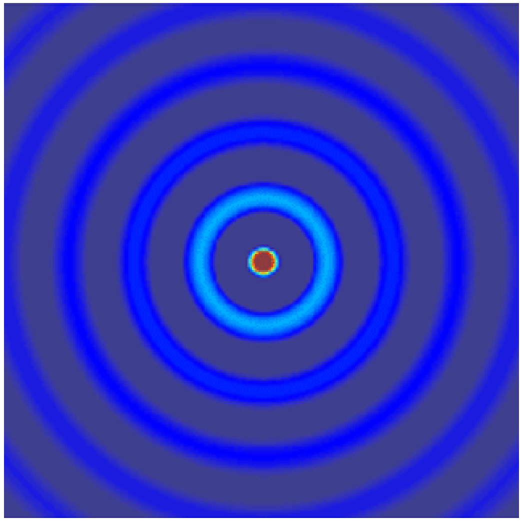
ECE 6341

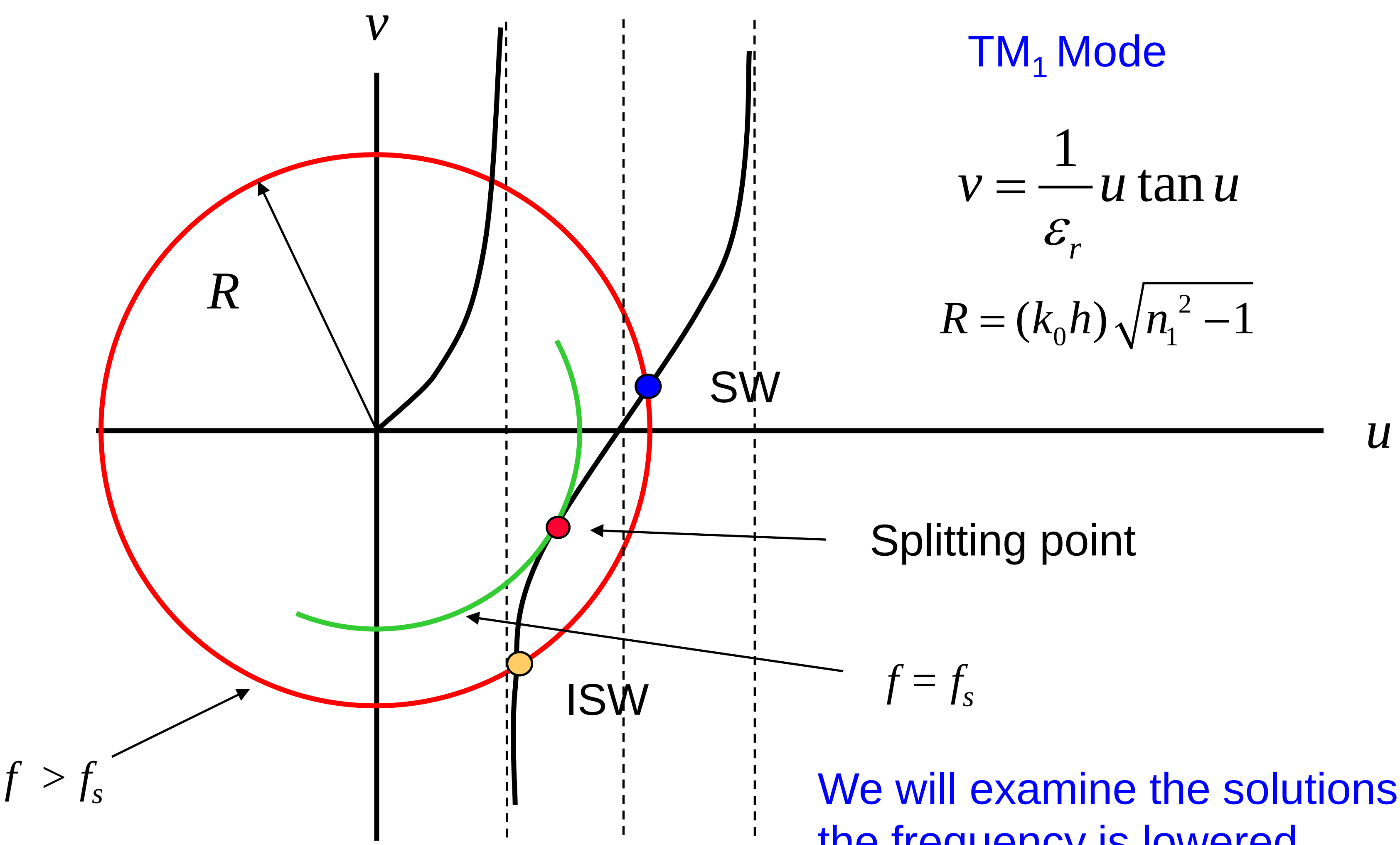
Spring 2016

Prof. David R. Jackson
ECE Dept.

Notes 6



Leaky Modes



TM$_1$ Mode

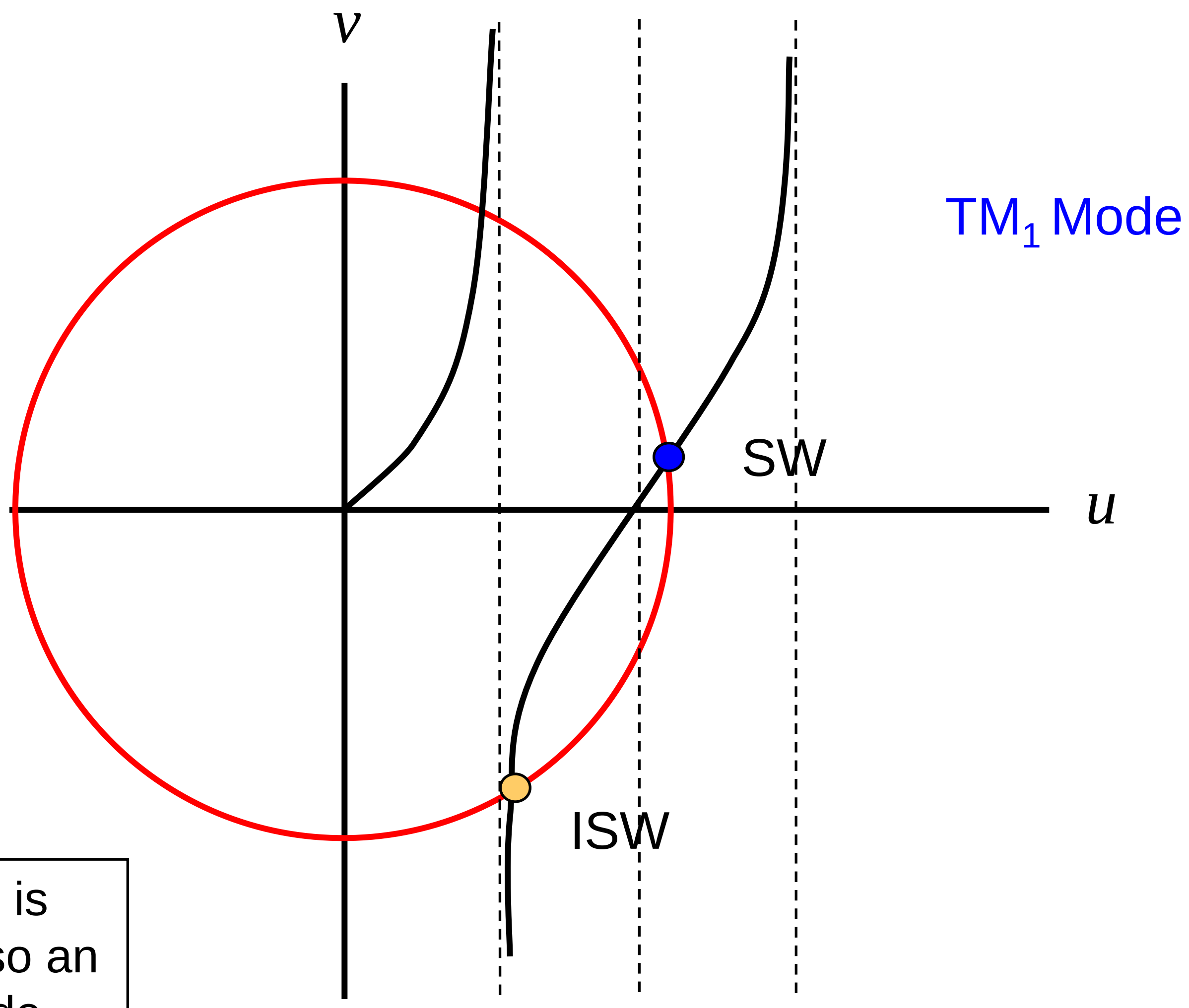
$$v = \frac{1}{\varepsilon_r} u \tan u$$

$$R = (k_0 h)\sqrt{n_1^2 - 1}$$

We will examine the solutions as the frequency is lowered.

Leaky Modes (cont.)

a) $f > f_c$ SW+ISW



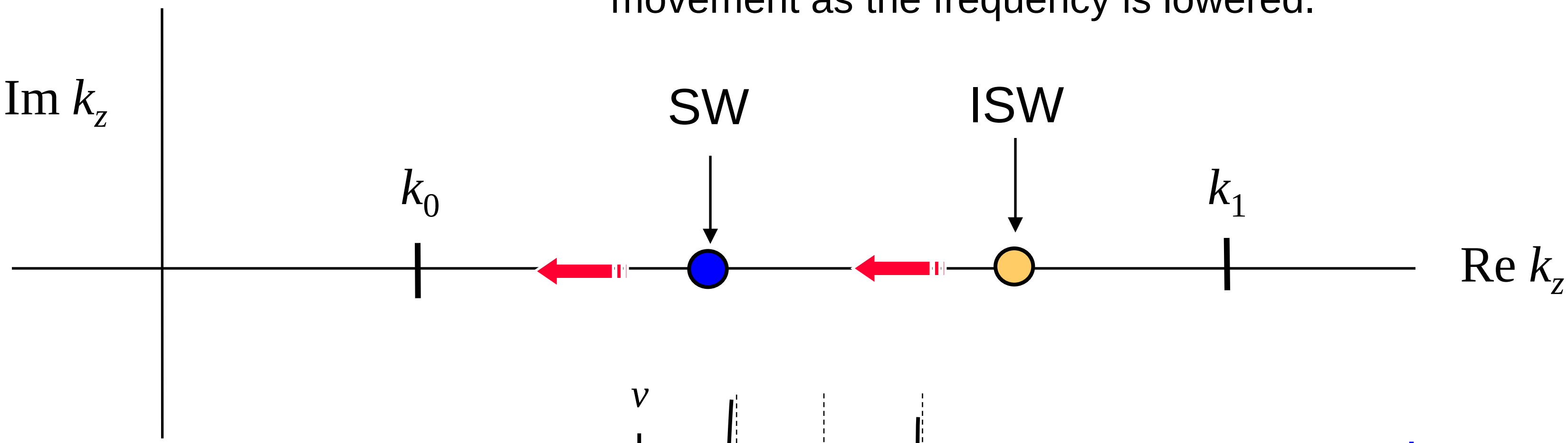
The TM_1 surface wave is above cutoff. There is also an improper TM_1 SW mode.

Note: There is also a TM_0 mode, but this is not shown

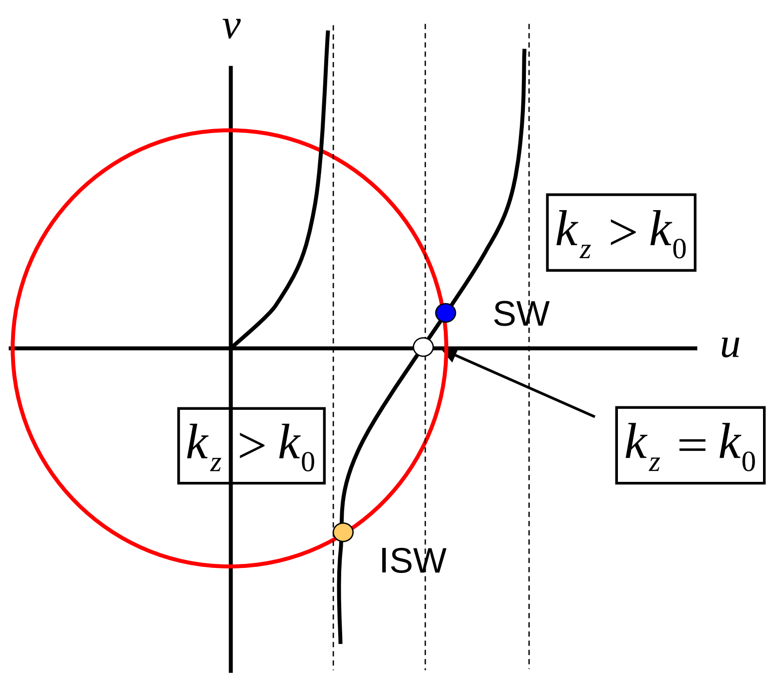
Leaky Modes (cont.)

a) $f > f_c$ SW+ISW

The red arrows indicate the direction of movement as the frequency is lowered.



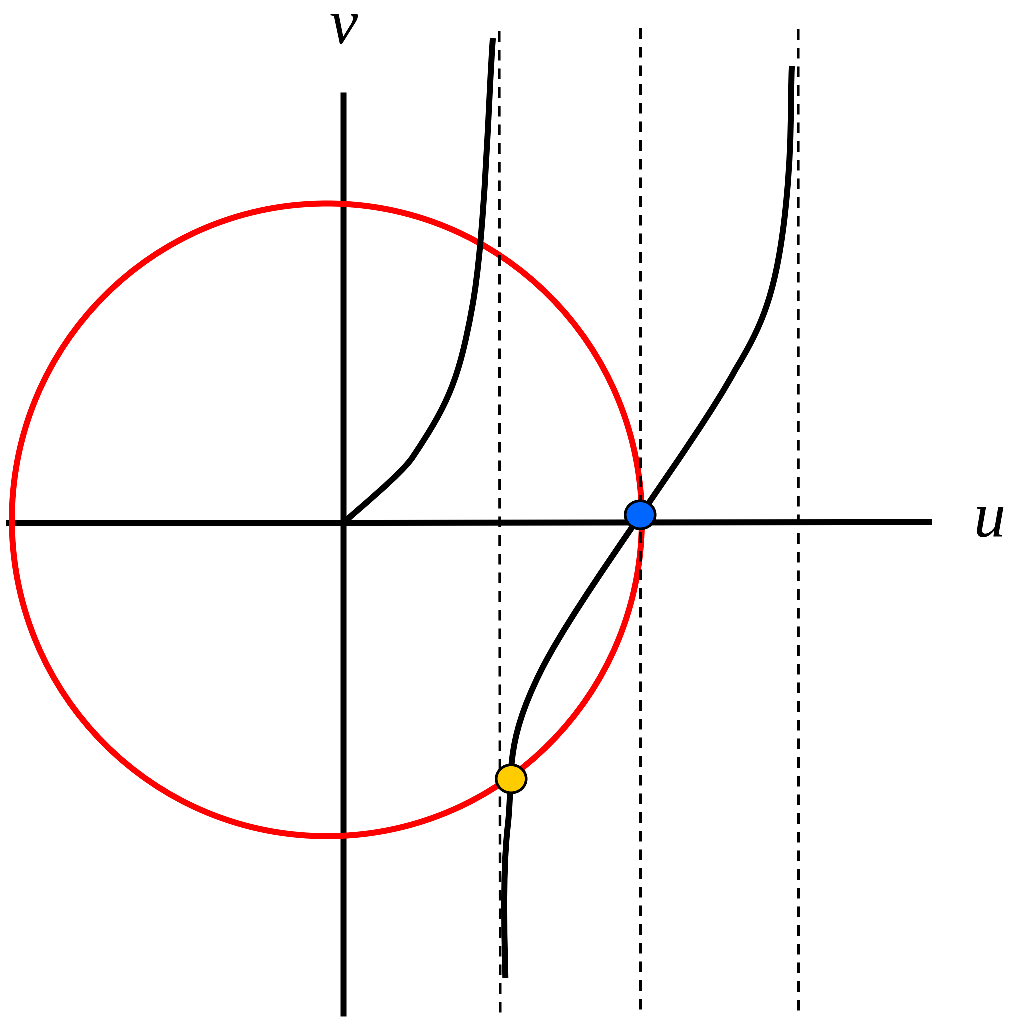
TM$_1$ Mode



$$v = h\left(k_z^2 - k_0^2\right)^{1/2}$$
$$= \pm h\sqrt{k_z^2 - k_0^2}$$

Leaky Modes (cont.)

b) $f = f_c$

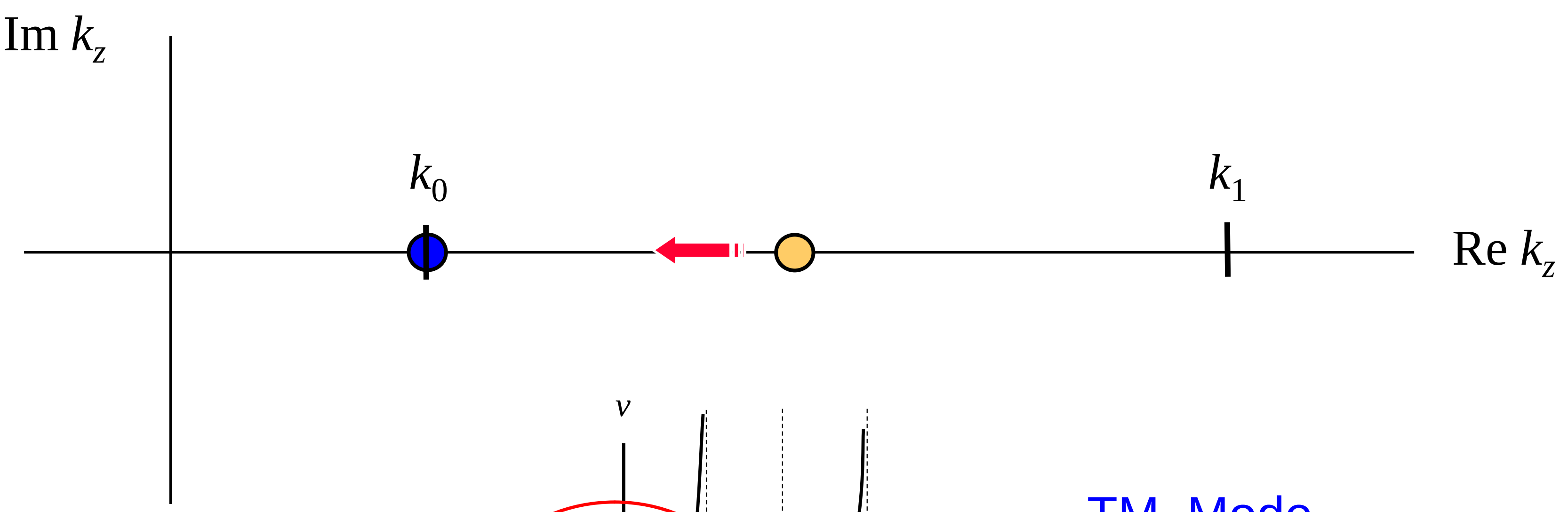


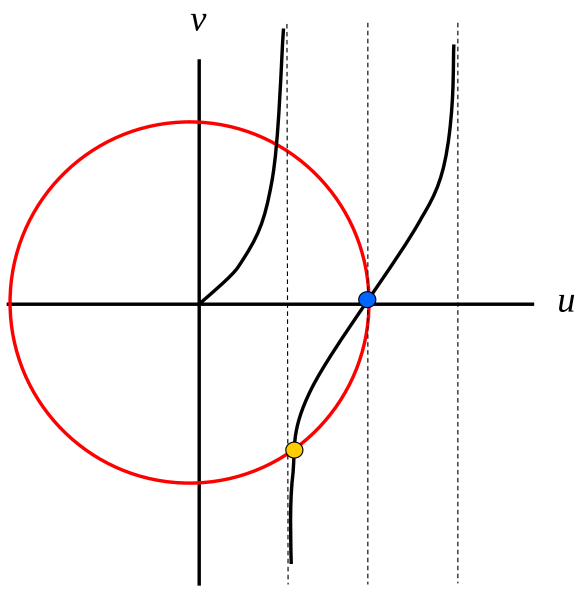
TM$_1$ Mode

The TM$_1$ surface wave is now at cutoff. There is also an improper SW mode.

Leaky Modes (cont.)

b) $f = f_c$

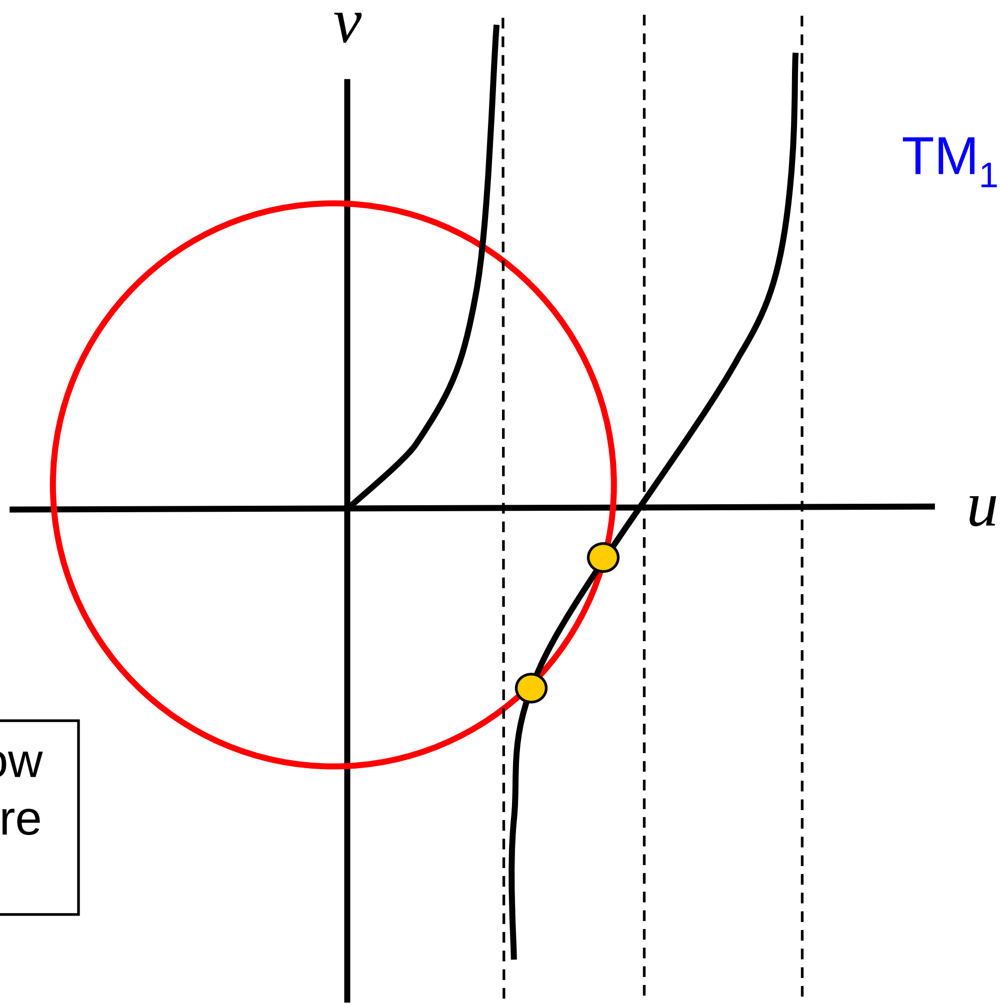




TM$_1$ Mode

Leaky Modes (cont.)

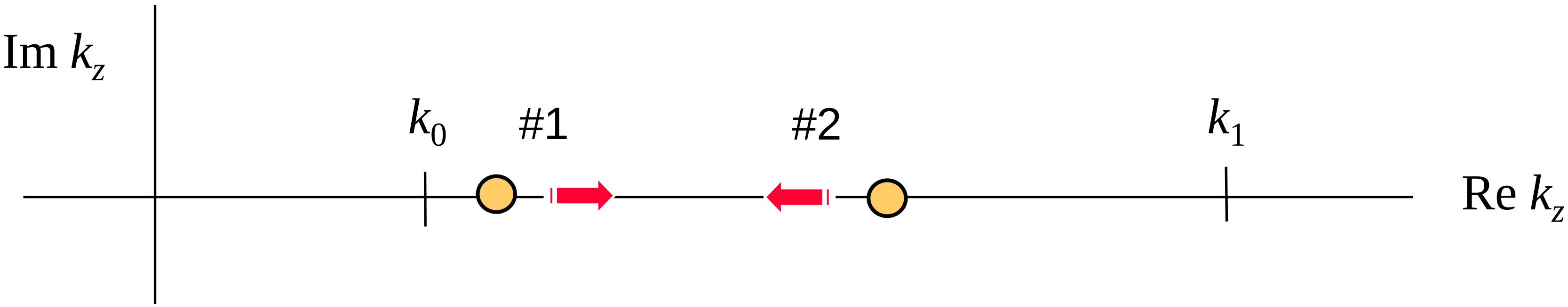
c) $f < f_c$ 2 ISWs

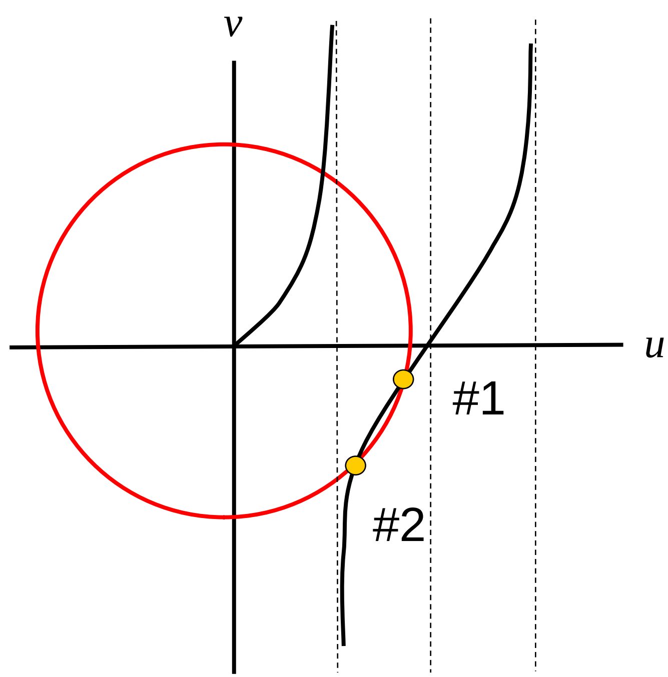


The TM_1 surface wave is now an improper SW, so there are two improper SW modes.

Leaky Modes (cont.)

c) $f < f_c$ 2 ISWs



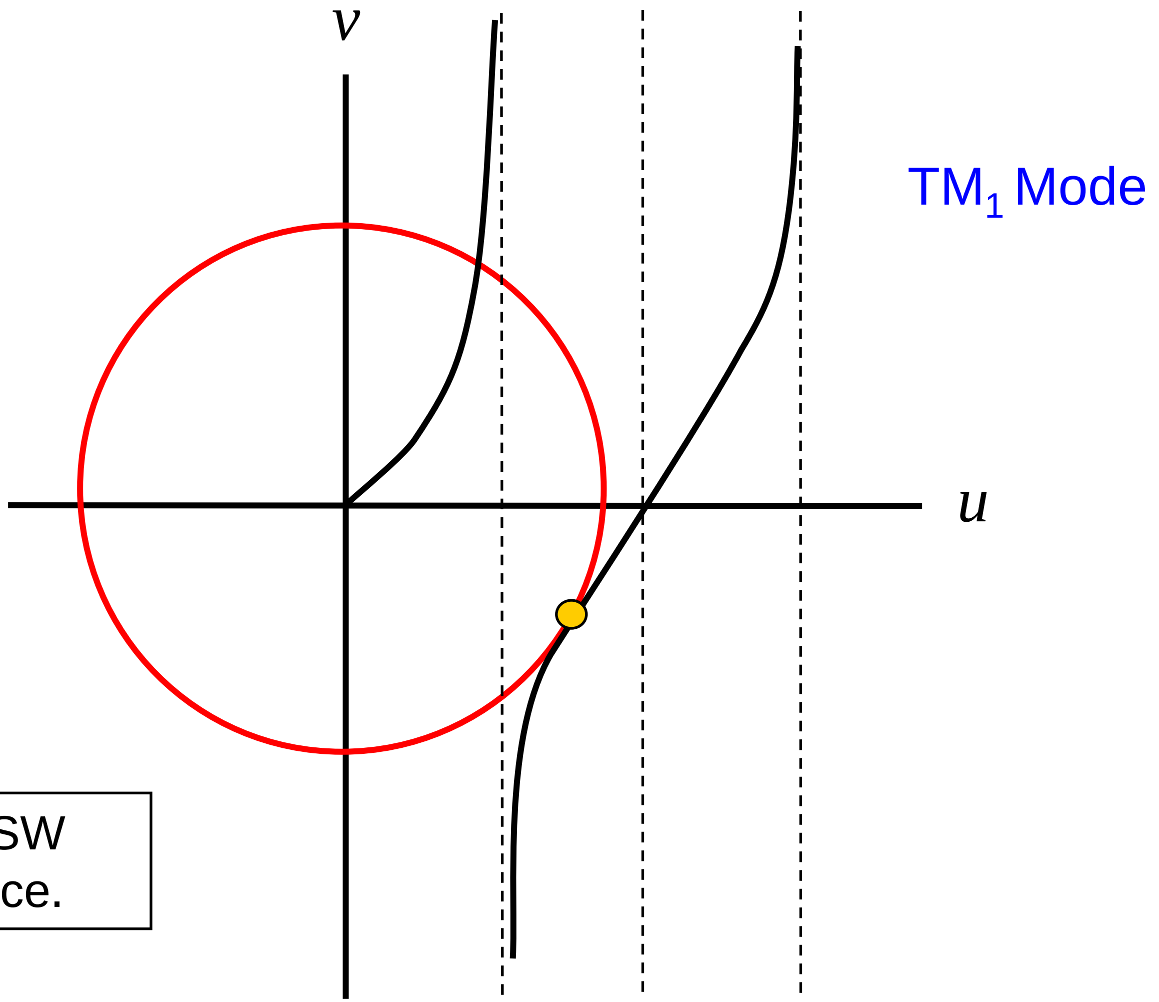


TM$_1$ Mode

The two improper SW modes approach each other.

Leaky Modes (cont.)

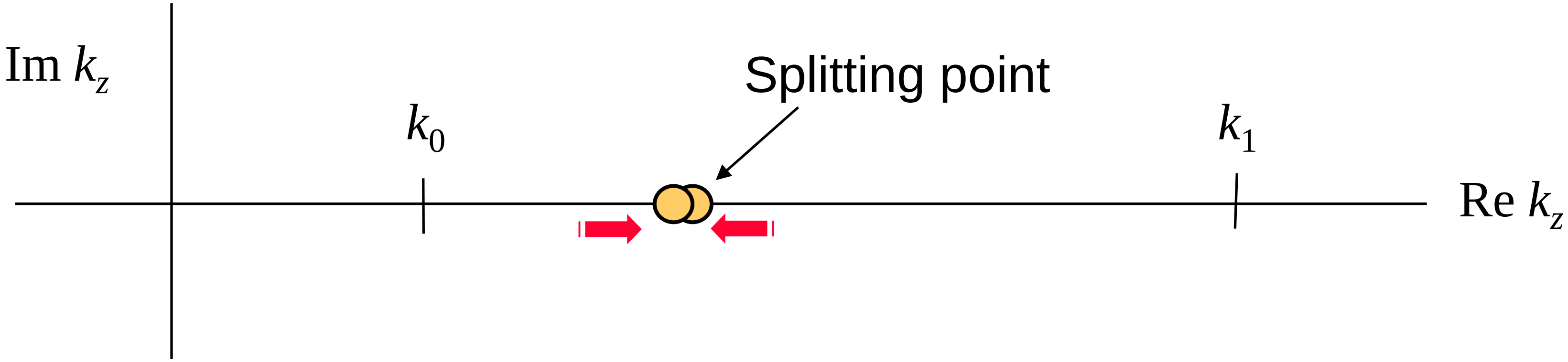
d) $f = f_s$

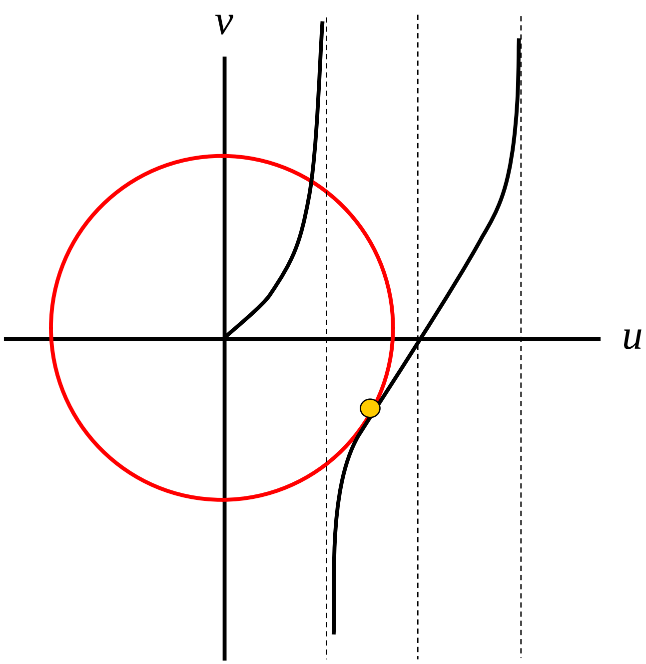


The two improper SW modes now coalesce.

Leaky Modes (cont.)

d) $f = f_s$

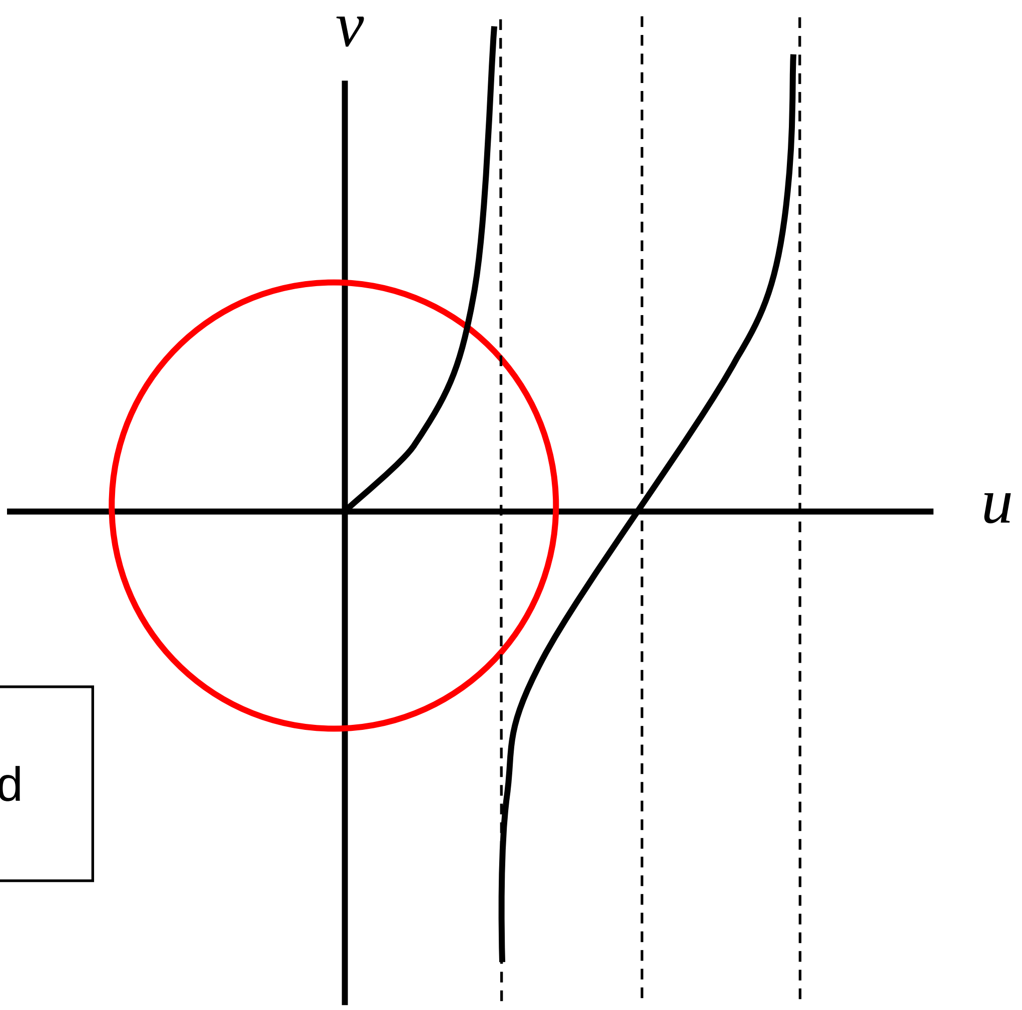




TM$_1$ Mode

Leaky Modes (cont.)

e) $f < f_s$



The wavenumber k_z becomes complex (and hence so do u and v).

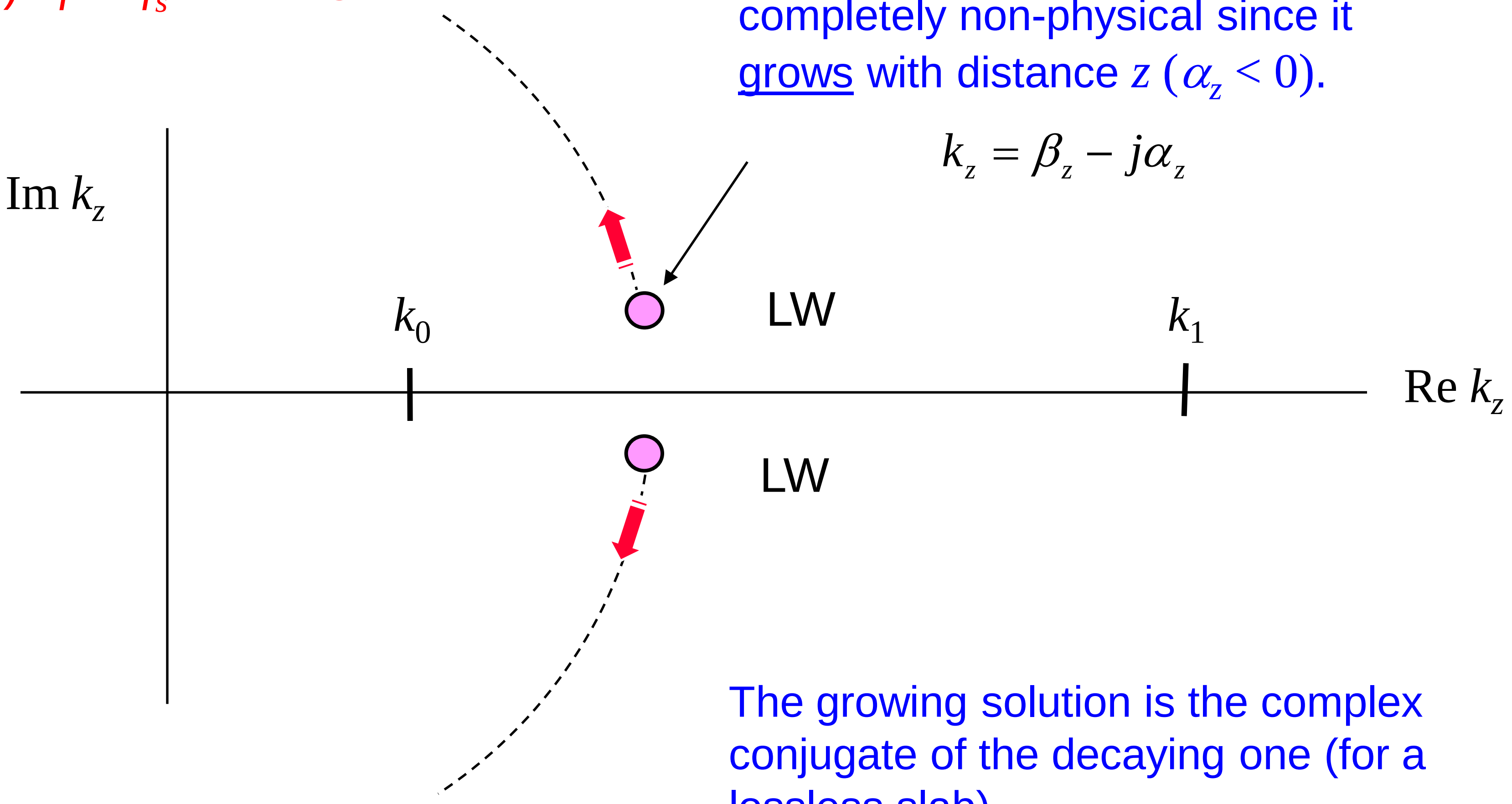
The graphical solution fails! (It cannot show us complex leaky-wave modal solutions.)

Leaky Modes (cont.)

e) $f < f_s$ 2 LWs

This solution is rejected as completely non-physical since it grows with distance z ($\alpha_z < 0$).

$$k_z = \beta_z - j\alpha_z$$



The growing solution is the complex conjugate of the decaying one (for a lossless slab).

Leaky Modes (cont.)

Proof of conjugate property (lossless slab)

TRE: $$\varepsilon_r = \frac{(k_1^2 - k_z^2)^{1/2}}{(k_z^2 - k_0^2)^{1/2}} \tan\left[\left((k_1^2 - k_z^2)^{1/2} h\right)\right]$$ TM_x Mode

Take conjugate of both sides:

$$\varepsilon_r = \frac{(k_1^2 - k_z^{*2})^{1/2}}{(k_z^{*2} - k_0^2)^{1/2}} \tan\left[\left((k_1^2 - k_z^{*2})^{1/2} h\right)\right]$$

Hence, the conjugate is a valid solution.

Leaky Modes (cont.)

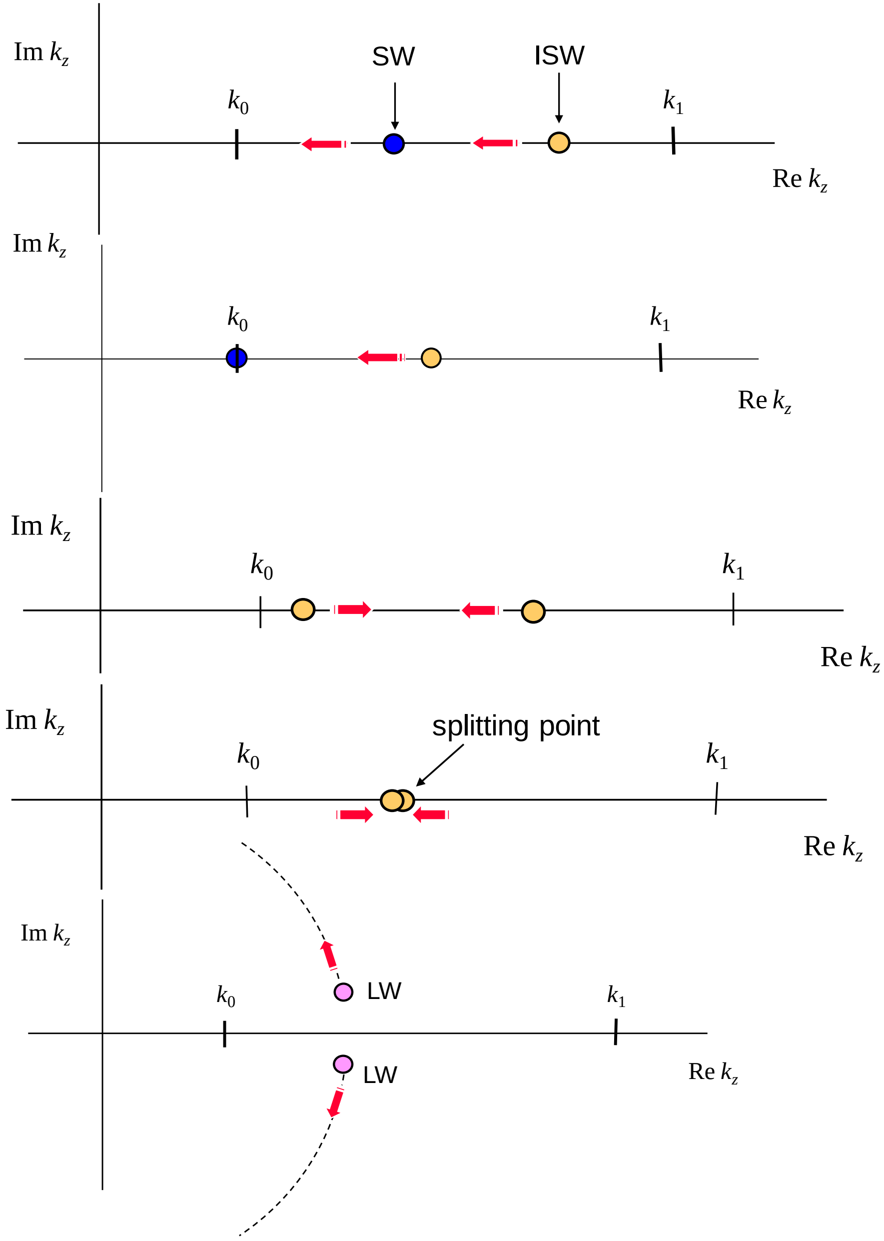
a) $f > f_c$

b) $f = f_c$

c) $f < f_c$

d) $f = f_s$

e) $f < f_s$



Here we see a summary of the frequency behavior for a typical surface-wave mode (e.g., TM_1).

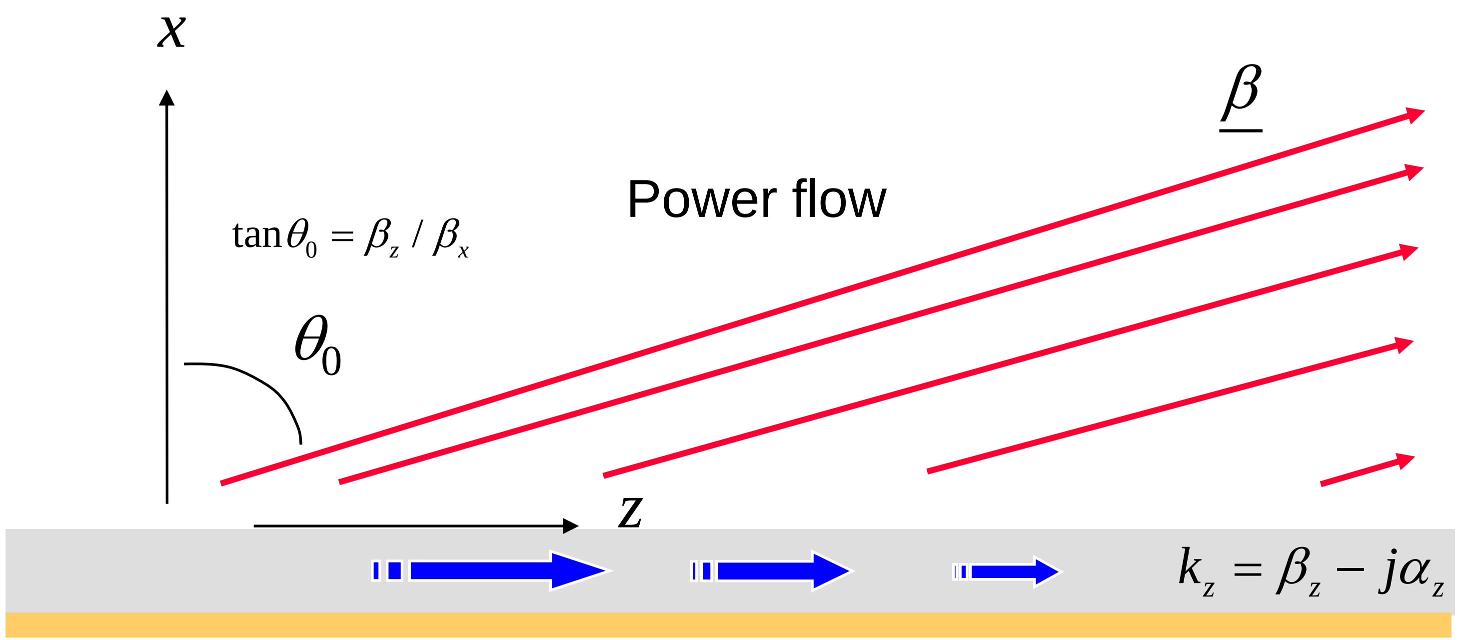
Exceptions:

- TM_0: Always remains a proper physical SW mode.
- TE_1: Goes from proper physical SW to nonphysical ISW; remains nonphysical ISW down to zero frequency.

Leaky Modes (cont.)

A leaky mode is a mode that has a complex wavenumber (even for a lossless structure). It loses energy as it propagates due to radiation.

$$\underline{\beta} = \mathrm{Re}\left(\underline{k}\right) = \mathrm{Re}\left(\underline{\hat{x}}k_x + \underline{\hat{z}}k_z\right) = \underline{\hat{x}}\beta_x + \underline{\hat{z}}\beta_z$$



Leaky Modes (cont.)

One interesting aspect: The fields of the leaky mode must be improper (exponentially increasing).

Proof:

$$k_{x0} = (k_0^2 - k_z^2)^{1/2}$$

Notes:
$\beta_z > 0$ (propagation in $+z$ direction)
$\alpha_z > 0$ (propagation in $+z$ direction)
$\beta_x > 0$ (outward radiation)

$$\Rightarrow \quad k_{x0}^2 = k_0^2 - k_z^2$$

$$\Rightarrow \quad (\beta_x - j\alpha_x)^2 = k_0^2 - (\beta_z - j\alpha_z)^2$$

$$\Rightarrow \quad \beta_x^2 - \alpha_x^2 - j2\beta_x\alpha_x = k_0^2 - \beta_z^2 + \alpha_z^2 + j2\alpha_z\beta_z$$

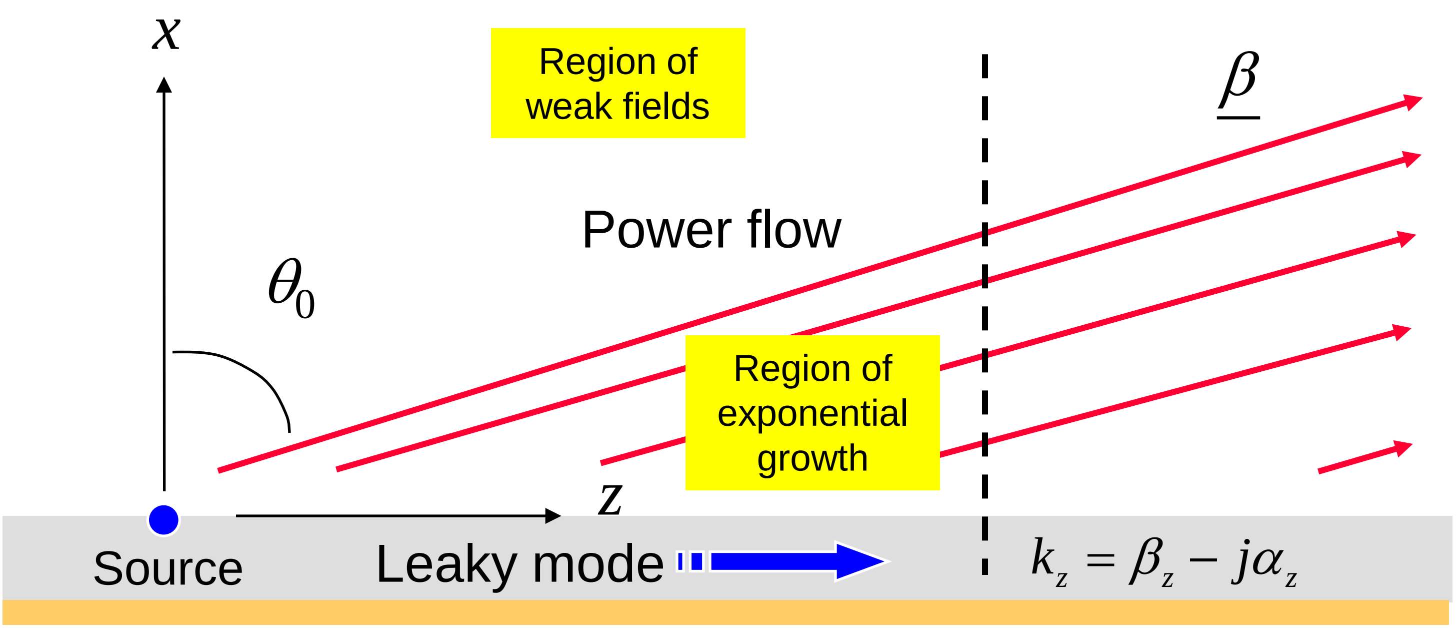
Taking the imaginary part of both sides: $\beta_x\alpha_x = -\alpha_z\beta_z$

Leaky Modes (cont.)

For a leaky wave excited by a source, the exponential growth will only persist out to a "shadow boundary" once a source is considered.

This is justified later in the course by an asymptotic analysis:

In the source problem, the LW pole is only captured when the observation point lies within the leakage region (region of exponential growth).



A hypothetical source launches a leaky wave going in one direction.

Leaky Modes (cont.)

A leaky-mode is considered to be "physical" if we can measure a significant contribution from it along the interface $(\theta_0 = 90^{\mathrm{o}})$.

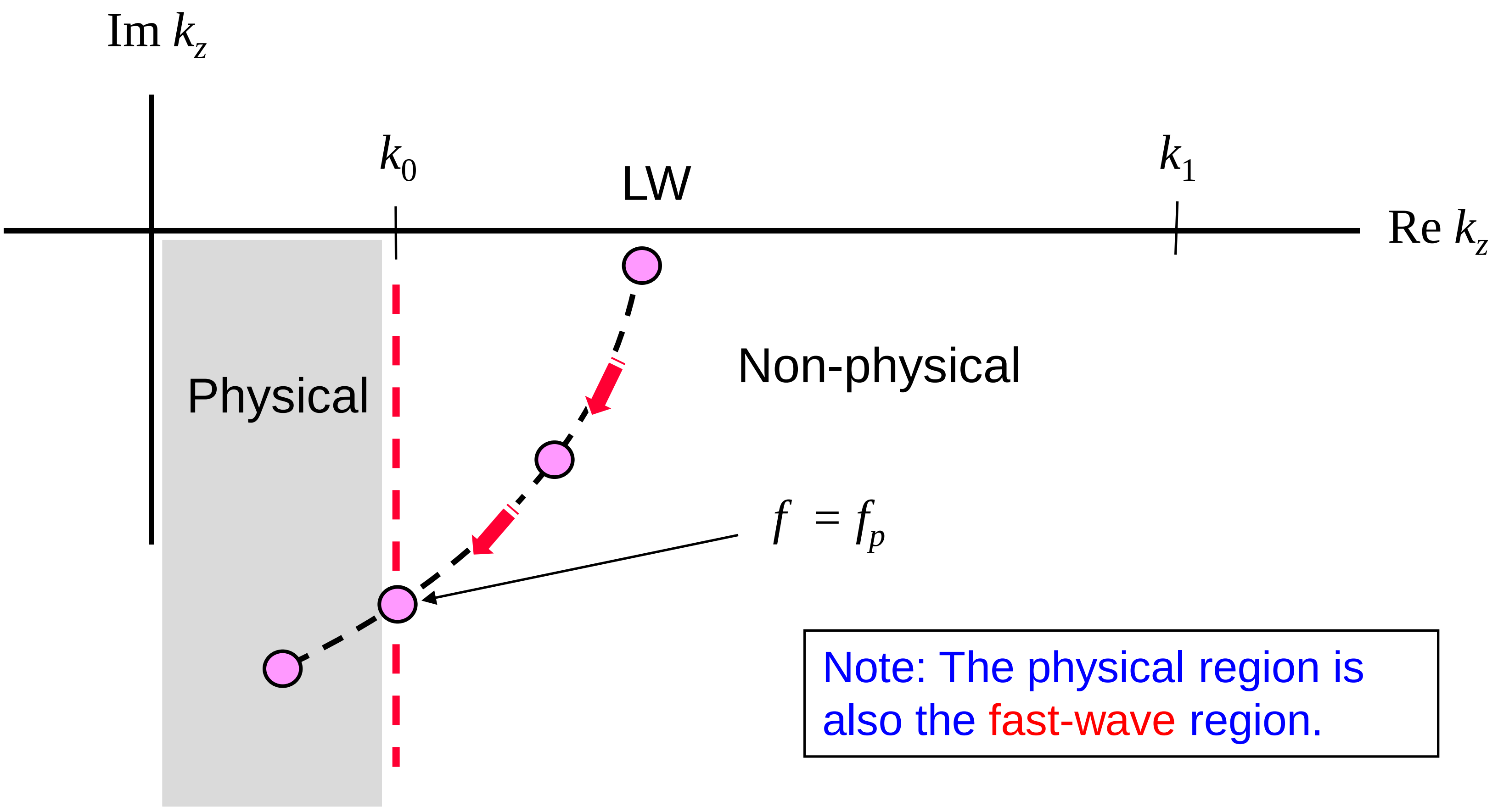
A requirement for a leaky mode to be strongly physical is that the wavenumber must lie within the "physical region" where is wave is a fast wave* $(\beta_z = \mathrm{Re}\, k_z < k_0)$.

Basic reason: The LW pole is not captured in the complex plane in the source problem if the LW is a slow wave.

* This is justified by asymptotic analysis, given later.

Leaky Modes (cont.)

f) $f < f_p$ Physical LW



Note: The physical region is also the fast-wave region.

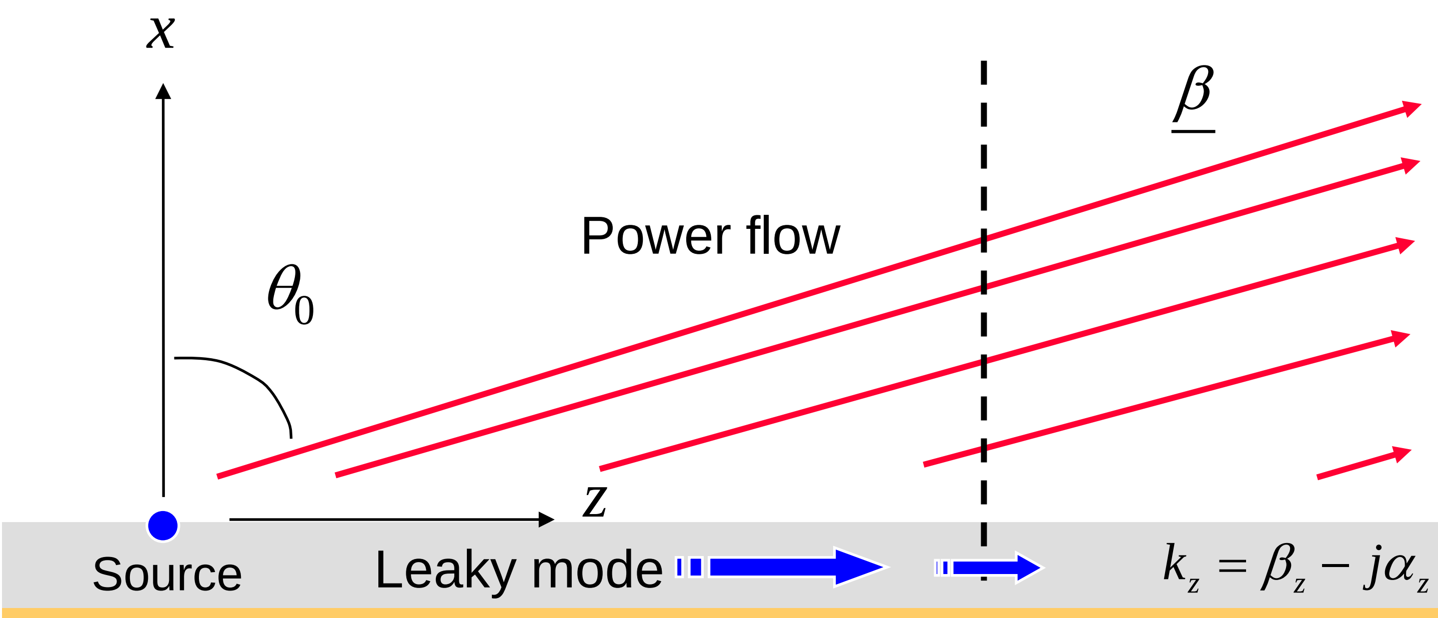
Physical leaky wave region ($\text{Re}\,k_z < k_0$)

Leaky Modes (cont.)

If the leaky mode is within the physical (fast-wave) region, a wedge-shaped radiation region will exist.

This is illustrated on the next two slides.

Leaky Modes (cont.)



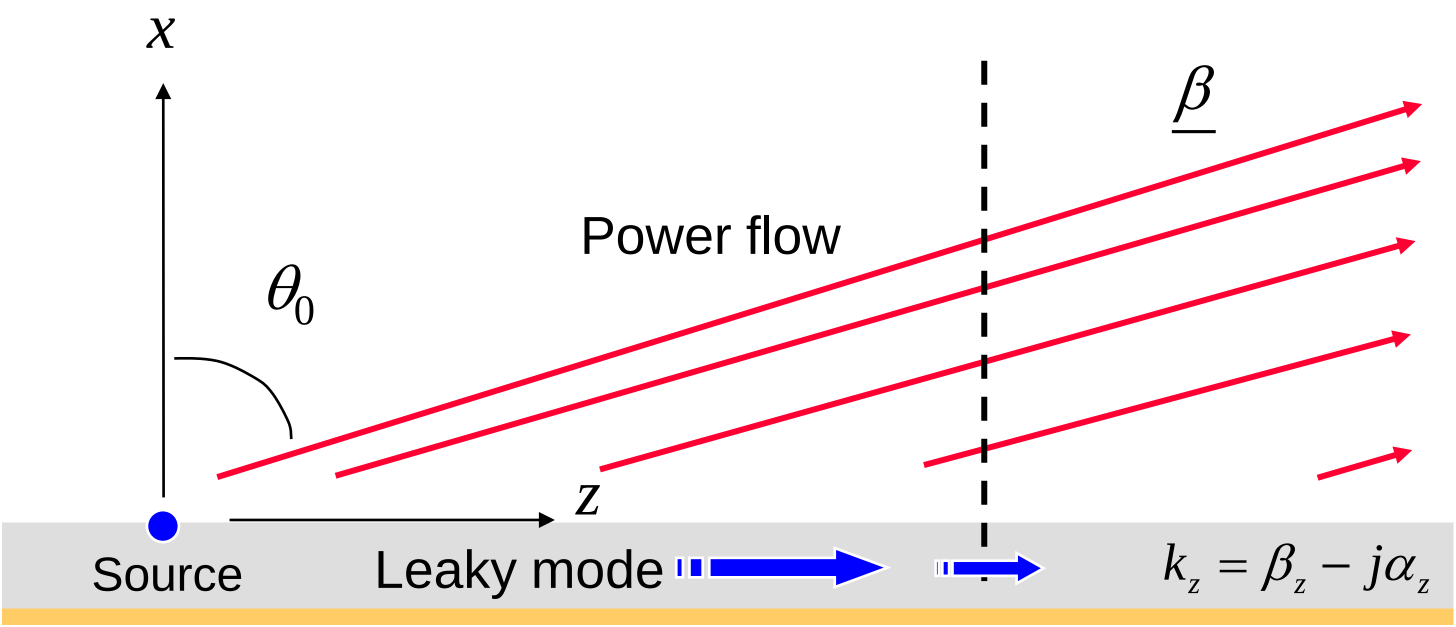
$$\underline{\beta} = \underline{\hat{x}}\beta_x + \underline{\hat{z}}\beta_z \qquad \beta_z = \left|\underline{\beta}\right| \sin\theta_0$$

$$\left|\underline{\beta}\right|^2 = \beta_x^2 + \beta_z^2 \approx k_x^2 + k_z^2 = k_0^2$$ (assuming small attenuation)

Hence $\beta_z \approx k_0 \sin\theta_0$

Significant radiation requires $\beta_z < k_0$.

Leaky Modes (cont.)

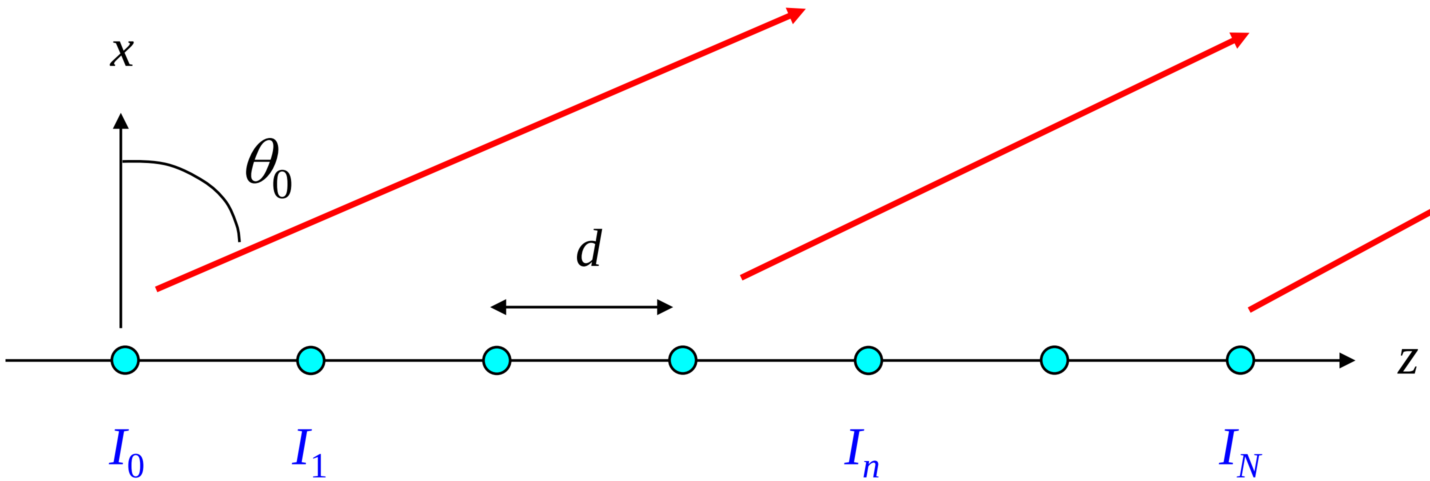


$$\theta_0 \approx \sin^{-1}\left(\beta_z / k_0\right)$$

As the mode approaches a slow wave ($\beta_z \to k_0$), the leakage region shrinks to zero ($\theta_0 \to 90^o$).

Leaky Modes (cont.)

Phased-array analogy



$$I_n = I_0\, e^{+j\phi_n} \qquad \phi_n = -\left(k_0 d \sin\theta_0\right) n$$

Equivalent phase constant:

$$e^{-jk_z z}\Big|_{z=nd} = e^{-j\left(k_0 d \sin\theta_0\right)n}$$

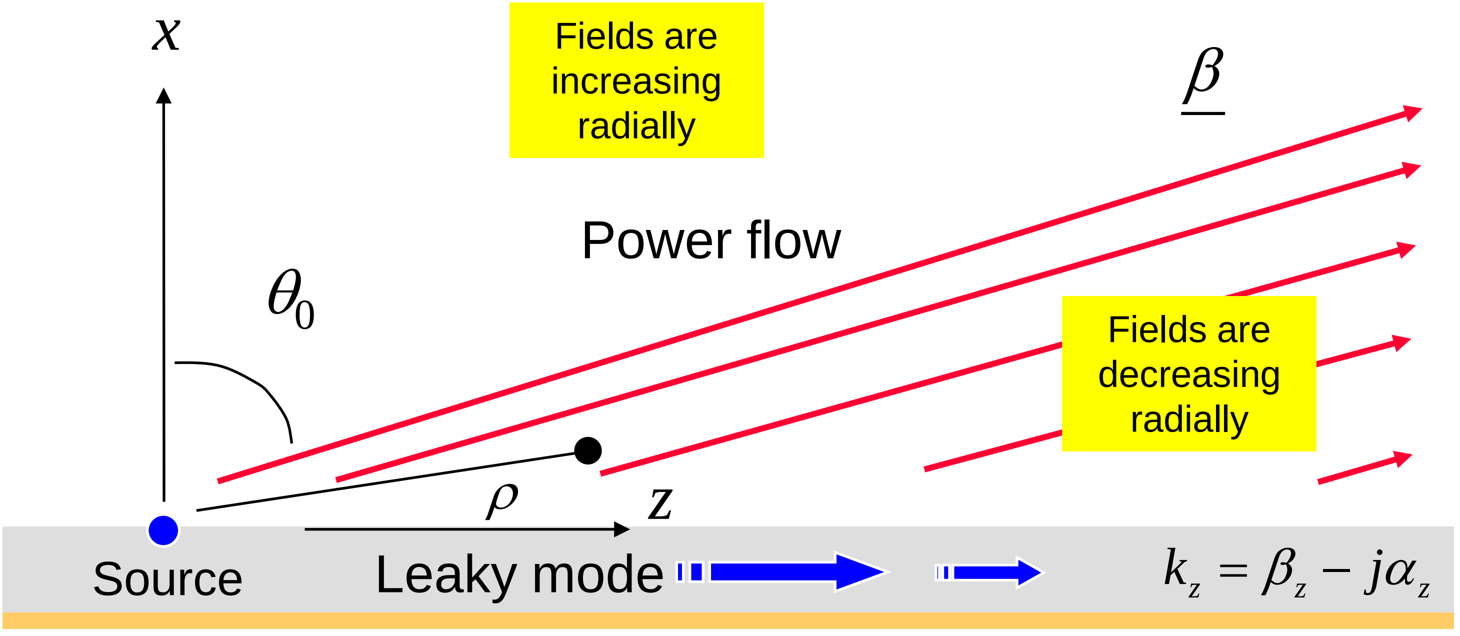
$$\Rightarrow \quad k_z = k_0 \sin\theta_0$$

Note:
A beam pointing at an angle in "visible space" requires that $k_z < k_0$.

Leaky Modes (cont.)

The angle θ_0 also forms the boundary between regions where the leaky-wave field <u>increases</u> and *decreases* with radial distance ρ in cylindrical coordinates (proof omitted*).

Recall: For a plane wave in a lossless region, the $\underline{\alpha}$ vector is perpendicular to the $\underline{\beta}$ vector.

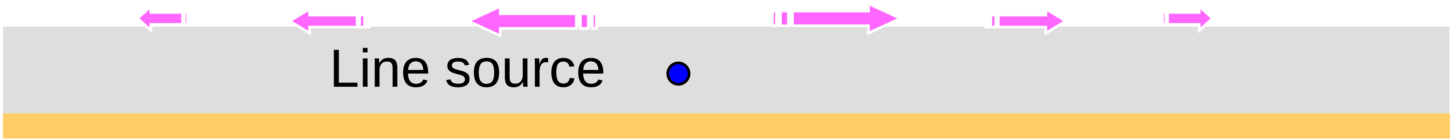


*Please see one of the homework problems.

Leaky Modes (cont.)

Excitation problem:

Bi-directional Leaky wave: $k_z = \beta_z - j\alpha_z$



The aperture field may strongly resemble the field of the leaky wave (creating a good leaky-wave antenna).

Requirements:

1) The LW should be in the physical region (i.e., a fast wave).
2) The amplitude of the LW should be strong.
3) The attenuation constant of the LW should be small.

A non-physical LW usually does not contribute significantly to the aperture field (this is seen from asymptotic theory, discussed later).

Leaky Modes (cont.)

Summary of frequency regions:

a) $f > f_c$ physical SW (non-radiating, proper)

b) $f_s < f < f_c$ non-physical ISW (non-radiating, improper)

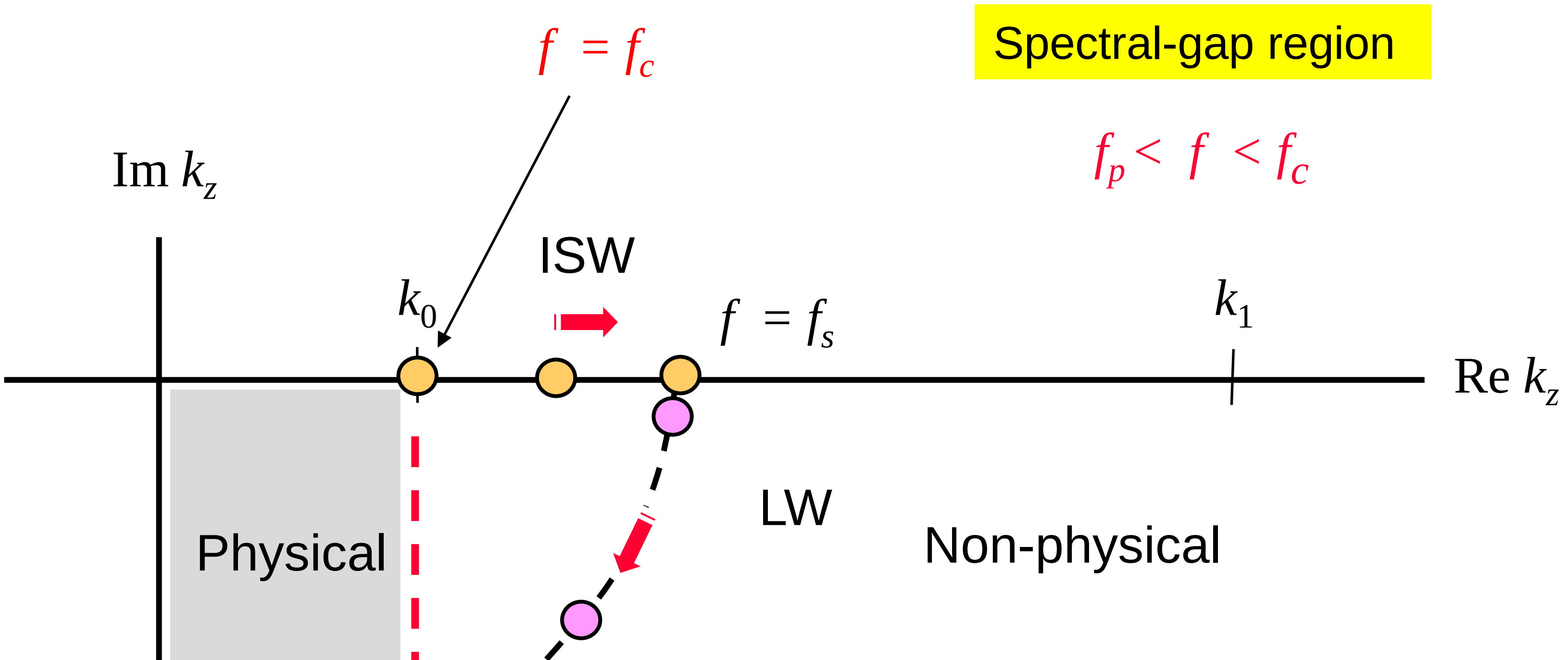
c) $f_p < f < f_s$ non-physical LW (radiating somewhat, improper)

d) $f < f_p$ physical LW (strong focused radiation, improper)

The frequency region $f_p < f < f_c$ is called the "spectral-gap" region (a term coined by Prof. A. A. Oliner).

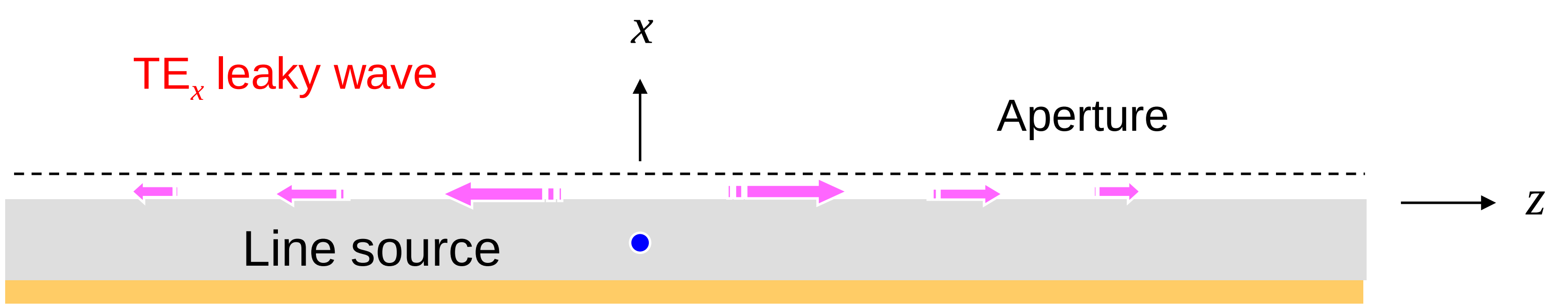
The LW mode is usually considered to be <u>nonphysical</u> in the <u>spectral-gap region</u>.

Leaky Modes (cont.)



Prof. A. A. Oliner

Field Radiated by Leaky Wave



For $x > 0$:

$$E_y(x,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{E}_y(0,k_z)\, e^{-jk_x x} e^{-jk_z z}\, dk_z,$$

$$k_x = \left(k_0^2 - k_z^2\right)^{1/2}$$

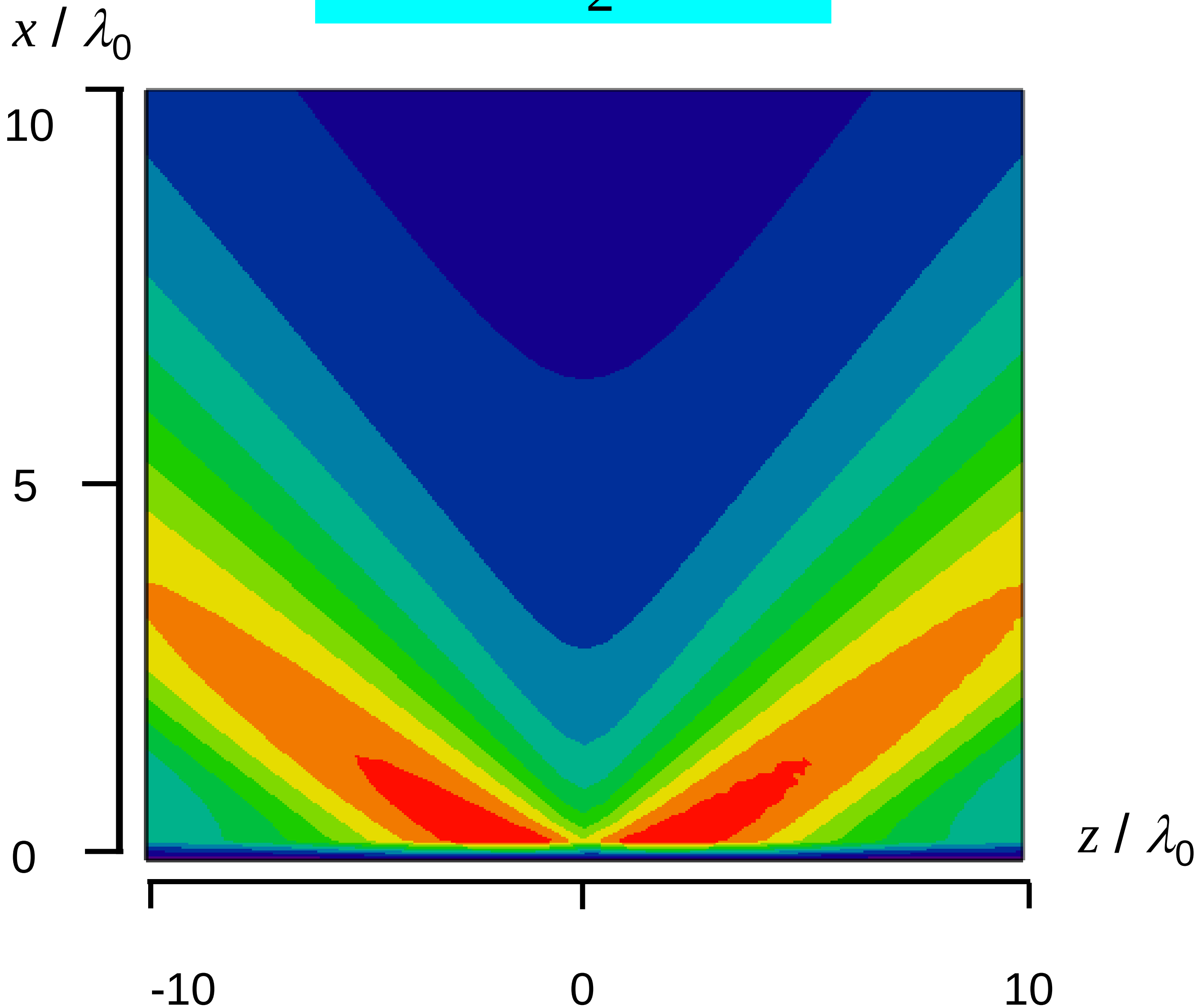
Note: The wavenumber k_x is chosen to be either positive real or negative imaginary.

Assume: $$E_y(0,z) = e^{-jk_z^{LW}|z|}$$

Then $$\tilde{E}_y(0,k_z) = 2j\left(\frac{k_z^{LW}}{k_z^2 - \left(k_z^{LW}\right)^2}\right)$$

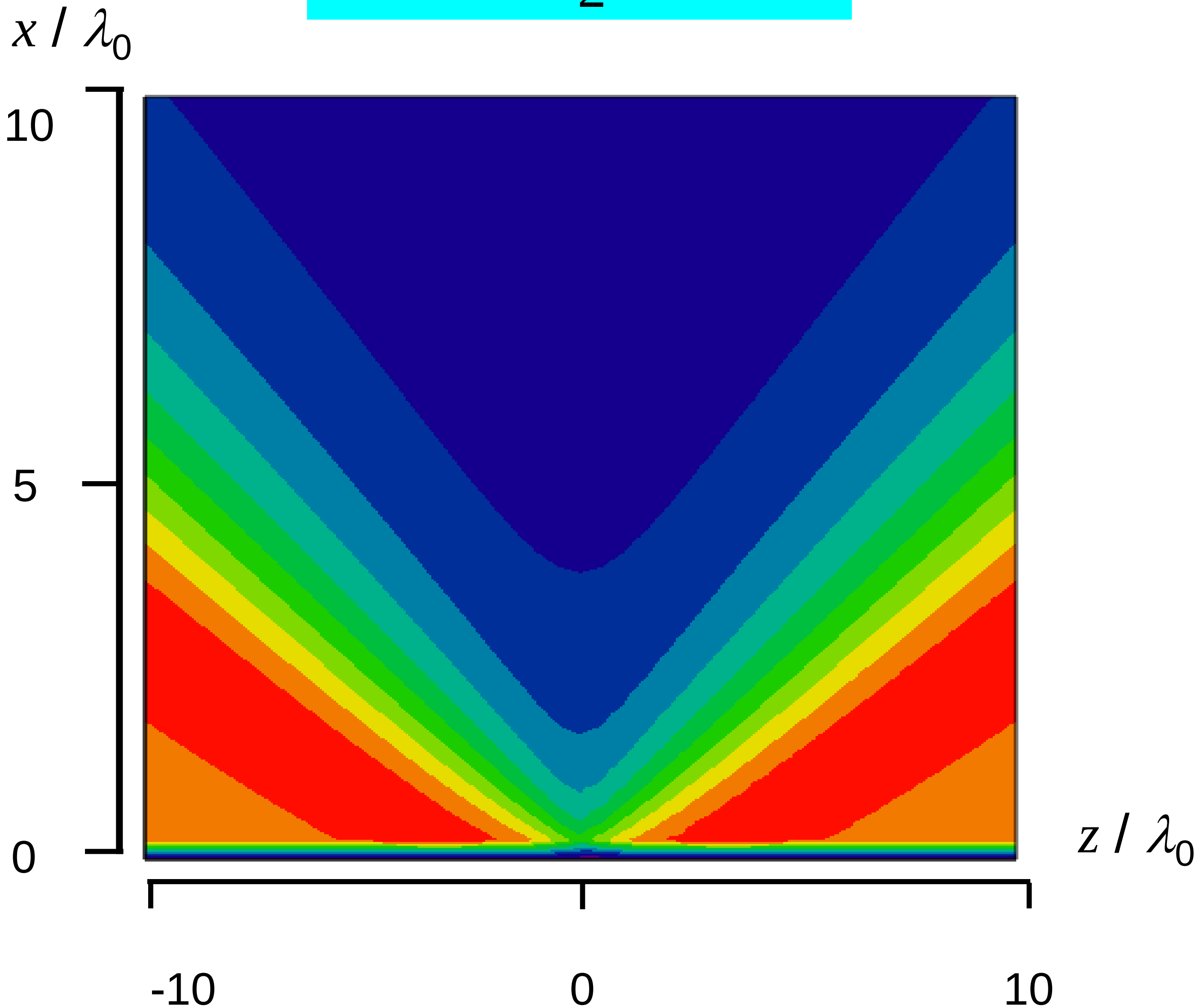
$$k_z^{LW} / k_0 = \frac{\sqrt{3}}{2} - j0.02$$

Radiation occurs at 60^o.



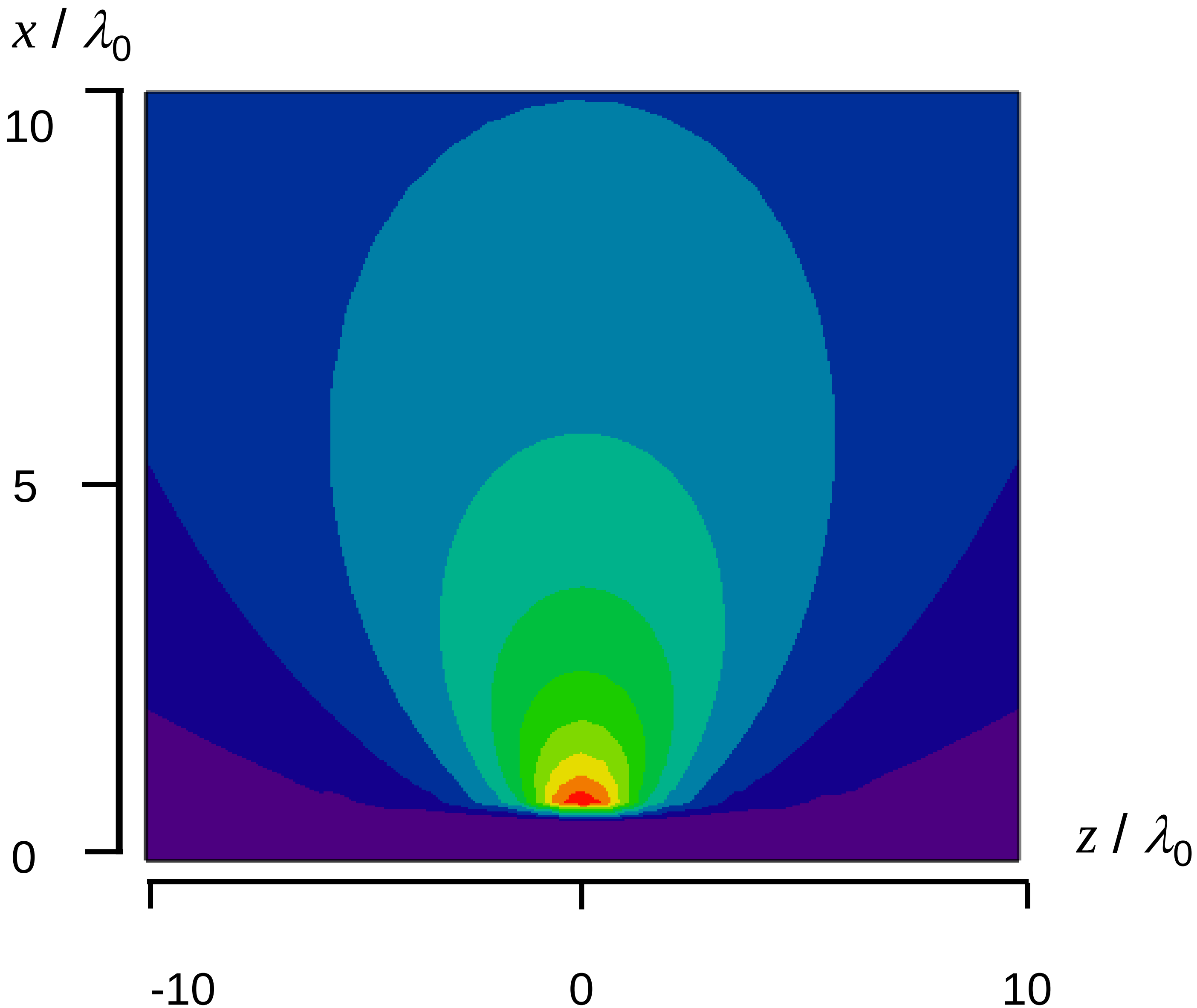
$$k_z^{LW} / k_0 = \frac{\sqrt{3}}{2} - j0.002$$

Radiation occurs at 60°.

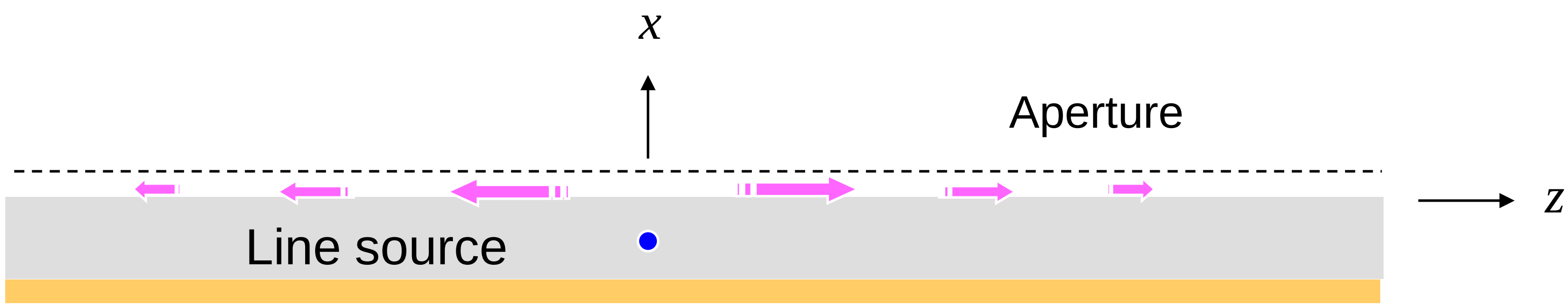


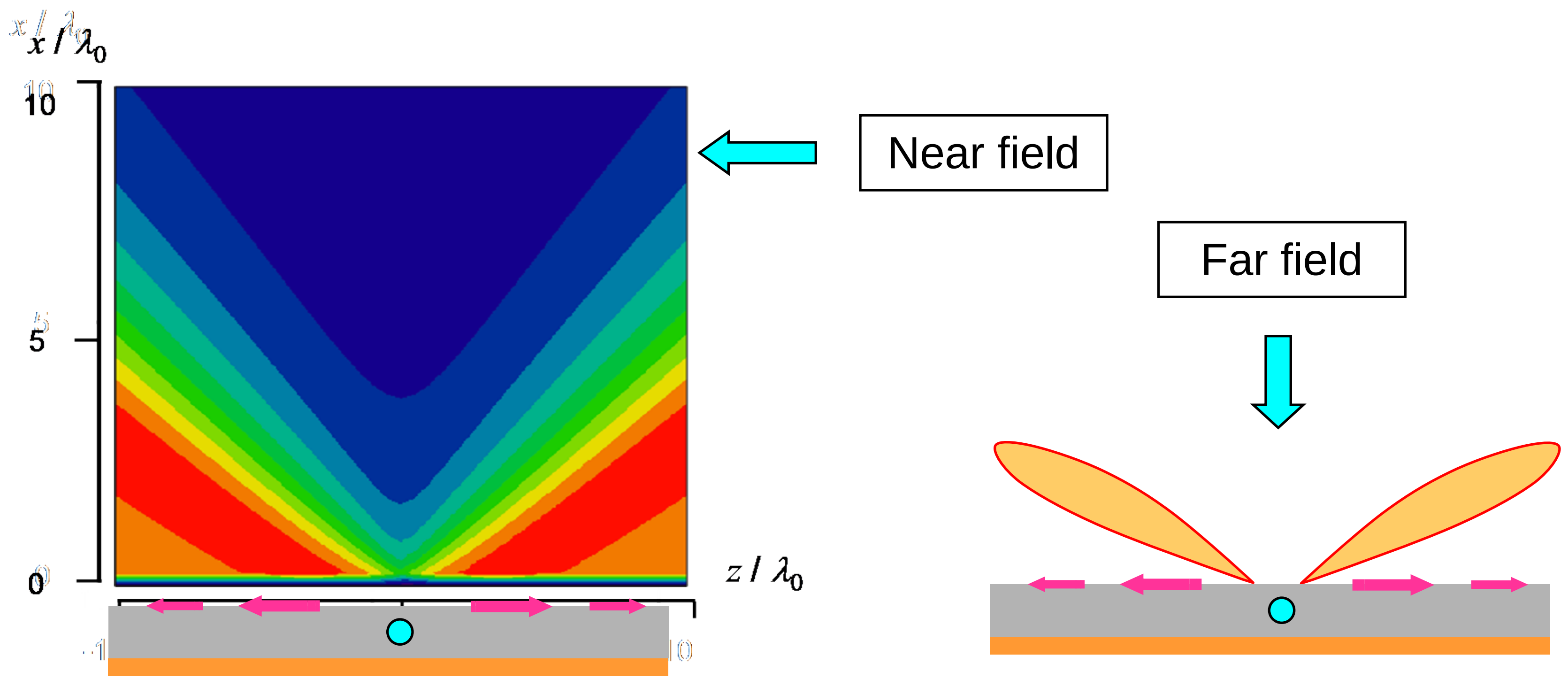
$$k_z^{LW} / k_0 = 1.5 - j0.02$$

The LW is nonphysical.

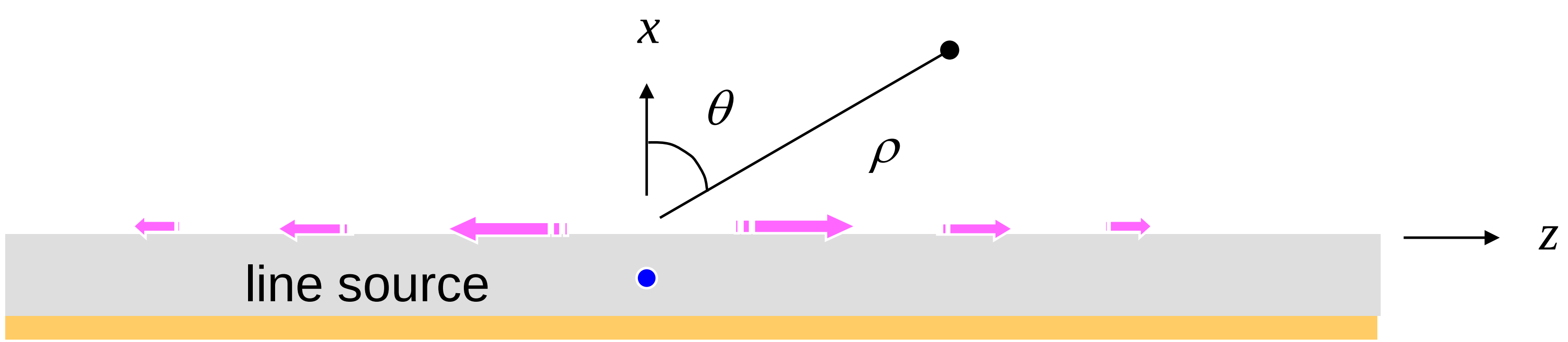


Leaky-Wave Antennas





Leaky-Wave Antennas (cont.)



Far-Field Array Factor (*AF*)

$$AF(\theta) = \int_{-\infty}^{\infty} E_y(0,z)\, e^{+j(k_0 \sin\theta) z}\, dz$$

$$= \int_{-\infty}^{\infty} e^{-jk_z^{LW}|z|}\, e^{+j(k_0 \sin\theta) z}\, dz$$

$$AF(\theta) = 2j\left(\frac{k_z^{LW}}{k_0^2 \sin^2\theta - \left(k_z^{LW}\right)^2}\right)$$

Leaky-Wave Antennas (cont.)

$$AF(\theta) = 2j\left(\frac{\beta_z - j\alpha_z}{k_0^2 \sin^2\theta - (\beta_z - j\alpha_z)^2}\right)$$

$$\Downarrow$$

$$|AF(\theta)| = 2\left(\frac{|\beta_z - j\alpha_z|}{|k_0^2 \sin^2\theta - \beta_z^2 + \alpha_z^2 + j2\alpha_z\beta_z|}\right)$$

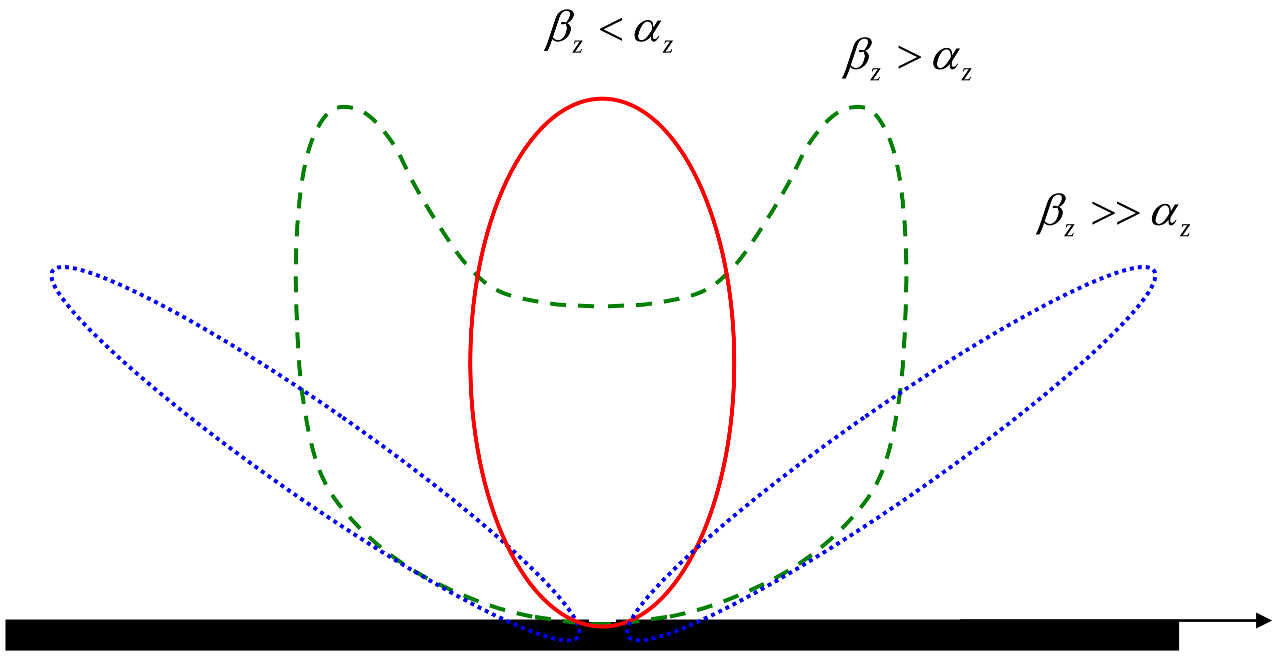
$$\Downarrow$$

$$|AF(\theta)| = 2\left(\frac{\beta_z^2 + \alpha_z^2}{(k_0^2 \sin^2\theta - \beta_z^2 + \alpha_z^2)^2 + (2\alpha_z\beta_z)^2}\right)^{1/2}$$

A sharp beam occurs at $k_0 \sin\theta_0 \approx \beta_z$

Leaky-Wave Antennas (cont.)

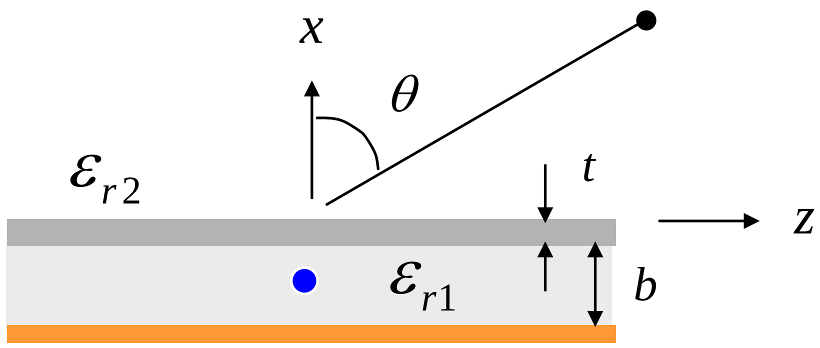
The two beams merge to becomes a broadside beam when $\beta_z < \alpha_z$



$$AF(\theta) = 2j\left(\frac{\beta_z - j\alpha_z}{k_0^2 \sin^2\theta - (\beta_z - j\alpha_z)^2}\right)$$

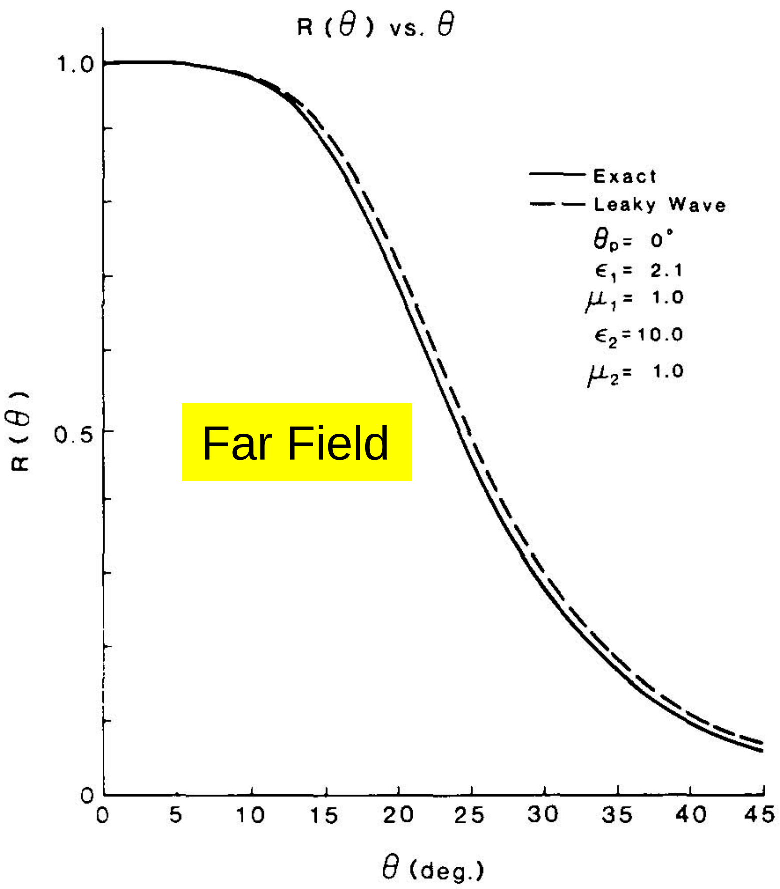
Leaky-Wave Antennas (cont.)

Two-layer (substrate/superstrate) structure excited by a line source.



$$\varepsilon_{r2} >> \varepsilon_{r1}$$

$$b / \lambda_1 = 0.5$$
$$t / \lambda_2 = 0.25$$



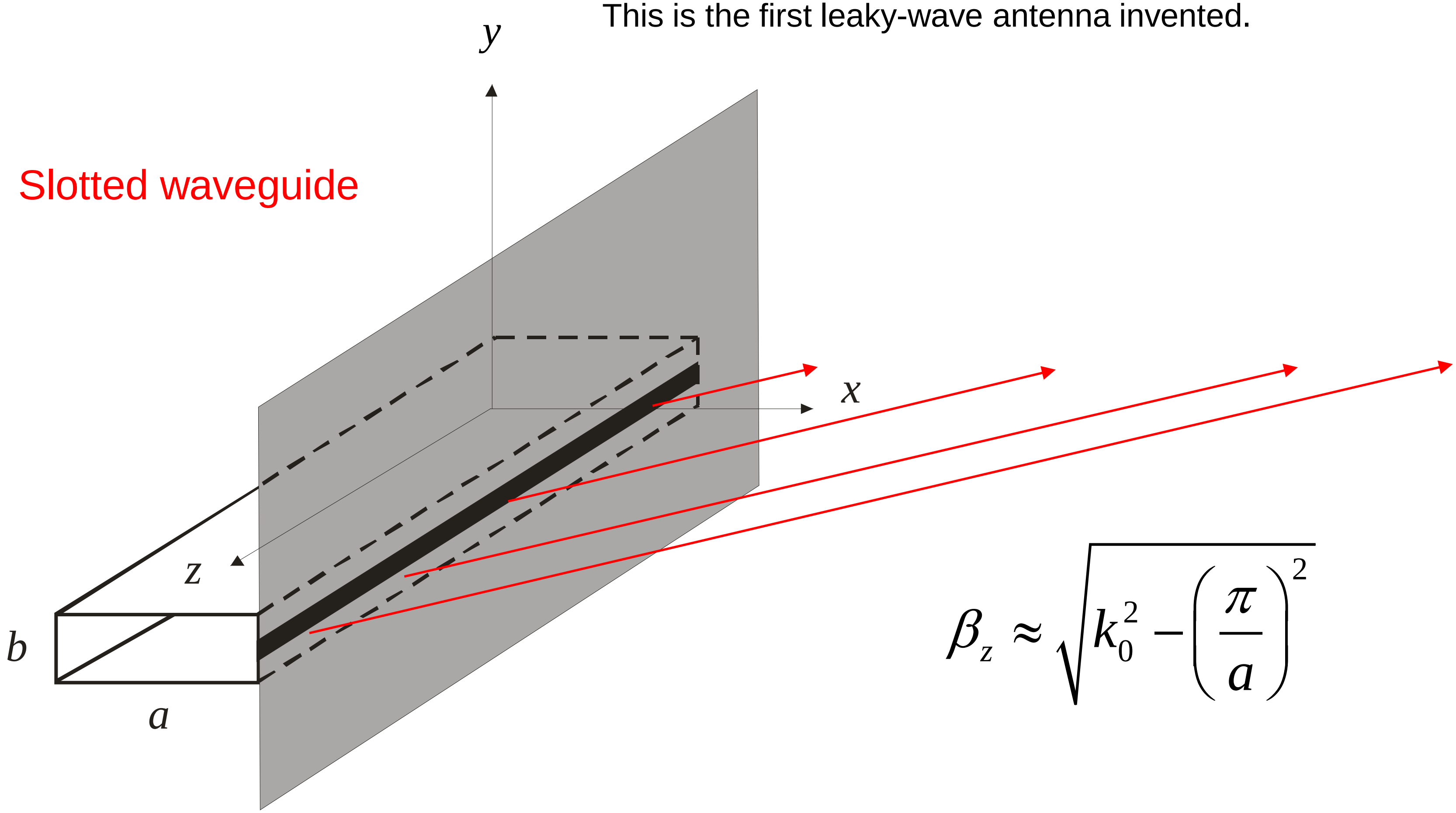
D. R. Jackson and A. A. Oliner, "A Leaky-Wave Analysis of the High-Gain Printed Antenna Configuration," *IEEE Trans. Antennas and Propagation*, vol. 36, pp. 905-910, July 1988.

Leaky-Wave Antennas (cont.)

W. W. Hansen, "Radiating electromagnetic waveguide," Patent, 1940, U.S. Patent No. 2.402.622.

This is the first leaky-wave antenna invented.

Slotted waveguide

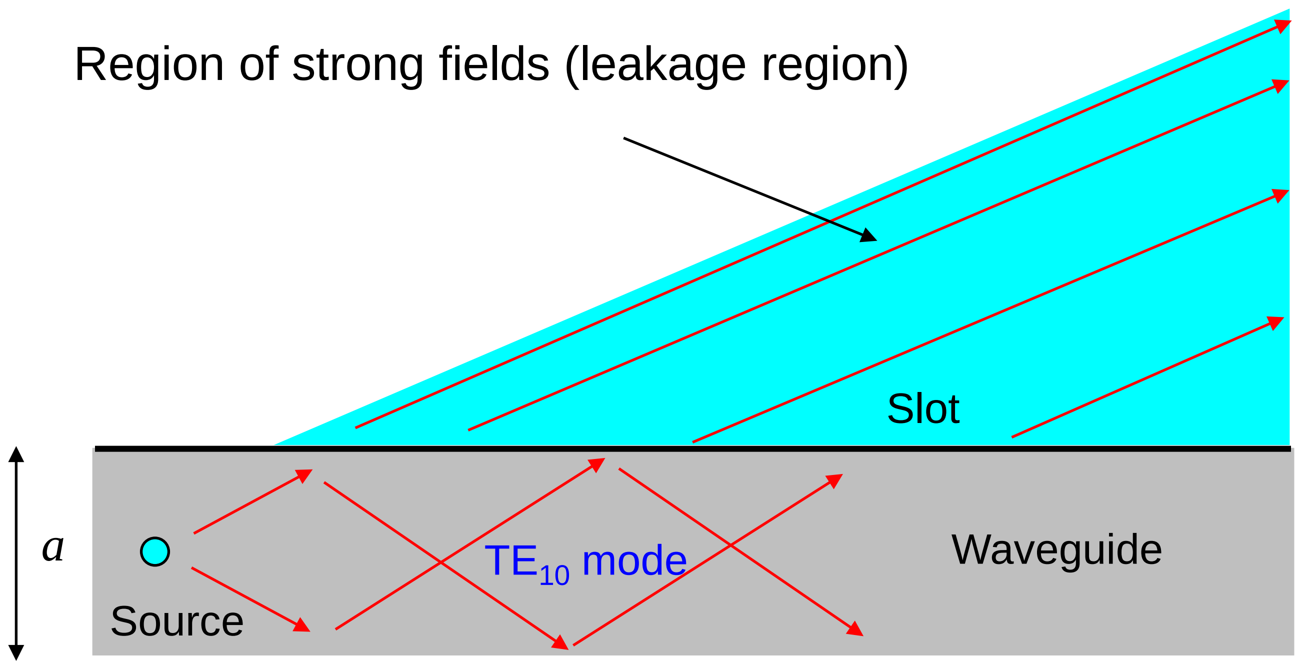


$$\beta_z \approx \sqrt{k_0^2 - \left(\frac{\pi}{a}\right)^2}$$

Note: $\beta_z < k_0$

Leaky-Wave Antennas (cont.)

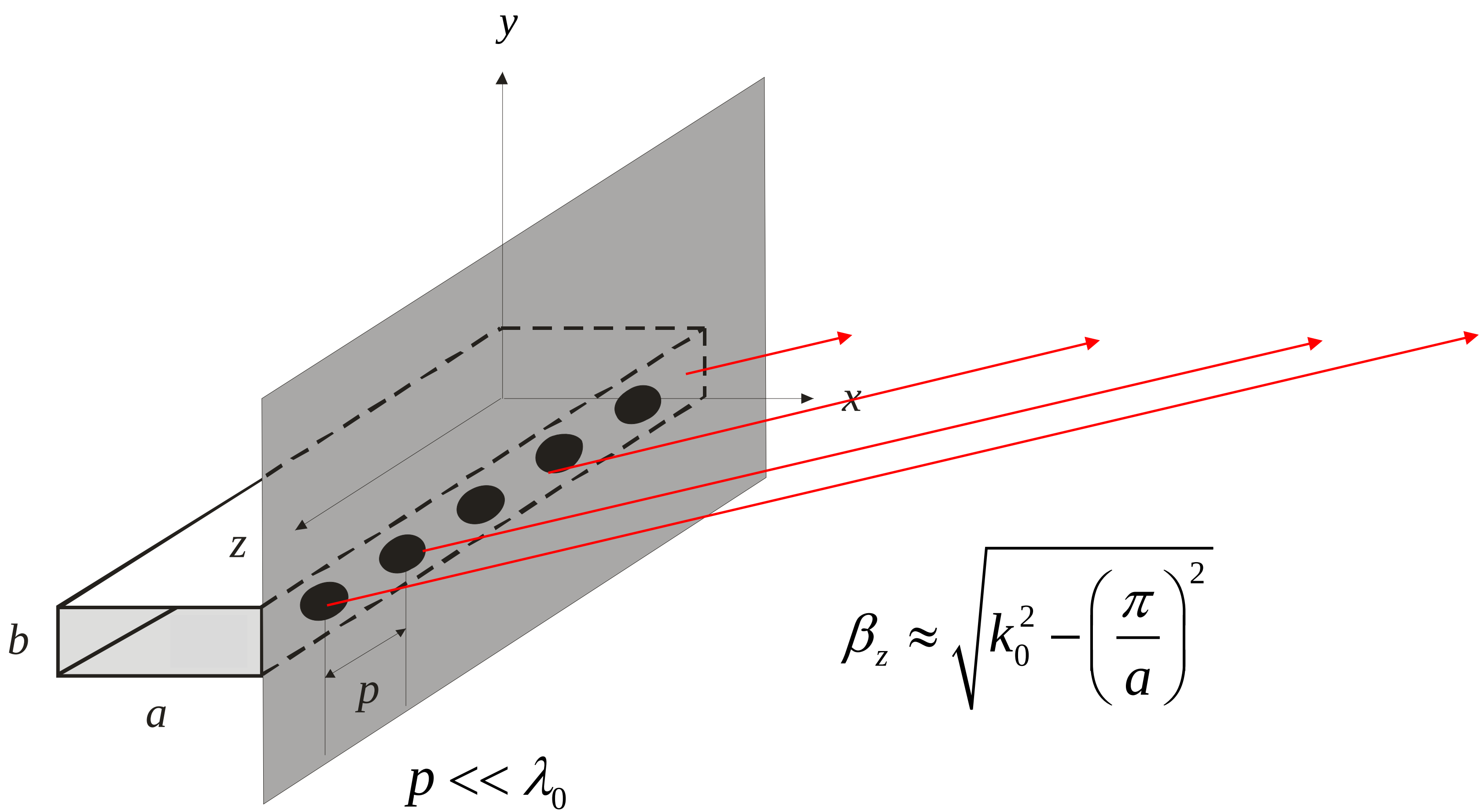
The slotted waveguide illustrates in a simple way why the field is weak outside of the "leakage region."



Top view

Leaky-Wave Antennas (cont.)

Another variation: Holey waveguide

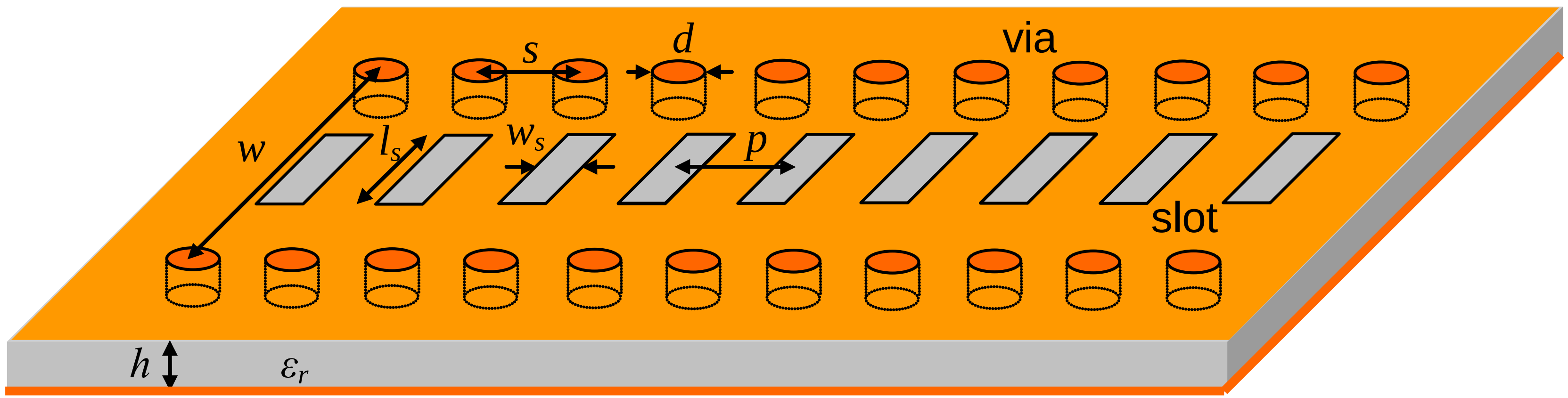


$$p << \lambda_0$$

$$\beta_z \approx \sqrt{k_0^2 - \left(\frac{\pi}{a}\right)^2}$$

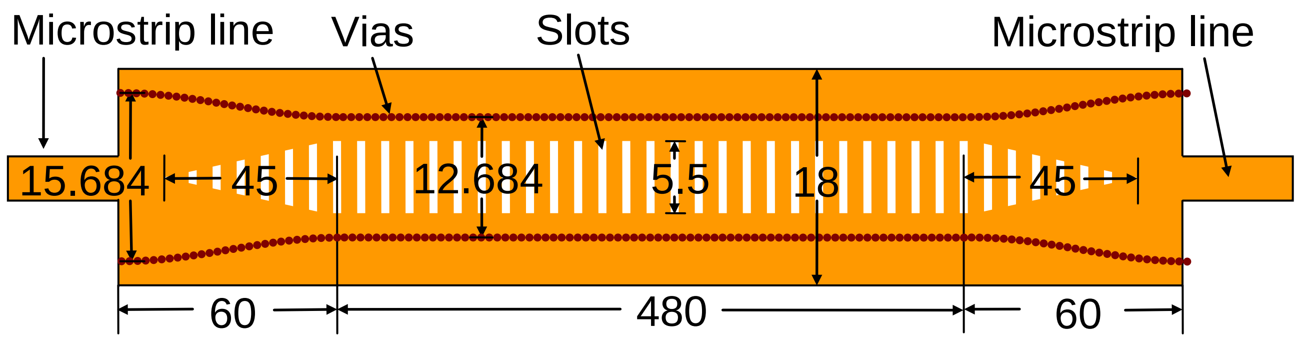
Leaky-Wave Antennas (cont.)

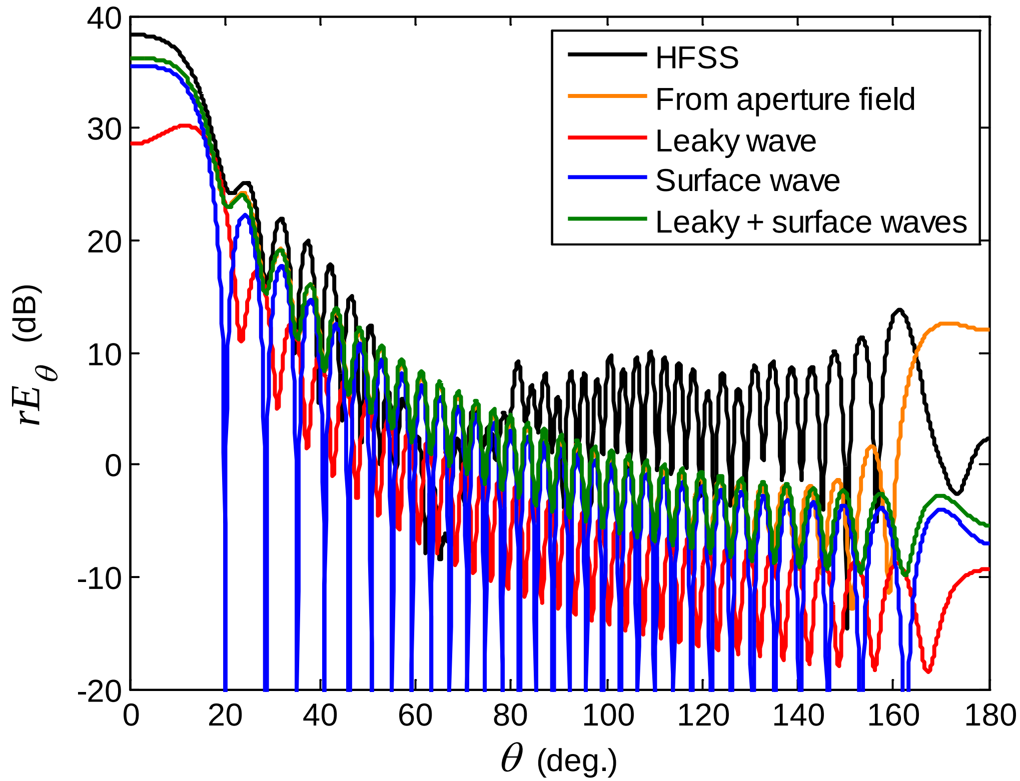
Another type of leaky-wave antenna, based on
substrate-integrated waveguide



Substrate-integrated waveguide (SIW)

Leaky-Wave Antennas (cont.)





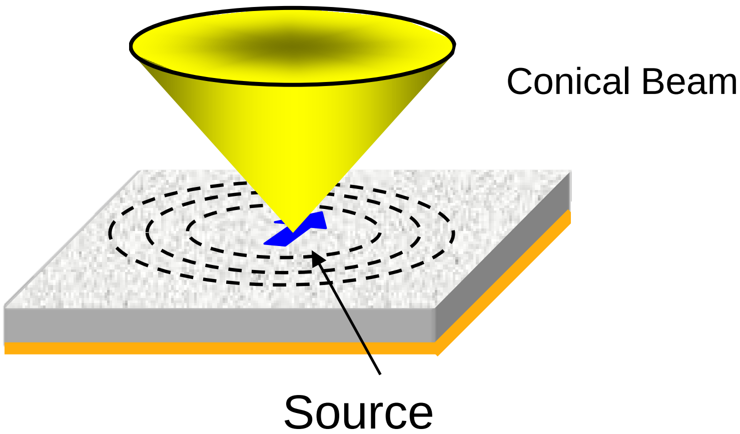
The transverse slots allow for magnetic currents that are transverse, which can radiate at endfire.

J. Liu, D. R. Jackson, and Y. Long, "Substrate Integrated Waveguide (SIW) Leaky-Wave Antenna With Transverse Slots," *IEEE Trans. Antennas and Propagation*, vol. 60, pp. 20-29, Jan. 2012.

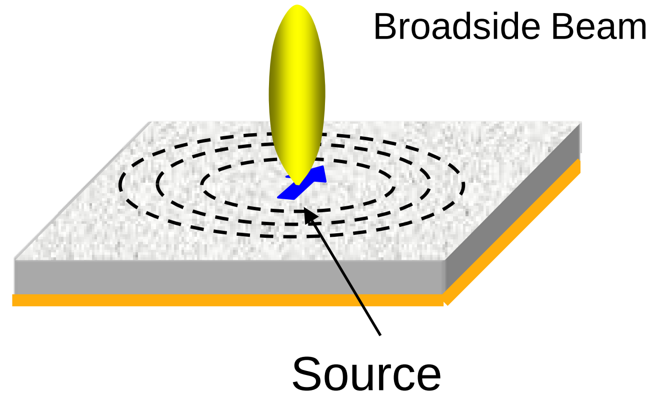
Leaky-Wave Antennas (cont.)

2-D Leaky-Wave Antenna

$\beta_\rho > \alpha_\rho$



$\beta_\rho < \alpha_\rho$



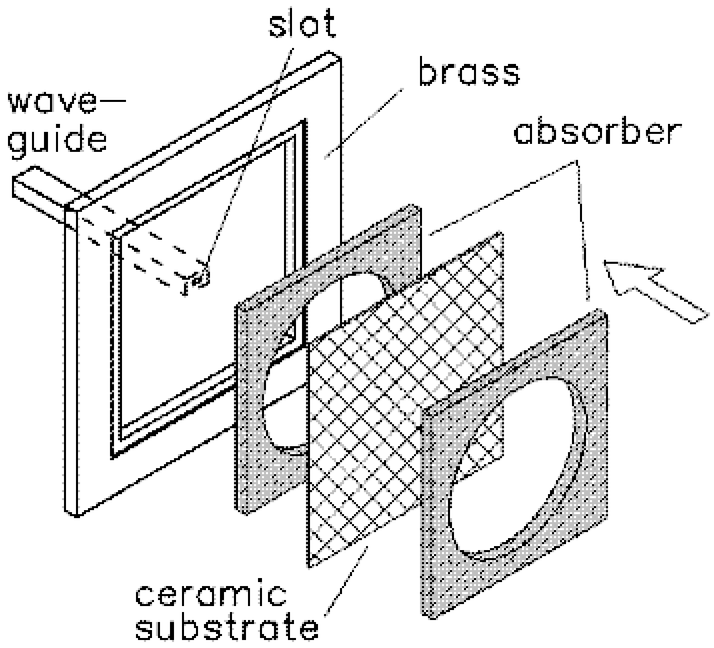
In the air region: $\psi(\rho, z) = H_0^{(2)}(k_\rho \rho) e^{-jk_{z0}z}$

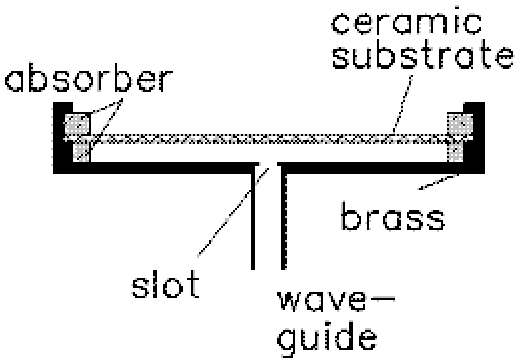
$$k_\rho = \beta_\rho - j\alpha_\rho$$

$$k_{z0} = \left(k_0^2 - k_\rho^2\right)$$

Leaky-Wave Antennas (cont.)

2-D Leaky-Wave Antenna



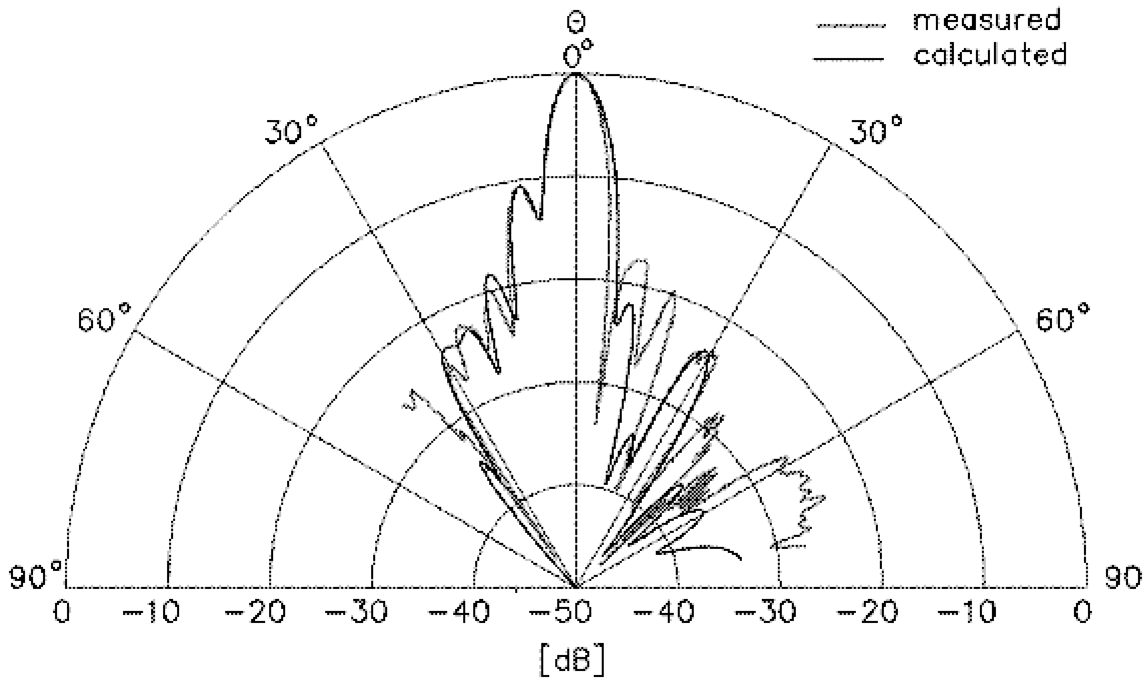


Implementation at millimeter-wave frequencies (62.2 GHz)

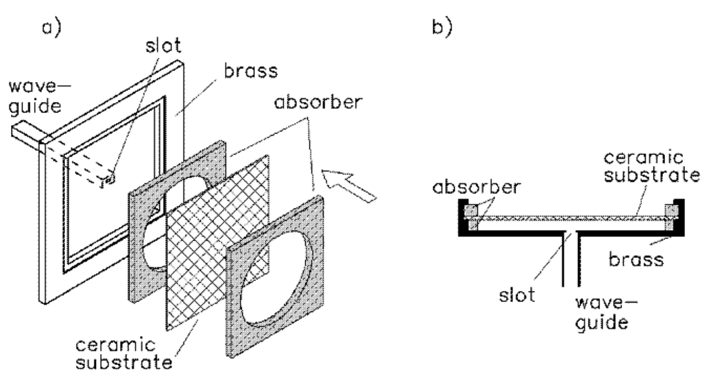
$\varepsilon_{r1} = 1.0$, $\varepsilon_{r2} = 55$, $h = 2.41$ mm, $t = 0.484$ mm, $a = 3.73\ \lambda_0$ (radius)

Leaky-Wave Antennas (cont.)

2-D Leaky-Wave Antenna



(E-plane shown on one side, H-plane on the other side)



Leaky-Wave Antennas (cont.)

2-D Leaky-Wave Antenna

The concept of using a "partially reflecting surface" (PRS) to create narrow beams goes back to von Trentini in 1956.

It was not understood that this is a leaky-wave effect.

Partially Reflecting Sheet Arrays*

GISWALT VON TRENTINI†

Summary—Multiple reflections of electromagnetic waves between two planes are studied, and the increase in directivity that results by placing a partially reflecting sheet in front of an antenna with a reflecting screen is investigated at a wavelength of 3.2 cm. The construction and performance of various models of such arrays is discussed. Thus, for example, a "reflex-cavity antenna" with an outer diameter of 1.88 λ and an over-all length of only 0.65 λ is described which has half-power beamwidths of 34° and 41° in the *E* and *H* planes, respectively, and a gain of approximately 14 db. It is shown that larger systems produce considerably greater directivity but that their efficiency is poor.

Introduction

A LARGE, plane, conducting screen placed behind an antenna serves as a shield against backward radiation and also affects, depending on its spacing, the forward pattern.[1] Improved directivity in the normal direction is possible only to a limited degree because the illumination of the reflecting screen is not optimum and the rays reflected from more distant zones produce, in part, out-of-phase contributions.

Improved illumination and an increase in directivity, and therefore in gain, can be obtained by adding a partially reflecting sheet in front of the antenna and parallel to the reflecting screen, causing multiple reflections between the sheet and screen. The distance between the sheet and screen must be such that the partial rays projected through the sheet into space have equal phases in the normal direction.

This paper presents the results of tests with various types of sheets and discusses a few simple forms of

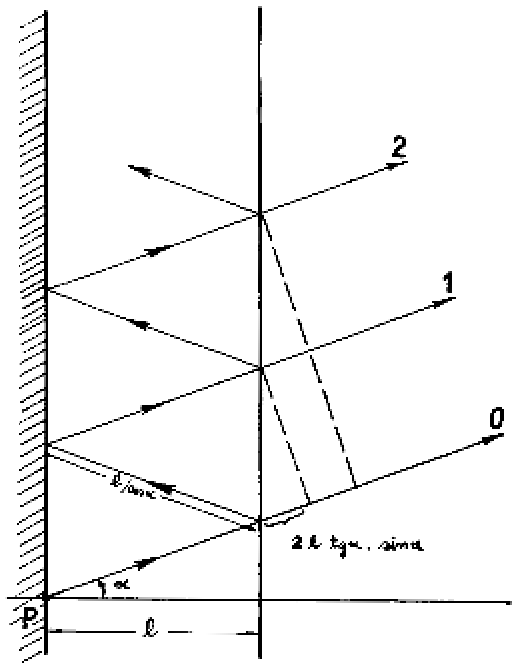


Fig. 1—Multiple reflections between screen and sheet.

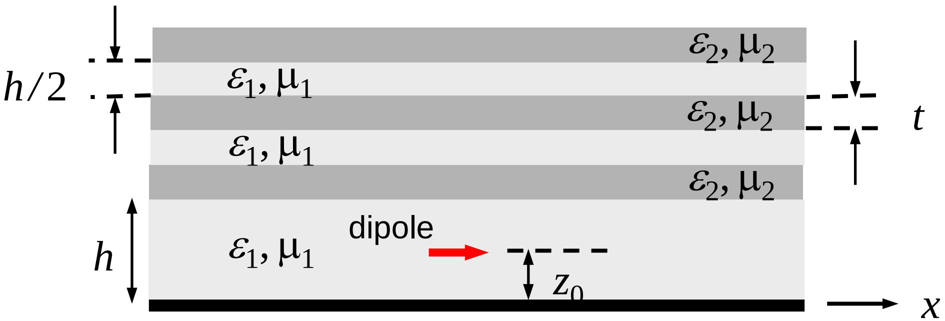
G. von Trentini, "Partially Reflecting Sheet Arrays," *IEEE Trans. Antennas Propagat.*, vol. 4, pp. 666-671, Oct. 1956.

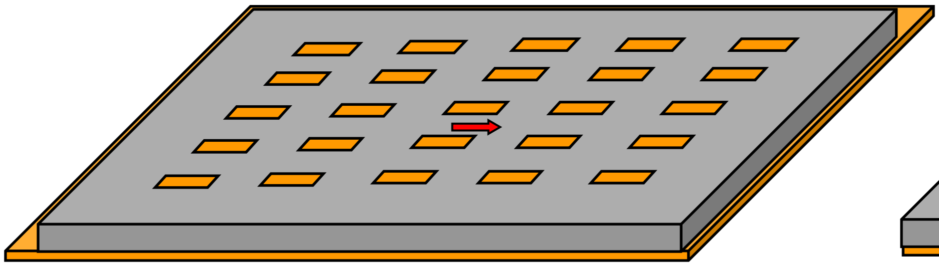
Leaky-Wave Antennas (cont.)

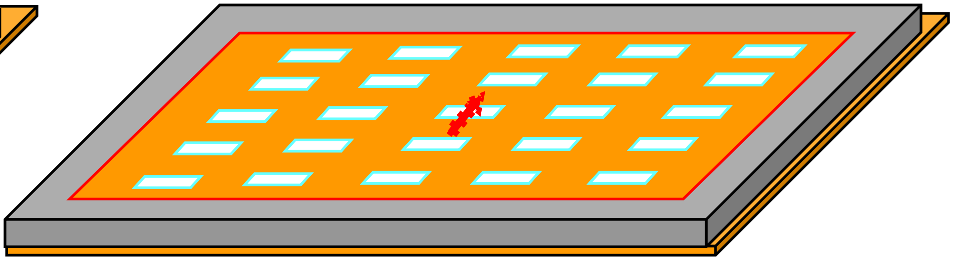
2-D Leaky-Wave Antenna

Working with Prof. Oliner, results were extended to other planar 2D leaky-wave antennas using different PRS structures.

Today these structures are often called "Fabry-Pérot resonant cavity antennas."



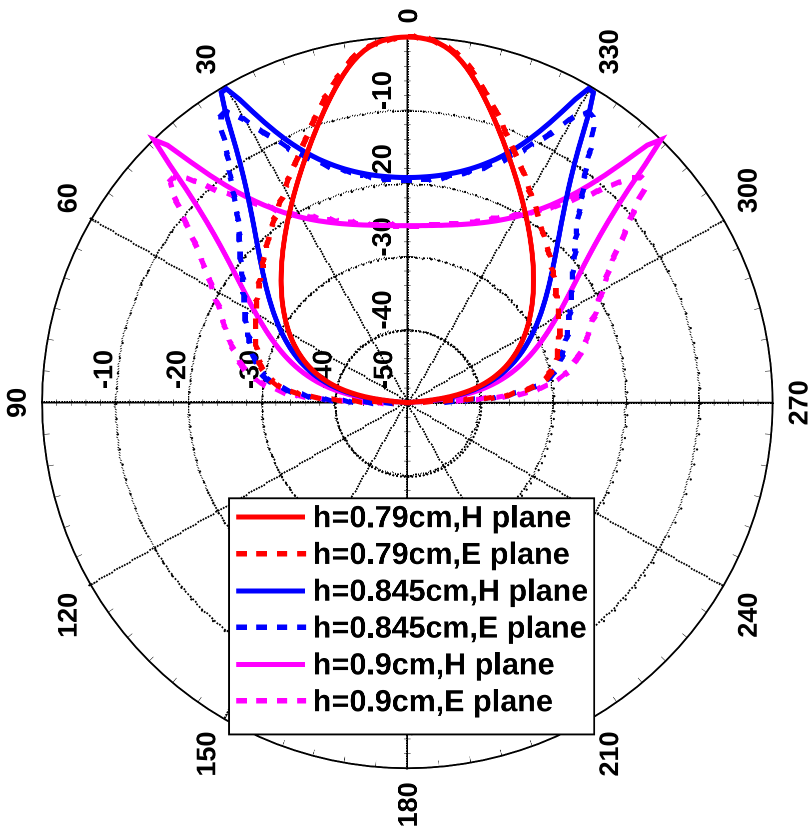




Leaky-Wave Antennas (cont.)

2-D Leaky-Wave Antenna

Directive pencil beams at broadside or conical beams can be produced.



Slot-PRS structure

Table showing beamwidth properties

	E-plane	H-plane
General Scan case	$\dfrac{2\varepsilon_r\sqrt{\varepsilon_r-\sin^2\theta_p}}{\pi\,\bar{B}_L^2\sin\theta_p\cos^2\theta_p}$	$\dfrac{2\left(\sqrt{\varepsilon_r-\sin^2\theta_p}\right)^3}{\pi\,\bar{B}_L^2\sin\theta_p}$
Broadside	$\dfrac{2}{\lvert\bar{B}_L\rvert}\sqrt{\dfrac{2\varepsilon_r^{3/2}}{\pi}}$	$\dfrac{2}{\lvert\bar{B}_L\rvert}\sqrt{\dfrac{2\varepsilon_r^{3/2}}{\pi}}$
Endfire	Narrow beam not possible	$\dfrac{2\left(\sqrt{\varepsilon_r-1}\right)^3}{\pi\bar{B}_L^2}$

$$\left(\bar{B}_L = B_L\eta_0\right)$$

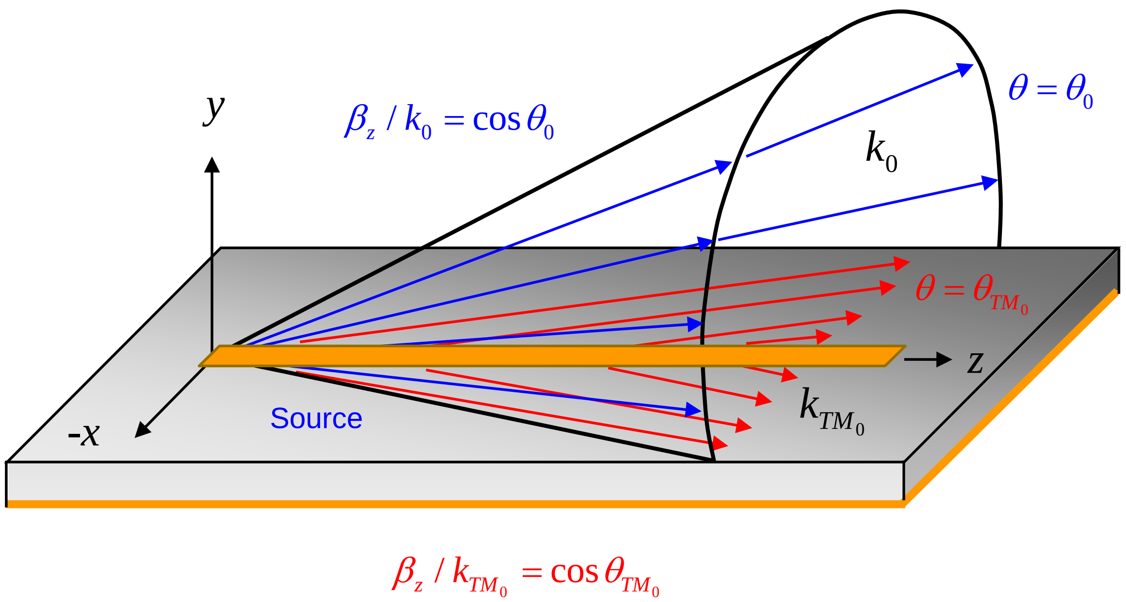
(normalized susceptance of PRS)

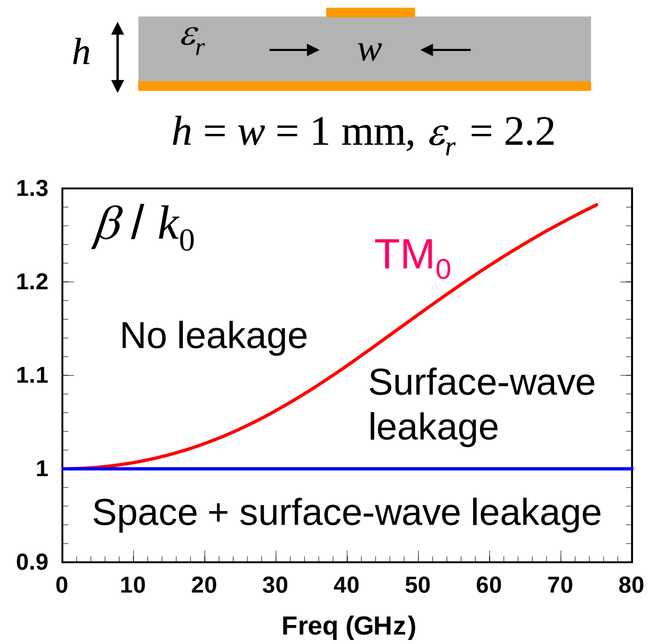
T. Zhao, D. R. Jackson, and J. T. Williams,“ General formulas for 2D leaky wave antennas,” *IEEE Trans. Antennas and Propagation*, vol. 53, pp. 3525-3533, Nov. 2005.

Leaky Waves on MIC Lines

It was found that two different types of leaky modes could exist on microwave integrated circuit (MIC) (i.e., printed-circuit) lines:

- leakage into the TM_0 surface wave (SW)
- leakage into SW + space





Physical surface-wave leakage: $k_0 < \beta < k_{TM_0}$

Physical space + surface-wave leakage: $\beta < k_0 < k_{TM_0}$

Leaky Waves on MIC Lines (cont.)

Leaky modes have been found on a variety of printed-circuit lines.

Leakage occurs at high frequency

- Microstrip line*
- Coplanar waveguide
- Coplanar strips
- Slotline

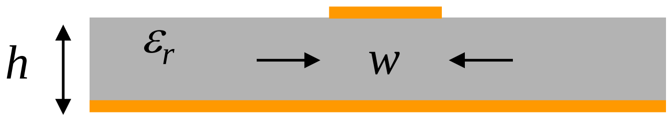
Leakage occurs at any frequency

- Stripline with an air gap*
- Microstrip with a top cover (low enough cover)
- Conductor-backed coplanar waveguide
- Conductor-backed slotline

*Illustrated here with examples

Leaky Waves on MIC Lines (cont.)

Microstrip

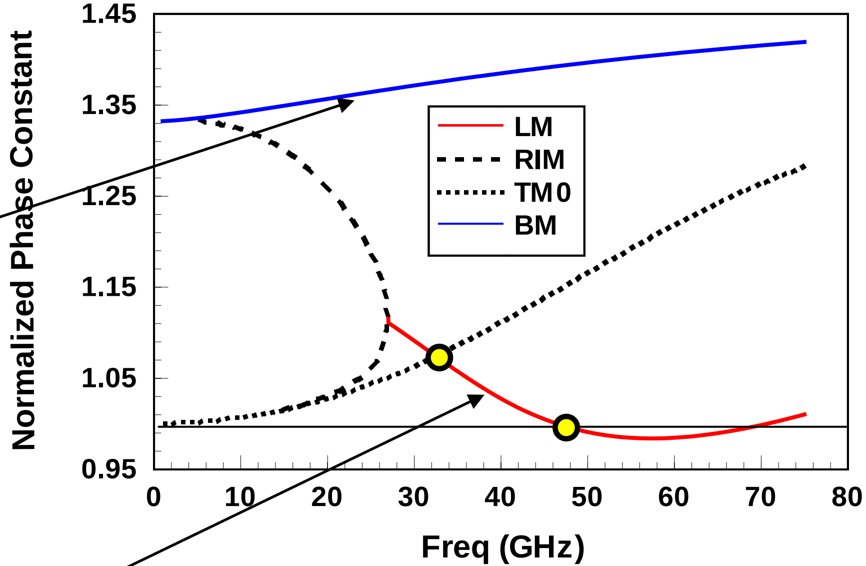


$h = w = 1$ mm, $\varepsilon_r = 2.2$

The bound mode is the usual quasi-TEM microstrip mode.

A physical leaky mode exists at high frequencies.

Normalized phase constant (β / k_0)



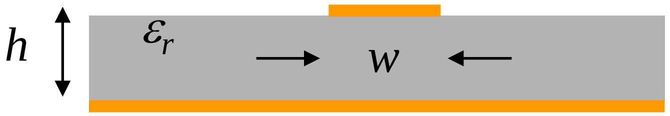
The leaky mode plotted here leaks into the TM_0 surface wave.

RIM: real improper mode (similar to ISW of slab)

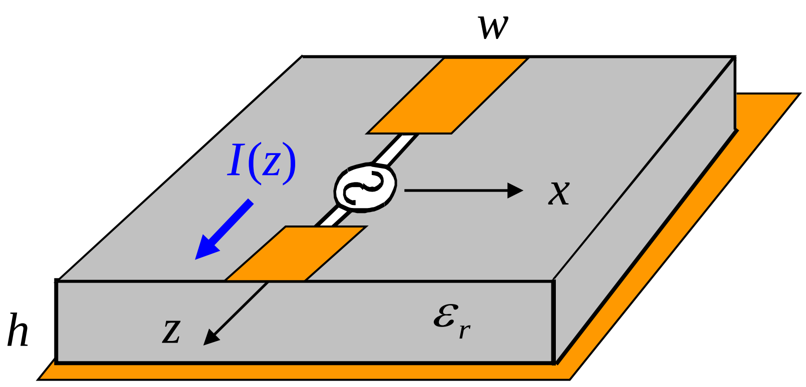
Leaky Waves on MIC Lines (cont.)

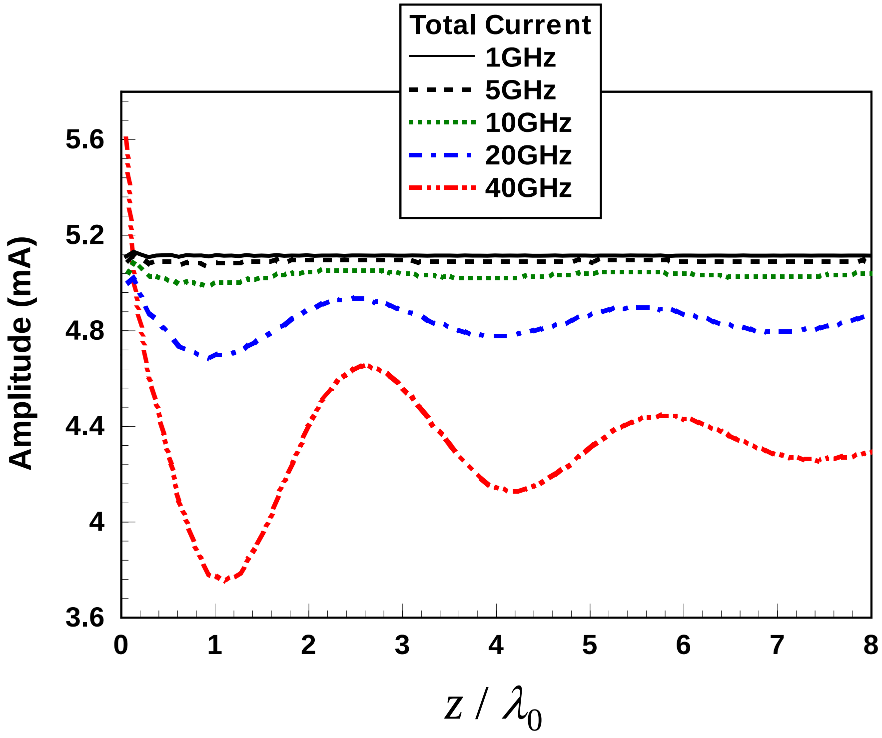
Microstrip

Source on Line



$h = w = 1$ mm, $\varepsilon_r = 2.2$

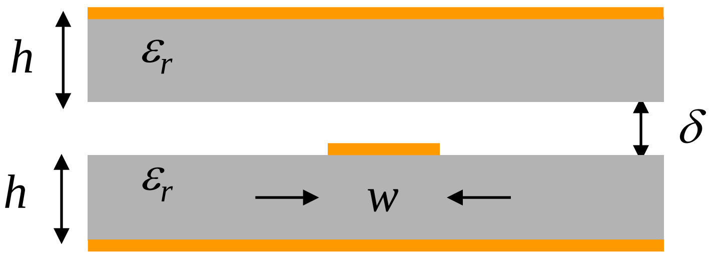




The leaky mode interferes with the bound mode at high frequency, causing spurious oscillations in the current on the line.

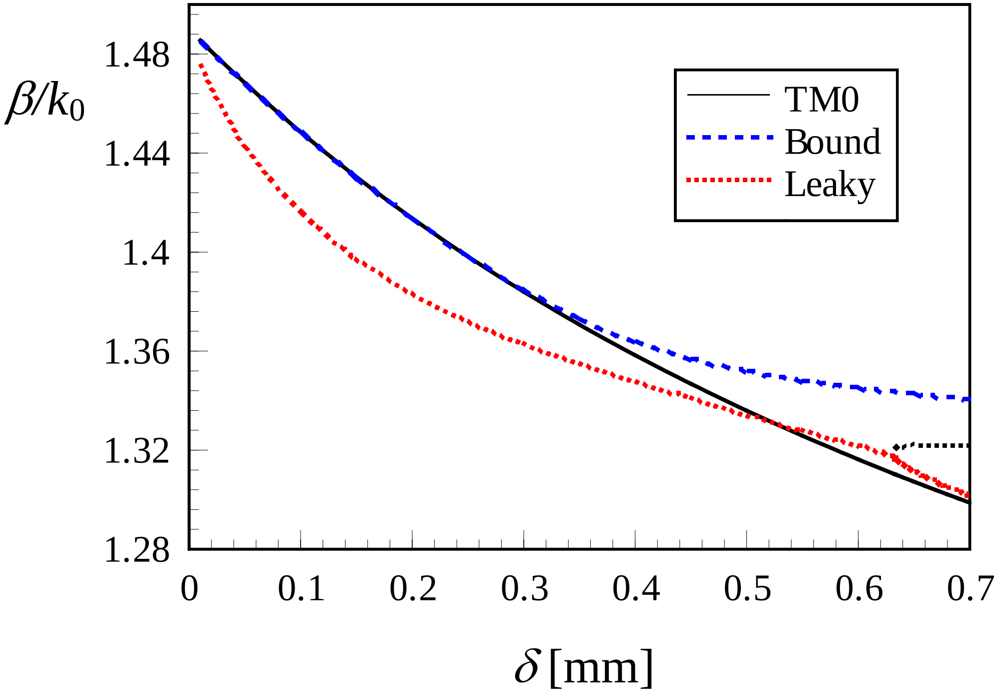
Leaky Waves on MIC Lines (cont.)

Stripline with an air gap



$h = w = 1$ mm, $\varepsilon_r = 2.22$

$f = 3.0$ GHz



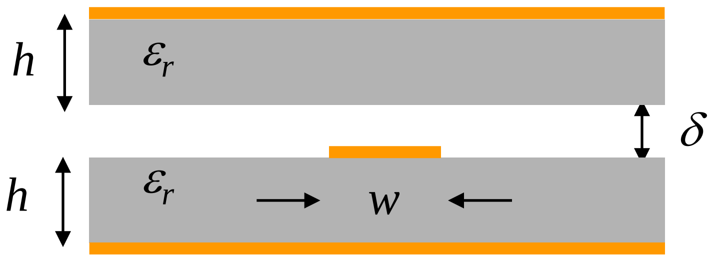
A physical leaky mode exists even at low frequency, when the air gap is small.

- The leaky mode is the one that turns into the TEM stripline mode as the air gap vanishes.
- The bound mode has a field that resembles a parallel-plate mode.

Leaky Waves on MIC Lines (cont.)

Stripline with an air gap

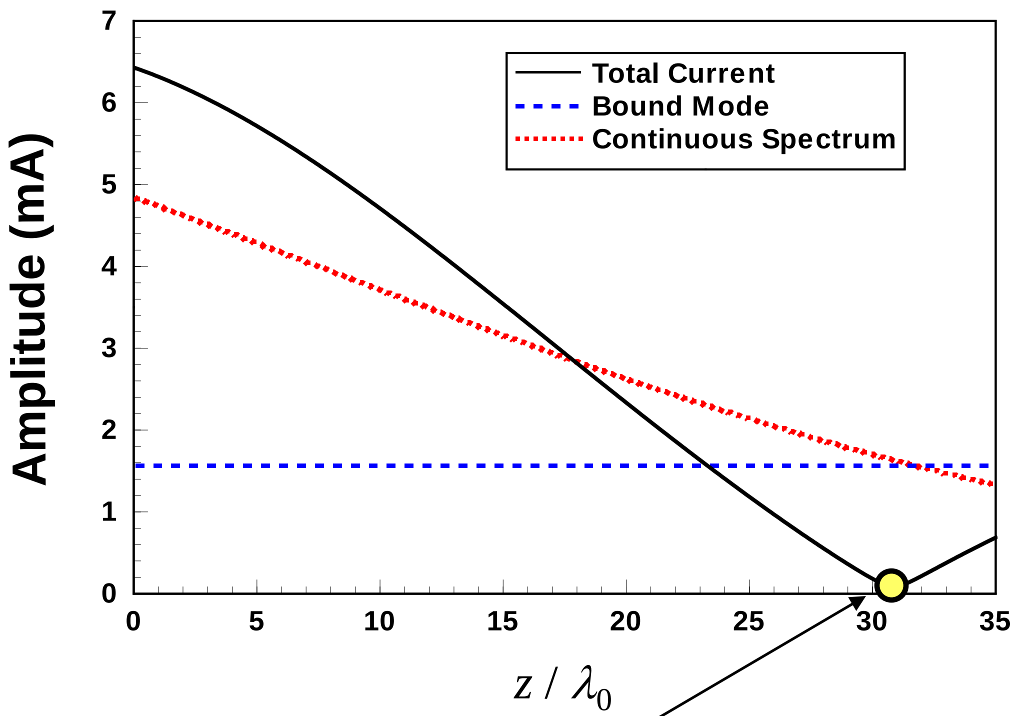
Source on Line



$h = w = 1$ mm, $\varepsilon_r = 2.22$

$f = 3.0$ GHz

$\delta = 0.353$ mm



The destructive interference between the bound mode and the leaky mode causes a null in the current on the strip.

Leaky Waves on MIC Lines (cont.)

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❖ D. Nghiem, J. T. Williams, D. R. Jackson, and A. A. Oliner, “Leakage of Dominant Mode on Stripline with a Small Air Gap,” *IEEE Trans. Microwave Theory and Techniques*, Vol. 43, No. 11, pp. 2549-2556, Nov. 1995.

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❖ D. Nghiem, J. T. Williams, D. R. Jackson, and A. A. Oliner, “Existence of a Leaky Dominant Mode on Microstrip Line with an Isotropic Substrate: Theory and Measurements,” *IEEE Trans. Microwave Theory and Techniques*, Vol. 44, No. 10, pp. 1710-1715, Oct. 1996.

❖ F. Mesa, D. R. Jackson, and M. Freire, “High Frequency Leaky-Mode Excitation on a Microstrip Line,” *IEEE Trans. Microwave Theory and Techniques*, Vol. 49, No. 12, pp. 2206-2215, Dec. 2001.