ECE 6341

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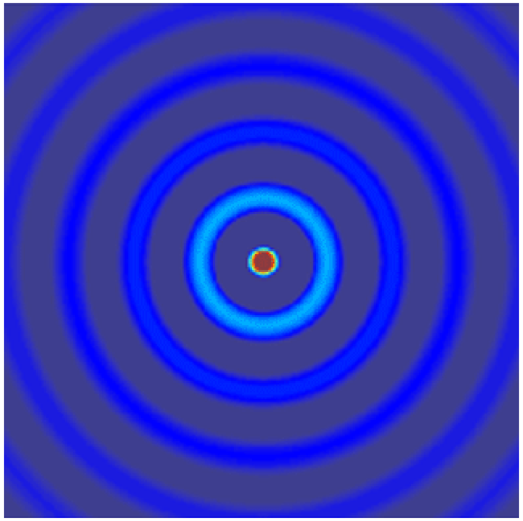
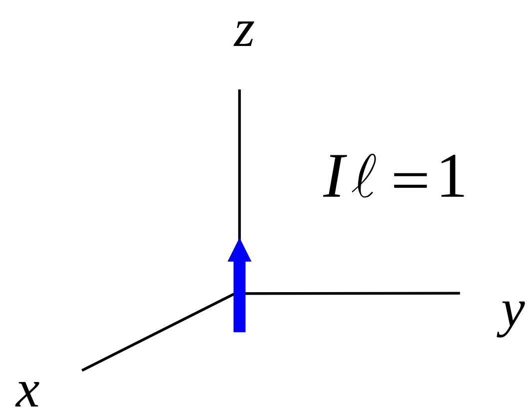


Notes 24

Dipole Radiation



Using the Lorenz gauge (the solution from ECE 6340):

$$\nabla^2 \underline{A}^{LG} + k^2 \underline{A}^{LG} = -\mu \underline{J}^{i}$$

$$\nabla^2 A_z^{LG} + k^2 A_z^{LG} = -\mu J_z = -\mu\, \delta(x)\,\delta(y)\,\delta(z)$$

$$\underline{A}^{LG} = \underline{\hat{z}}\, A_z^{LG}$$

$$A_z^{LG} = \mu \frac{e^{-jkr}}{4\pi r}$$

Note: This is not the same magnetic vector potential that we get using the Debye potential!

Dipole Radiation (cont.)

The exact fields are (from ECE 6340):

$$E_r = \frac{1}{2\pi} e^{-jkr} \left(\frac{\eta}{r^2} + \frac{1}{j\omega\varepsilon r^3} \right) \cos\theta$$

$$E_\theta = \frac{1}{4\pi} e^{-jkr} \left(\frac{j\omega\mu}{r} + \frac{\eta}{r^2} + \frac{1}{j\omega\varepsilon r^3} \right) \sin\theta$$

$$H_\phi = \frac{1}{4\pi} e^{-jkr} \left(\frac{jk}{r} + \frac{1}{r^2} \right) \sin\theta$$

Observation: $H_r = 0$ (TM$_r$)

Dipole Radiation (cont.)

TM$_r$

$$A_r = A P_n(\cos\theta)\,\hat{H}_n^{(2)}(kr)$$

Note: $m = 0$ (no ϕ variation)

No Q_n ($\pm z$ axis included)

$\upsilon = n$ ($-z$ axis included)

Examine some values of n:

$$P_0(\cos\theta) = 1$$

$$P_1(\cos\theta) = \cos\theta$$

$$P_2(\cos\theta) = \frac{1}{4}(3\cos 2\theta + 1)$$

Dipole Radiation (cont.)

From the TM$_r$ table: $$E_r = \frac{1}{j\omega\mu\varepsilon}\left(\frac{\partial^2}{\partial r^2}+k^2\right)A_r$$

From the ECE 6340 solution, $E_r \propto \cos\theta$

Hence $A_r \propto \cos\theta$

Hence, choose $n = 1$

Recall: $P_1\left(\cos\theta\right) = \cos\theta$

Dipole Radiation (cont.)

Therefore, we have:

$$A_r = A\cos\theta \hat{H}_1^{(2)}(kr)$$

Next, simplify the Schelkunoff Hankel function:

$$\hat{H}_1^{(2)}(x) = x\sqrt{\frac{\pi}{2x}} H_{3/2}^{(2)}(x) = \sqrt{\frac{\pi x}{2}}\left[J_{3/2}(x) - jY_{3/2}(x)\right]$$

$$Y_\upsilon(x) = \frac{J_\upsilon(x)\cos\upsilon\pi - J_{-\upsilon}(x)}{\sin\upsilon\pi} \quad \Longrightarrow \quad Y_{3/2}(x) = J_{-3/2}(x)$$

Dipole Radiation (cont.)

We have (from the Schaum's Math Handbook):

$$J_{3/2}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{\sin x}{x} - \cos x\right)$$

$$J_{-3/2}(x) = \sqrt{\frac{2}{\pi x}}\left(-\frac{\cos x}{x} - \sin x\right)$$

Hence

$$\hat{H}_1^{(2)}(x) = \sqrt{\frac{\pi x}{2}}\left[\sqrt{\frac{2}{\pi x}}\left(\frac{\sin x}{x} - \cos x\right) - j\sqrt{\frac{2}{\pi x}}\left(-\frac{\cos x}{x} - \sin x\right)\right]$$

Dipole Radiation (cont.)

$$\hat{H}_1^{(2)}(x)=\sqrt{\frac{\pi x}{2}}\left[\sqrt{\frac{2}{\pi x}}\left(\frac{\sin x}{x}-\cos x\right)-j\sqrt{\frac{2}{\pi x}}\left(-\frac{\cos x}{x}-\sin x\right)\right]$$

Hence

$$\begin{aligned}\hat{H}_1^{(2)}(x)&=\left(\frac{\sin x}{x}-\cos x\right)-j\left(-\frac{\cos x}{x}-\sin x\right)\\&=(-\cos x+j\sin x)+\frac{j}{x}(\cos x-j\sin x)\\&=-\left[(\cos x-j\sin x)-\frac{j}{x}(\cos x-j\sin x)\right]\\&=-\left(1-\frac{j}{x}\right)e^{-jx}\end{aligned}$$

so

$$\hat{H}_1^{(2)}(x)=-\left(1-\frac{j}{x}\right)e^{-jx}$$

Dipole Radiation (cont.)

Hence

$$A_r = A\cos\theta\left[-\left(1-\frac{j}{kr}\right)e^{-jkr}\right]$$

The final step is to determine the coefficient A.

Dipole Radiation (cont.)

Compare the E_r field: $E_r = \frac{1}{j\omega\mu\varepsilon}\left(\frac{\partial^2}{\partial r^2} + k^2\right)A_r$

$$E_r = \frac{1}{j\omega\mu\varepsilon}\left(\frac{\partial^2}{\partial r^2} + k^2\right)\left[A\cos\theta(-1)\left(1 - \frac{j}{kr}\right)e^{-jkr}\right] \quad \text{(from TM}_r\text{ table)}$$

$$= \frac{1}{2\pi}e^{-jkr}\left(\frac{\eta}{r^2} + \frac{1}{j\omega\varepsilon r^3}\right)\cos\theta \quad \text{(from ECE 6340 solution)}$$

The final result is

$$A = -\frac{j\mu k}{4\pi}$$

Dipole Radiation (cont.)

Hence we have

$$A_r = \frac{j\mu k}{4\pi}\cos\theta\left(1-\frac{j}{kr}\right)e^{-jkr}$$

Dipole Radiation (cont.)

Comparison

Debye potentials (using Debye Gauge):

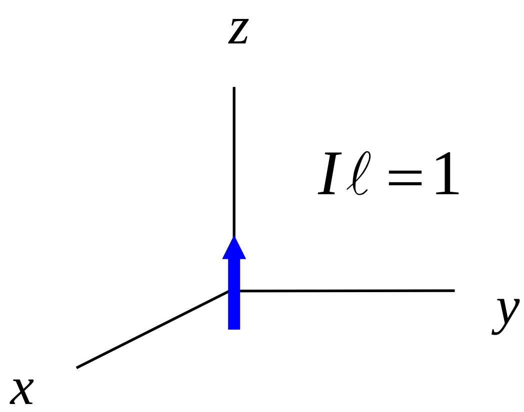
$$\underline{A} = \underline{\hat{r}} \left(\frac{j\mu k}{4\pi} \right) \cos\theta \left(1 - \frac{j}{kr} \right) e^{-jkr}$$

Lorenz Gauge:

$$\underline{A} = \underline{\hat{z}} \left(\frac{\mu}{4\pi r} \right) e^{-jkr}$$

Dipole Radiation (cont.)

Although the potentials are completely different, the fields must be the same!



$$\underline{H} = \frac{1}{\mu} \nabla \times \underline{A}$$

$$\underline{E} = \frac{1}{j\omega\varepsilon} \nabla \times \underline{H}$$

$$\underline{A} = \underline{\hat{z}} \left(\frac{\mu}{4\pi r} \right) e^{-jkr}$$

$$\underline{A} = \underline{\hat{r}} \left(\frac{j\mu k}{4\pi} \right) \cos\theta \left(1 - \frac{j}{kr} \right) e^{-jkr}$$