ECE 6341

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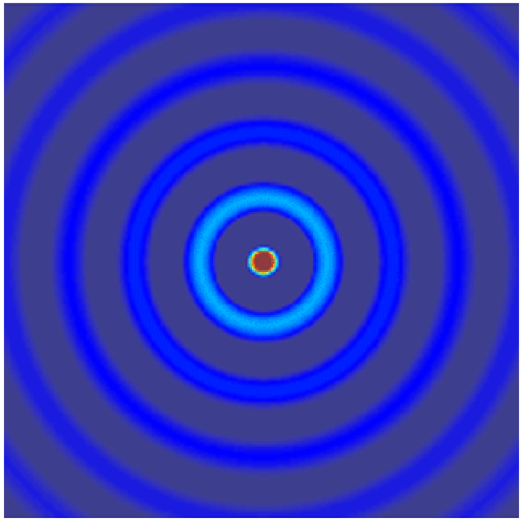
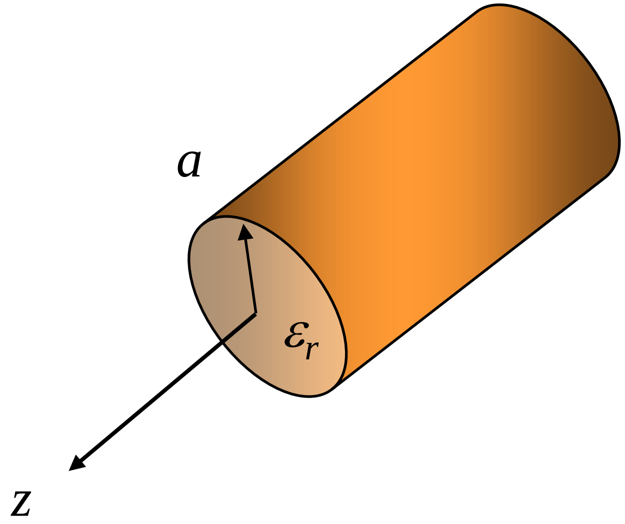
Notes 9



Circular Waveguide

The waveguide is homogeneously filled,
so we have independent TE_z and TM_z modes.



TM$_z$ mode:

$$A_z = \psi\left(\rho, \phi, z\right)$$

$$\psi = \begin{Bmatrix} J_\upsilon(k_\rho \rho) \\ Y_\upsilon(k_\rho \rho) \end{Bmatrix} \begin{Bmatrix} \sin(\upsilon\phi) \\ \cos(\upsilon\phi) \end{Bmatrix} e^{-jk_z z}$$

$$k_\rho^2 = k^2 - k_z^2$$

Circular Waveguide (cont.)

(1) ϕ variation $\quad \phi \in [0, 2\pi]$

$$\psi(\rho, \phi + 2\pi, z) = \psi(\rho, \phi, z)$$

(uniqueness of solution)

$$\Rightarrow \upsilon = n$$

Choose $\cos(n\phi)$

$$\psi = \begin{Bmatrix} J_n(k_\rho \rho) \\ Y_n(k_\rho \rho) \end{Bmatrix} \cos(n\phi)\, e^{-jk_z z}$$

Circular Waveguide (cont.)

(2) The field should be finite on the z axis $\psi\left(0,\phi,z\right)$

⇨ $Y_n(k_\rho \rho)$ is not allowed

$$\psi = \cos(n\phi)\, J_n(k_\rho \rho)\, e^{-jk_z z}$$

$$k_\rho^2 = k^2 - k_z^2$$

Circular Waveguide (cont.)

(3) B.C.'s: $E_z\left(a,\phi,z\right)=0$

$$E_z=\frac{1}{j\omega\mu\varepsilon}\left(k^2+\frac{\partial^2}{\partial z^2}\right)\psi$$

$$=\frac{1}{j\omega\mu\varepsilon}\left(k^2-k_z^2\right)\psi$$

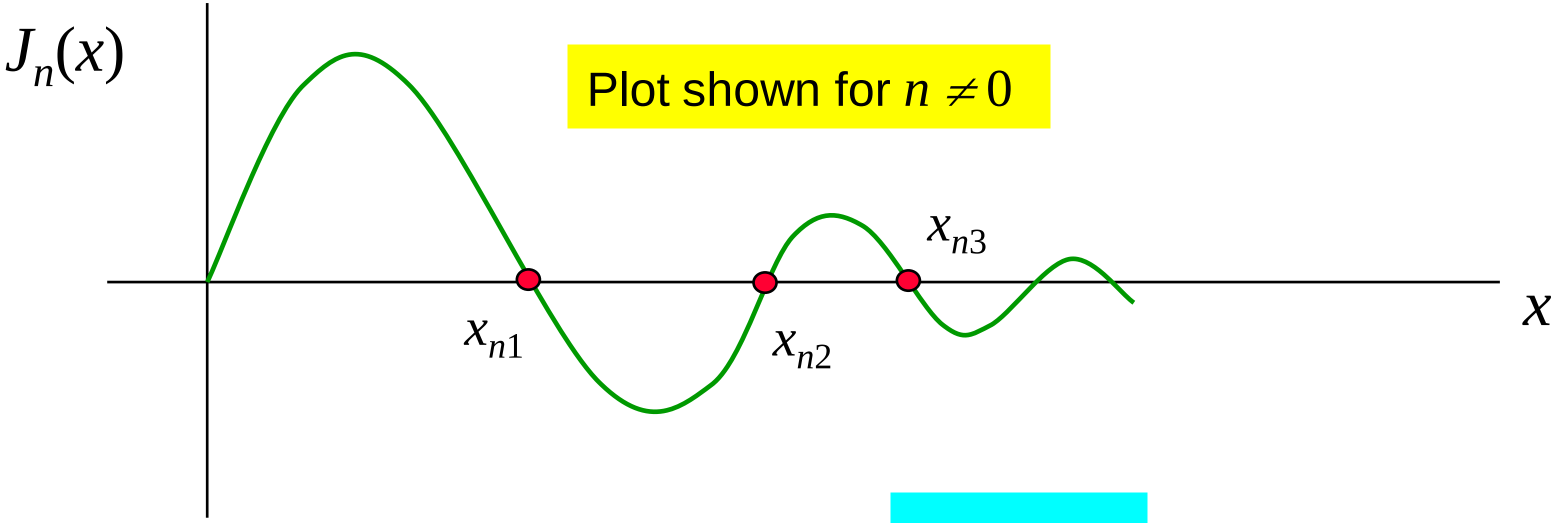
$$=\left(\frac{k_\rho^2}{j\omega\mu\varepsilon}\right)\psi$$

so $\psi(a,\phi,z)=0$

Hence $J_n(k_\rho a)=0$

Circular Waveguide (cont.)

$$J_n(k_\rho a) = 0$$



$$k_\rho a = x_{np} \Rightarrow k_\rho = \frac{x_{np}}{a}$$

Note: $x_{n0} = 0$ is not included since $J_n\left(x_{np}\frac{\rho}{a}\right) = 0$ (trivial soln.)

Circular Waveguide (cont.)

TM$_{np}$ mode:

$$A_z = \cos(n\phi)\, J_n\left(x_{np}\frac{\rho}{a}\right) e^{-jk_z z} \qquad n = 0, 1, 2\ldots$$

$$k_z = \left(k^2 - \left(\frac{x_{np}}{a}\right)^2\right)^{1/2} \qquad p = 1, 2, 3, \ldots\ldots\ldots$$

Cutoff Frequency: TM$_z$

$$k_z^2 = k^2 - k_\rho^2$$

$$k_z = 0 \quad \Rightarrow \quad k = k_\rho = \frac{x_{np}}{a}$$

$$2\pi f_c \sqrt{\mu\varepsilon} = \frac{x_{np}}{a}$$

$$f_c^{\,TM} = \left(\frac{c}{2\pi a \sqrt{\varepsilon_r}} \right) x_{np}$$

Cutoff Frequency: TM_z (cont.)

x_{np} values

$p \setminus n$	0	1	2	3	4	5
1	2.405	3.832	5.136	6.380	7.588	8.771
2	5.520	7.016	8.417	9.761	11.065	12.339
3	8.654	10.173	11.620	13.015	14.372	
4	11.792	13.324	14.796			

TM_{01}, TM_{11}, TM_{21}, TM_{02}, ……..

TE_z Modes

$$F_z = \psi\left(\rho, \phi, z\right)$$

$$\psi = \cos(n\phi)\, J_n(k_\rho \rho)\, e^{-jk_z z}$$

$$H_z = \frac{k_\rho^2}{j\omega\mu\varepsilon}\psi$$

Note: $\psi\left(a, \phi, z\right) \neq 0$

TE_z Modes (cont.)

Set

$$E_\phi\left(a,\phi,z\right)=0$$

$$E_\phi=\frac{1}{\varepsilon}\frac{\partial\psi}{\partial\rho}$$

so

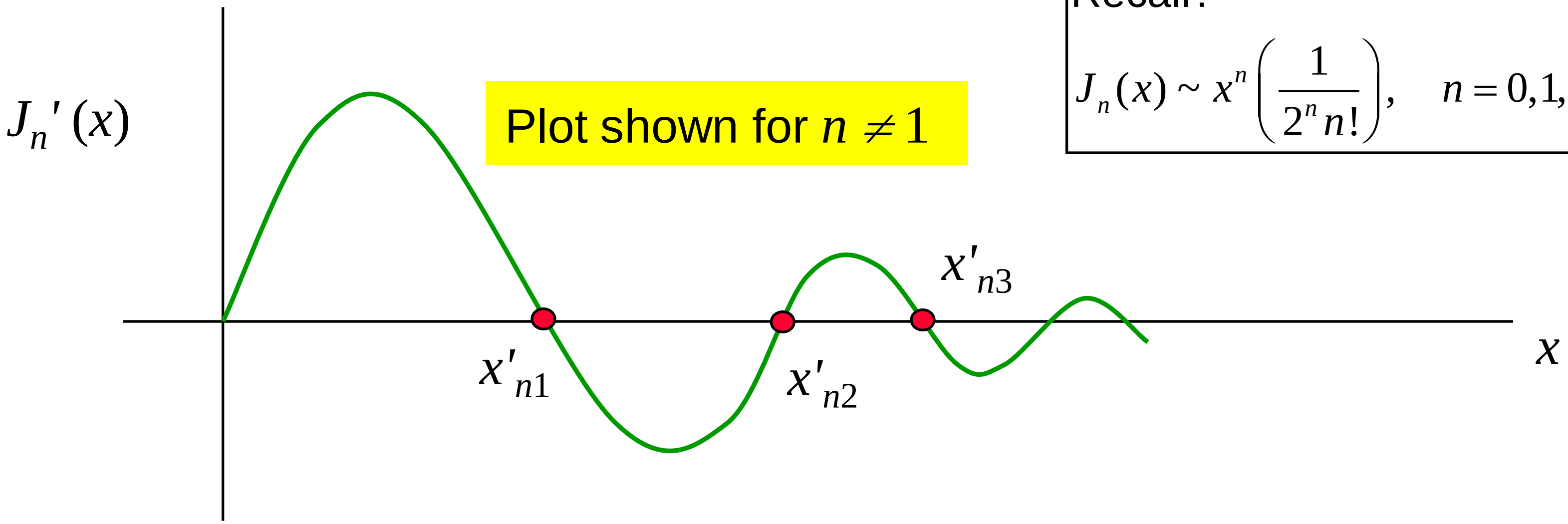
$$\left.\frac{\partial\psi}{\partial\rho}\right|_{\rho=a}=0$$

Hence

$$J_n'(k_\rho a)=0$$

TE_z Modes (cont.)

$$J_n'(k_\rho a) = 0$$



Recall:

$$J_n(x) \sim x^n \left(\frac{1}{2^n n!} \right), \quad n = 0, 1, 2, \ldots.$$

$$k_\rho a = x'_{np}$$

$$k_\rho = \frac{x'_{np}}{a} \qquad p = 1, 2, 3, \ldots..$$

Note: $p = 0$ is not included (see next slide).

TE_z Modes (cont.)

$$\psi = \cos(n\phi)\, J_n\left(x'_{np}\,\frac{\rho}{a} \right) e^{-jk_z z} \qquad p = 1, 2, \ldots$$

Note: If $p = 0$ $\quad x'_{np} = 0$

$$n \neq 0 \qquad J_n\left(x'_{np}\,\frac{\rho}{a} \right) = J_n(0) = 0 \qquad \text{(trivial soln.)}$$

$$n = 0 \qquad J_0\left(x'_{np}\,\frac{\rho}{a} \right) = J_0(0) = 1$$

$$\Rightarrow \quad \psi = \underbrace{e^{-jk_z z} = e^{-jkz}}_{k_\rho = 0} \qquad \text{(trivial fields)}$$

Cutoff Frequency: TE$_z$

$$k_z^2 = k^2 - k_\rho^2$$

$$k_z = 0 \quad \Rightarrow \quad k_\rho = k = \frac{x'_{np}}{a}$$

$$2\pi f_c \sqrt{\mu\varepsilon} = \frac{x'_{np}}{a}$$

$$f_c^{TE} = \left(\frac{c}{2\pi a \sqrt{\varepsilon_r}} \right) x'_{np}$$

Cutoff Frequency:TE$_z$

x'_{np} values

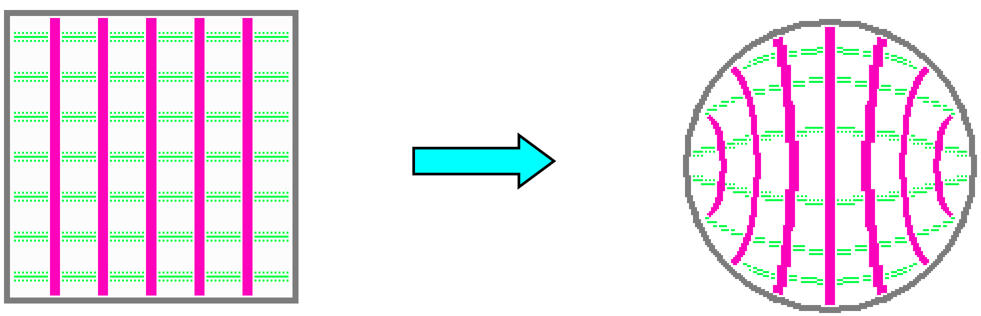
$p \backslash n$	0	1	2	3	4	5
1	3.832	1.841	3.054	4.201	5.317	5.416
2	7.016	5.331	6.706	8.015	9.282	10.520
3	10.173	8.536	9.969	11.346	12.682	13.987
4	13.324	11.706	13.170			

TE_{11}, TE_{21}, TE_{01}, TE_{31}, ……..

TE_{11} Mode

The dominant mode of circular waveguide is the TE_{11} mode.

Electric field
Magnetic field



(from Wikipedia)

TE_{10} mode of rectangular waveguide

TE_{11} mode of circular waveguide

The TE_{11} mode can be thought of as an evolution of the TE_{10} mode of rectangular waveguide as the boundary changes shape.

Attenuation Property of TE_{01} Mode

Goal: We wish to study the high-frequency dependence of attenuation on frequency for circular waveguide modes, and show the interesting behavior of the TE_{01} mode (the loss decreases as frequency increases).

$$\alpha_c = \frac{<\mathscr{P}_d>}{2<\mathscr{P}_f>}$$

$$R_s = \frac{1}{\sigma\delta} = \frac{1}{\sigma\sqrt{\frac{2}{\omega\mu\sigma}}} = \sqrt{\frac{\omega\mu}{2\sigma}} = \mathscr{O}\left(\sqrt{\omega}\right)$$

$$<\mathscr{P}_d> = \oint_C \frac{1}{2} R_s \left|\underline{H}_t\right|^2 dl$$

TE_z Mode:

$$H_\phi = \frac{1}{j\omega\mu\varepsilon}\frac{1}{\rho}\frac{\partial^2 F_z}{\partial\phi\,\partial z}$$

$$H_z = \frac{1}{j\omega\mu\varepsilon}k_\rho^2 F_z$$

Assume that F_z is order 1 as the frequency increases.

Recall that k_ρ is a constant.

Attenuation Property (cont.)

$$H_z = \mathscr{O}\left(\frac{1}{\omega}\right)$$

$$H_\phi = \begin{cases} \mathscr{O}(1) & n \neq 0 \\ 0 & n = 0 \end{cases}$$

Note:

$$k_z = \mathscr{O}(k) = \mathscr{O}(\omega)$$

$n \neq 0$

$$< \mathscr{P}_d > = \mathscr{O}\left(\sqrt{\omega}\right)\left[\mathscr{O}\left(\frac{1}{\omega^2}\right) + \mathscr{O}(1)\right]$$
$$= \mathscr{O}\left(\omega^{1/2}\right)$$

$n = 0$

$$< \mathscr{P}_d > = \mathscr{O}\left(\sqrt{\omega}\right)\left[\mathscr{O}\left(\frac{1}{\omega^2}\right) + 0\right]$$
$$= \mathscr{O}\left(\omega^{-3/2}\right)$$

Attenuation Property (cont.)

From the TE_z table:

$$\underline{E} = \mathcal{O}(1) \qquad \text{e.g.} \qquad E_\phi = \frac{1}{\varepsilon}\frac{\partial F_z}{\partial \rho}$$

$$\underline{H} = \mathcal{O}(1) \qquad \text{e.g.} \qquad H_\rho = \frac{1}{j\omega\mu\varepsilon}\frac{\partial^2 F_z}{\partial \rho\, \partial z}$$

Hence $\left\langle \mathscr{P}_f \right\rangle = \mathcal{O}\left(1\right)$

Attenuation Property (cont.)

Hence

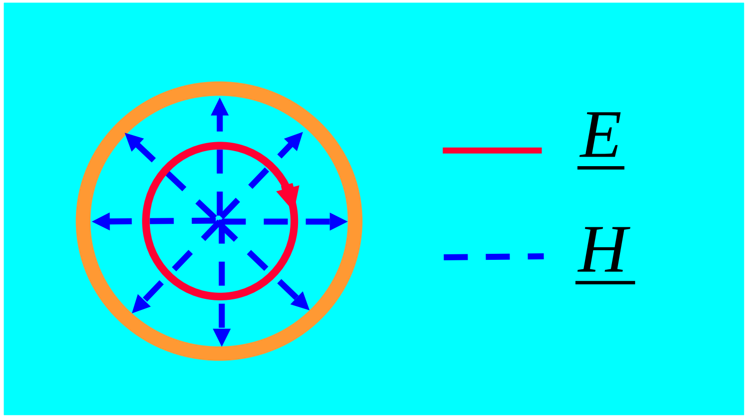
Usual behavior for rectangular waveguides

If $n \neq 0$: $$\alpha_c = \mathcal{O}\left(\omega^{1/2}\right)$$

If $n = 0$: $$\alpha_c = \mathcal{O}\left(\omega^{-3/2}\right)$$

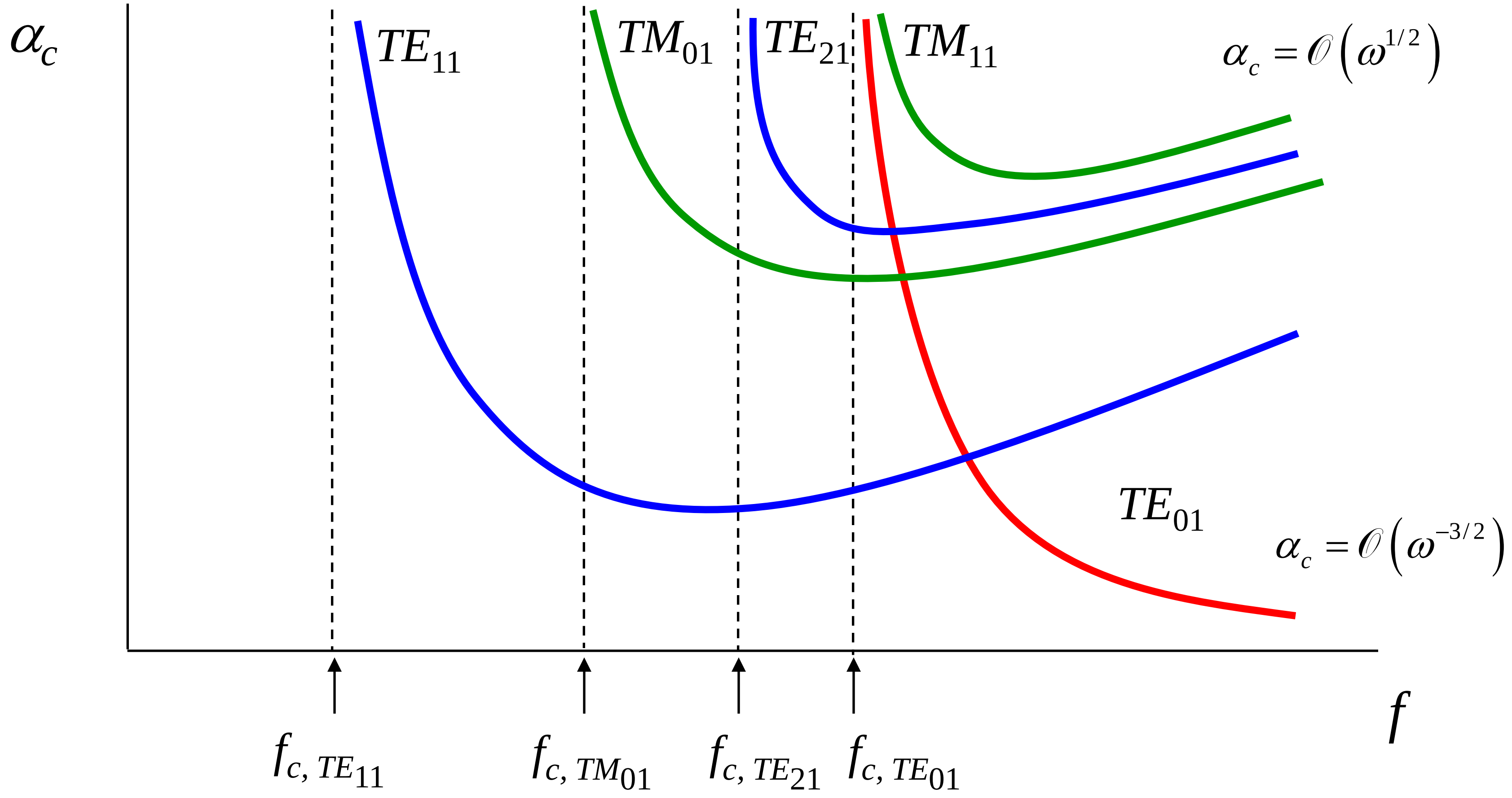
Decreases with frequency!

$n = 0$:



Note: The mode TE_{0p} mode can be supported by a series of concentric rings, since there is no longitudinal (z-directed) current ($H_\phi = 0$).

Attenuation Property (cont.)



Attenuation Property (cont.)

The TE_{01} mode was studied extensively as a candidate for long-range communications – but was not competitive with antennas. Also, fiber-optic cables eventually became available with lower loss than the TE_{01} mode. It is still useful for some applications (e.g., high power).

From the beginning, the most obvious application of waveguides had been as a communications medium. It had been determined by both Schelkunoff and Mead, independently, in July 1933, that an axially symmetric electric wave (TE_{01}) in circular waveguide would have an attenuation factor that decreased with increasing frequency [44]. This unique characteristic was believed to offer a great potential for wide-band, multichannel systems, and for many years to come the development of such a system was a major focus of work within the waveguide group at BTL. It is important to note, however, that the use of waveguide as a long transmission line never did prove to be practical, and Southworth eventually began to realize that the role of waveguide would be somewhat different than originally expected. In a memorandum dated October 23, 1939, he concluded that microwave radio with highly directive antennas was to be preferred to long transmission lines. “Thus,” he wrote, “we come to the conclusion that the hollow, cylindrical conductor is to be valued primarily as a new circuit element, but not yet as a new type of toll cable” [45]. It was as a circuit element in military radar that waveguide technology was to find its first major application and to receive an enormous stimulus to both practical and theoretical advance.

K. S. Packard, “The Origins of Waveguide: A Case of Multiple Rediscovery,” *IEEE Trans. MTT*, pp. 961-969, Sept. 1984.

Attenuation Property (cont.)

VertexRSI's Torrance Facility is a leading supplier of antenna feed components for the various commercial and military bands. A patented circular polarized 4-port diplexer meeting all Intelsat specifications leads a full array of products.

Products include:
4-Port Diplexers, CP or Linear;
3-Port Diplexers, 2xRx & 1xTx;
2-Port Diplexers, RxTx, X-Pol or Co-Pol, CP or Linear;
TE21 Monopulse Tracking Couplers;
TE01 Mode Components; Transitions;
Filters; Flex Waveguides;
Waveguide Bends; Twists; Runs; etc.

Many of the items are "off the shelf products".

Products can be custom tailored to a customer's application.

Many of the products can be supplied with standard feed horns for prime or offset antennas.