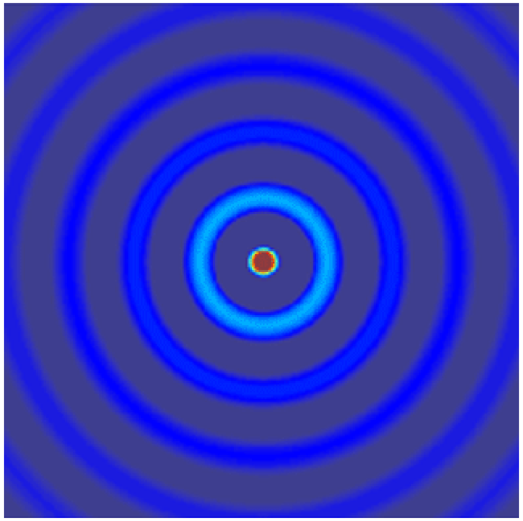
ECE 6341

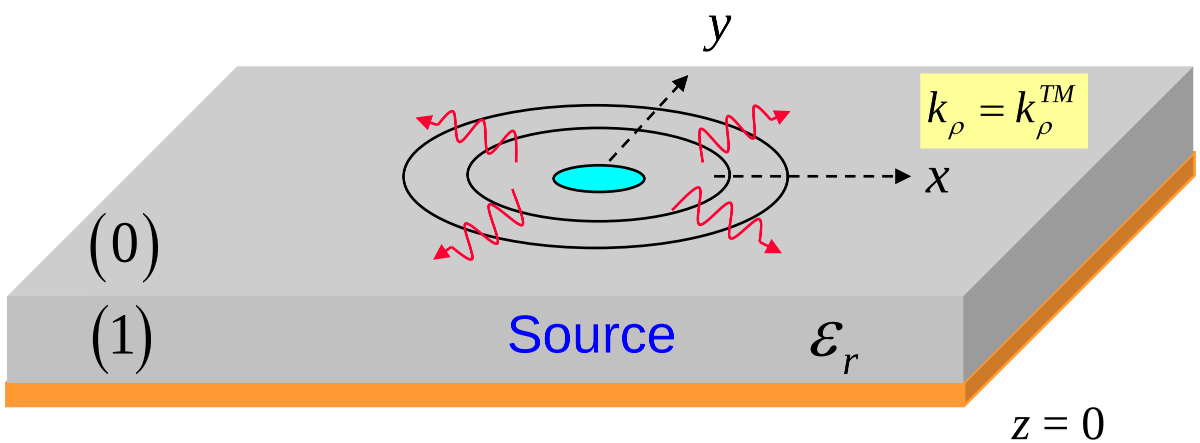
Spring 2016

Prof. David R. Jackson
ECE Dept.

Notes 19



Cylindrical Surface Waves



TM$_z$ SW:

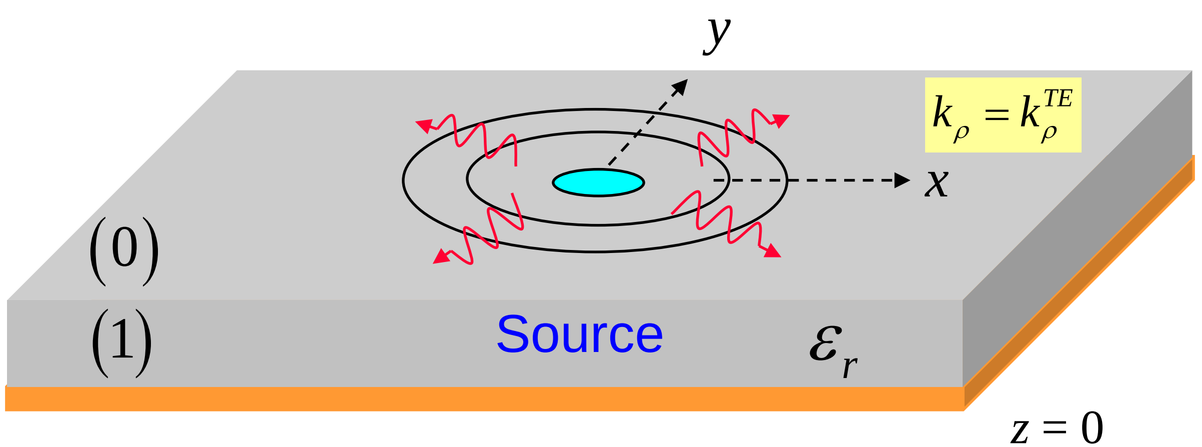
$$A_{z1} = \cos(n\phi) H_n^{(2)}(k_\rho \rho)\cos(k_{z1} z)$$

$$A_{z0} = A\cos(n\phi) H_n^{(2)}(k_\rho \rho) e^{-\alpha_{z0} z}$$

$$k_{z1} = \sqrt{k_1^2 - k_\rho^2} \qquad \alpha_{z0} = \sqrt{k_\rho^2 - k_0^{\,2}}$$

Note: The sin/cos choice is arbitrary here.

Cylindrical Surface Waves (cont.)



TE$_z$ SW:

$$F_{z1} = \sin(n\phi) H_n^{(2)}(k_\rho \rho) \sin(k_{z1} z)$$

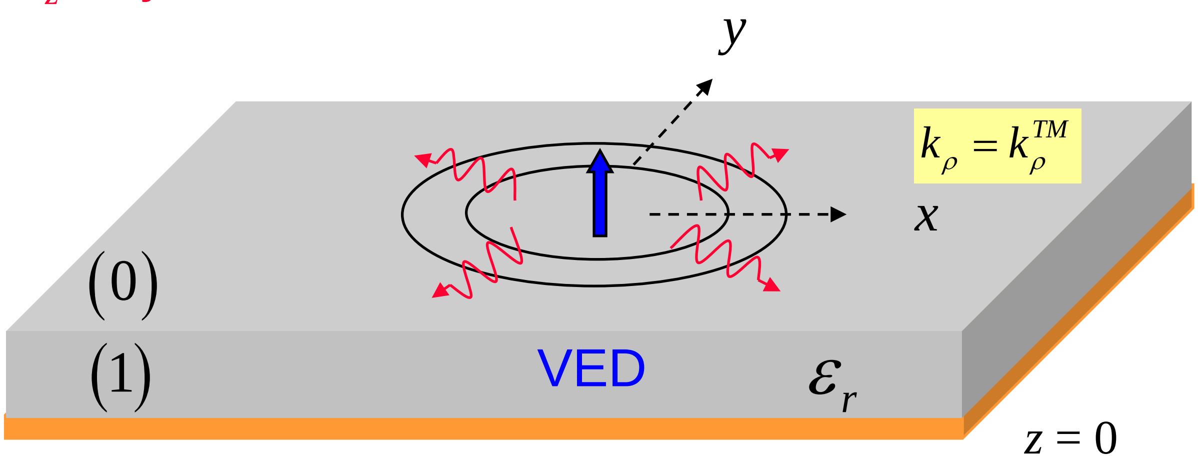
$$F_{z0} = B \sin(n\phi) H_n^{(2)}(k_\rho \rho) e^{-\alpha_{z0} z}$$

$$k_{z1} = \sqrt{k_1^2 - k_\rho^2} \qquad \alpha_{z0} = \sqrt{k_\rho^2 - k_0{}^2}$$

Note: The sin/cos choice is arbitrary here.

Cylindrical Surface Waves (cont.)

VED: TM_z only



$$A_{z1} = H_0^{(2)}(k_\rho \rho)\cos(k_{z1} z)$$

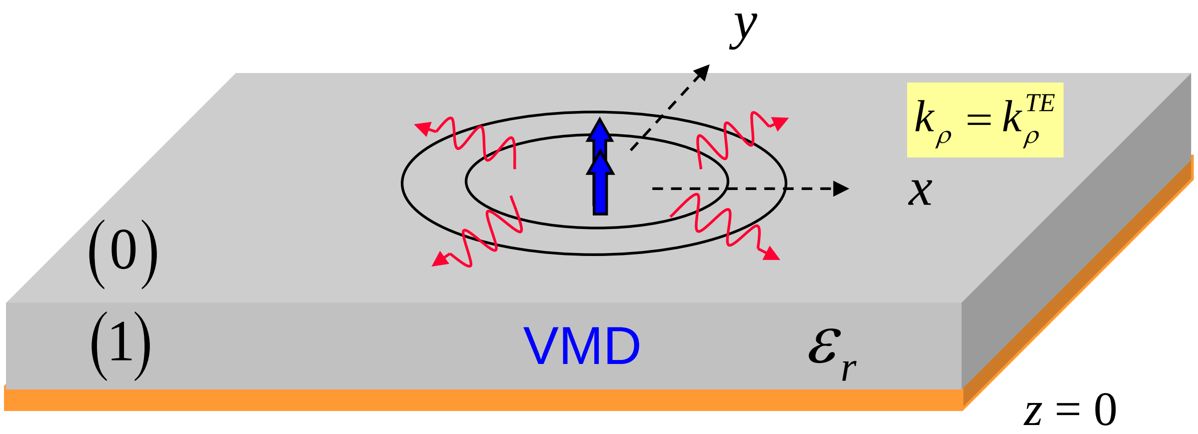
$$A_{z0} = A H_0^{(2)}(k_\rho \rho)\, e^{-\alpha_{z0} z}$$

$$k_{z1} = \sqrt{k_1^2 - k_\rho^2}$$

$$\alpha_{z0} = \sqrt{k_\rho^2 - k_0^{\,2}}$$

Cylindrical Surface Waves (cont.)

VMD: TE_z only



$$F_{z1} = H_0^{(2)}(k_\rho \rho)\sin(k_{z1} z)$$

$$F_{z0} = B\,H_0^{(2)}(k_\rho \rho)\,e^{-\alpha_{z0} z}$$

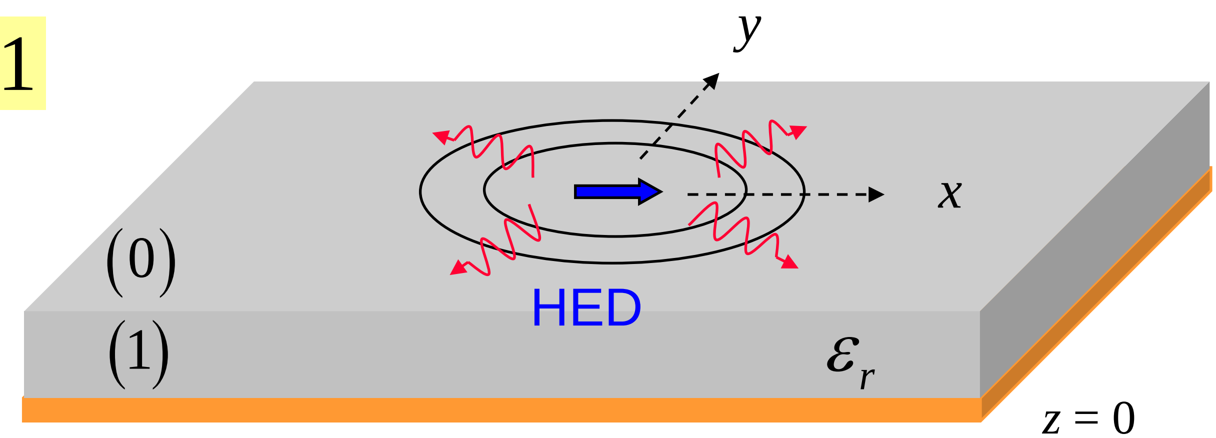
$$k_{z1} = \sqrt{k_1^2 - k_\rho^2}$$

$$\alpha_{z0} = \sqrt{k_\rho^2 - k_0^{\,2}}$$

Cylindrical Surface Waves (cont.)

HED: TM_z+TE_z (if there is a TE_z surface wave above cutoff).

$$n = 1$$



TM_z SW:

$$A_{z1} = \cos\phi\, H_1^{(2)}(k_\rho^{TM}\rho)\cos(k_{z1}z)$$

$$A_{z0} = A\cos\phi H_1^{(2)}(k_\rho^{TM}\rho)e^{-\alpha_{z0}z}$$

TE_z SW:

$$F_{z1} = \sin(\phi) H_1^{(2)}(k_\rho^{TE}\rho)\sin(k_{z1}z)$$

$$F_{z0} = B\sin(\phi) H_1^{(2)}(k_\rho^{TE}\rho)e^{-\alpha_{z0}z}$$

$$k_{z1} = \sqrt{k_1^2 - k_\rho^2} \qquad \alpha_{z0} = \sqrt{k_\rho^2 - k_0^{\,2}}$$

Cylindrical Surface Waves (cont.)

Consider a TM$_z$ surface wave that varies in a region as $e^{\mp jk_z z}$

$$E_\rho = \frac{1}{j\omega\mu\varepsilon}\frac{\partial^2 A_z}{\partial\rho\,\partial z}$$

$$H_\phi = -\frac{1}{\mu}\frac{\partial A_z}{\partial\rho}$$

(We could also examine E_ϕ and H_ρ.)

$$\frac{E_\rho}{H_\phi} = \frac{\frac{1}{j\omega\mu\varepsilon}\left(\mp jk_z\right)\frac{\partial A_z}{\partial\rho}}{-\frac{1}{\mu}\frac{\partial A_z}{\partial\rho}}$$

$$= \pm\frac{k_z}{\omega\varepsilon}$$

Note: The characteristic impedance is independent of the azimuthal index n and the radial distance ρ.

Hence

$$Z_0^{TM} = \frac{k_z}{\omega\varepsilon}$$

(Similarly for TE$_z$)

$$Z_0^{TE} = \frac{\omega\mu}{k_z}$$

Cylindrical Surface Waves (cont.)

Hence the same TRE is found as in the case of a 1D SW, with

$$k_z^2 = \left(k^2 - k_\rho^2\right)^{1/2}$$

instead of

$$k_z^2 = \left(k^2 - k_x^2\right)^{1/2}$$

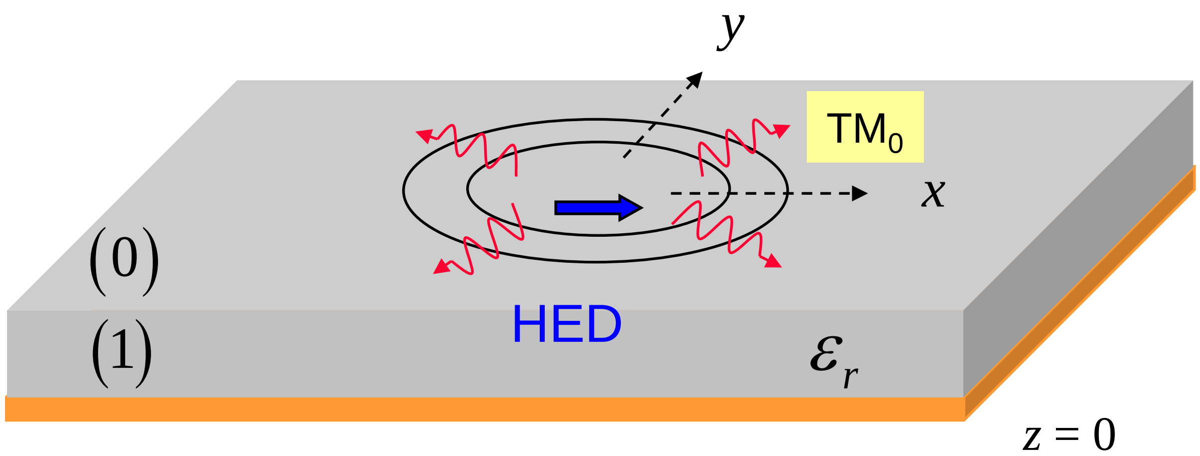
(Here x is the direction of propagation for a 1D SW.)

Hence

$$k_\rho^{\mathrm{cyl}} = k_x^{\mathrm{1D}}$$

Cylindrical Surface Waves (cont.)

Example: TM_0 surface wave from an HED source



$$k_\rho = k_\rho^{TM_0}$$

TM_0 SW:

$$A_{z1} = \cos\phi\, H_1^{(2)}(k_\rho \rho)\cos(k_{z1} z)$$

$$A_{z0} = A\cos\phi\, H_1^{(2)}(k_\rho \rho)\, e^{-\alpha_{z0} z}$$

$$k_{z1} = \sqrt{k_1^2 - k_\rho^2} \qquad \alpha_{z0} = \sqrt{k_\rho^2 - k_0^2}$$

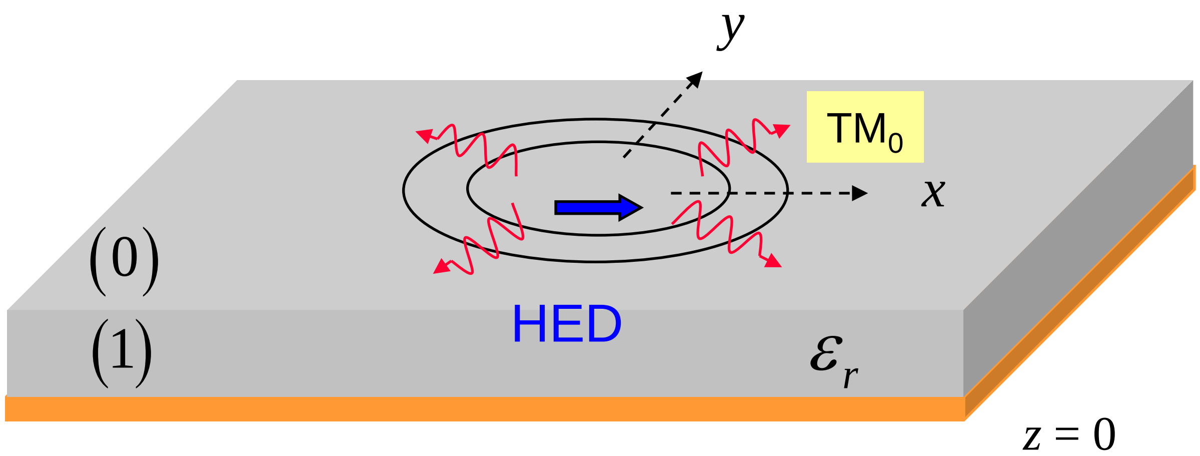
TRE:

$$\varepsilon_r = \frac{(k_1^2 - k_\rho^2)^{1/2}}{\sqrt{k_\rho^2 - k_0^2}} \tan\left((k_1^2 - k_\rho^2)^{1/2} h\right)$$

Use 1D TRE : $k_x \to k_\rho$

Cylindrical Surface Waves (cont.)

Example: TM_0 surface wave from an HED source



Far away from the source:

$$A_{z1} = \left(\sqrt{\frac{2}{\pi k_\rho}} e^{j\pi/4} \right) \cos\phi \left(\frac{e^{-jk_\rho \rho}}{\sqrt{\rho}} \right) \cos(k_{z1} z)$$

$$A_{z0} = A \left(\sqrt{\frac{2}{\pi k_\rho}} e^{j\pi/4} \right) \cos\phi \left(\frac{e^{-jk_\rho \rho}}{\sqrt{\rho}} \right) e^{-\alpha_{z0} z}$$

Recall:

$$H_\upsilon^{(2)}(x) \sim \sqrt{\frac{2}{\pi x}} e^{-j\left(x - \upsilon\frac{\pi}{2} - \frac{\pi}{4}\right)}$$