ECE 6341

Spring 2016

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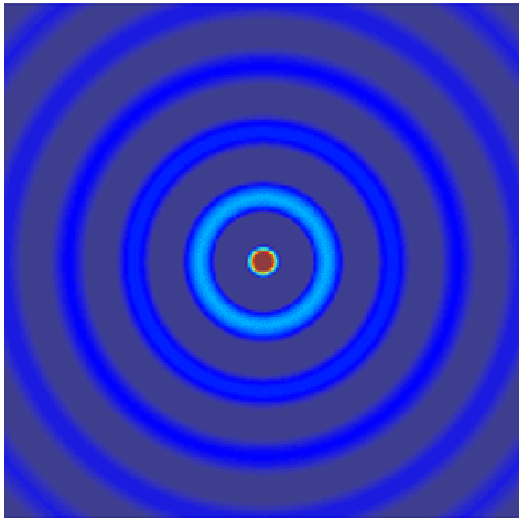
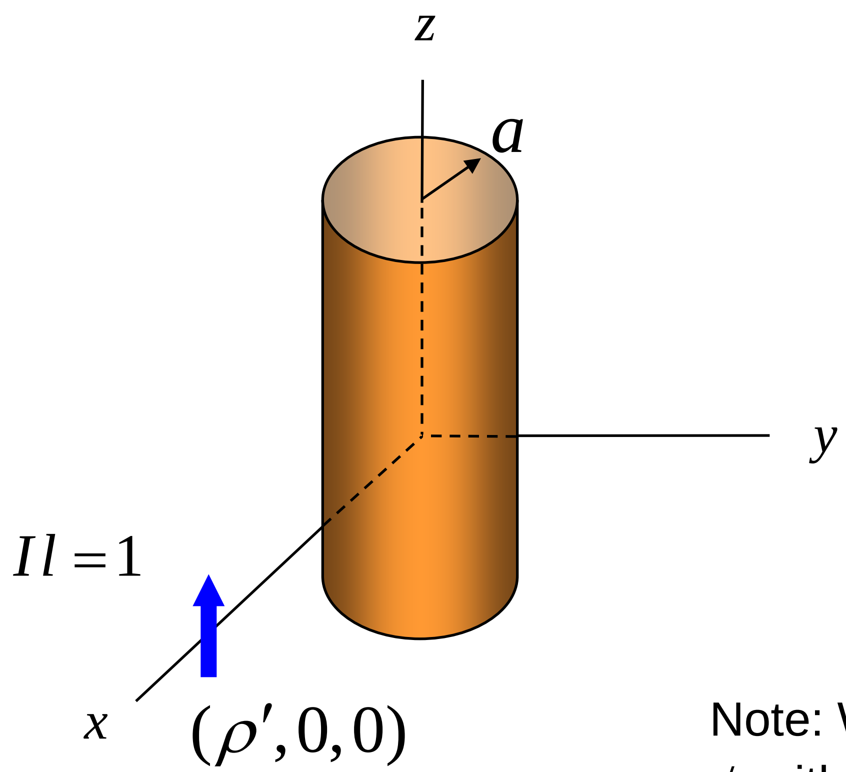


Notes 16

Dipole Scattering



Note: We can replace z with $z - z'$ and ϕ with $\phi - \phi'$ at the end, if we wish.

Dipole at origin (from Notes 15):

$$A_z^i = \frac{\mu_0}{8\pi j} \int_{-\infty}^{+\infty} H_0^{(2)}(k_\rho \rho)\, e^{-jk_z z}\, dk_z$$

Dipole Scattering (cont.)

Dipole at $(\rho', 0, 0)$

$$A_z^i = \frac{\mu_0}{8\pi j} \int_{-\infty}^{+\infty} H_0^{(2)}\left(k_\rho \left|\underline{\rho} - \underline{\rho}'\right|\right) e^{-jk_z z} dk_z$$

Now use the addition theorem: $(\rho < \rho')$

$$H_0^{(2)}\left(k_\rho \left|\underline{\rho} - \underline{\rho}'\right|\right) = \sum_{n=-\infty}^{+\infty} H_n^{(2)}(k_\rho \rho') J_n(k_\rho \rho) e^{jn\phi}$$

Hence

$$A_z^i = \frac{\mu_0}{8\pi j} \int_{-\infty}^{+\infty} \left(\sum_{n=-\infty}^{+\infty} H_n^{(2)}(k_\rho \rho') J_n(k_\rho \rho) e^{jn\phi} \right) e^{-jk_z z} dk_z$$

Valid for $\rho < \rho'$

Dipole Scattering (cont.)

Assume a scattered field (see the note below):

$$A_z^s = \frac{\mu_0}{8\pi j}\int_{-\infty}^{+\infty}\left(\sum_{n=-\infty}^{+\infty} f_n(k_z)H_n^{(2)}(k_\rho \rho)e^{jn\phi}\right)e^{-jk_z z}dk_z$$

B.C.'s: $A_z^s = -A_z^i$ at $\rho = a$

Hence

$$f_n(k_z)H_n^{(2)}(k_\rho a) = -H_n^{(2)}(k_\rho \rho')J_n(k_\rho a)$$

Note: We assume that the scattered field is TM_z. This follows from the fact that we can successfully satisfy the boundary conditions.

Dipole Scattering (cont.)

or

$$f_n(k_z) = -J_n(k_\rho a)\left(\frac{H_n^{(2)}(k_\rho \rho')}{H_n^{(2)}(k_\rho a)}\right)$$

We then have

$$A_z^s = \frac{\mu_0}{8\pi j}\int_{-\infty}^{+\infty}\left(\sum_{n=-\infty}^{+\infty} f_n(k_z)\, H_n^{(2)}(k_\rho \rho)\, e^{jn\phi}\right) e^{-jk_z z}\, dk_z$$

Dipole Scattering (cont.)

Far-Field Calculation

$$A_z = A_z^i + A_z^s$$

$$A_z^i = \frac{\mu_0}{4\pi} \frac{e^{-jk_0\left|\underline{r}-\underline{r}'\right|}}{\left|\underline{r}-\underline{r}'\right|}$$

From ECE 6340 (far-field approximation):

$$A_z^i \sim \frac{\mu_0}{4\pi}\left(\frac{e^{-jk_0 r}}{r}\right)e^{+jk_x x'}$$

or

$$A_z^i \sim \frac{\mu_0}{4\pi}\left(\frac{e^{-jk_0 r}}{r}\right)e^{+jk_0\rho'\sin\theta\cos\phi}$$

since $k_x = k_0 \sin\theta\cos\phi, \quad x' = \rho'$

Dipole Scattering (cont.)

Note: Instead of taking the curl of the incident vector potential, we can also evaluate the incident electric field directly, since we know how a single dipole in free space radiates.

From ECE 6340:

$$E_{\theta}^{i} \sim \frac{j\omega\mu_0}{4\pi}\left(\frac{e^{-jk_0 r}}{r}\right)\sin\theta\, e^{+jk_0\rho'\sin\theta\cos\phi}$$

Dipole Scattering (cont.)

Now we need to evaluate the scattered field in the far-zone.

$$A_z^s = \frac{\mu_0}{8\pi j} \int_{-\infty}^{+\infty} \left(\sum_{n=-\infty}^{+\infty} f_n(k_z) H_n^{(2)}(k_\rho \rho) e^{jn\phi} \right) e^{-jk_z z} dk_z$$

Switch the order of summation and integration:

$$A_z^s = \frac{\mu_0}{8\pi j} \sum_{n=-\infty}^{+\infty} e^{jn\phi} \int_{-\infty}^{+\infty} f_n(k_z) H_n^{(2)}(k_\rho \rho) \, e^{-jk_z z} dk_z$$

Dipole Scattering (cont.)

$$A_z^s = \frac{\mu_0}{8\pi j} \sum_{n=-\infty}^{+\infty} e^{jn\phi} \int_{-\infty}^{+\infty} f_n(k_z)\, H_n^{(2)}(k_\rho \rho)\, e^{-jk_z z} dk_z$$

Use the far-field Identity:

$$\int_{-\infty}^{+\infty} f(k_z)\, H_n^{(2)}(k_\rho \rho) e^{-jk_z z} dk_z \sim 2 j^{n+1} \left(\frac{e^{-jk_0 r}}{r} \right) f(k_0 \cos\theta)$$

Then we have

$$A_z^s \sim \frac{\mu_0}{8\pi j} \sum_{n=-\infty}^{+\infty} e^{jn\phi}\, 2 j^{n+1} \left(\frac{e^{-jk_0 r}}{r} \right) f_n(k_0 \cos\theta)$$

Dipole Scattering (cont.)

$$A_z^s \sim \frac{\mu_0}{8\pi j} \sum_{n=-\infty}^{+\infty} e^{jn\phi} 2 j^{n+1} \left(\frac{e^{-jk_0 r}}{r} \right) f_n(k_0 \cos\theta)$$

Hence, we have

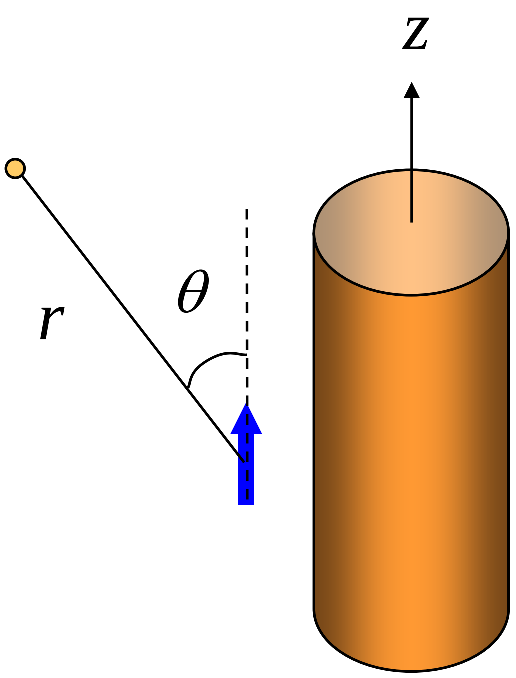
$$A_z^s \sim \frac{\mu_0}{4\pi} \left(\frac{e^{-jk_0 r}}{r} \right) \sum_{n=-\infty}^{+\infty} e^{jn\phi} j^n f_n(k_0 \cos\theta)$$

The scattered field is clearly in the form of a spherical wave.

Dipole Scattering (cont.)

We now calculate the scattered electric field as

$$r \to \infty$$



From ECE 6340:

In the far field of any antenna, we have

$$\underline{E} \sim -j\omega \underline{A}_t$$

so

$$\underline{E} \sim -j\omega\left(\underline{\hat{\theta}}\left(\underline{\hat{\theta}} \cdot \underline{A}\right) + \underline{\hat{\phi}}\left(\underline{\hat{\phi}} \cdot \underline{A}\right)\right)$$

or

$$\underline{E} \sim -j\omega\left(\underline{\hat{\theta}}\left(\underline{\hat{\theta}} \cdot \underline{\hat{z}} A_z\right) + \underline{\hat{\phi}}\left(\underline{\hat{\phi}} \cdot \underline{\hat{z}} A_z\right)\right)$$

$$\underline{\hat{z}} \cdot \underline{\hat{\theta}} = -\sin\theta$$

Dipole Scattering (cont.)

Hence

$$E_\theta^s \sim j\omega \sin\theta A_z^s$$

We thus have

$$E_\theta^s \sim j\omega\mu_0 \left(\sin\theta\right)\frac{1}{4\pi}\left(\frac{e^{-jk_0 r}}{r}\right)\sum_{n=-\infty}^{+\infty} e^{jn\phi} j^n f_n(k_0\cos\theta)$$

with

$$f_n(k_z) = -J_n(k_\rho a)\left(\frac{H_n^{(2)}(k_\rho \rho')}{H_n^{(2)}(k_\rho a)}\right)$$