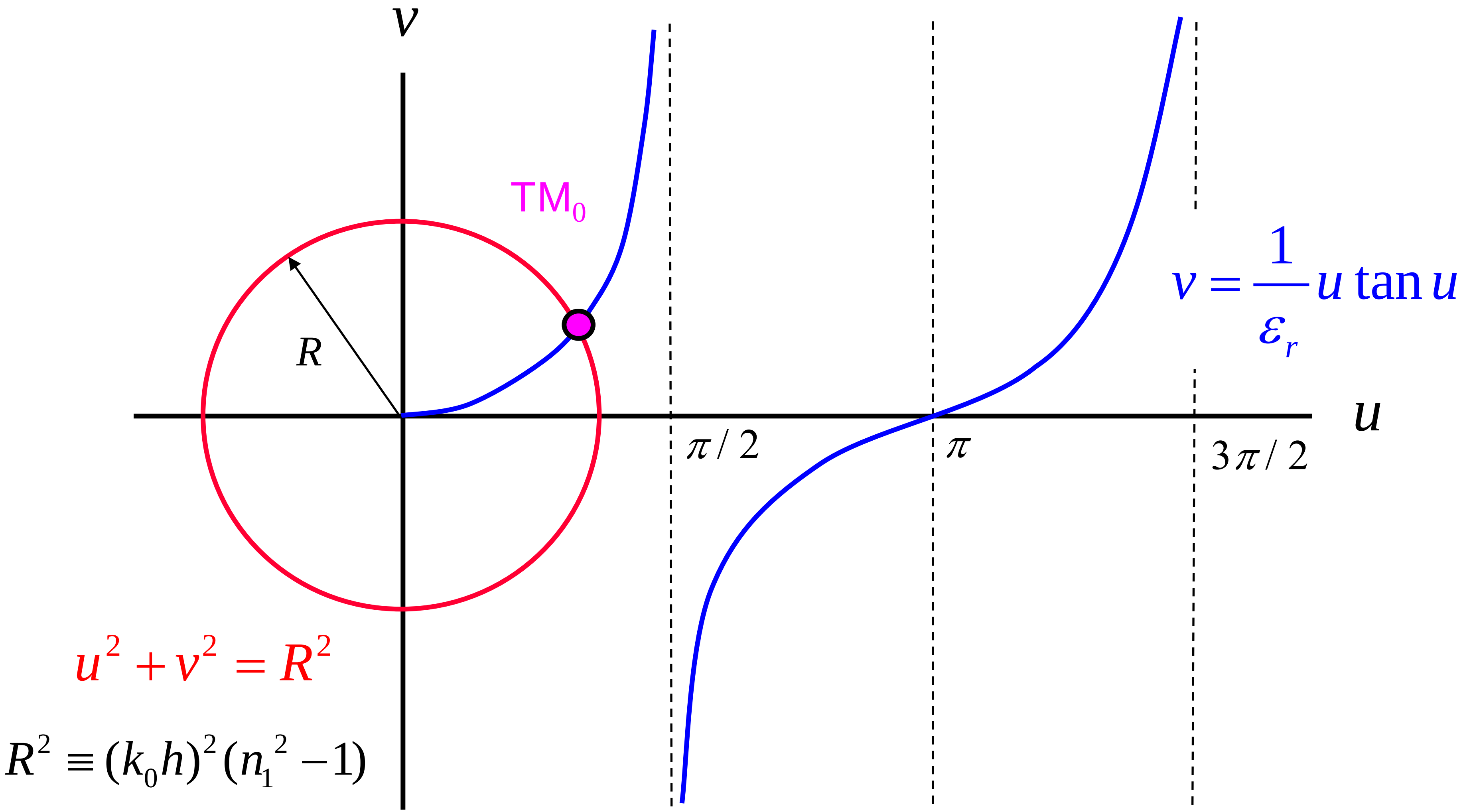
$$R \equiv (k_0 h)\sqrt{n_1^{\,2} - 1}$$

Note:
R is proportional to frequency.

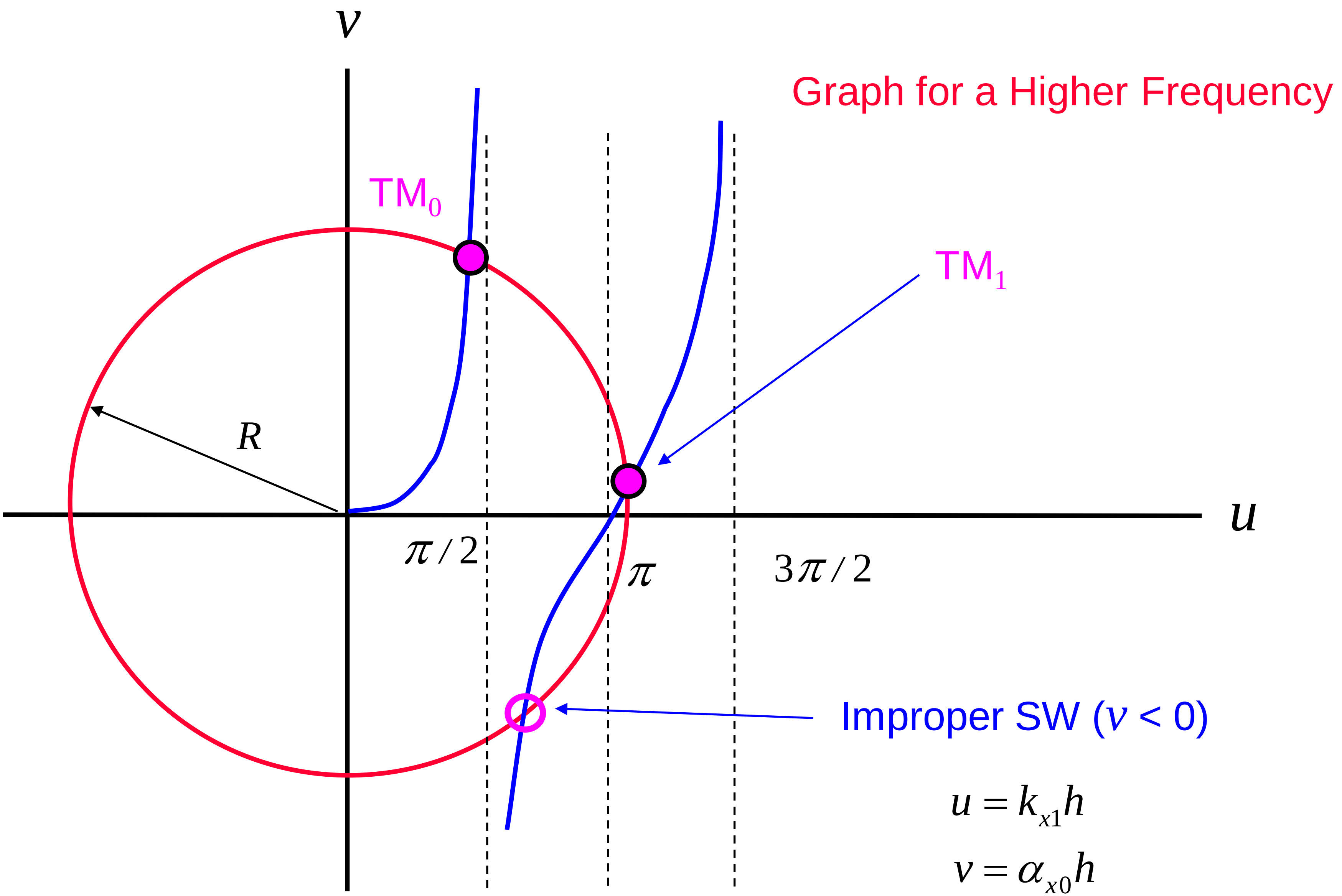
Then

$$u^2 + v^2 = R^2$$

Graphical Solution (cont.)



Graphical Solution (cont.)



$$u = k_{x1} h$$

$$v = \alpha_{x0} h$$

Proper vs. Improper

Recall: $v = \alpha_{x0} h$

If $v > 0$: "proper SW" (fields decrease in x direction)

If $v < 0$: "improper SW" (fields increase in x direction)

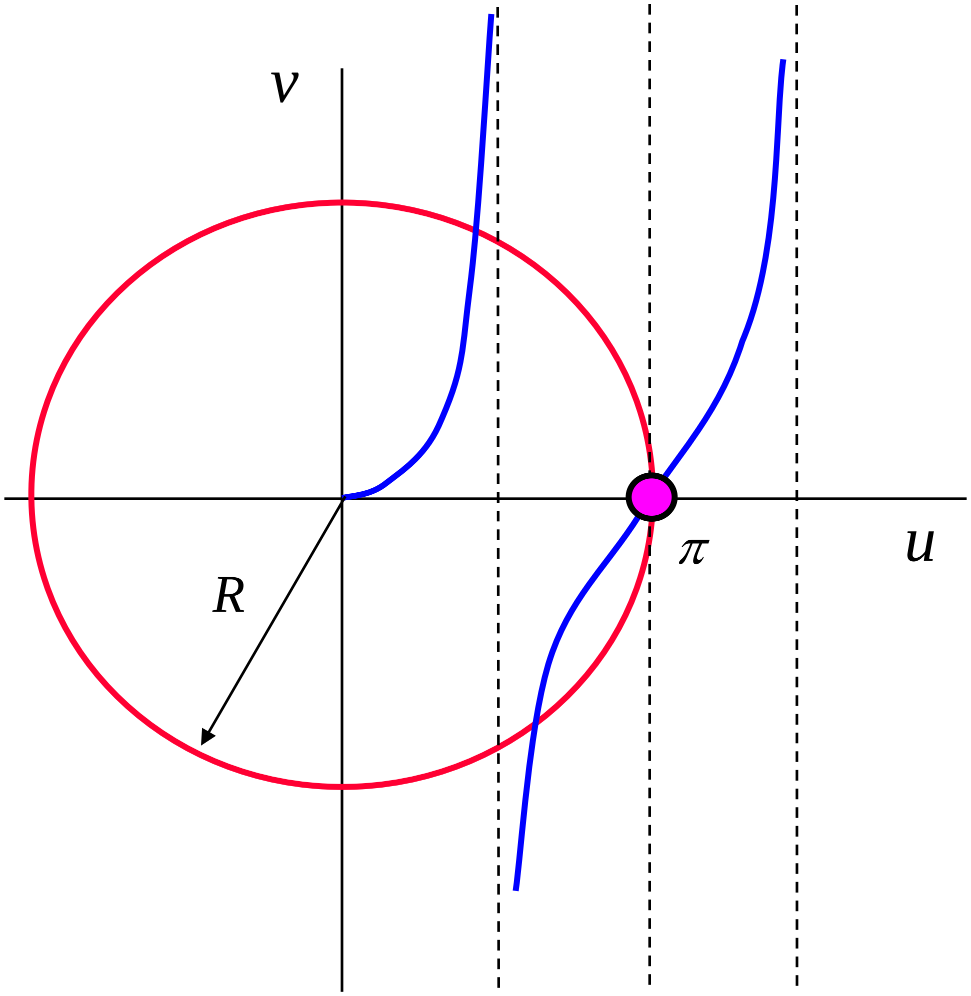
Cutoff frequency for an open structure:
the transition frequency between a proper and improper mode

$\Rightarrow$ $k_z = k_0$ at cutoff

Note: This definition is different from that for a closed waveguide structure (where $k_z = 0$ at the cutoff frequency).

Cutoff frequency: TM_1 mode: $v = 0, \;\; u = \pi$

TM_x Cutoff Frequency



TM_1: $R = \pi$

$$k_0 h \sqrt{n_1^2 - 1} = \pi$$

$$\frac{h}{\lambda_0} = \frac{1/2}{\sqrt{n_1^2 - 1}}$$

For other TM_n modes:

$$\text{TM}_n: \quad \frac{h}{\lambda_0} = \frac{n/2}{\sqrt{n_1^2 - 1}}$$

$$n = 0, 1, 2, \ldots$$

Further Properties of SW Solutions
(obtained from the graphical solution)

Property 1) $k_z = k_0$ at f_c

Proof:

$$v = \alpha_{x0} h = h\sqrt{k_z{}^2 - k_0{}^2}$$

At $f = f_c$: $v = 0 \Rightarrow k_z = k_0$

Properties of SW Solutions (cont.)

Property 2) $k_z \to k_1 \text{ at } f \to \infty$

Proof:

$$u \to \frac{\pi}{2} + n\pi \quad (\text{a constant})$$

Hence

$$u = k_{x1} h = h\sqrt{k_1^2 - k_z^2} \to \text{constant}$$

so

$$k_1 h \sqrt{1 - \left(\frac{k_z}{k_1}\right)^2} \to \text{constant}$$

Therefore,

$$\frac{k_z}{k_1} \to 1$$

TM_0 Mode

The TM_0 mode has two special properties:

TM_0 #1)   No cut-off $\left(f_c \rightarrow 0 \right)$

Proof: see graphical solution

TM_0 Mode (cont.)

TM_0 #2) $$\frac{k_z}{k_0} \to 1 \text{ as } f \to 0$$

Proof: $$v = \frac{1}{\varepsilon_r} u \tan u \approx \frac{1}{\varepsilon_r} u^2$$

Hence

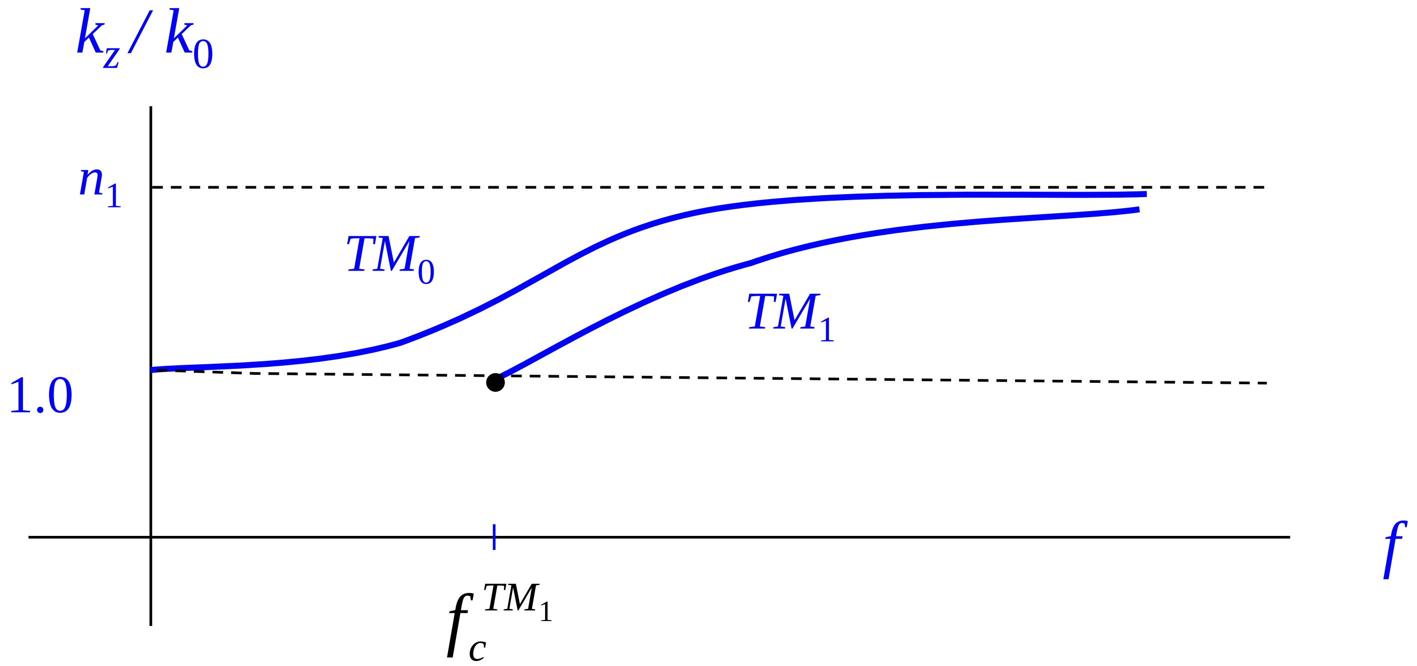
$$h\sqrt{k_z^2 - k_0^2} \approx \frac{1}{\varepsilon_r} h^2 (k_1^2 - k_z^2)$$

$$\Rightarrow k_0 h \sqrt{\left(\frac{k_z}{k_0}\right)^2 - 1} \approx \frac{1}{\varepsilon_r} (k_0 h)^2 \left(n_1^2 - \left(\frac{k_z}{k_0}\right)^2 \right)$$

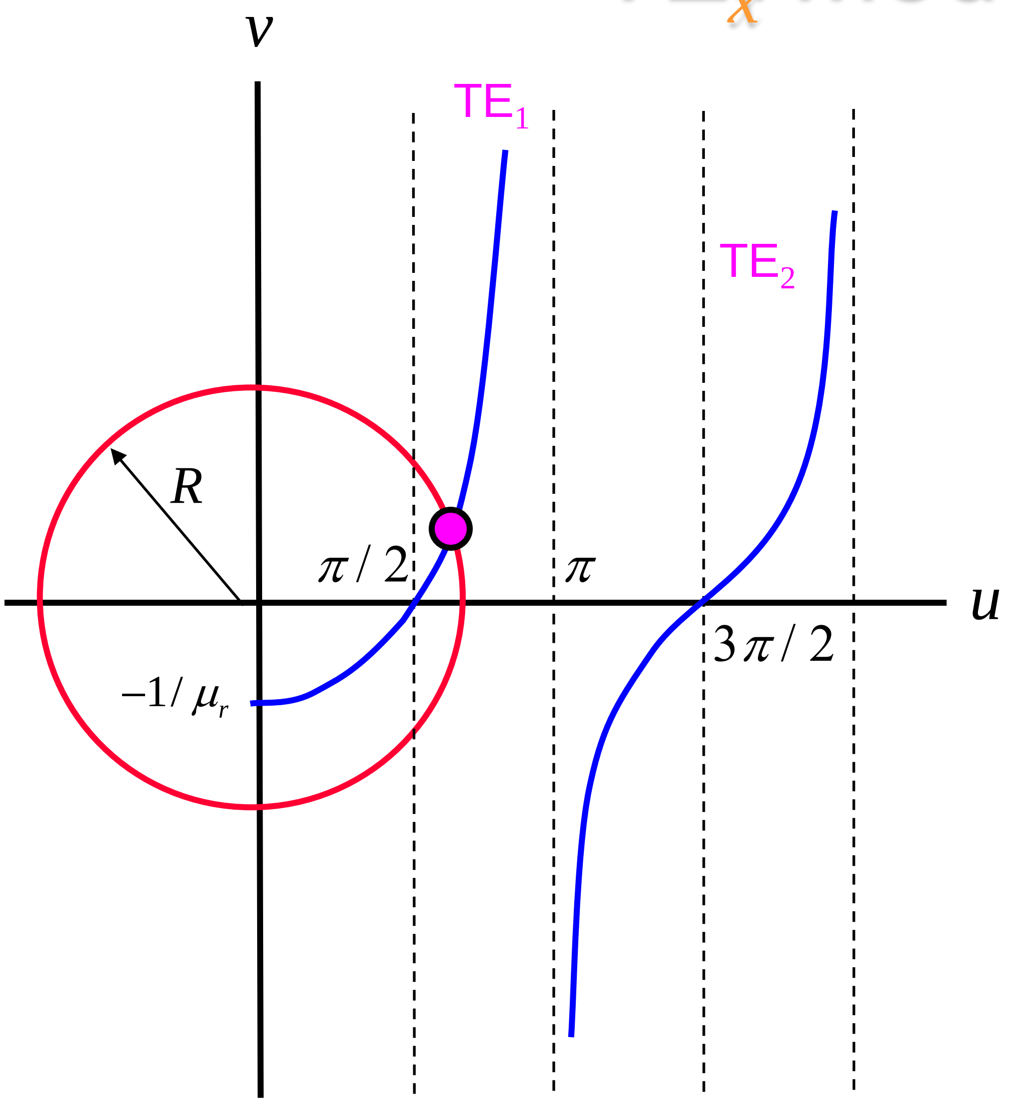
$$\Rightarrow \sqrt{\left(\frac{k_z}{k_0}\right)^2 - 1} \approx \frac{1}{\varepsilon_r} (k_0 h) \left(n_1^2 - \left(\frac{k_z}{k_0}\right)^2 \right)$$

$$\to 0$$

Dispersion Plot



TE_x Modes



$$\frac{1}{\mu_r} = -\left(\frac{\alpha_{x0}}{k_{x1}}\right)\tan(k_{x1}h)$$

or

$$\alpha_{x0}h = -\frac{1}{\mu_r}(k_{x1}h)\cot(k_{x1}h)$$

$$v = -\frac{1}{\mu_r}u\cot u$$

TE_x Modes (cont.)

No TE_0 mode ($f_c = 0$)

TE_1 cut-off frequency at ($R = \pi / 2$):

$$(k_0 h)\sqrt{n_1^2 - 1} = \frac{\pi}{2}$$

$$\frac{h}{\lambda_0} = \frac{1/4}{\sqrt{n_1^2 - 1}}$$

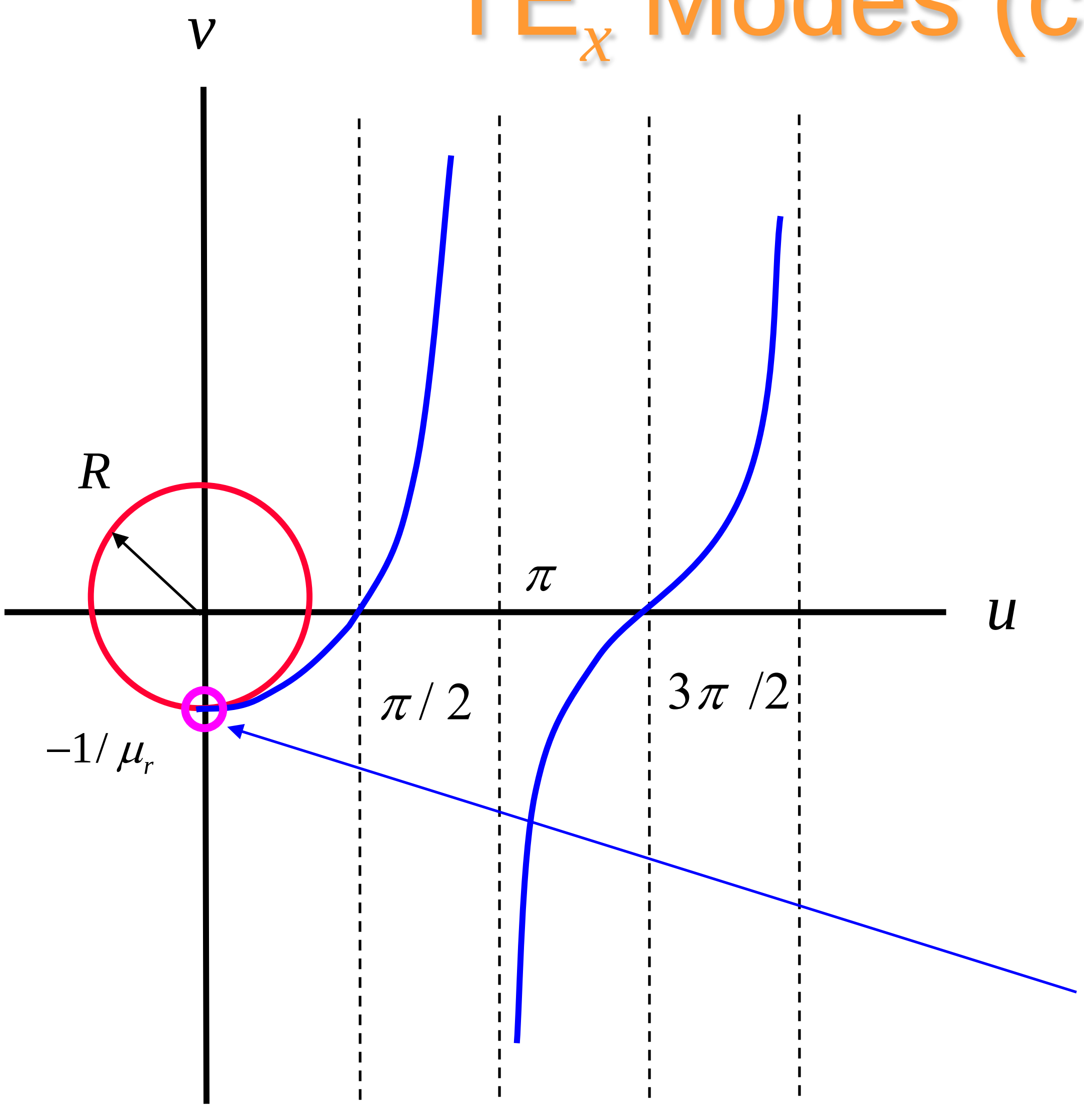
In general,

TE_n:

$$\frac{h}{\lambda_0} = \frac{(2n-1)/4}{\sqrt{n_1^2 - 1}}$$

$$n = 1, 2, 3, \ldots\ldots\ldots$$

TE_x Modes (cont.)



$$v = -\frac{1}{\mu_r} u \cot u$$

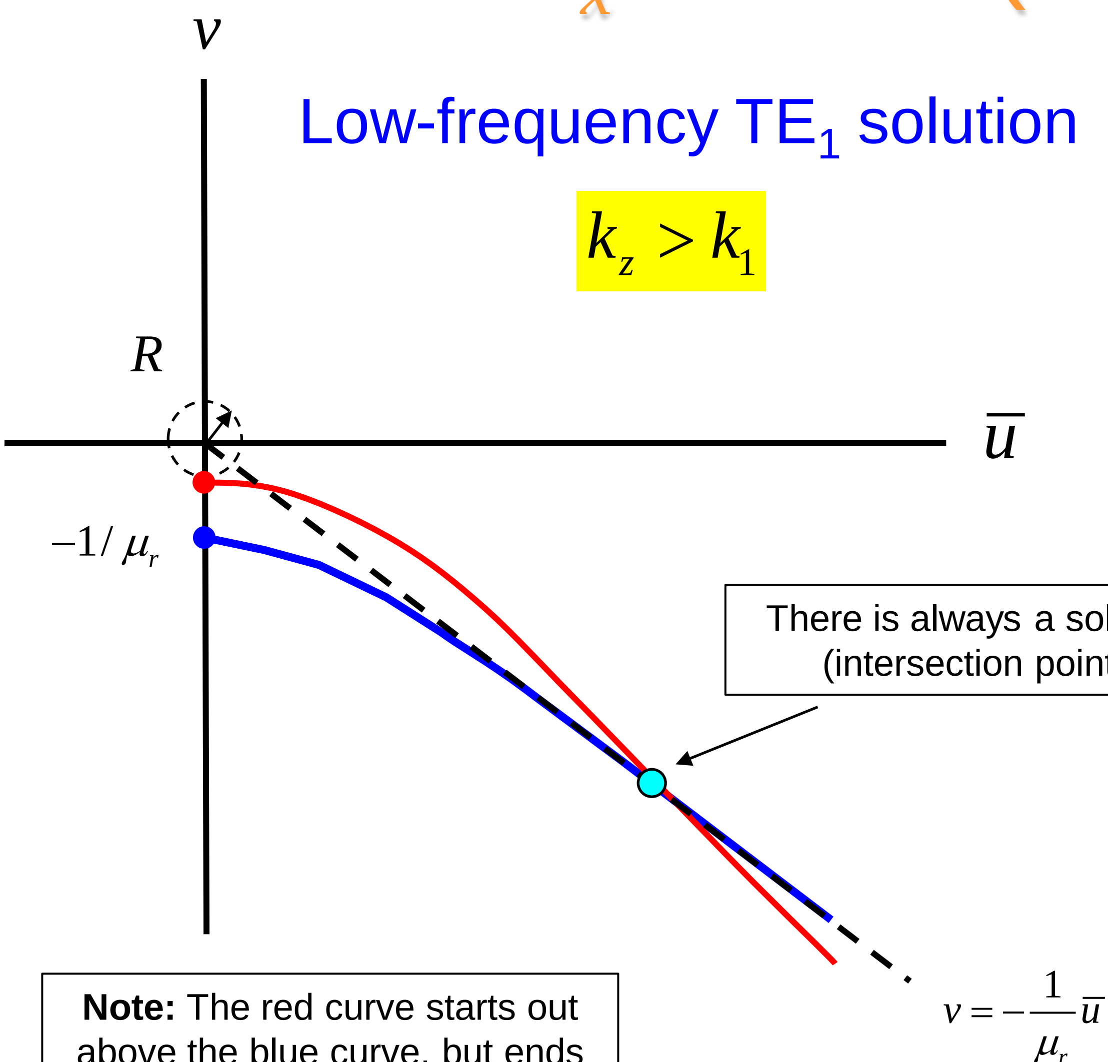
$$u = h\sqrt{k_1^2 - k_z^2}$$

$$v = h\sqrt{k_z^2 - k_0^2}$$

At this frequency, $u = 0$.
For lower frequencies,
u becomes imaginary ($k_z > k_1$).

If we wish to track the TE_1 ISW for lower frequencies, we need to re-formulate the graphical solution.

TE_x Modes (cont.)



Let

$$u = -jh\sqrt{k_z^2 - k_1^2} = -j\bar{u}$$

$$\bar{u} \equiv \sqrt{k_z^2 - k_1^2}$$

$$v = -\frac{1}{\mu_r} u \cot u$$

$$v = -\frac{1}{\mu_r} \bar{u} \coth \bar{u}$$

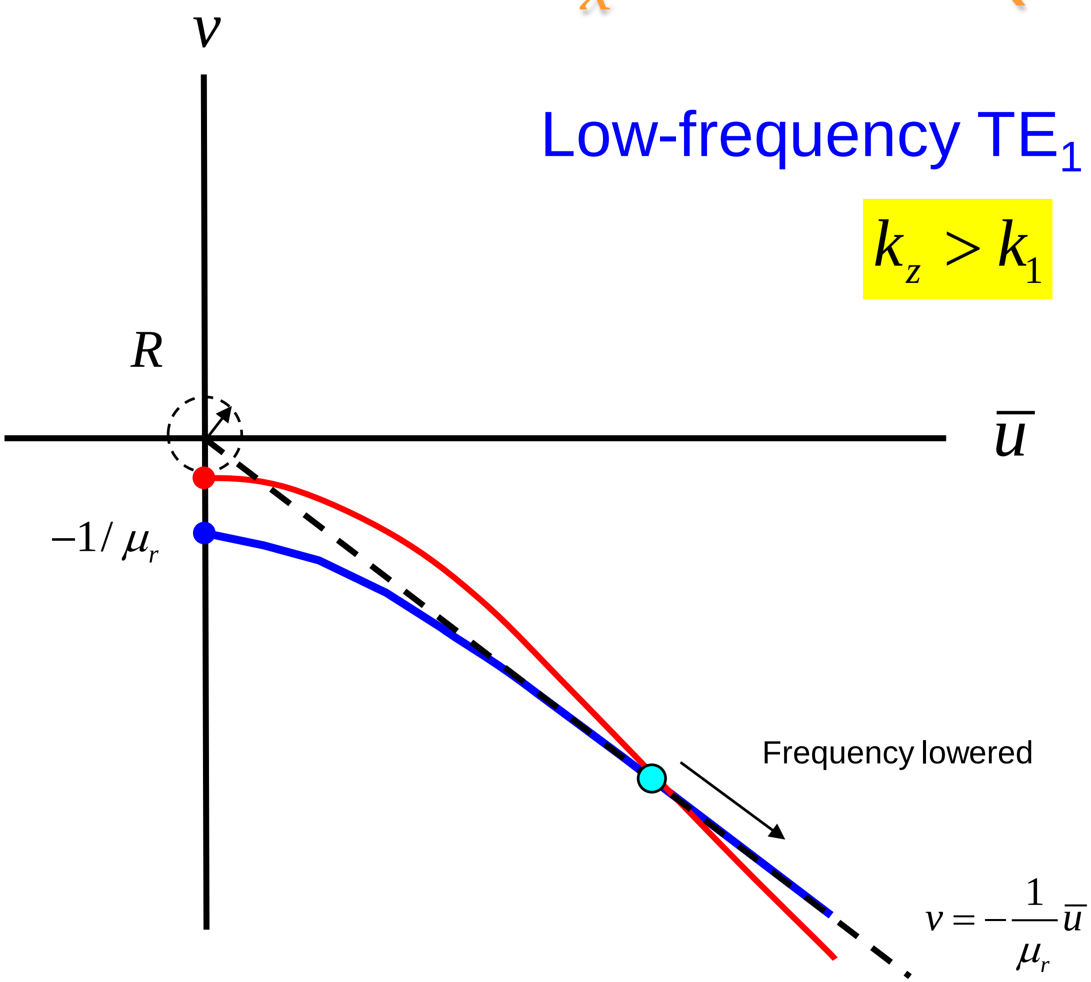
Note: The red curve starts out above the blue curve, but ends up below the blue curve.

$$u^2 + v^2 = R^2 \quad \Rightarrow \quad v = -\sqrt{R^2 + \bar{u}^2} \quad (v < 0)$$

TE_x Modes (cont.)

Low-frequency TE_1 solution

$$k_z > k_1$$



$$v = -\frac{1}{\mu_r}\bar{u}\coth\bar{u}$$

$$v = -\sqrt{R^2 + u^2}$$

As the frequency is lowered, the point of intersection moves further out, making the mode increase more rapidly in the air region.