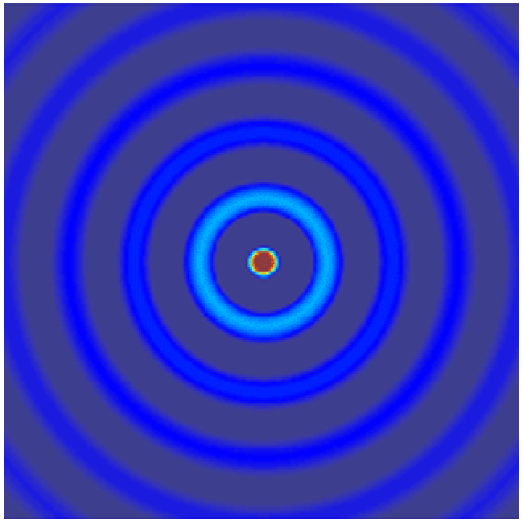
ECE 6341

Spring 2016

Prof. David R. Jackson
ECE Dept.



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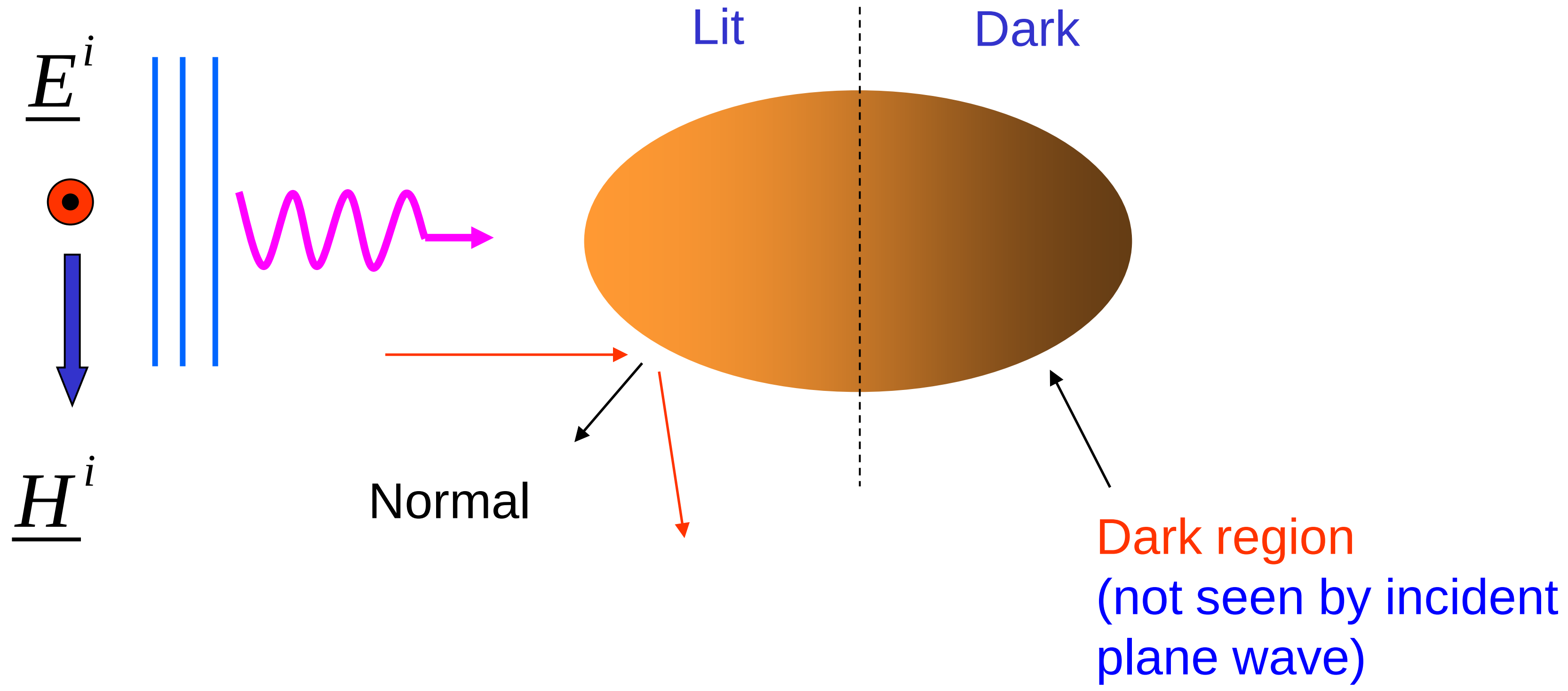
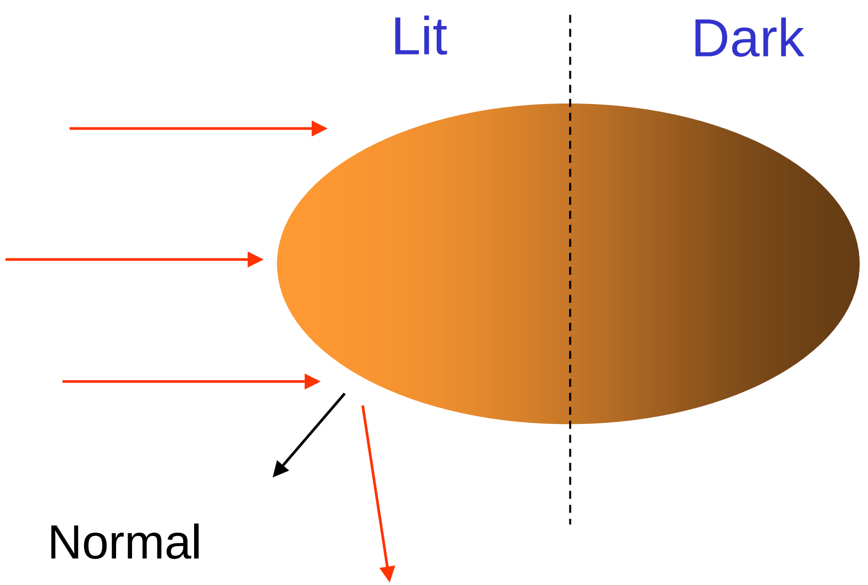
Physical Optics



$\underline{J}_s \approx \underline{0}$ on dark region

Physical Optics (cont.)



Locally, the reflection acts like plane-wave reflection from a flat surface.

$$\Gamma = -1 \ \text{(PEC)}$$

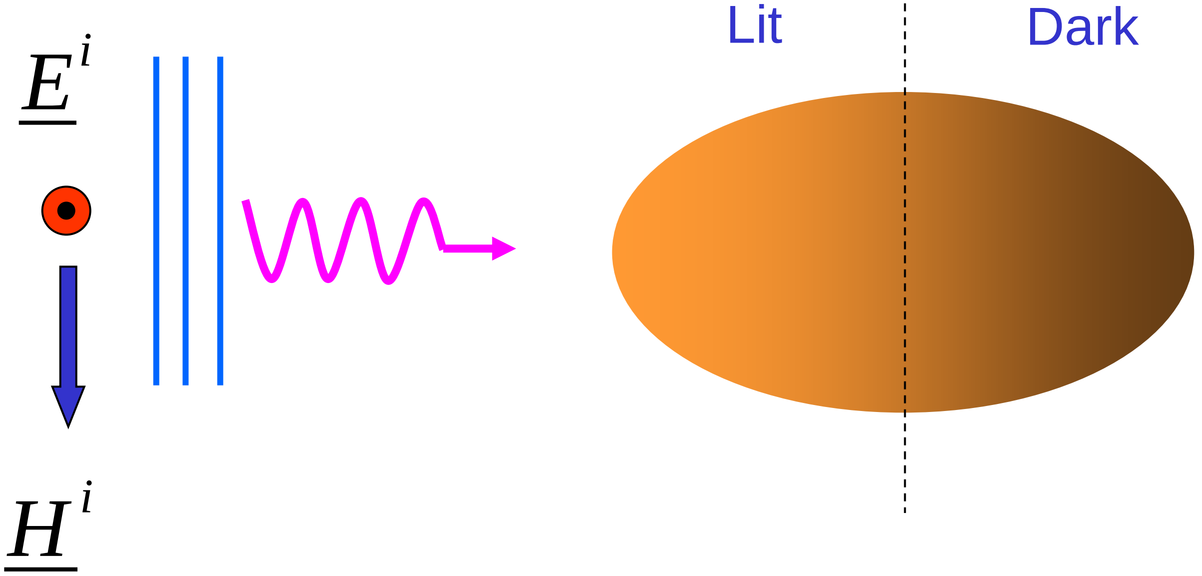
$$\underline{J}_s = \hat{\underline{n}} \times \underline{H} = \hat{\underline{n}} \times \left(\underline{H}^i + \underline{H}^r \right)$$

$$\hat{\underline{n}} \times \underline{H}^r \approx \Gamma_I \, \hat{\underline{n}} \times \underline{H}^i = -\Gamma_V \, \hat{\underline{n}} \times \underline{H}^i = -\Gamma \, \hat{\underline{n}} \times \underline{H}^i = \hat{\underline{n}} \times \underline{H}^i$$

Γ_I = reflection coefficient for current in the TEN

Γ_V = reflection coefficient for voltage in the TEN

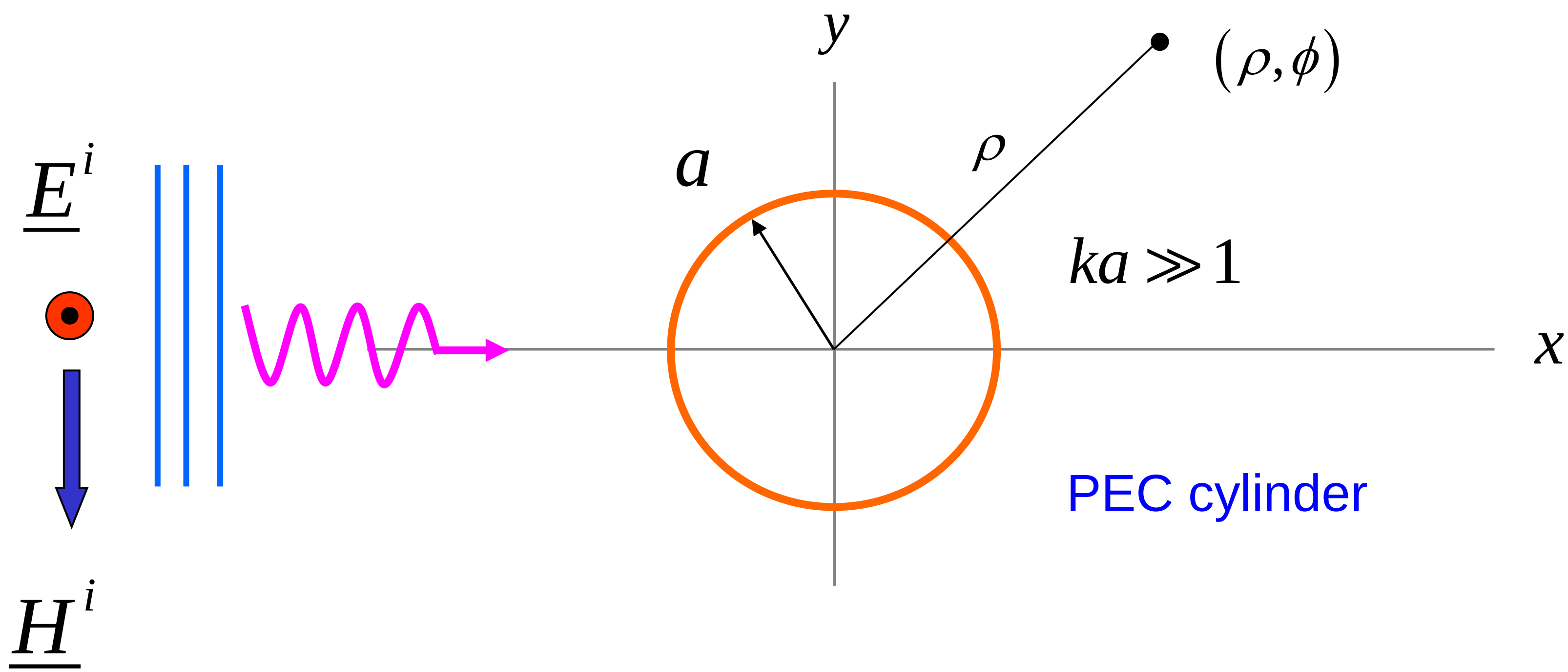
Physical Optics (cont.)



Lit region: $\underline{J}_s \approx 2\underline{\hat{n}} \times \underline{H}^i$

Dark region: $\underline{J}_s \approx \underline{0}$

High-Frequency Scattering by Cylinder



$$\underline{E}^i = \underline{\hat{z}}\, e^{-jkx}$$

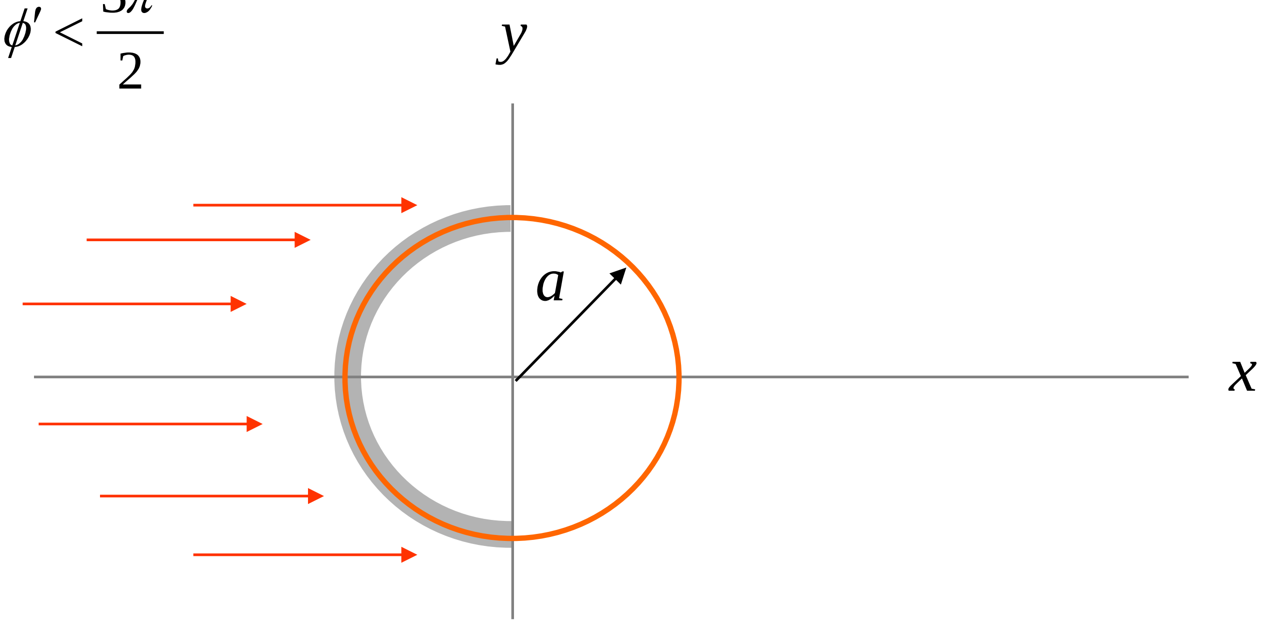
$$\underline{H}^i = \underline{\hat{y}} \left(-\frac{1}{\eta} \right) e^{-jkx}$$

High-Frequency Scattering by Cylinder (cont.)

$$\underline{J}_s = \begin{cases} 2\underline{\hat{n}} \times \underline{H}^i, & \text{Lit region} \\ \underline{0}, & \text{Dark region} \end{cases}$$

Lit region:

$$\frac{\pi}{2} < \phi' < \frac{3\pi}{2}$$



High-Frequency Scattering by Cylinder (cont.)

$$\underline{J}_s \approx 2\underline{\hat{\rho}} \times \underline{H}^i = 2\left[\underline{\hat{x}}\cos\phi' + \underline{\hat{y}}\sin\phi'\right] \times \left(\underline{\hat{y}}\left(\frac{-1}{\eta}\right)e^{-jkx}\right)$$

$$= \underline{\hat{z}}\left(\frac{-2}{\eta}\right)\cos\phi' \, e^{-jkx}$$

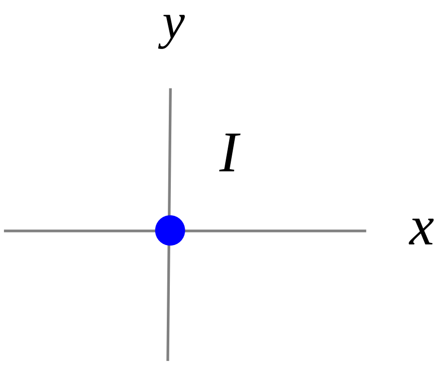
$$\underline{J}_s \approx \underline{\hat{z}}\left(\frac{-2}{\eta}\right)\cos\phi' e^{-jka\cos\phi'}$$

$$\frac{\pi}{2} < \phi' < \frac{3\pi}{2}$$

We next calculate the radiation from this current.

High-Frequency Scattering by Cylinder (cont.)

Consider first a z-directed line source at the origin:



$$A_z = \frac{\mu I}{4j} H_0^{(2)}(k\rho)$$

$$E_z = \frac{1}{j\omega\mu\varepsilon}\left(\frac{\partial^2}{\partial z} + k^2\right)A_z = -j\omega A_z = -\frac{\omega\mu}{4} I\, H_0^{(2)}(k\rho)$$

High-Frequency Scattering by Cylinder (cont.)

Far field:

$$E_z \sim -\frac{\omega\mu}{4} I \sqrt{\frac{2}{\pi k \rho}}\, e^{-jk\rho} e^{j\frac{\pi}{4}}$$

Next, consider the line source to be located at (x', y'):

$$E_z \sim -\frac{\omega\mu}{4} I \sqrt{\frac{2}{\pi k \rho}}\, e^{-jk\rho} e^{j\frac{\pi}{4}} \Psi$$

$$\Psi = \text{phase term} = e^{j\left(k_x x' + k_y y'\right)}$$

$$k_x = k\cos\phi \qquad x' = a\cos\phi'$$

$$k_y = k\sin\phi \qquad y' = a\sin\phi'$$

Hence

$$\Psi = e^{jka\cos\left(\phi - \phi'\right)}$$

High-Frequency Scattering by Cylinder (cont.)

$$E_z \sim -\frac{\omega\mu}{4} I \sqrt{\frac{2}{\pi k \rho}}\, e^{-jk\rho} e^{j\frac{\pi}{4}} \Psi$$

$$I \to dI = J_{sz}(\phi')\, a\, d\phi'$$

Hence

$$dE_z^s = -\frac{\omega\mu\, dI}{4} \sqrt{\frac{2}{\pi k \rho}} e^{-jk\rho} e^{j\frac{\pi}{4}} e^{jka\cos(\phi-\phi')}$$

or

$$dE_z^s = -\frac{\omega\mu}{4}\left[J_{sz}(\phi')\, a\, d\phi'\right] \sqrt{\frac{2}{\pi k \rho}} e^{-jk\rho} e^{j\frac{\pi}{4}} e^{jka\cos(\phi-\phi')}$$

High-Frequency Scattering by Cylinder (cont.)

or

$$dE_z^s = -\frac{\omega\mu}{4}\left[\left(\frac{-2}{\eta}\right)\cos\phi' e^{-jka\cos\phi'} a\,d\phi'\right]\sqrt{\frac{2}{\pi k\rho}}e^{-jk\rho}e^{j\frac{\pi}{4}}e^{jka\cos(\phi-\phi')}$$

Integrating over the lit region, we have,

$$E_z^s = \frac{\omega\mu}{2\eta}\sqrt{\frac{2}{\pi k\rho}}e^{-jk\rho}e^{j\frac{\pi}{4}}a\int_{\pi/2}^{3\pi/2}\cos\phi' e^{j(ka)\left[\cos(\phi-\phi')-\cos\phi'\right]}d\phi'$$

High-Frequency Scattering by Cylinder (cont.)

This may be written as

$$E_z^s = \frac{\omega\mu}{2\eta}\sqrt{\frac{2}{\pi k\rho}}\, e^{-jk\rho} e^{j\frac{\pi}{4}} a \; I(ka)$$

where

$$I(ka) \equiv \int_{\pi/2}^{3\pi/2} \cos\phi' e^{j(ka)\left[\cos(\phi-\phi')-\cos\phi'\right]} d\phi'$$

Compare with $$I(\Omega) = \int_a^b f(x) e^{j\Omega g(x)} dx$$

Hence, we can identify

$$\Omega = ka$$

$$f(\phi') = \cos\phi'$$

$$g(\phi') = \cos(\phi-\phi') - \cos\phi'$$

High-Frequency Scattering by Cylinder (cont.)

Find the stationary-phase point (SPP):

$$g(\phi') = \cos(\phi - \phi') - \cos\phi'$$

$$g'(\phi_0') = 0$$

$$\Rightarrow \sin(\phi - \phi_0') + \sin\phi_0' = 0$$

$$\Rightarrow \sin\underbrace{(\phi - \phi_0')}_{A} = -\sin\underbrace{\phi_0'}_{B}$$

$$\sin A = -\sin B$$

$$A = -B + 2\pi n$$

or

$$A = B + \pi + 2\pi n$$

$$\phi - \phi_0' = -\phi_0' + 2\pi n$$

$$\Rightarrow \phi = 2\pi n \Rightarrow g(\phi') \equiv 0$$

(No SPP. Assume $\phi \neq 2\pi n$)

or

$$\phi - \phi_0' = \phi_0' + \pi + 2\pi n$$

$$\phi_0' = \frac{\phi}{2} - \frac{\pi}{2} - \pi n$$

High-Frequency Scattering by Cylinder (cont.)

We require the restriction that

$$\phi_0' \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$$

From the previous slide,

$$\phi_0' = \frac{\phi}{2} - \frac{\pi}{2} - \pi n$$

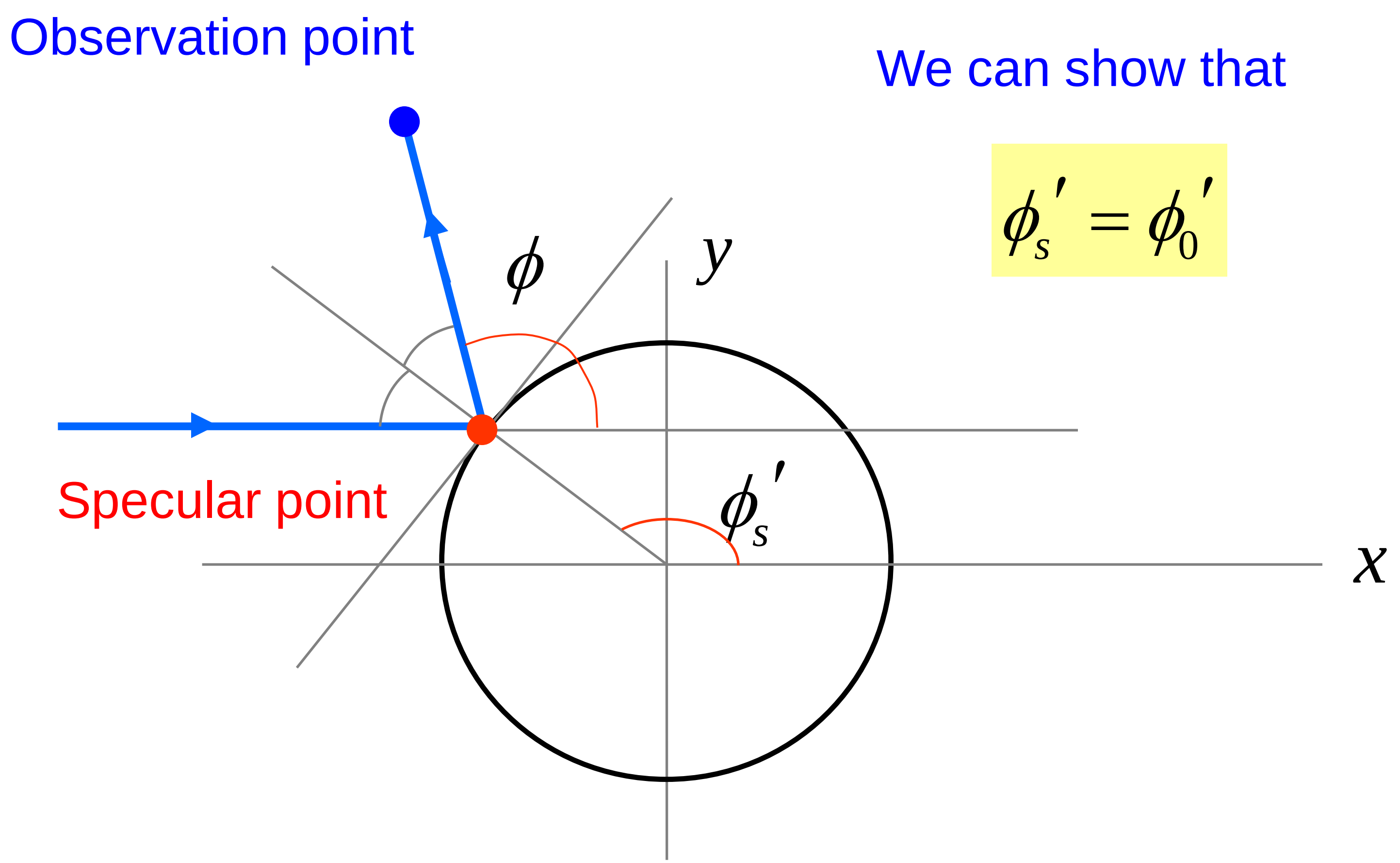
Also, $0 < \phi < 2\pi \quad \Rightarrow \quad 0 < \frac{\phi}{2} < \pi$

Hence, choose $n = -1$:

$$\phi_0' = \frac{\phi}{2} + \frac{\pi}{2}$$

Geometrical Optics

The specular point of reflection is the point at which the ray reflects off and travels to the observation point.

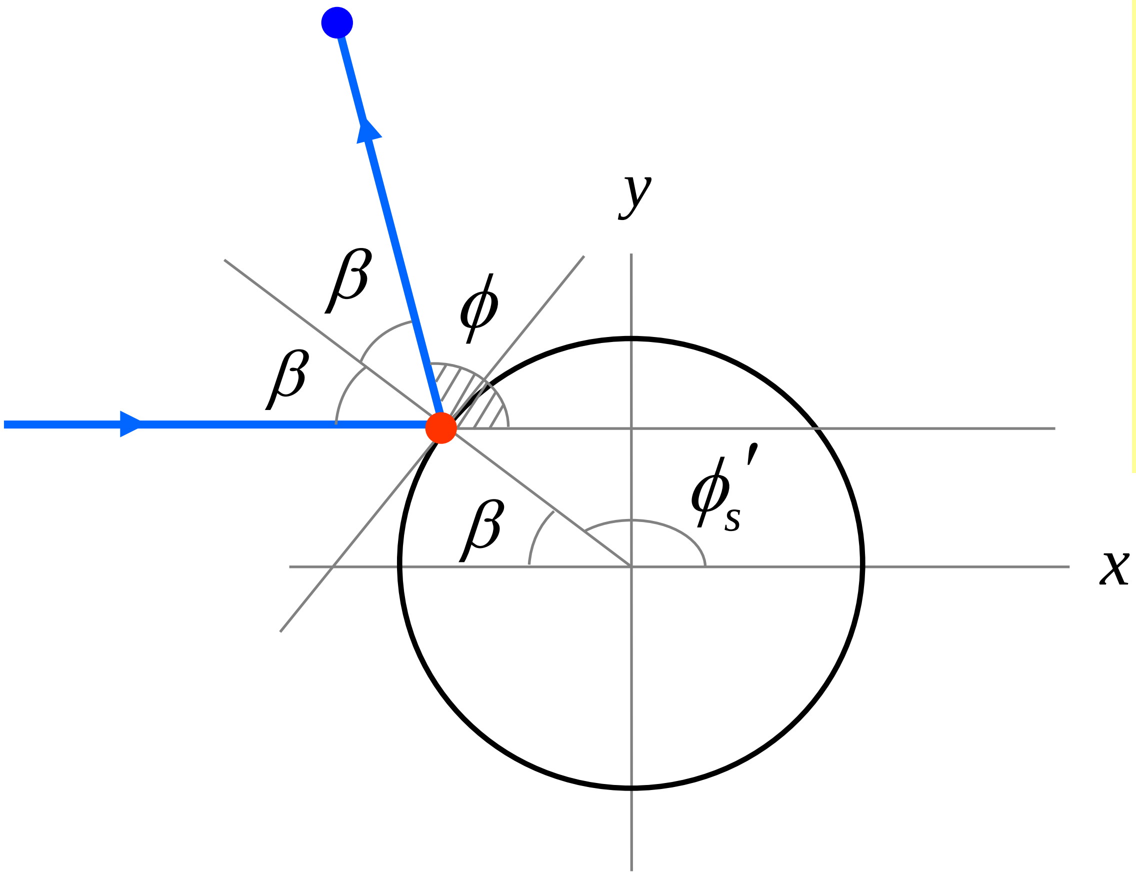


We can show that

$$\phi_s' = \phi_0'$$

Geometrical Optics (cont.)

Proof $\left(\phi_s' = \phi_0'\right)$

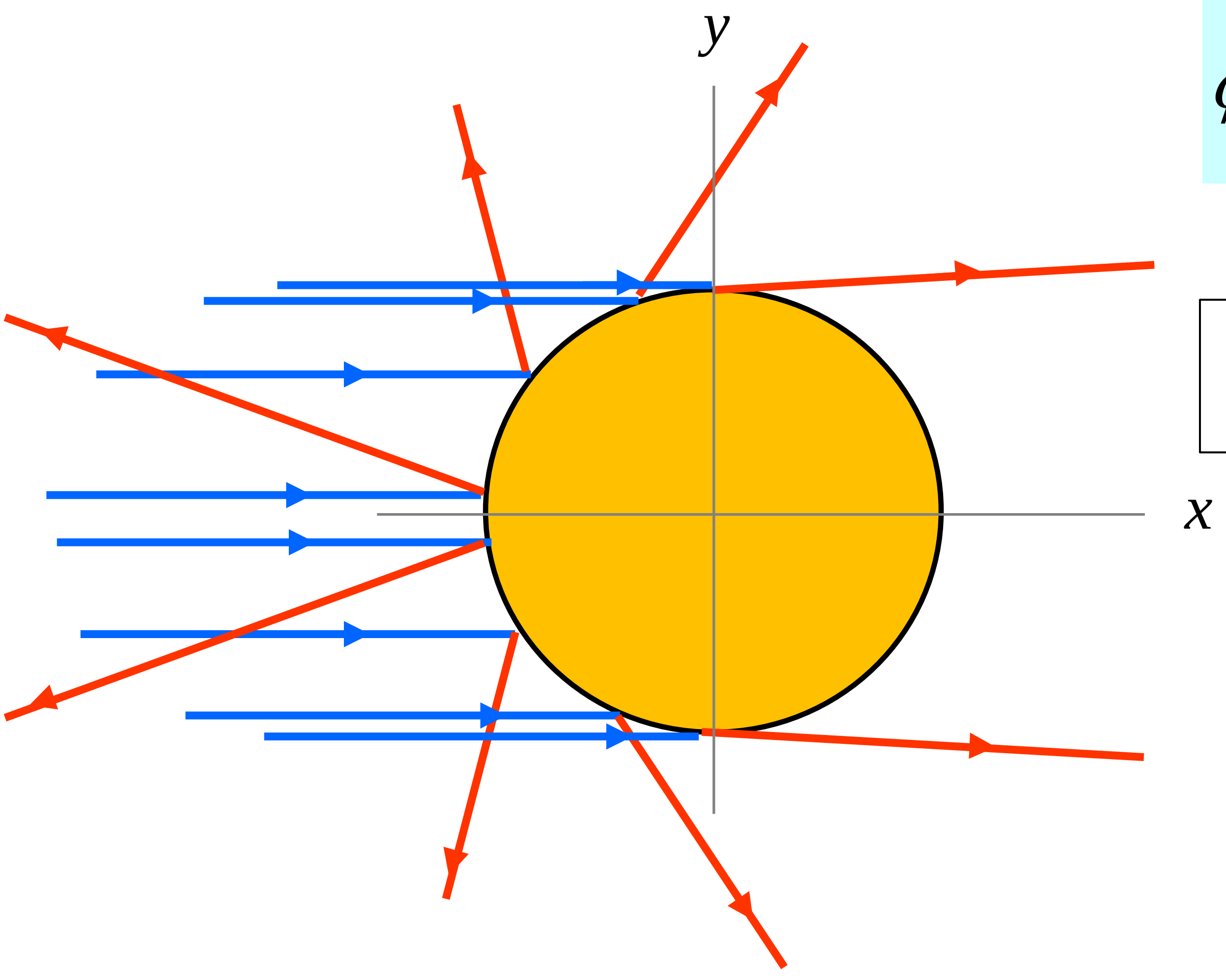


$$\begin{aligned}\phi &= \pi - 2\beta \\ &= \pi - 2\left(\pi - \phi_s'\right) \\ &= -\pi + 2\phi_s'\end{aligned}$$

$$\Rightarrow \phi_s' = \left(\frac{\phi}{2} + \frac{\pi}{2}\right)$$

High-Frequency Scattering by Cylinder (cont.)

Note that there is always a stationary-phase point, for all observation angles (except $\phi = 0$).



$$\phi_0' = \frac{\phi}{2} + \frac{\pi}{2}$$

Note:
The stationary-phase method will fail for $\phi = \pi / 2$.

High-Frequency Scattering by Cylinder (cont.)

Next, calculate the g function at the stationary-phase point:

$$g\left(\phi'\right) = \cos\left(\phi - \phi'\right) - \cos\phi'$$

Recall : $\phi_0{}' = \dfrac{\phi}{2} + \dfrac{\pi}{2}$

$$\begin{aligned}
g\left(\phi_0{}'\right) &= \cos\left(\phi - \phi_0{}'\right) - \cos\phi_0{}' \\
&= \cos\left(\phi - \left(\frac{\phi}{2} + \frac{\pi}{2}\right)\right) - \cos\left(\frac{\phi}{2} + \frac{\pi}{2}\right) \\
&= \cos\left(\frac{\phi}{2} - \frac{\pi}{2}\right) - \cos\left(\frac{\phi}{2} + \frac{\pi}{2}\right) \\
&= \sin\left(\frac{\phi}{2}\right) + \sin\left(\frac{\phi}{2}\right) = 2\sin\left(\frac{\phi}{2}\right)
\end{aligned}$$

Hence, we have

$$g\left(\phi_0{}'\right) = 2\sin\left(\frac{\phi}{2}\right)$$

High-Frequency Scattering by Cylinder (cont.)

Next, calculate the second derivative of the g function:

$$g(\phi') = \cos(\phi - \phi') - \cos\phi'$$

$$g''(\phi') = -\cos(\phi - \phi') + \cos\phi'$$

At SPP:

$$g''(\phi_0') = -\cos(\phi - \phi_0') + \cos\phi_0'$$
$$= -g(\phi_0')$$

Hence, we have

$$g''(\phi_0') = -2\sin\left(\frac{\phi}{2}\right)$$

Note:

$$g''(\phi_0') < 0$$

High-Frequency Scattering by Cylinder (cont.)

Recall:

$$I(\Omega) \approx f(x_0)\, e^{j\Omega g(x_0)} \sqrt{\frac{2\pi}{\Omega \left| g''(x_0) \right|}}\, e^{\pm j\frac{\pi}{4}} \qquad \begin{array}{ll} +, & g''(x_0) > 0 \\ -, & g''(x_0) < 0 \end{array}$$

$$\Omega = ka$$

$$f(\phi') = \cos\phi'$$

$$g(\phi') = \cos(\phi - \phi') - \cos\phi'$$

$$\phi_0' = \frac{\phi}{2} + \frac{\pi}{2}$$

$$g(\phi_0') = 2\sin\left(\frac{\phi}{2}\right)$$

$$g''(\phi_0') = -2\sin\left(\frac{\phi}{2}\right)$$

Hence

$$I(\Omega) \sim \cos\left(\frac{\phi}{2} + \frac{\pi}{2}\right) e^{j(ka)2\sin\left(\frac{\phi}{2}\right)} \sqrt{\frac{2\pi}{ka\left|-2\sin\frac{\phi}{2}\right|}}\, e^{-j\frac{\pi}{4}}$$

High-Frequency Scattering by Cylinder (cont.)

$$I(\Omega) \sim \cos\left(\frac{\phi}{2}+\frac{\pi}{2}\right) e^{j(ka)2\sin\left(\frac{\phi}{2}\right)} \sqrt{\frac{2\pi}{ka\left|-2\sin\frac{\phi}{2}\right|}}\, e^{-j\frac{\pi}{4}}$$

Simplify using

$$\cos\left(\frac{\phi}{2}+\frac{\pi}{2}\right) = \sin\left(\frac{\phi}{2}\right)$$

Then we have

$$I(\Omega) \sim -\sqrt{\frac{\pi}{ka}} \sqrt{\sin\left(\frac{\phi}{2}\right)}\, e^{j(ka)2\sin\left(\frac{\phi}{2}\right)} e^{-j\frac{\pi}{4}}$$

High-Frequency Scattering by Cylinder (cont.)

Recall:

$$E_z^s = \frac{\omega\mu}{2\eta}\sqrt{\frac{2}{\pi k\rho}}e^{-jk\rho}e^{j\frac{\pi}{4}}a\; I(\Omega)$$

Therefore, we have

$$E_z^s \sim \frac{\omega\mu}{2\eta}\sqrt{\frac{2}{\pi k\rho}}e^{-jk\rho}e^{j\frac{\pi}{4}}a\left[\left(-\sqrt{\frac{\pi}{ka}}\right)\sqrt{\sin\left(\frac{\phi}{2}\right)}e^{j(ka)2\sin\left(\frac{\phi}{2}\right)}e^{-j\frac{\pi}{4}}\right]$$

Use $\omega\mu/\eta = k$

Simplifying, we have

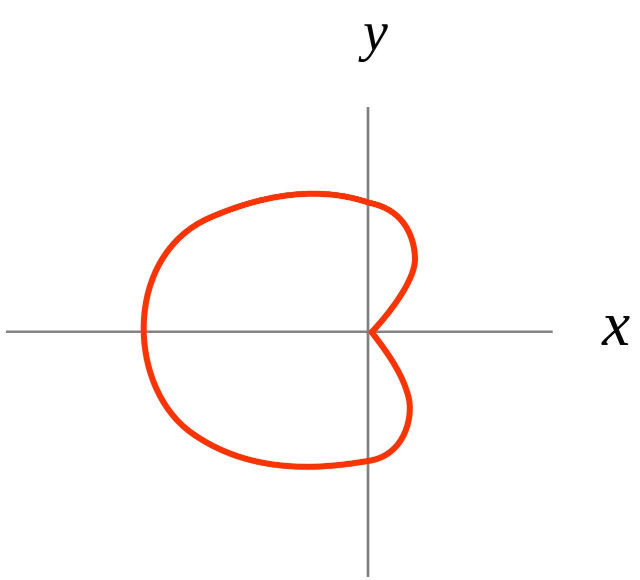
$$E_z^s \sim -\sqrt{\frac{a}{2\rho}}\sqrt{\sin\left(\frac{\phi}{2}\right)}e^{-jk\rho}e^{j(ka)2\sin\left(\frac{\phi}{2}\right)}$$

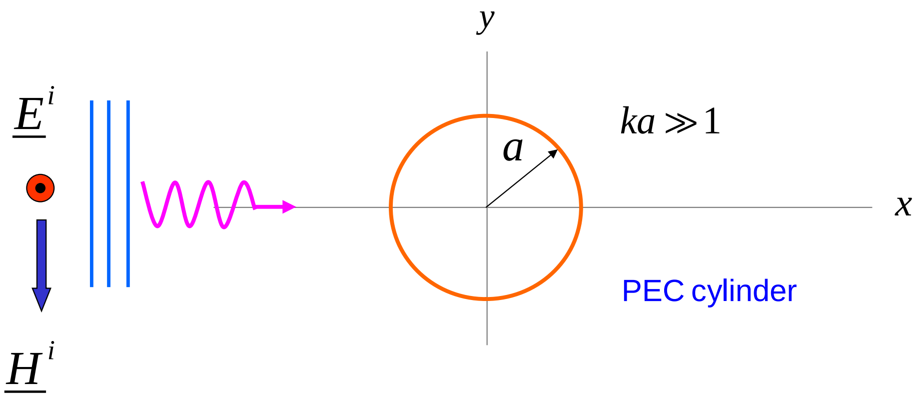
High-Frequency Scattering by Cylinder (cont.)

Final high-frequency radiation pattern of cylinder (scattered field)

$$E_z^s \sim -\sqrt{\frac{a}{2\rho}}\sqrt{\sin\left(\frac{\phi}{2}\right)}\, e^{-jk\rho} e^{j(ka)2\sin\left(\frac{\phi}{2}\right)}$$

$$\phi \neq 0$$





High-Frequency Scattering by Cylinder (cont.)

Solution for $\phi = 0$

In this case we have:

$$I(ka) \equiv \int_{\pi/2}^{3\pi/2} \cos\phi' e^{j(ka)\left[\cos(\phi-\phi')-\cos\phi'\right]} d\phi'$$

$$= \int_{\pi/2}^{3\pi/2} \cos\phi' d\phi'$$

$$= -2$$

Recall:

$$E_z^s = \frac{\omega\mu}{2\eta}\sqrt{\frac{2}{\pi k\rho}} e^{-jk\rho} e^{j\frac{\pi}{4}} a\, I(\Omega)$$

Hence:

$$E_z^s = -ka\sqrt{\frac{2}{\pi k\rho}} e^{-jk\rho} e^{j\frac{\pi}{4}}$$

High-Frequency Scattering by Cylinder (cont.)

In the backscattered direction ($\phi = \pi$):

$$E_z^{sb} \sim -\sqrt{\frac{a}{2\rho}}\, e^{-jk\rho} e^{j2(ka)}$$

Echo width (monostatic RCS):

$$W_e \left(\frac{\left|E_0\right|^2}{2\eta} \right) \left(\frac{1}{2\pi\rho} \right) = \frac{\left|E_z^{sb}\right|^2}{2\eta} \quad \Rightarrow \quad W_e = \lim_{\rho \to \infty} \frac{2\pi\rho \left|E_z^s\right|^2}{\left|E_0\right|^2}$$

Note: $E_0 = 1$ in our case.

High-Frequency Scattering by Cylinder (cont.)

$$W_e = 2\pi\rho \left| E_z^{sb} \right|^2$$

$$E_z^s \sim -\sqrt{\frac{a}{2\rho}}\, e^{-jk\rho} e^{j2(ka)}$$

Hence

$$W_e = 2\pi\rho \left(\sqrt{\frac{a}{2\rho}} \right)^2 = \pi a$$

We then have

$$W_e = \pi a$$

(circumference of lit region)