ECE 6341

Spring 2016

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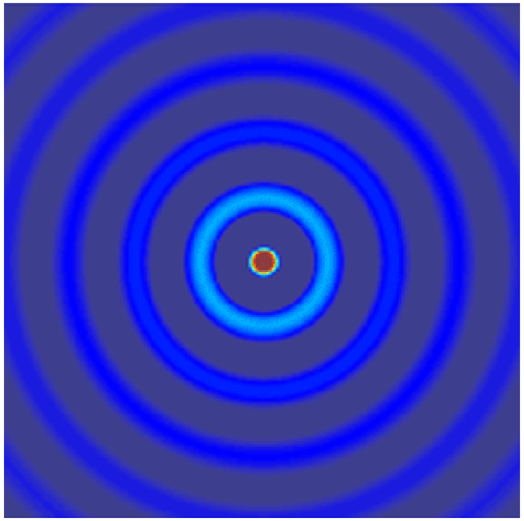
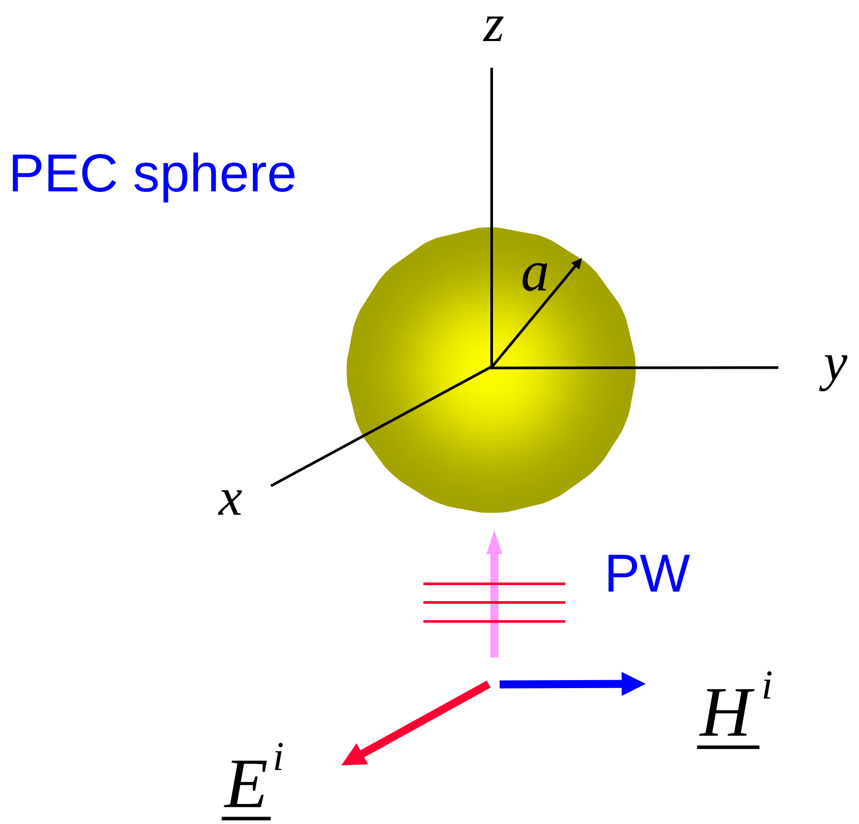


Notes 27

Scattering by Sphere



$$\underline{E}^i = \underline{\hat{x}}\, E_0 e^{-jkz} \qquad \underline{H}^i = \underline{\hat{y}} \frac{E_0}{\eta} e^{-jkz}$$

Scattering by Sphere (cont.)

$$A_r^i = E_0 \cos\phi \sum_{n=0}^{\infty} c_n \hat{J}_n\left(kr\right) P_n^1\left(\cos\theta\right)$$

$$F_r^i = \frac{E_0}{\eta} \sin\phi \sum_{n=0}^{\infty} c_n \hat{J}_n\left(kr\right) P_n^1\left(\cos\theta\right)$$

where

$$c_n = \frac{1}{\omega} \frac{\left(-j\right)^n \left(2n+1\right)}{n(n+1)}$$

Scattering by Sphere (cont.)

Let

$$A_r = A_r^i + A_r^s$$

$$F_r = F_r^i + F_r^s$$

where

$$A_r^s = E_0 \cos\phi \sum_{n=0}^{\infty} d_n \hat{H}_n^{(2)}(kr) P_n^1(\cos\theta)$$

$$F_r^s = \frac{E_0}{\eta} \sin\phi \sum_{n=0}^{\infty} e_n \hat{H}_n^{(2)}(kr) P_n^1(\cos\theta)$$

Examine the field components to determine the boundary conditions.

Scattering by Sphere (cont.)

For example,

$$E_\theta = \frac{-1}{\varepsilon r \sin\theta}\frac{\partial F_r}{\partial \phi} + \frac{1}{j\omega\mu\varepsilon r}\frac{\partial^2 A_r}{\partial r \partial \theta}$$

$$\Rightarrow \qquad F_r = 0\Big|_{r=a} \qquad \frac{\partial A_r}{\partial r} = 0\Big|_{r=a}$$

Scattering by Sphere (cont.)

$$A_r^i = E_0 \cos\phi \sum_{n=0}^{\infty} c_n \hat{J}_n(kr) P_n^1(\cos\theta)$$

$$F_r^i = \frac{E_0}{\eta} \sin\phi \sum_{n=0}^{\infty} c_n \hat{J}_n(kr) P_n^1(\cos\theta)$$

$$A_r^s = E_0 \cos\phi \sum_{n=0}^{\infty} d_n \hat{H}_n^{(2)}(kr) P_n^1(\cos\theta)$$

$$F_r^s = \frac{E_0}{\eta} \sin\phi \sum_{n=0}^{\infty} e_n \hat{H}_n^{(2)}(kr) P_n^1(\cos\theta)$$

$$F_r = 0\Big|_{r=a} \qquad \frac{\partial A_r}{\partial r} = 0\Big|_{r=a}$$

Hence

$$d_n = -c_n \frac{\hat{J}_n{}'(ka)}{\hat{H}_n^{(2)}{}'(ka)}$$

$$e_n = -c_n \frac{\hat{J}_n(ka)}{\hat{H}_n^{(2)}(ka)}$$

$$c_n = \frac{1}{\omega} \frac{(-j)^n (2n+1)}{n(n+1)}$$

Scattering by Sphere (cont.)

The exact solution to scattering by spherical particles is called the Mie† series solution (first published in 1908).

† **Gustav Adolf Feodor Wilhelm Ludwig Mie** (Sept. 29, 1869 – Feb. 13, 1957) was a German physicist.

He was the first to publish the solution to scattering by a dielectric sphere.

He received a doctorate degree in mathematics at the age of 22.

http://en.wikipedia.org/wiki/Gustav_Mie

Scattering by Sphere (cont.)

From the TE$_r$ / TM$_r$ table:

$$E_\theta = \frac{-1}{\varepsilon r \sin\theta}\frac{\partial F_r}{\partial \phi} + \frac{1}{j\omega\mu\varepsilon r}\frac{\partial^2 A_r}{\partial r \partial \theta}$$

Recall:

$$A_r^s = E_0 \cos\phi \sum_{n=0}^{\infty} d_n \hat{H}_n^{(2)}(kr) P_n^1(\cos\theta)$$

$$F_r^s = \frac{E_0}{\eta}\sin\phi \sum_{n=0}^{\infty} e_n \hat{H}_n^{(2)}(kr) P_n^1(\cos\theta)$$

Hence, we have

$$E_\theta^s = -\frac{1}{\varepsilon r \sin\theta}\frac{E_0}{\eta}\cos\phi \sum_{n=0}^{\infty} e_n \hat{H}_n^{(2)}(kr) P_n^1(\cos\theta)$$
$$+\frac{1}{j\omega\mu\varepsilon r} E_0 \cos\phi \sum_{n=0}^{\infty} d_n\, k\, \hat{H}_n^{(2)\prime}(kr)(-\sin\theta) P_n^{1\,\prime}(\cos\theta)$$

Scattering by Sphere (cont.)

Similarly,

$$E_{\phi} = \frac{1}{\varepsilon r}\frac{\partial F_r}{\partial \theta} + \frac{1}{j\omega\mu\varepsilon r \sin\theta}\frac{\partial^2 A_r}{\partial r \partial \phi}$$

Recall:

$$A_r^s = E_0 \cos\phi \sum_{n=0}^{\infty} d_n \hat{H}_n^{(2)}(kr) P_n^1(\cos\theta)$$

$$F_r^s = \frac{E_0}{\eta} \sin\phi \sum_{n=0}^{\infty} e_n \hat{H}_n^{(2)}(kr) P_n^1(\cos\theta)$$

Hence, we have

$$E_{\phi} = \frac{1}{\varepsilon r}\frac{E_0}{\eta} \sin\phi \sum_{n=0}^{\infty} e_n \hat{H}_n^{(2)}(kr)(-\sin\theta) P_n^{1\prime}(\cos\theta)$$

$$-\frac{1}{j\omega\mu\varepsilon r \sin\theta} E_0 \sin\phi \sum_{n=0}^{\infty} d_n k \hat{H}_n^{(2)\prime}(kr) P_n^1(\cos\theta)$$

Scattering by Sphere (cont.)

Also,

$$E_r = \frac{1}{j\omega\mu\varepsilon}\left(\frac{\partial^2}{\partial r^2} + k^2\right)A_r$$

Recall:

$$A_r^s = E_0 \cos\phi \sum_{n=0}^{\infty} d_n \hat{H}_n^{(2)}\left(kr\right) P_n^1\left(\cos\theta\right)$$

$$F_r^s = \frac{E_0}{\eta} \sin\phi \sum_{n=0}^{\infty} e_n \hat{H}_n^{(2)}\left(kr\right) P_n^1\left(\cos\theta\right)$$

Hence, we have

$$E_r^s = \frac{1}{j\omega\mu\varepsilon} E_0 \cos\phi \sum_{n=0}^{\infty} d_n k^2 \left(\hat{H}_n^{(2)\prime\prime}\left(kr\right) + \hat{H}_n^{(2)}\left(kr\right)\right) P_n^1\left(\cos\theta\right)$$

Summary

Exact Scattered Field

$$E_\theta^s = -\frac{1}{\varepsilon r \sin\theta}\frac{E_0}{\eta}\cos\phi\sum_{n=0}^{\infty} e_n \hat{H}_n^{(2)}(kr)P_n^1(\cos\theta) + \frac{1}{j\omega\mu\varepsilon r}E_0\cos\phi\sum_{n=0}^{\infty} d_n\, k\,\hat{H}_n^{(2)\prime}(kr)(-\sin\theta)P_n^{1\prime}(\cos\theta)$$

$$E_\phi^s = \frac{1}{\varepsilon r}\frac{E_0}{\eta}\sin\phi\sum_{n=0}^{\infty} e_n \hat{H}_n^{(2)}(kr)(-\sin\theta)P_n^{1\prime}(\cos\theta) - \frac{1}{j\omega\mu\varepsilon r\sin\theta}E_0\sin\phi\sum_{n=0}^{\infty} d_n k\hat{H}_n^{(2)\prime}(kr)P_n^1(\cos\theta)$$

Summary (cont.)

$$E_r^s = \frac{1}{j\omega\mu\varepsilon} E_0 \cos\phi \sum_{n=0}^{\infty} d_n k^2 \left(\hat{H}_n^{(2)\prime\prime}(kr) + \hat{H}_n^{(2)}(kr) \right) P_n^1(\cos\theta)$$

where

$$d_n = -c_n \frac{\hat{J}_n{}'(ka)}{\hat{H}_n^{(2)}{}'(ka)} \qquad e_n = -c_n \frac{\hat{J}_n(ka)}{\hat{H}_n^{(2)}(ka)}$$

$$c_n = \frac{1}{\omega} \frac{(-j)^n (2n+1)}{n(n+1)}$$

Note on Legendre Functions

The associated Legendre functions can be calculated in terms of the regular Legendre functions, if we wish:

$$P_n^m(x) = (-1)^m \left(1-x^2\right)^{m/2} \frac{d^m}{dx^m} P_n(x)$$

Harrington notation

so

$$P_n^1(x) = -\sqrt{1-x^2}\,\frac{d}{dx} P_n(x) = -\sqrt{1-x^2}\,P_n'(x)$$

$$P_n^{1\,\prime}(x) = \frac{-\frac{1}{2}(-2x)}{\sqrt{1-x^2}} P_n'(x) - \sqrt{1-x^2}\,P_n''(x) = \frac{x}{\sqrt{1-x^2}} P_n'(x) - \sqrt{1-x^2}\,P_n''(x)$$

And thus

$$P_n^1(\cos\theta) = -\sin\theta\, P_n'(\cos\theta)$$

$$P_n^{1\,\prime}(\cos\theta) = \cot\theta\, P_n'(x) - \sin\theta\, P_n''(x)$$

Note: Recurrence formulas can be used to calculate the derivatives of the Legendre polynomials.

Far Field

In the far field we have

$$\hat{H}_n^{(2)}(x) = x\sqrt{\frac{\pi}{2x}} H_{n+1/2}^{(2)}(x)$$

$$\sim x\sqrt{\frac{\pi}{2x}} \left[\sqrt{\frac{2j}{\pi x}} e^{-jx} j^{n+1/2} \right]$$

$$= j^{n+1} e^{-jx}$$

So we have

$$\hat{H}_n^{(2)}(x) \sim j^{n+1} e^{-jx}$$

$$\hat{H}_n^{(2)\prime}(x) \sim (-j)\, j^{n+1} e^{-jx} = j^n e^{-jx}$$

Far Field (cont.)

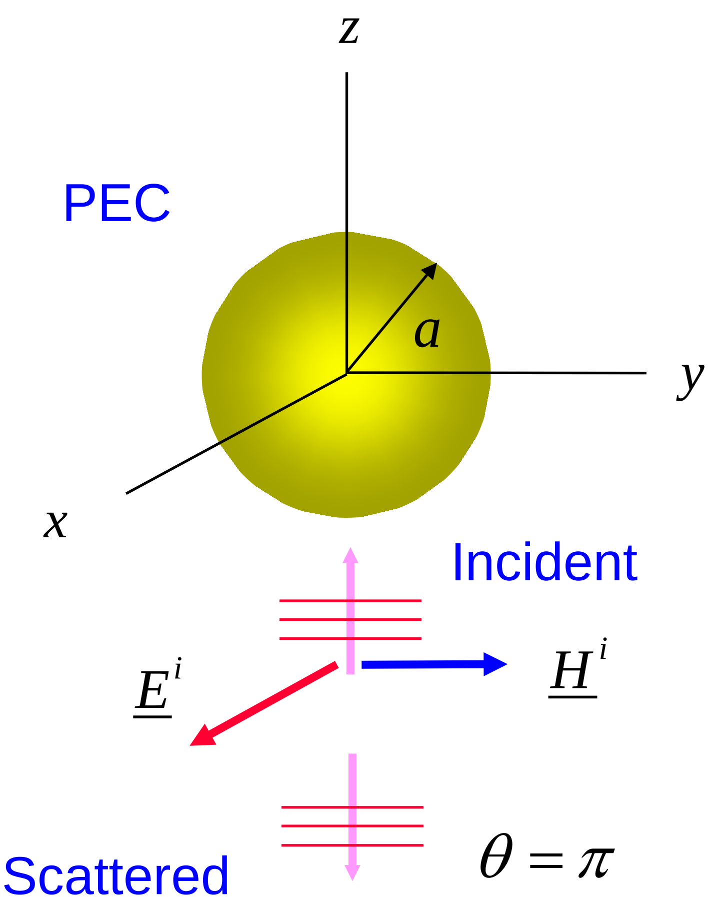
Exact Scattered Far Field

$$E_\theta^s = -\frac{1}{\varepsilon \sin\theta}\frac{E_0}{\eta}\cos\phi\left(\frac{e^{-jkr}}{r}\right)\sum_{n=0}^{\infty} e_n j^{n+1} P_n^1(\cos\theta) + \frac{1}{j\omega\mu\varepsilon}E_0\cos\phi\left(\frac{e^{-jkr}}{r}\right)\sum_{n=0}^{\infty} d_n k\, j^n(-\sin\theta)P_n^{1\prime}(\cos\theta)$$

$$E_\phi^s = \frac{1}{\varepsilon}\frac{E_0}{\eta}\sin\phi\left(\frac{e^{-jkr}}{r}\right)\sum_{n=0}^{\infty} e_n j^{n+1}(-\sin\theta)P_n^{1\prime}(\cos\theta) - \frac{1}{j\omega\mu\varepsilon\sin\theta}E_0\sin\phi\left(\frac{e^{-jkr}}{r}\right)\sum_{n=0}^{\infty} d_n k\, j^n P_n^1(\cos\theta)$$

Note: We can ignore E_r in the far field.

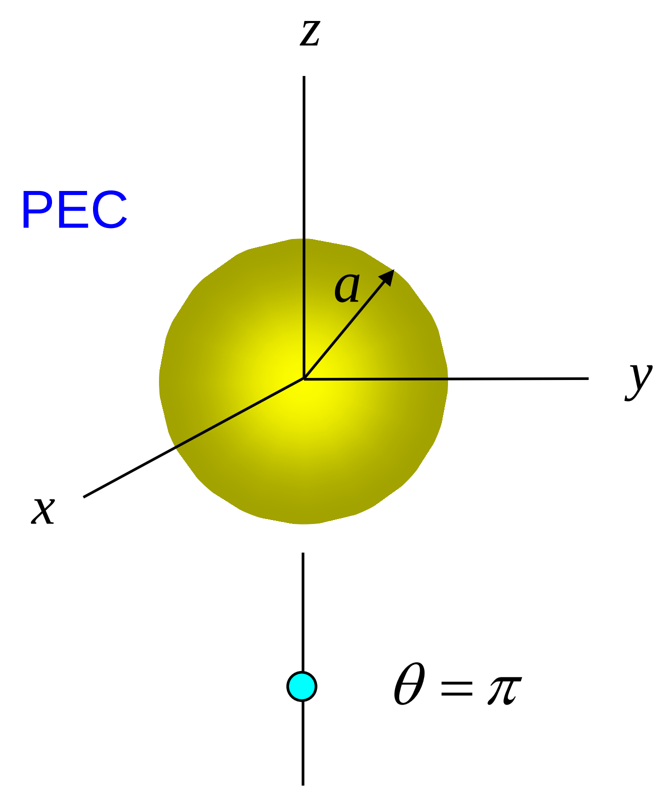
Backscattered Field



From symmetry,

$$\underline{E}^s(r, \pi, \phi) = \underline{\hat{x}}\, E_x^s(r, \pi, \phi)$$

Backscattered Field (cont.)



Note: The backscattered field can be represented in different ways, depending on the angle ϕ that is chosen.

Choose this one $\longrightarrow$ $\underline{E}^s(r,\pi,\phi) = \underline{\hat{\theta}}\, E_\theta^s(r,\pi,0)$

or

$$\underline{E}^s(r,\pi,\phi) = \underline{\hat{\phi}}\, E_\phi^s(r,\pi,\pi/2)$$

Backscattered Field (cont.)

Set $\phi = 0$:

$$E_\theta^{sb} = -\frac{1}{\varepsilon \sin\theta}\frac{E_0}{\eta}\left(\frac{e^{-jkr}}{r}\right)\sum_{n=0}^{\infty} e_n j^{n+1} P_n^1(\cos\theta)$$

$$+\frac{1}{j\omega\mu\varepsilon}E_0\left(\frac{e^{-jkr}}{r}\right)\sum_{n=0}^{\infty} d_n\, k\, j^n (-\sin\theta) P_n^{1\,\prime}(\cos\theta)$$

Next, use $\quad P_n^1(\cos\theta) = -\sin\theta P_n'(\cos\theta)$

$$P_n^{1\,\prime}(\cos\theta) = \cot\theta P_n'(x) - \sin\theta P_n''(x)$$

so that

$$E_\theta^{sb} = \frac{1}{\varepsilon}\frac{E_0}{\eta}\left(\frac{e^{-jkr}}{r}\right)\sum_{n=0}^{\infty} e_n j^{n+1} P_n'(\cos\theta)$$

$$+\frac{1}{j\omega\mu\varepsilon}E_0\left(\frac{e^{-jkr}}{r}\right)\sum_{n=0}^{\infty} d_n\, k\, j^n \left((-\cos\theta)P_n'(\cos\theta) + \sin^2\theta P_n''(\cos\theta)\right)$$

$$\theta \to \pi$$

Backscattered Field (cont.)

We then have, letting $\theta \to \pi$

$$E_\theta^{sb} = \frac{1}{\varepsilon}\frac{E_0}{\eta}\left(\frac{e^{-jkr}}{r}\right)\sum_{n=0}^{\infty} e_n\, j^{n+1} P_n'(-1)$$

$$+\frac{1}{j\omega\mu\varepsilon}E_0\left(\frac{e^{-jkr}}{r}\right)\sum_{n=0}^{\infty} d_n\, k\, j^{n} P_n'(-1)$$

We can also write this as

Note: $E_x\big|_{\theta=\pi} = -E_\theta(r,\pi,0)$

$$E_x^{sb} = -\frac{1}{\varepsilon}\frac{E_0}{\eta}\left(\frac{e^{-jkr}}{r}\right)\sum_{n=0}^{\infty} e_n\, j^{n+1} P_n'(-1)$$

$$-\frac{1}{j\omega\mu\varepsilon}E_0\left(\frac{e^{-jkr}}{r}\right)\sum_{n=0}^{\infty} d_n\, k\, j^{n} P_n'(-1)$$

Backscattered Field (cont.)

Final Backscattered Far Field

$$E_x^{sb} = -\frac{1}{\sqrt{\mu\varepsilon}} E_0 \left(\frac{e^{-jkr}}{r}\right) \sum_{n=0}^{\infty} e_n \, j^{n+1} P_n'(-1)$$
$$+\frac{1}{\sqrt{\mu\varepsilon}} E_0 \left(\frac{e^{-jkr}}{r}\right) \sum_{n=0}^{\infty} d_n \, j^{n+1} P_n'(-1)$$

Low-Frequency Approximation

Let

$$\boxed{\frac{a}{\lambda_0} \to 0}$$

Keep the dominant term in the series, which is the $n = 1$ term

$$\left(\text{not } n \;=\; 0\text{, since } P_0^1(x) = -\sqrt{1-x^2}\,P_0'(x) = -\sqrt{1-x^2}\,\frac{d}{dx}(1) = 0 \right)$$

Please see the formulas for d_n and e_n to verify that lower terms are more dominant.

Low-Frequency Backscattered Field

We thus have

$$E_x^{sb} = \frac{1}{\sqrt{\mu\varepsilon}} E_0 \left(\frac{e^{-jkr}}{r} \right) e_1 P_1' (-1)$$

$$- \frac{1}{\sqrt{\mu\varepsilon}} E_0 \left(\frac{e^{-jkr}}{r} \right) d_1 P_1' (-1)$$

where

$$P_1(x) = x, \quad P_1'(x) = 1$$

Low-Frequency Backscattered Field (cont.)

Hence, we have

$$E_x^{sb} = \frac{1}{\sqrt{\mu\varepsilon}} E_0 \left(\frac{e^{-jkr}}{r} \right) e_1 - \frac{1}{\sqrt{\mu\varepsilon}} E_0 \left(\frac{e^{-jkr}}{r} \right) d_1$$

We next evaluate the coefficients e_1 and d_1 at low frequency.

Low-Frequency Backscattered Field (cont.)

Examine e_1:

$$e_1 = -c_1 \frac{\hat{J}_1(ka)}{\hat{H}_1^{(2)}(ka)}$$

Recall that

$$c_n = \frac{1}{\omega} \frac{(-j)^n (2n+1)}{n(n+1)}$$

so

$$c_1 = \frac{1}{\omega} \frac{(-j)(3)}{1(2)} = -j \frac{3}{2} \frac{1}{\omega}$$

Low-Frequency Backscattered Field (cont.)

Next, examine the Bessel function terms as $x = ka \to 0$:

$$\begin{aligned}
\hat{J}_1(x) &= x\, j_1(x) \\
&= x\sqrt{\frac{\pi}{2x}}\, J_{3/2}(x) \\
&\approx x\sqrt{\frac{\pi}{2x}}\left[\frac{x^{3/2}}{\left(\frac{3}{2}\right)!\,2^{3/2}}\right] \\
&= \left[\frac{\sqrt{\frac{\pi}{2}}}{\left(\frac{3}{2}\right)!\,2^{3/2}}\right] x^2
\end{aligned}$$

Low-Frequency Backscattered Field (cont.)

Also

$$Y_{\upsilon}(x) = \frac{J_{\upsilon}(x)\cos\upsilon\pi - J_{-\upsilon}(x)}{\sin\upsilon\pi}$$

$$Y_{3/2}(x) = J_{-3/2}(x)$$

$$\begin{aligned}
\hat{H}_1^{(2)}(x) &= x\,h_1^{(2)}(x) \\
&= x\sqrt{\frac{\pi}{2x}}\,H_{3/2}^{(2)}(x) \\
&\approx x\sqrt{\frac{\pi}{2x}}\left[-jY_{3/2}(x)\right] \\
&= x\sqrt{\frac{\pi}{2x}}(-j)\left[J_{-3/2}(x)\right] \\
&= x\sqrt{\frac{\pi}{2x}}(-j)\left[-\sqrt{\frac{2}{\pi x}}\left(\underbrace{\cancel{\sin x}}_{\approx 0} + \frac{\cos x}{x}\right)\right] \sim \frac{j}{x}
\end{aligned}$$

Low-Frequency Backscattered Field (cont.)

Hence

$$e_1 = -c_1 \frac{\hat{J}_1(ka)}{\hat{H}_1^{(2)}(ka)} \sim \left(j \frac{3}{2} \frac{1}{\omega} \right) \frac{\left[\dfrac{\sqrt{\dfrac{\pi}{2}} x^2}{\left(\dfrac{3}{2} \right)! \, 2^{3/2}} \right]}{\dfrac{j}{x}}$$

or

$$e_1 \sim \left(\frac{3}{2} \frac{1}{\omega} \right) \frac{\sqrt{\dfrac{\pi}{2}}}{\left(\dfrac{3}{2} \right)! \, 2^{3/2}} x^3$$

Recall: $x = ka$

Low-Frequency Backscattered Field (cont.)

$$\frac{3}{2}! = \frac{3}{2}\left(\frac{1}{2}!\right) = \frac{3}{2}\left(\frac{\sqrt{\pi}}{2}\right) = \frac{3}{4}\sqrt{\pi}$$

so

$$e_1 \sim \left(\frac{3}{2}\frac{1}{\omega}\right)\frac{\sqrt{\dfrac{\pi}{2}}}{\left(\dfrac{3}{4}\sqrt{\pi}\right)2^{3/2}}x^3$$

or

$$e_1 \sim \frac{1}{2}\frac{1}{\omega}x^3 \qquad (x = ka)$$

Hence

$$e_1 \sim \frac{1}{2\omega}(ka)^3$$

Low-Frequency Backscattered Field (cont.)

Examine $\boldsymbol{d_1}$:

$$d_1 = -c_1 \frac{\hat{J}_1'(x)}{\hat{H}_1^{(2)\prime}(x)}$$

As a shortcut, compare with what we just did for e_1:

$$e_1 = -c_1 \frac{\hat{J}_1(x)}{\hat{H}_1^{(2)}(x)}$$

Low-Frequency Backscattered Field (cont.)

From the previous approximations for e_1, we note that

$$\frac{\hat{J}_1(x)}{\hat{H}_1^{(2)}(x)} \approx \frac{c_1 x^2}{c_2 \dfrac{1}{x}}$$

Hence

$$\frac{\hat{J}_1{}'(x)}{\hat{H}_1^{(2)}{}'(x)} \approx \frac{c_1 2x}{c_2\left(-\dfrac{1}{x^2}\right)}$$

$$= -2\,\frac{c_1 x^2}{c_2 \dfrac{1}{x}}$$

$$\approx -2\left(\frac{\hat{J}_1(x)}{\hat{H}_1^{(2)}(x)}\right)$$

Therefore

$$\boxed{d_1 \sim -2e_1}$$

Low-Frequency Backscattered Field (cont.)

Recall:

$$E_x^{sb} = \frac{1}{\sqrt{\mu\varepsilon}} E_0 \left(\frac{e^{-jkr}}{r} \right) e_1 - \frac{1}{\sqrt{\mu\varepsilon}} E_0 \left(\frac{e^{-jkr}}{r} \right) d_1$$

and

$$d_1 \sim -2e_1$$

The total backscattered field is then:

$$\boxed{E_x^{sb} = 3 \frac{1}{\sqrt{\mu\varepsilon}} E_0 \left(\frac{e^{-jkr}}{r} \right) e_1}$$

with

$$\boxed{e_1 \sim \frac{1}{2\omega} (ka)^3}$$

Low-Frequency Backscattered Field (cont.)

The total low-frequency backscattered field is then:

$$E_x^{sb} = 3\frac{E_0}{\sqrt{\mu\varepsilon}}\left(\frac{e^{-jkr}}{r}\right)\left[\left(\frac{1}{2\omega}\right)(ka)^3\right]$$

Use $k = k_0, \quad \omega = 2\pi f = 2\pi\frac{c}{\lambda_0} = \frac{2\pi}{\lambda_0}\frac{1}{\sqrt{\mu_0\varepsilon_0}}$

Low-Frequency Backscattered Field (cont.)

Final Low-frequency Backscattered Field

$$E_x^{sb} = \left(\frac{E_0}{\pi}\right)\lambda_0\left(k_0 a\right)^3\left(\frac{e^{-jk_0 r}}{r}\right)\left(\frac{3}{4}\right)$$

Echo Area (Radar Cross Section)

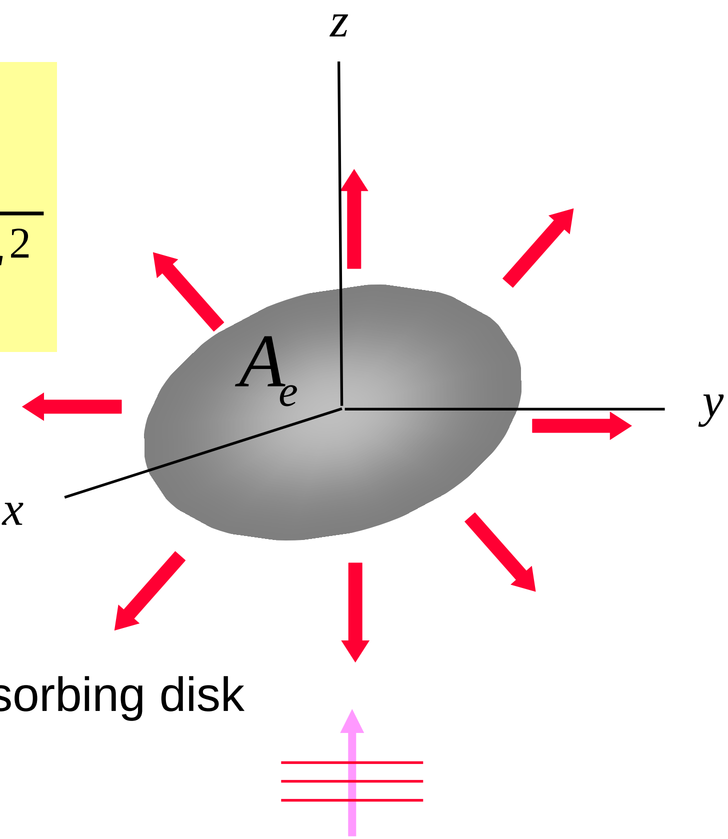
$$A_e = \lim_{r \to \infty} \frac{4\pi r^2 \left| E_x^{sb} \right|^2}{\left| E_0 \right|^2}$$

Monostatic RCS

Physical interpretation:

$$\frac{\left| E_x^{sb} \right|^2}{2\eta_0} = \left(\frac{\left| E_0 \right|^2}{2\eta_0} A_e \right) \frac{1}{4\pi r^2}$$

This is the radiated power of an isotropic radiating "black body" disk that absorbs all incident power.



Perfectly absorbing disk

Echo Area (Radar Cross Section) (cont.)

$$A_e = \left(\frac{4\pi r^2}{\left|E_0\right|^2}\right)\left|\left(\frac{E_0}{\pi}\right)\lambda_0\left(k_0 a\right)^3\left(\frac{e^{-jk_0 r}}{r}\right)\left(\frac{3}{4}\right)\right|^2$$

Simplifying, we have

$$A_e = \frac{9}{4\pi}\lambda_0^2\left(k_0 a\right)^6$$

Echo Area (Radar Cross Section) (cont.)

Final Result

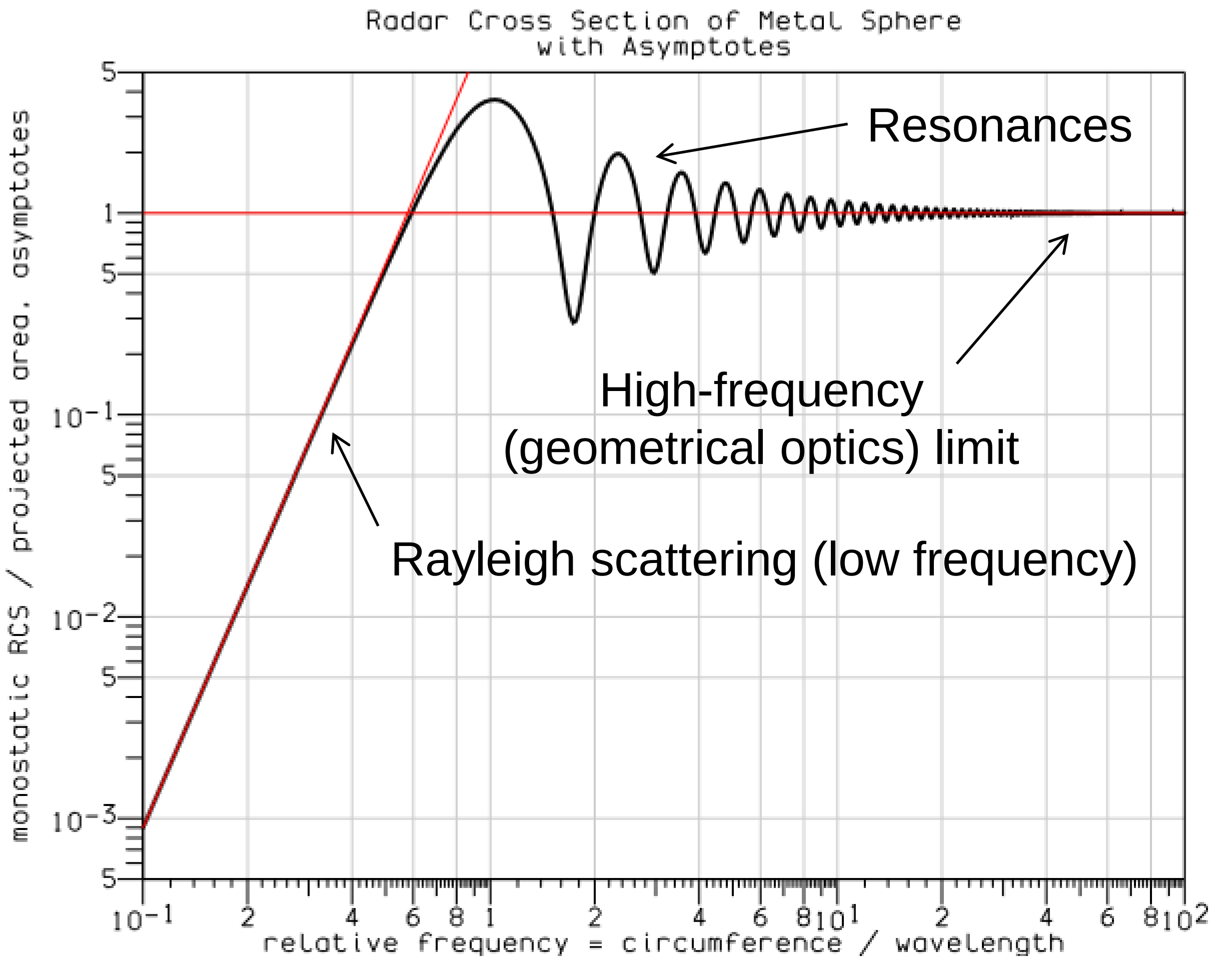
$$A_e \sim \frac{9}{4\pi}\lambda_0^2\left(k_0 a\right)^6$$

For electrically small particles, there is a much stronger scattering from the particles as the size increases.

Scattering from small dielectric or metallic particles is called "Rayleigh scattering."

$$A_e \sim \omega^4$$

Echo Area (Radar Cross Section) (cont.)



http://en.wikipedia.org/wiki/Mie_scattering

Echo Area (Radar Cross Section) (cont.)

Result for Dielectric Sphere (derivation omitted)

$$A_e \sim \frac{9}{4\pi}\lambda_0^2 \left(k_0 a\right)^6 \left(\frac{\varepsilon_r - 1}{\varepsilon_r + 2}\right)^2$$

Extra factor added

Dipole Moment of Particle

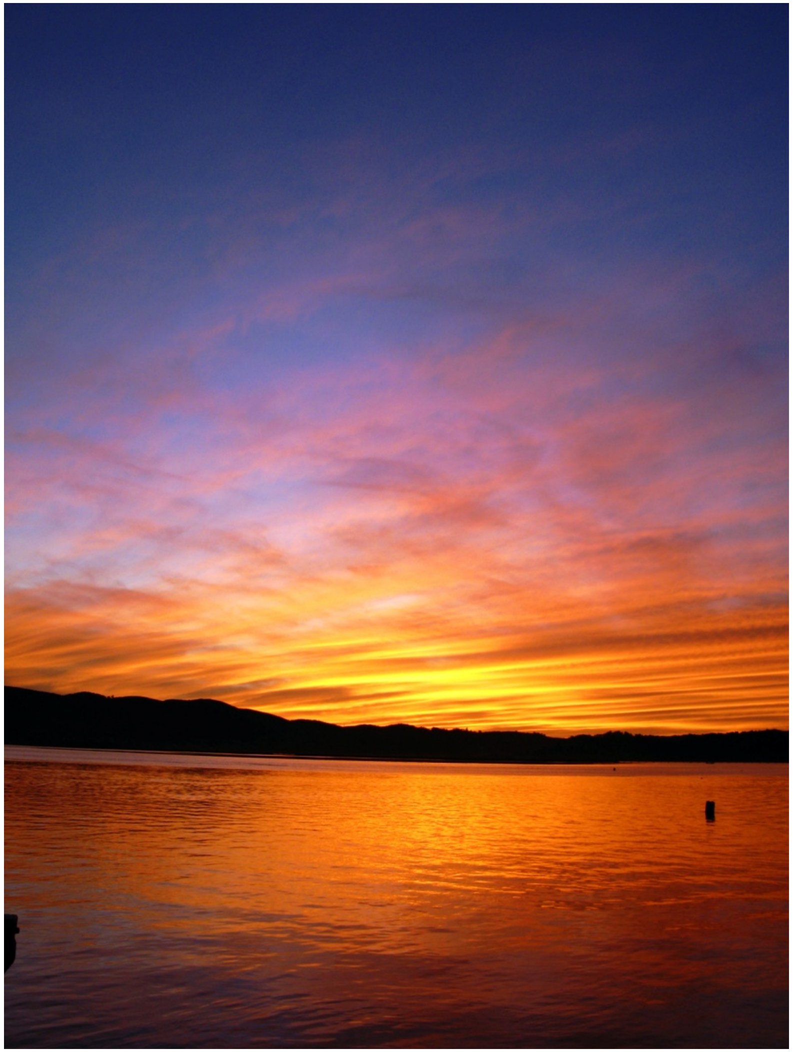
Dipole Approximation

The sphere acts as a small radiating dipole in the x direction (the direction of the incident electric field).

The dipole moment that is given by:

$$Il = -j\frac{3\lambda_0 (k_0 a)^3}{\omega\mu_0}\left(\frac{\varepsilon_r - 1}{\varepsilon_r + 2}\right)E_0$$

Rayleigh Scattering in the Sky



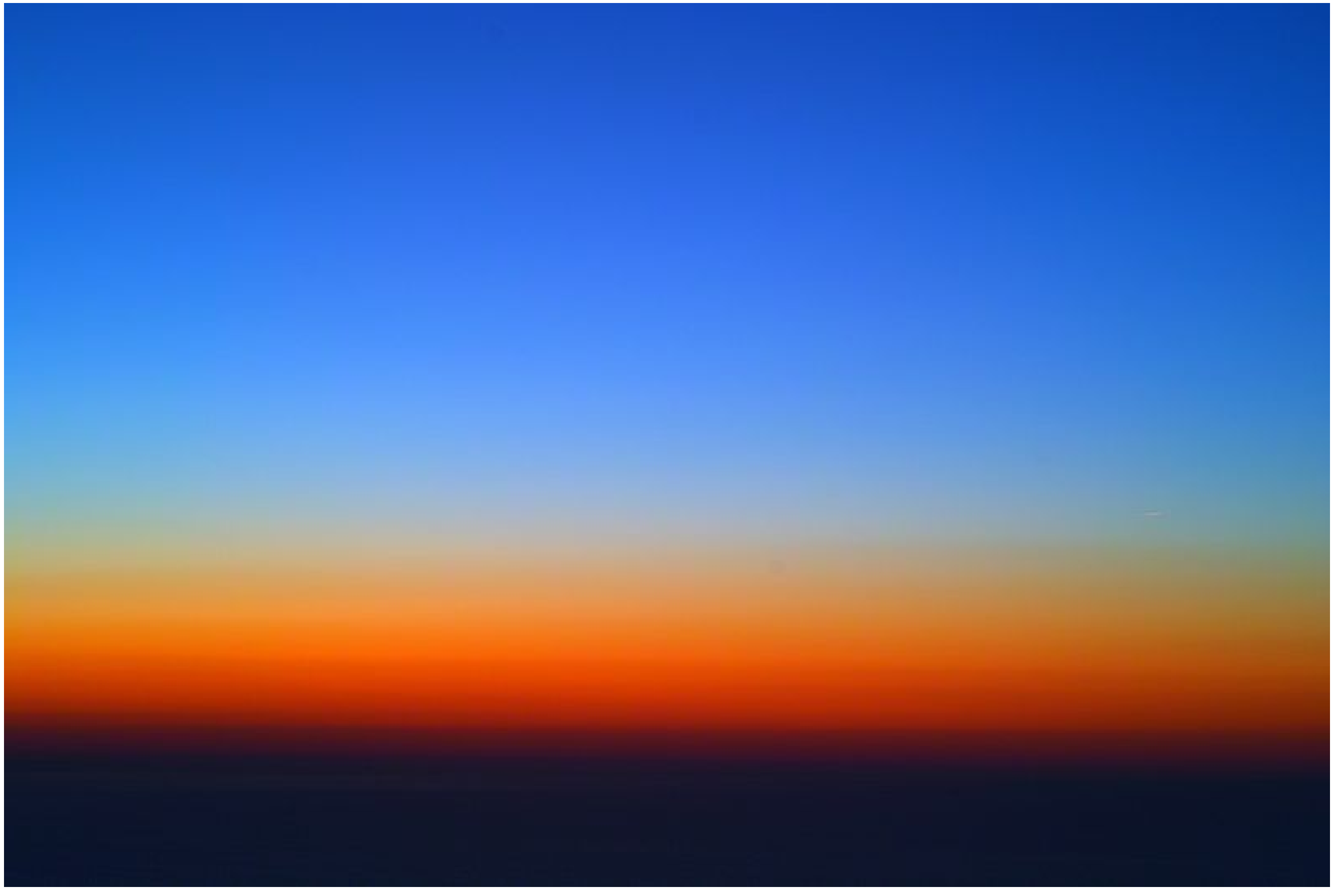
The change of sky color at sunset (red nearest the sun, blue furthest away) is caused by Rayleigh scattering from atmospheric gas particles which are much smaller than the wavelengths of visible light. The grey/white color of the clouds is caused by Mie scattering by water droplets which are of a comparable size to the wavelengths of visible light.

John Strutt (Lord Rayleigh), “On the transmission of light through an atmosphere containing small particles in suspension, and on the origin of the blue of the sky,” *Philosophical Magazine*, series 5, vol. 47, pp. 375-394, 1899.

http://en.wikipedia.org/wiki/Mie_scattering

Rayleigh Scattering in the Sky (cont.)

Rayleigh scattering is more evident after sunset. This picture was taken about one hour after sunset at a 500m altitude, looking at the horizon where the sun had set.



http://en.wikipedia.org/wiki/Rayleigh_scattering

Rayleigh Scattering in the Sky (cont.)

Scattered blue light is polarized. The picture on the right is shot through a polarizing filter which removes light that is linearly polarized in a specific direction

Note: When looking up at the sky at sunset or sunrise, the light scattered by the sky above you will be polarized in a north-south direction. (There is little radiated field from the dipole moment that is vertically polarized since you are looking at the axis of the dipole; therefore the horizontal (north-south) polarization dominates.)

http://en.wikipedia.org/wiki/Rayleigh_scattering