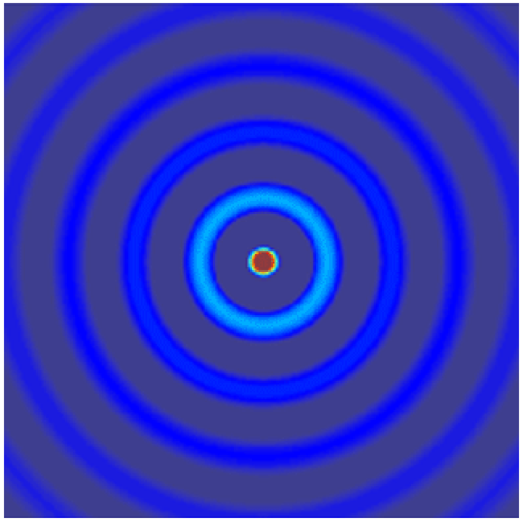
ECE 6341

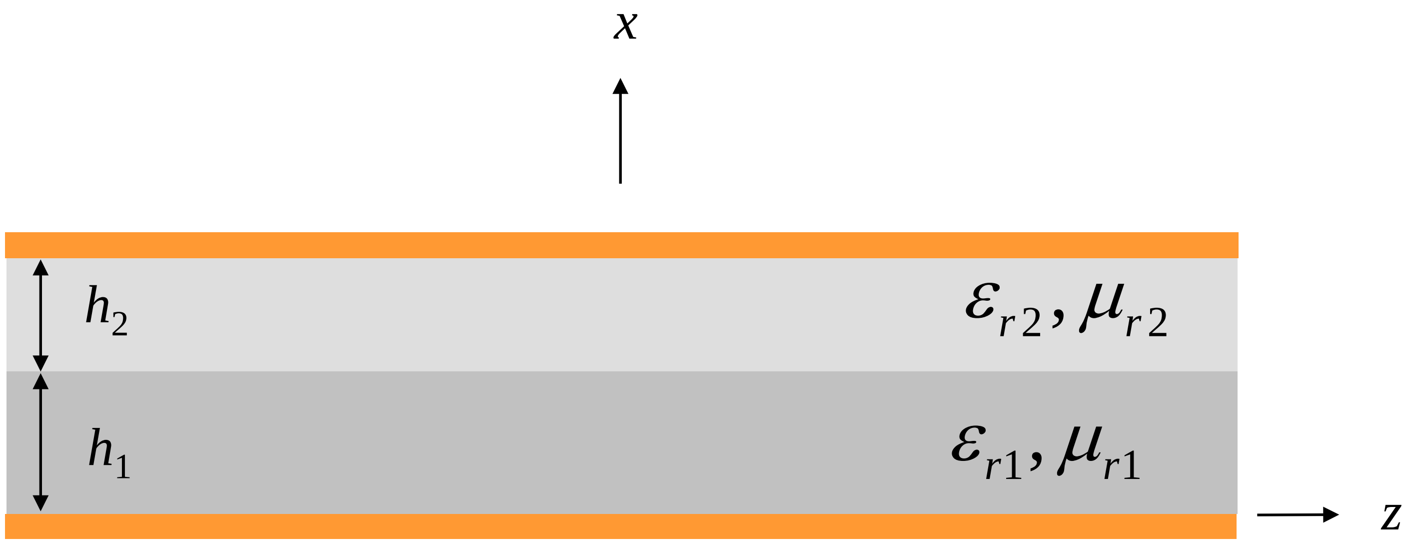
Spring 2016

Prof. David R. Jackson
ECE Dept.



Notes 7

Two-Layer Stripline Structure

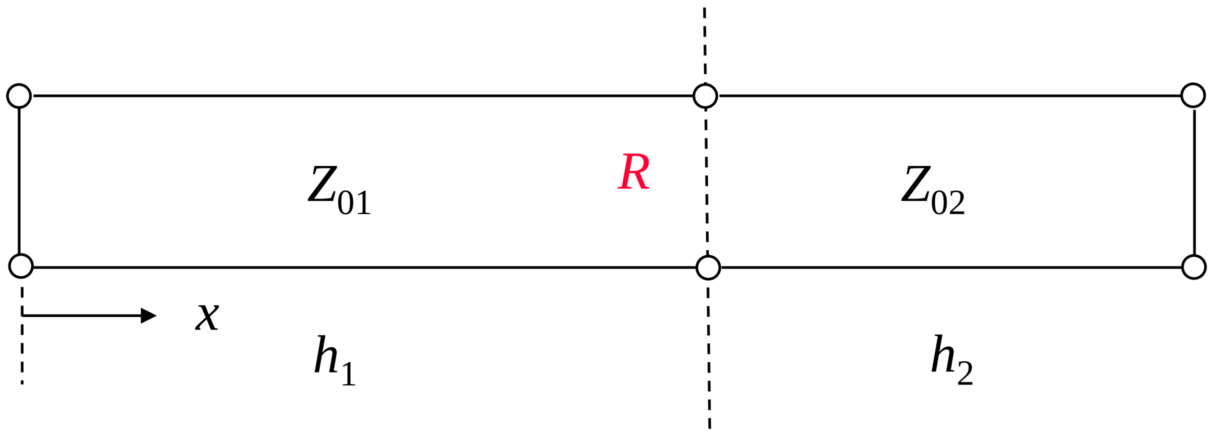


Goal: Derive a transcendental equation for the wavenumber k_z of the TM_x modes of propagation.

Assumption: There is no variation of the fields in the y direction, and propagation is along the z direction.

Two-Layer Stripline Structure (cont.)

TEN:



TM$_x$ impedances:

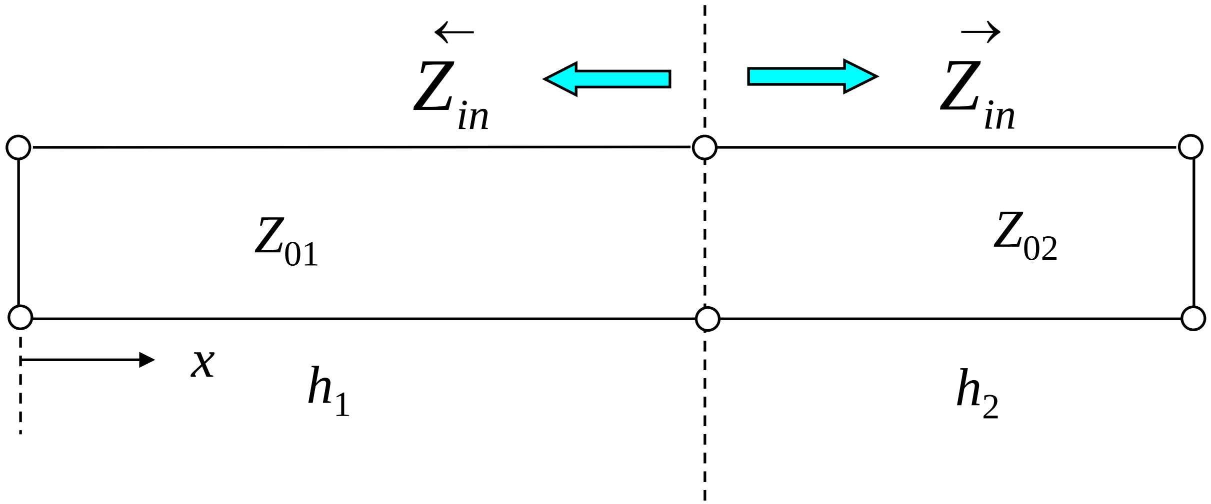
$$Z_{01} = \frac{k_{x1}}{\omega \varepsilon_1} \qquad Z_{02} = \frac{k_{x2}}{\omega \varepsilon_2}$$

Note: Because this is a lossless *closed structure*, k_z is either real (above cutoff) or imaginary (below cutoff).

$$k_{x1} = \left(k_1^2 - k_z^2\right)^{1/2} \qquad k_{x2} = \left(k_2^2 - k_z^2\right)^{1/2}$$

Transverse Resonance Equation (TRE): $\overleftarrow{Z}_{in} = -\overrightarrow{Z}_{in}$

Two-Layer Stripline Structure (cont.)



$$\overleftarrow{Z}_{in} = jZ_{01}\tan\left(k_{x1}h_1\right) \qquad \overrightarrow{Z}_{in} = jZ_{02}\tan\left(k_{x2}h_2\right)$$

TRE: $$Z_{01}\tan\left(k_{x1}h_1\right) = -Z_{02}\tan\left(k_{x2}h_2\right)$$

or

$$\frac{k_{x1}}{\omega\varepsilon_1}\tan\left(k_{x1}h_1\right) = -\frac{k_{x2}}{\omega\varepsilon_2}\tan\left(k_{x2}h_2\right)$$

TEN (cont.)

Hence

$$\varepsilon_{r2} k_{x1} \tan\left(k_{x1} h_1\right) = -\varepsilon_{r1} k_{x2} \tan\left(k_{x2} h_2\right)$$

Assuming nonmagnetic materials,

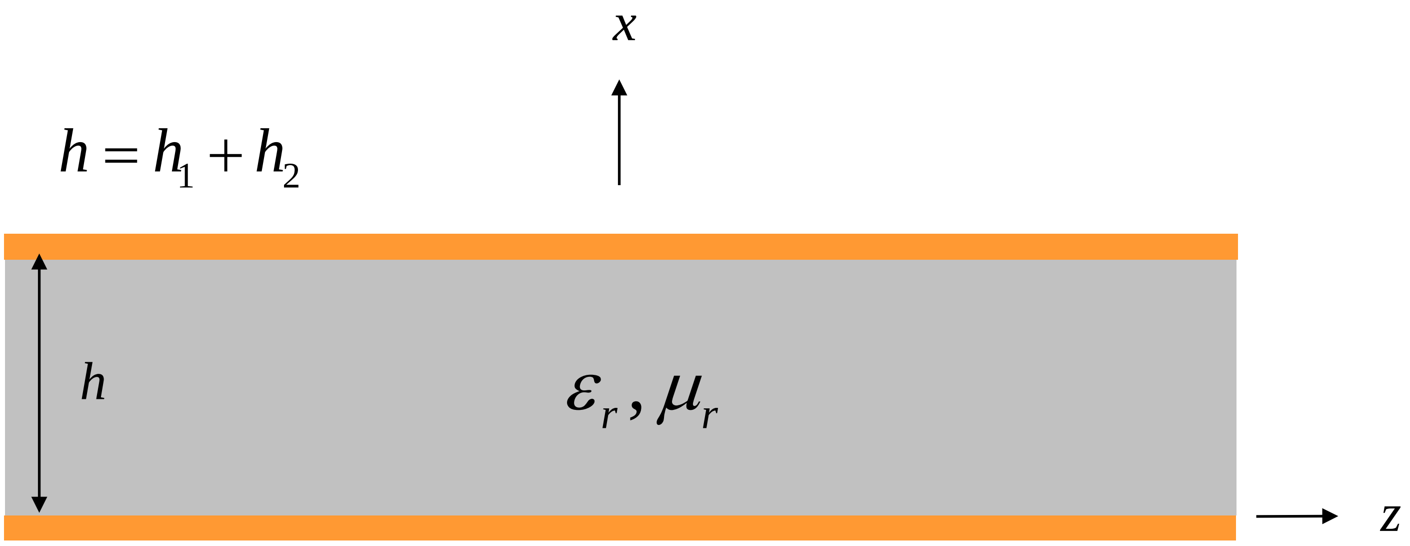
$$k_{x1} = \left(k_0^2 \varepsilon_{r1} - k_z^2\right)^{1/2} \qquad k_{x2} = \left(k_0^2 \varepsilon_{r2} - k_z^2\right)^{1/2}$$

A similar analysis could be applied for the TE_x modes.

Parallel-Plate Waveguide

Special case: parallel-plate waveguide

$$h = h_1 + h_2$$



$$k_{x1} = k_{x2} = k_x$$

Parallel-Plate Waveguide (cont.)

$$\varepsilon_r k_x \tan\left(k_x h_1\right) = -\varepsilon_r k_x \tan\left(k_x h_2\right)$$

or

$$\tan\left(k_x h_1\right) + \tan\left(k_x h_2\right) = 0$$

Helpful identity:

$$\tan\left(a+b\right) = \frac{\tan(a)+\tan(b)}{1-\tan(a)\tan(b)}$$

We then have

$$\tan\left(k_x h_1 + k_x h_2\right)\left[1-\tan\left(k_x h_1\right)\tan\left(k_x h_2\right)\right] = 0$$

This will be satisfied if

$$\tan\left(k_x h_1 + k_x h_2\right) = 0$$

Parallel-Plate Waveguide (cont.)

Hence we have

$$\tan\left(k_x h\right) = 0$$

so

$$k_x h = m\pi, \quad m = 0,1,2\ldots$$

We have (from the separation equation)

$$k_z = \sqrt{k_0^2 \varepsilon_r - k_x^2}$$

Hence

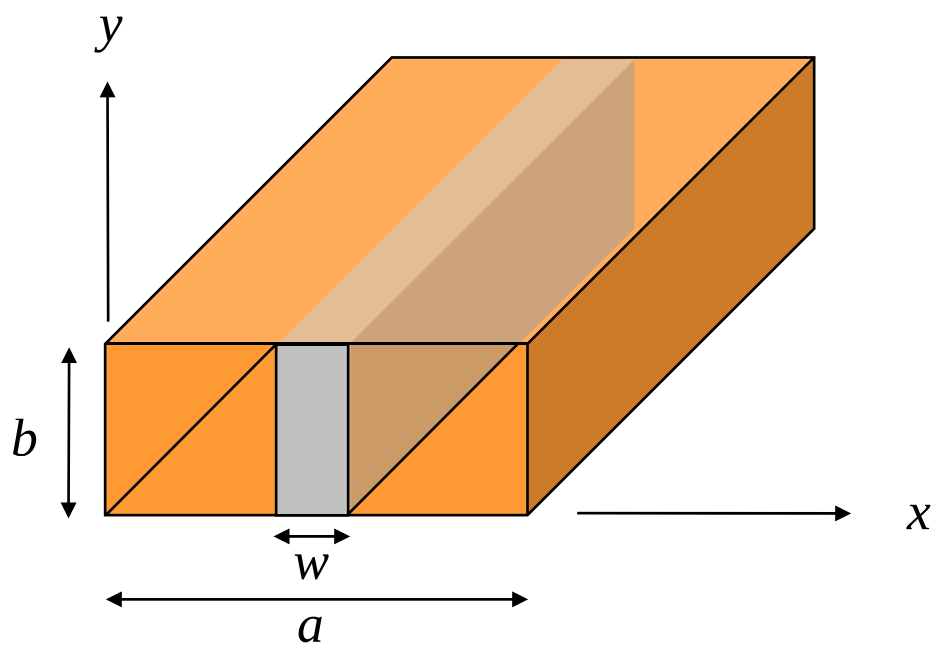
$$\boxed{k_z = \sqrt{k_0^2 \varepsilon_r - \left(\frac{m\pi}{h}\right)^2}}$$

Note: TM_x and TE_x modes have the same wavenumber, but only the TM_x mode can exist for $m = 0$. (the electric field is perpendicular to the metal walls).

Waveguide With Slab

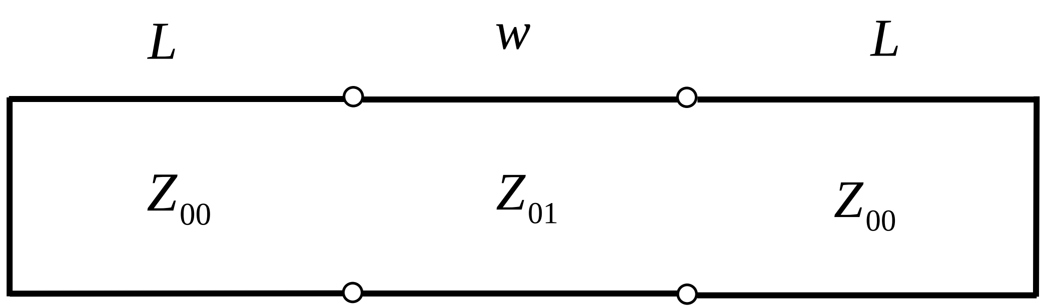
TE_x^{mn} modes

TM_x^{mn} modes



$$L = (a - w) / 2$$

TEN :



Waveguide With Slab (cont.)

$$Z_{00}^{TE} = \frac{\omega\mu_0}{k_{x0}} \qquad Z_{00}^{TM} = \frac{k_{x0}}{\omega\varepsilon_0}$$

$$Z_{01}^{TE} = \frac{\omega\mu_1}{k_{x1}} \qquad Z_{01}^{TM} = \frac{k_{x1}}{\omega\varepsilon_1}$$

Wavenumbers:

$$k_{x0} = \left[k_0^2 - k_z^2 - \left(\frac{n\pi}{b}\right)^2 \right]^{1/2}$$

$$k_{x1} = \left[k_0^2 \varepsilon_r - k_z^2 - \left(\frac{n\pi}{b}\right)^2 \right]^{1/2}$$

Note:

$$A_x = \sin\left(\frac{n\pi y}{b}\right) F(x) e^{-jk_z z}$$

$$F_x = \cos\left(\frac{n\pi y}{b}\right) G(x) e^{-jk_z z}$$

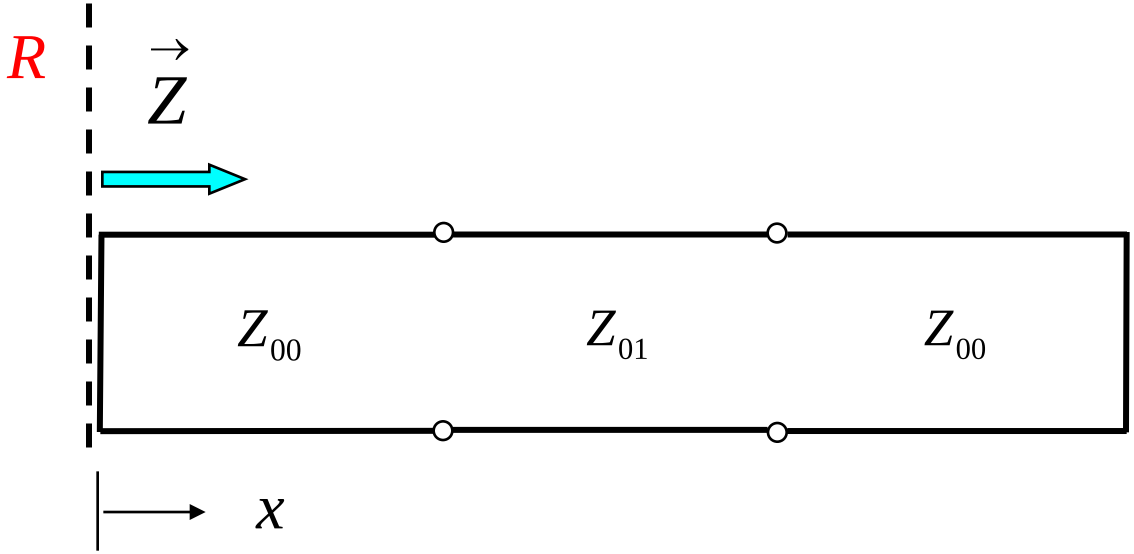
Note: In the BCs, A_x “acts” like E_x while F_x “acts” like H_x.

Waveguide With Slab (cont.)

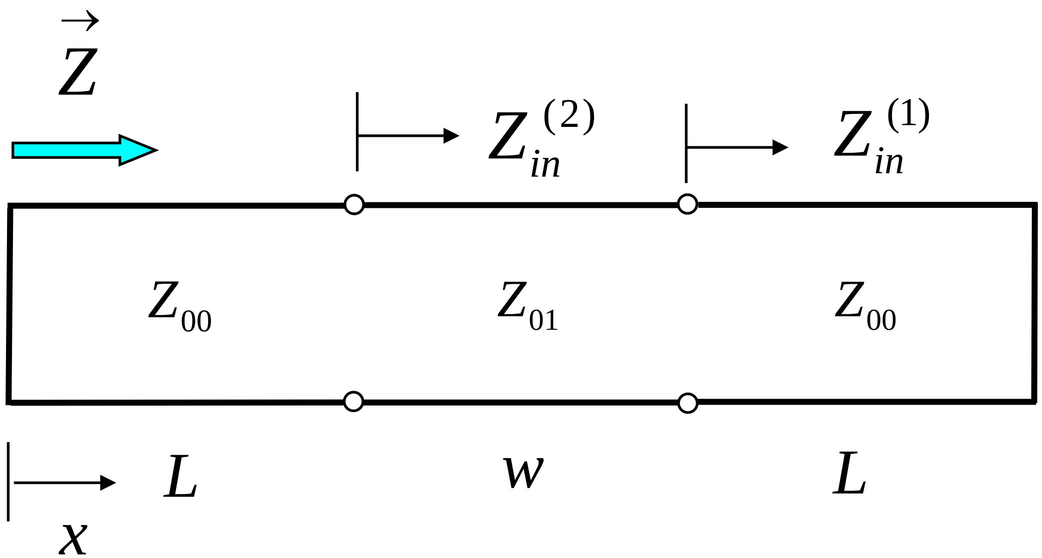
First try reference plane at $x = 0$:

$$\cancel{\overleftarrow{Z}} = -\overrightarrow{Z}$$

$$\Rightarrow \boxed{\overrightarrow{Z} = 0}$$



Waveguide With Slab (cont.)



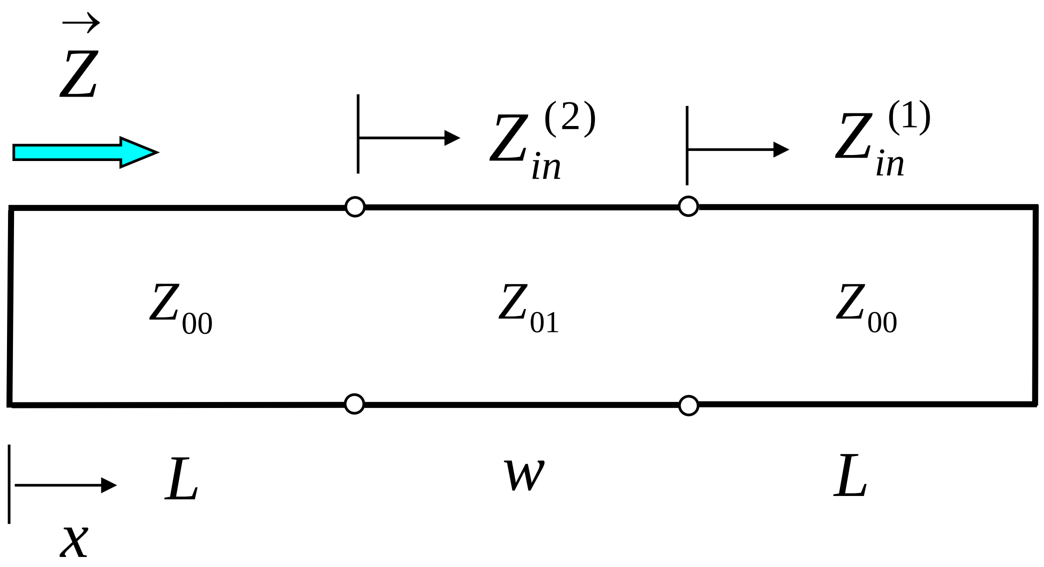
General formula:

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right]$$

Apply this twice:

$$\begin{cases} Z_{in}^{(1)} = jZ_{00} \tan(k_{x0} L) \\ Z_{in}^{(2)} = Z_{01} \left[\dfrac{Z_{in}^{(1)} + jZ_{01} \tan(k_{x1} w)}{Z_{01} + jZ_{in}^{(1)} \tan(k_{x1} w)} \right] \end{cases}$$

Waveguide With Slab (cont.)



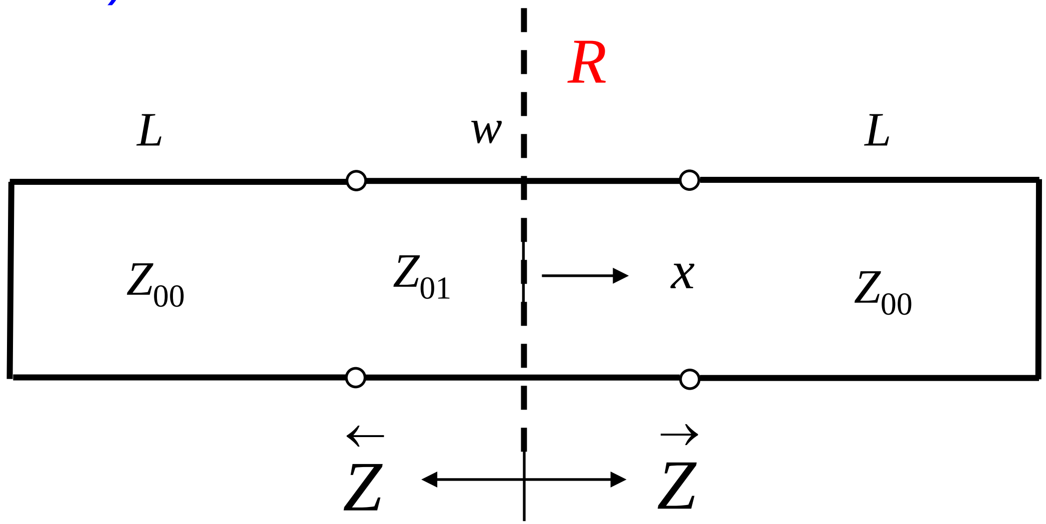
Apply again:

$$\vec{Z} = Z_{00}\left[\frac{Z_{in}^{(2)} + jZ_{00}\tan(k_{x0}L)}{Z_{00} + jZ_{in}^{(2)}\tan(k_{x0}L)}\right]$$

A mess !

Waveguide With Slab (cont.)

Now try a reference plane at the <u>center</u> of the structure (the origin is now re-defined here).



TRE:

$$\overleftarrow{Z} = -\overrightarrow{Z}$$

But (from symmetry):

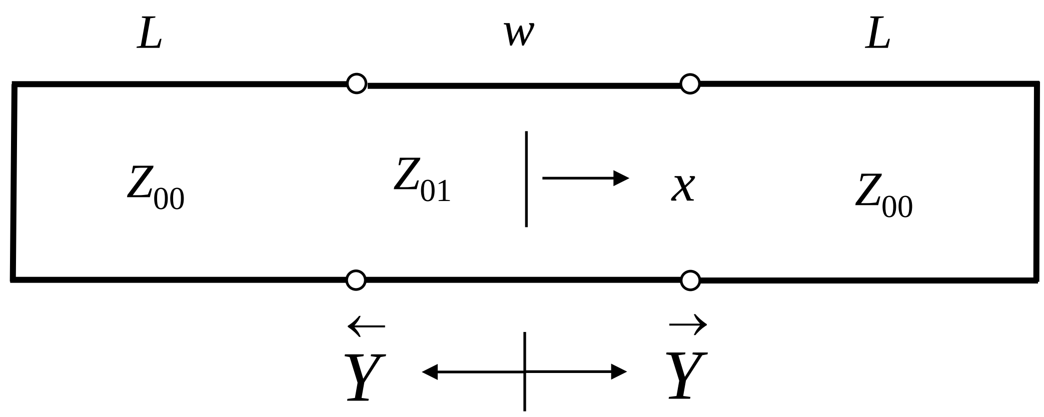
$$\overleftarrow{Z} = \overrightarrow{Z}$$

Hence $\overrightarrow{Z} = 0$

(This is the only <u>finite</u> number that will work.)

Waveguide With Slab (cont.)

Now use an <u>admittance</u> formulation



TRE:

$$\overleftarrow{Y} = -\overrightarrow{Y}$$

But (from symmetry):

$$\overleftarrow{Y} = \overrightarrow{Y}$$

Hence $\overrightarrow{Y} = 0$

(only <u>finite</u> number that will work)

Hence $\overrightarrow{Z} = \infty$

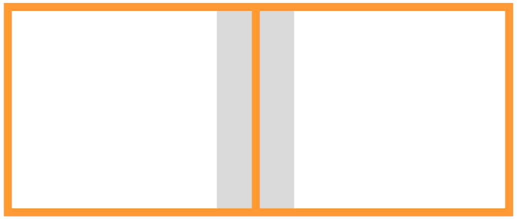
Waveguide With Slab (cont.)

Hence, there are two valid solutions:

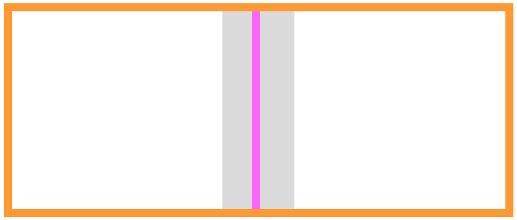
$$\vec{Z} = \begin{cases} 0 \\ \infty \end{cases}$$

$$\vec{Z} = 0$$ PEC wall

$$\vec{Z} = \infty$$ PMC wall



PEC wall



PMC wall

Waveguide With Slab (cont.)

From symmetry:

$$E_z(-x) = AE_z(x)$$

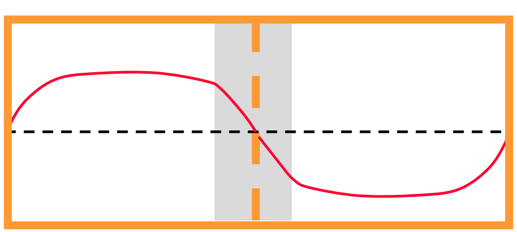
Odd mode: $A = -1$

Even mode: $A = +1$

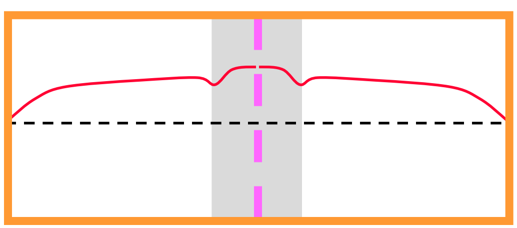
$$E_z = 0$$

$$\frac{\partial E_z}{\partial x} = 0$$

Plots of E_z



Odd mode: PEC wall



Even mode: PMC wall

Waveguide With Slab (cont.)

PEC wall: odd mode

PMC wall: even mode

$$\text{PEC}: \ \underline{E}_t = 0, \ \ H_n = 0, \ \ \frac{\partial \underline{H}_t}{\partial n} = \underline{0}$$

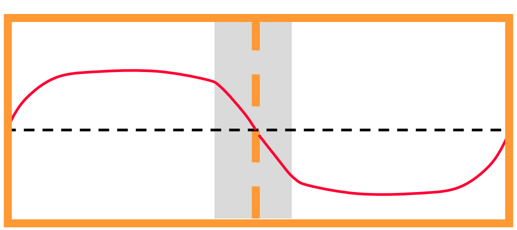
$$\text{PMC}: \ \underline{H}_t = 0, \ \ E_n = 0, \ \ \frac{\partial \underline{E}_t}{\partial n} = \underline{0}$$

Note: even/odd is classification is based on the E_z field.

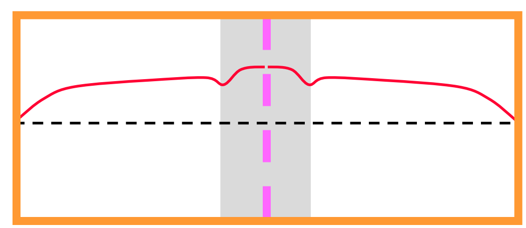
$$E_z = 0$$

$$\frac{\partial E_z}{\partial x} = 0$$

Plots of E_z



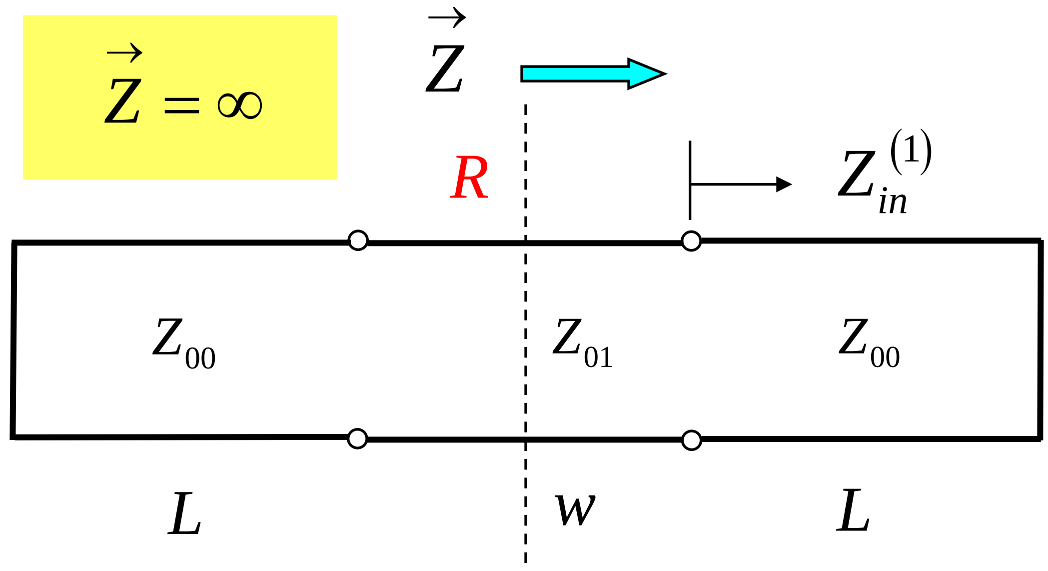
Odd mode: PEC wall



Even mode: PMC wall

Example: TE$_x$ Even Modes

$$\overrightarrow{Z} = Z_{01}\left[\frac{Z_{in}^{(1)} + jZ_{01}\tan\left(k_{x1}\frac{w}{2}\right)}{Z_{01} + jZ_{in}^{(1)}\tan\left(k_{x1}\frac{w}{2}\right)}\right]$$



Set

$$Z_{01} + jZ_{in}^{(1)}\tan\left(k_{x1}\frac{w}{2}\right) = 0$$

$$\Rightarrow \quad Z_{01} + j\left[jZ_{00}\tan(k_{x0}L)\right]\tan\left(k_{x1}\frac{w}{2}\right) = 0$$

$$\Rightarrow \quad Z_{01} - Z_{00}\tan(k_{x0}L)\tan\left(k_{x1}\frac{w}{2}\right) = 0$$

TE$_x$ Even Modes (cont.)

$$\frac{\omega\mu_0}{k_{x1}} - \frac{\omega\mu_0}{k_{x0}} \tan\left(k_{x0}L\right) \tan\left(k_{x1}\frac{w}{2}\right) = 0$$

or

$$k_{x0} - k_{x1} \tan(k_{x0}L) \tan\left(k_{x1}\frac{w}{2}\right) = 0$$

TE$_x$ even

where $k_{x0} = \left[k_0^{\,2} - k_z^2 - \left(\frac{n\pi}{b}\right)^2\right]^{1/2}$ $\quad k_{x1} = \left[k_0^{\,2}\varepsilon_r - k_z^2 - \left(\frac{n\pi}{b}\right)^2\right]^{1/2}$

Hence, the transcendental equation is in the form

$$F(k_z) = 0$$

Four Possible Cases

(We have only done one of them: TE$_x$ even.)

TE$_x$ even	TM$_x$ even
TE$_x$ odd	TM$_x$ odd

Cutoff Frequency (TE_x Even)

Set $\boxed{k_z = 0}$ (closed structure)

Then

$$k_{x0} = \left[k_0^2 - \cancel{k_z^2} - \left(\frac{n\pi}{b} \right)^2 \right]^{\frac{1}{2}} \qquad k_{x1} = \left[k_0^2 \varepsilon_r - \cancel{k_z^2} - \left(\frac{n\pi}{b} \right)^2 \right]^{\frac{1}{2}}$$

Hence, the transcendental equation is now in the form

$$\boxed{F(k_0) = 0}$$

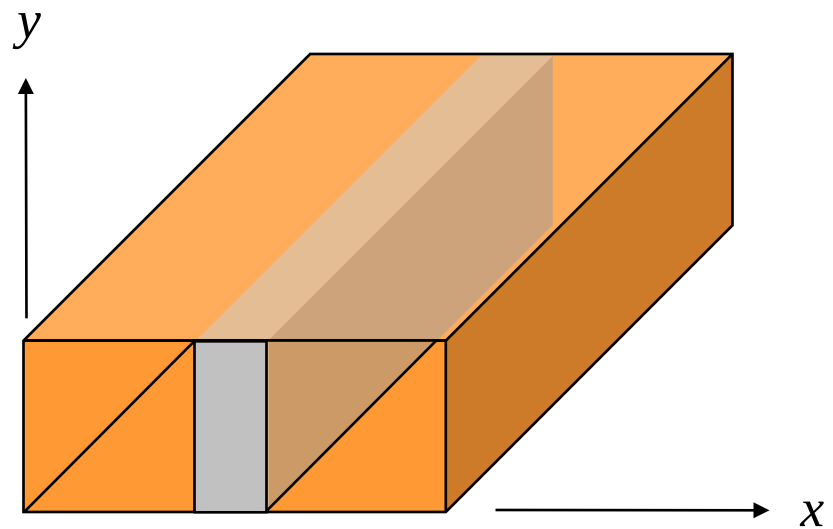
where

$$k_0 = 2\pi f_c \sqrt{\mu_0 \varepsilon_0}$$

f_c = cutoff frequency

LSE/LSM Terminology

This terminology is often used in the microwave community.



LSE: "Longitudinal Section Electric". This means the same thing as TE_x. The electric field vector of the mode has no x component, and hence it lies within the yz plane (This is called the "longitudinal plane," which means the plane parallel to the slab face.

LSM: "Longitudinal Section Magnetic". This means the same thing as TM_x. The magnetic field vector of the mode has no x component, and hence it lies within the yz plane (the "longitudinal plane).