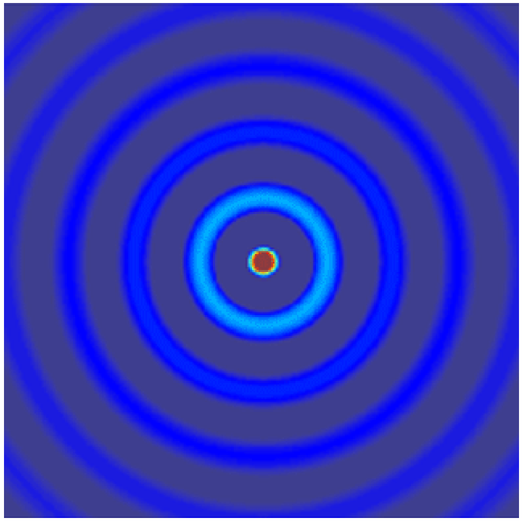
ECE 6341

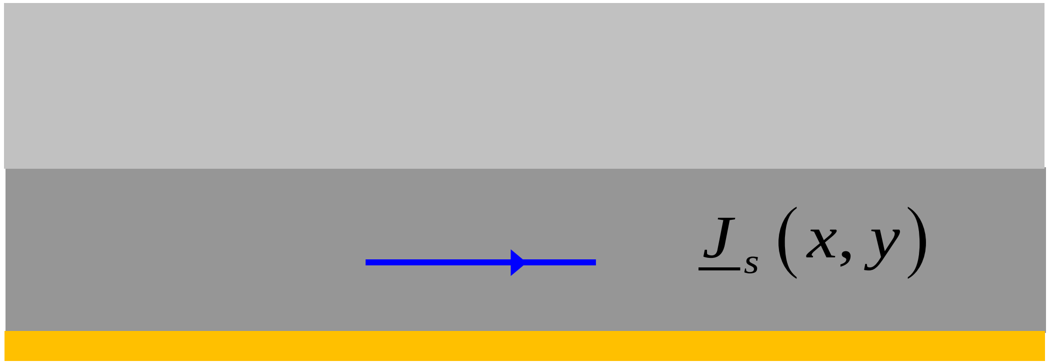
Spring 2016

Prof. David R. Jackson
ECE Dept.



Notes 39

Finite Source



For a phased current sheet: $\underline{J}_s^{\,p}(x,y) = \underline{J}_{s0}^{\,p}\, e^{-j(k_x x + k_y y)}$

The tangential electric field that is produced is:

$$\underline{E}_t(x,y,z) = \underline{\hat{u}}\, V_i^{TM}(z)\left(-\underline{J}_{s0}^{\,p} \cdot \underline{\hat{u}}\right) e^{-j(k_x x + k_y y)}$$

$$+\,\underline{\hat{v}}\left(-V_i^{TE}(z)\right)\left(+\underline{J}_{s0}^{\,p} \cdot \underline{\hat{v}}\right) e^{-j(k_x x + k_y y)}$$

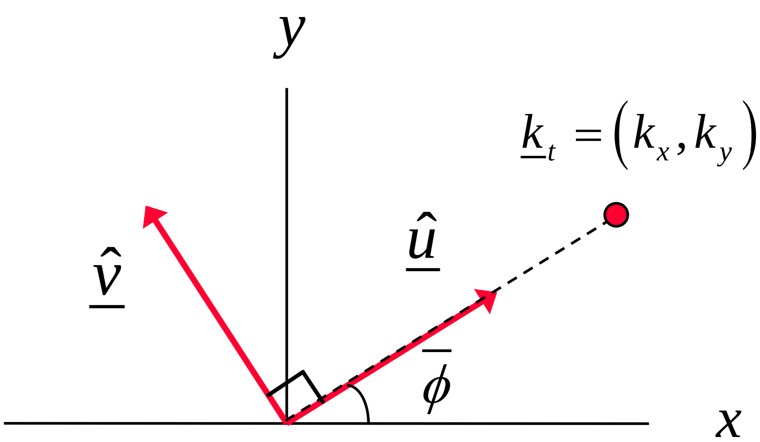
Recall that $\underline{J}_{s0}^{\,p} = \dfrac{1}{(2\pi)^2}\, \underline{\tilde{J}}_s(k_x, k_y)\, dk_x\, dk_y$

Finite Source (cont.)

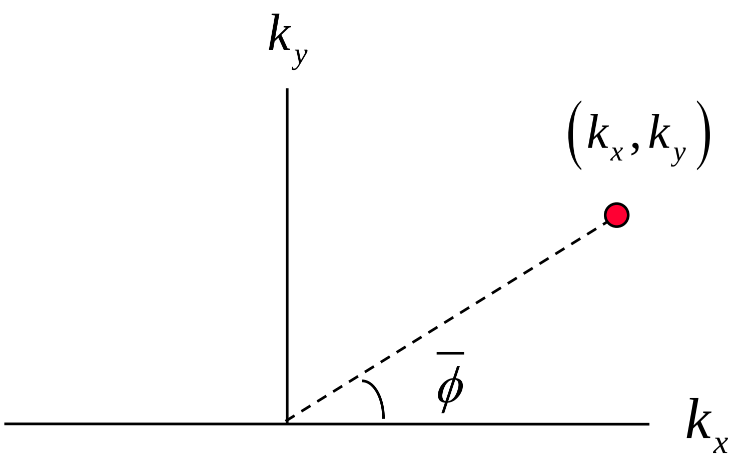
Hence

$$\underline{E}_t(x,y,z)=\frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left\{\underline{\hat{u}}V_i^{TM}(z)\left[-\underline{\tilde{J}}_s\cdot\underline{\hat{u}}\right]-\underline{\hat{v}}V_i^{TE}(z)\left[\underline{\tilde{J}}_s\cdot\underline{\hat{v}}\right]\right\}$$
$$\cdot e^{-j(k_x x+k_y y)}dk_x dk_y$$

Note: $\underline{\hat{u}}=\underline{\hat{u}}(k_x,k_y),\;\; \underline{\hat{v}}=\underline{\hat{v}}(k_x,k_y)$



Spatial coordinates



Wavenumber plane

TEN Model for Transform of Fields

We can also write

$$\underline{E}_t(x,y,z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \underline{\tilde{E}}_t(k_x,k_y,z)\, e^{-j(k_x x + k_y y)}\, dk_x\, dk_y$$

Comparing with the previous result, we have

$$\underline{\tilde{E}}_t(k_x,k_y,z) = \underline{\hat{u}}\, V_i^{TM}(z)\left[-\underline{\tilde{J}}_s \cdot \underline{\hat{u}}\right] - \underline{\hat{v}}\, V_i^{TE}(z)\left[\underline{\tilde{J}}_s \cdot \underline{\hat{v}}\right]$$

Similarly,

$$\underline{\tilde{H}}_t(k_x,k_y,z) = \underline{\hat{u}}\, I_i^{TE}(z)\left[\underline{\tilde{J}}_s \cdot \underline{\hat{v}}\right] + \underline{\hat{v}}\, I_i^{TM}(z)\left[-\underline{\tilde{J}}_s \cdot \underline{\hat{u}}\right]$$

This motivates the following identifications:

TEN Model (cont.)

Modeling equations for horizontal electric surface current:

$$V^{TM}(z) = \underline{\tilde{E}}_t(k_x, k_y, z) \cdot \underline{\hat{u}}$$

$$V^{TE}(z) = -\underline{\tilde{E}}_t(k_x, k_y, z) \cdot \underline{\hat{v}}$$

$$I^{TE}(z) = \underline{\tilde{H}}_t(k_x, k_y, z) \cdot \underline{\hat{u}}$$

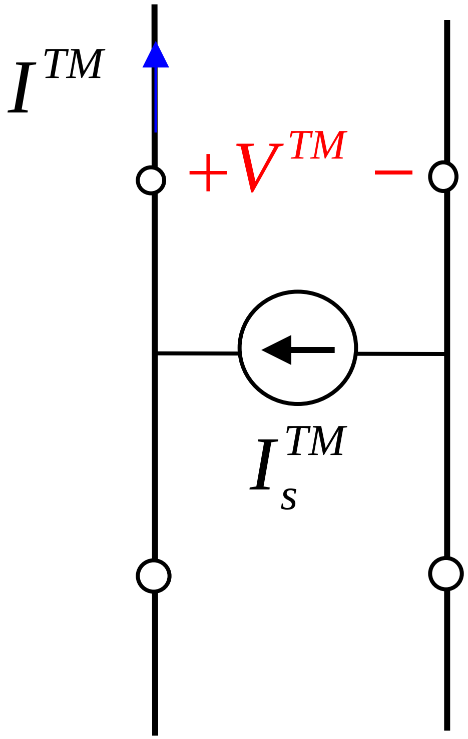
$$I^{TM}(z) = \underline{\tilde{H}}_t(k_x, k_y, z) \cdot \underline{\hat{v}}$$

$$I_s^{TM} = -\underline{\tilde{J}}_s(k_x, k_y) \cdot \underline{\hat{u}}$$

$$I_s^{TE} = +\underline{\tilde{J}}_s(k_x, k_y) \cdot \underline{\hat{v}}$$

TEN Model (cont.)

TM_z :



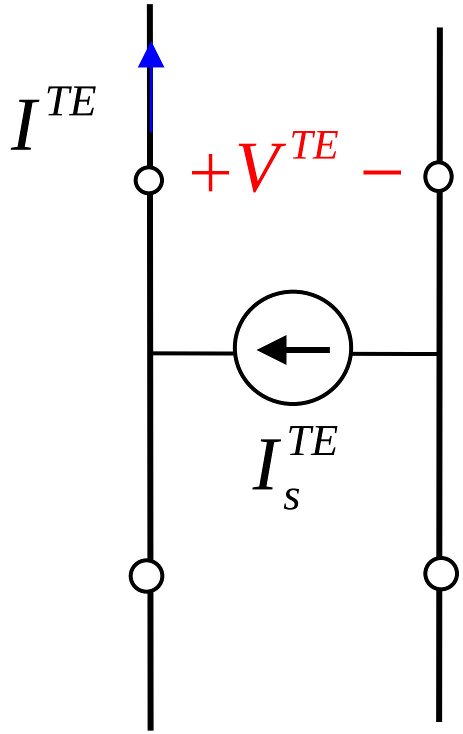
$$V^{TM} = \underline{\tilde{E}}_t \cdot \underline{\hat{u}}$$

$$I^{TM} = \underline{\tilde{H}}_t \cdot \underline{\hat{v}}$$

$$I_s^{TM} = -\underline{\tilde{J}}_s \cdot \underline{\hat{u}}$$

TEN Model (cont.)

TE_z :

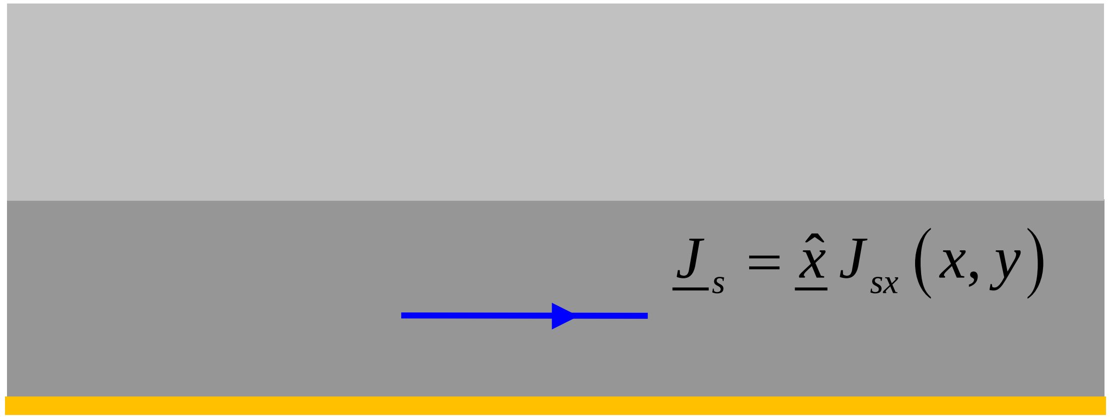


$$V^{TE} = -\underline{\tilde{E}}_t \cdot \underline{\hat{v}}$$

$$I^{TE} = \underline{\tilde{H}}_t \cdot \underline{\hat{u}}$$

$$I_s^{TE} = \underline{\tilde{J}}_s \cdot \underline{\hat{v}}$$

Example



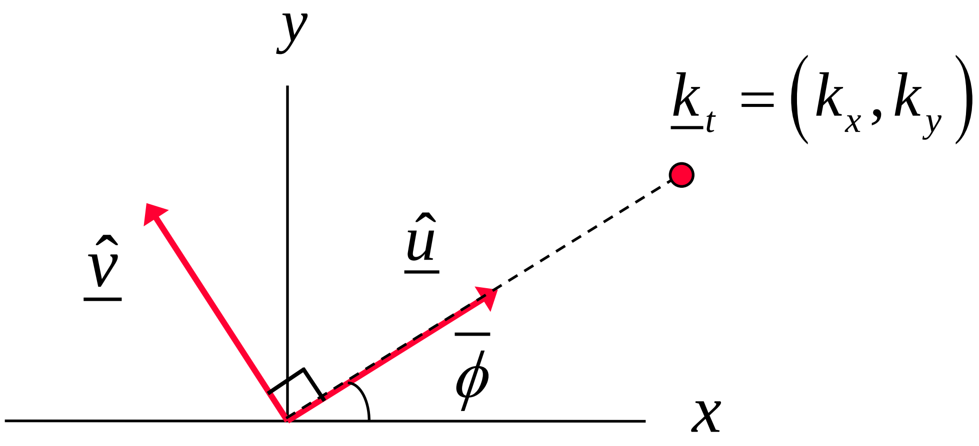
Find $E_x(x, y, z)$

$$\begin{aligned}\tilde{E}_x\left(k_x, k_y, z\right) &= \underline{\hat{x}} \cdot \left[\underline{\hat{u}} \left(\underline{\tilde{E}}_t \cdot \underline{\hat{u}} \right) + \underline{\hat{v}} \left(\underline{\tilde{E}}_t \cdot \underline{\hat{v}} \right) \right] \\ &= \left(\underline{\hat{x}} \cdot \underline{\hat{u}} \right) V^{TM}(z) + \left(\underline{\hat{x}} \cdot \underline{\hat{v}} \right) \left(-V^{TE}(z) \right)\end{aligned}$$

Example (cont.)

$$\underline{\hat{x}} \cdot \underline{\hat{u}} = \cos\overline{\phi} = \frac{k_x}{k_t}$$

$$\underline{\hat{x}} \cdot \underline{\hat{v}} = -\sin\overline{\phi} = -\frac{k_y}{k_t}$$



Hence

$$\tilde{E}_x(k_x, k_y, z) = \left(\frac{k_x}{k_t}\right) V^{TM}(z) + \left(\frac{k_y}{k_t}\right) V^{TE}(z)$$

$$= \left(\frac{k_x}{k_t}\right) V_i^{TM}(z)\left[-\underline{\tilde{J}}_s \cdot \underline{\hat{u}}\right] + \left(\frac{k_y}{k_t}\right) V_i^{TE}(z)\left[\underline{\tilde{J}}_s \cdot \underline{\hat{v}}\right]$$

$$\underline{\tilde{J}}_s \cdot \underline{\hat{u}} = \left(\tilde{J}_{sx}\underline{\hat{x}}\right) \cdot \underline{\hat{u}} = \tilde{J}_{sx}\cos\overline{\phi} = \tilde{J}_{sx}\left(\frac{k_x}{k_t}\right) \qquad \underline{\tilde{J}}_s \cdot \underline{\hat{v}} = \left(\tilde{J}_{sx}\underline{\hat{x}}\right) \cdot \underline{\hat{v}} = \tilde{J}_{sx}\left(-\sin\overline{\phi}\right) = \tilde{J}_{sx}\left(-\frac{k_y}{k_t}\right)$$

Example (cont.)

Hence

$$\tilde{E}_x\left(k_x,k_y,z\right)=\left(\frac{k_x}{k_t}\right)V_i^{TM}\left(z\right)\left[-\tilde{J}_{sx}\left(\frac{k_x}{k_t}\right)\right]+\left(\frac{k_y}{k_t}\right)V_i^{TE}\left(z\right)\left[\tilde{J}_{sx}\left(-\frac{k_y}{k_t}\right)\right]$$

$$=-\frac{1}{k_t^2}\tilde{J}_{sx}\left[k_x^2V_i^{TM}\left(z\right)+k_y^2V_i^{TE}\left(z\right)\right]$$

or

$$E_x(x,y,z)=\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}-\frac{1}{k_t^2}\tilde{J}_{sx}\left[k_x^2V_i^{TM}\left(z\right)+k_y^2V_i^{TE}\left(z\right)\right]\cdot e^{-j\left(k_xx+k_yy\right)}dk_xdk_y$$

Dyadic Green's Function

$$\underline{\underline{G}}\left(x-x', y-y'; z, z'\right) = \begin{bmatrix} G_{xx} & G_{xy} & G_{xz} \\ G_{yx} & G_{yy} & G_{yz} \\ G_{zx} & G_{zy} & G_{zz} \end{bmatrix}$$

where

$G_{ij} = E_i\left(x, y, z\right)$ due to the unit-amplitude electric dipole at $\left(x', y', z'\right)$

$$\underline{J}\left(x, y, z\right) = \underline{\hat{j}}\,\delta\left(x-x'\right)\delta\left(y-y'\right)\delta\left(z-z'\right)$$

From superposition:

Note: We have translational invariance due to the infinite substrate.

$$\underline{E}\left(x, y; z, z'\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \underline{\underline{G}}\left(x-x', y-y'; z, z'\right) \cdot \underline{J}_s\left(x', y'; z'\right) dx'\,dy'$$

where $\underline{E} = \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$ $\quad \underline{J}_s = \begin{bmatrix} J_{sx} \\ J_{sy} \\ J_{sz} \end{bmatrix}$

We assume here that the currents are located on a planar surface z'.

Dyadic Green's Function (cont.)

$$\underline{E}(x,y;z,z') = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \underline{\underline{G}}(x-x',y-y';z,z')\cdot \underline{J}_s(x',y';z')\,dx'\,dy'$$

This is recognized as a 2D convolution:

$$\underline{E} = \underline{\underline{G}} * \underline{J}_s$$

Taking the 2D Fourier transform of both sides,

$$\underline{\tilde{E}} = \underline{\underline{\tilde{G}}} \cdot \underline{\tilde{J}}_s$$

where $\underline{\underline{\tilde{G}}} = \underline{\underline{\tilde{G}}}(k_x,k_y;z,z')$

Dyadic Green's Function (cont.)

$$\underline{\tilde{E}} = \underline{\underline{\tilde{G}}} \cdot \underline{\tilde{J}}_s$$

$\underline{\underline{\tilde{G}}}\left(k_x, k_y; z, z'\right)$ is called the spectral-domain dyadic Green's function.

It is the Fourier transform of the spatial-domain dyadic Green's function.

Assuming we wish the x component of the electric field due to an x-directed current J_{sx} (x', y'), we have

$$\tilde{E}_x = \tilde{G}_{xx}\, \tilde{J}_{sx}$$

In order to indentify $\tilde{G}_{xx}$, we use

$$E_x(x, y; z, z') = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} -\frac{1}{k_t^2}\, \tilde{J}_{sx}\left(k_x, k_y\right)\left[k_x^2\, V_i^{TM}\left(z, z'\right) + k_y^2\, V_i^{TE}\left(z, z'\right)\right] \cdot e^{-j\left(k_x x + k_y y\right)} dk_x dk_y$$

Dyadic Green's Function (cont.)

$$E_x(x,y;z,z') = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\{-\frac{1}{k_t^2}\tilde{J}_{sx}\left[k_x^2 V_i^{TM}(z,z') + k_y^2 V_i^{TE}(z,z')\right]\right\} \cdot e^{-j\left(k_x x + k_y y\right)} dk_x dk_y$$

$$\Downarrow$$

$$\tilde{E}_x(k_x,k_y;z,z') = \left(-\frac{1}{k_t^2}\left[k_x^2 V_i^{TM}(z,z') + k_y^2 V_i^{TE}(z,z')\right]\right)\tilde{J}_{sx}\left(k_x,k_y\right)$$

Recall that $\tilde{E}_x = \tilde{G}_{xx}\,\tilde{J}_{sx}$

Hence

$$\tilde{G}_{xx}\left(k_x,k_y;z,z'\right) = -\frac{1}{k_t^2}\left[k_x^2 V_i^{TM}(z,z') + k_y^2 V_i^{TE}(z,z')\right]$$

Dyadic Green's Function (cont.)

We then have:

$$\underline{\underline{\tilde{G}}} = \begin{bmatrix} \tilde{G}_{xx} & \tilde{G}_{xy} & \tilde{G}_{xz} \\ \tilde{G}_{yx} & \tilde{G}_{yy} & \tilde{G}_{yz} \\ \tilde{G}_{zx} & \tilde{G}_{zy} & \tilde{G}_{zz} \end{bmatrix}$$

$$\tilde{G}_{xx}\left(k_x, k_y; z, z'\right) = -\frac{1}{k_t^2}\left[k_x^2 V_i^{TM}\left(z, z'\right) + k_y^2 V_i^{TE}\left(z, z'\right)\right]$$

The other eight components could be found in a similar way.

We could also find the magnetic field components.

We can also find the fields due to a magnetic current.

$\tilde{G}_{xx} = \tilde{G}_{xx}^{EJ}$ (the xx component of the electric field due to an electric durrent)

Dyadic Green's Function (cont.)

The different types of spectral-domain dyadic Green's functions are:

$\tilde{G}_{ij}^{EJ}$ Gives electric field due to electric current

$\tilde{G}_{ij}^{EM}$ Gives electric field due to magnetic current

$\tilde{G}_{ij}^{HJ}$ Gives magnetic field due to electric current

$\tilde{G}_{ij}^{HM}$ Gives magnetic field due to magnetic current

Note:
There are 36 terms here, though many are equal by reciprocity or symmetry.
There are 20 unique terms (five from each type of Green's function).

Summary of Results for All Sources

These results are derived in Notes 44.

$$V^{TM} = \tilde{E}_u$$
$$I^{TM} = \tilde{H}_v$$
$$V^{TE} = -\tilde{E}_v$$
$$I^{TE} = \tilde{H}_u$$

$$I_s^{TM} = -\tilde{J}_{su}$$
$$V_s^{TM} = -\tilde{M}_{sv} + \left(\frac{k_t}{\omega\varepsilon}\right)\tilde{J}_{sz}$$

$$V_s^{TE} = -\tilde{M}_{su}$$
$$I_s^{TE} = \tilde{J}_{sv} + \left(\frac{k_t}{\omega\mu}\right)\tilde{M}_{sz}$$

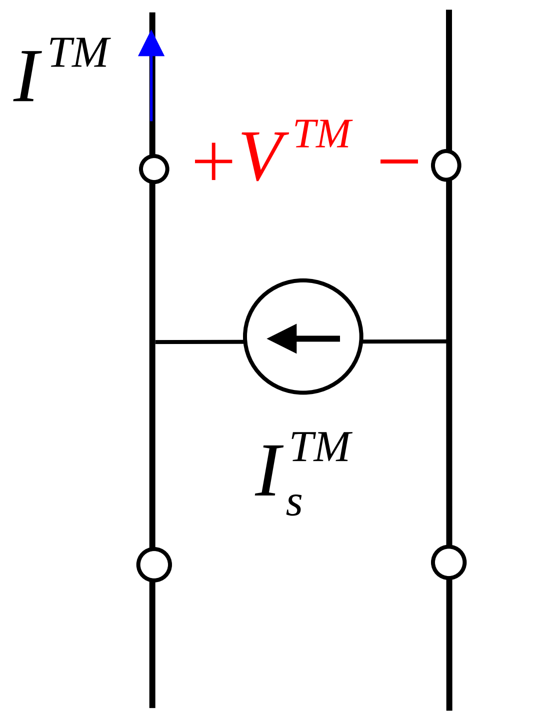
Definition of “vertical planar currents”:

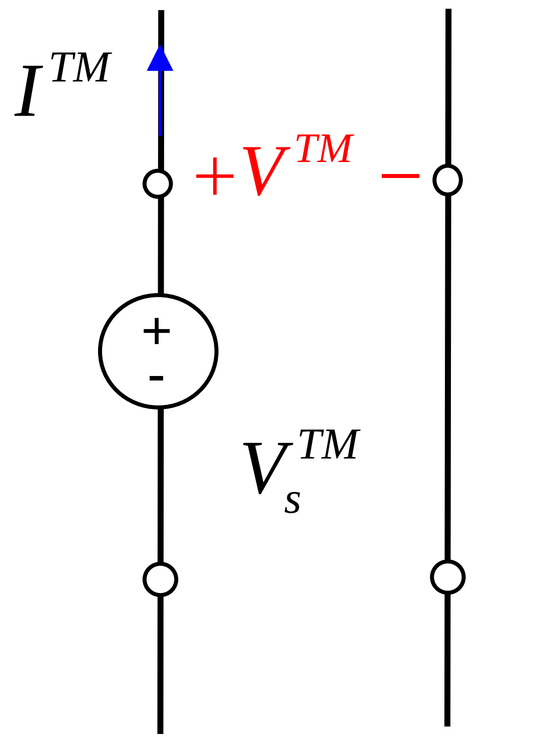
$$J_z(x,y,z) = J_{sz}(x,y)\,\delta(z)$$
$$M_z(x,y,z) = M_{sz}(x,y)\,\delta(z)$$

Sources used in Modeling

TM_z :

V_i = voltage due to 1[A] parallel current source
I_i = current due to 1[A] parallel current source
V_v = voltage due to 1[V] series voltage source
I_v = current due to 1[V] series voltage source





$$V^{TM}(z) = V_i^{TM}(z)\, I_s^{TM}$$

$$I^{TM}(z) = I_i^{TM}(z)\, I_s^{TM}$$

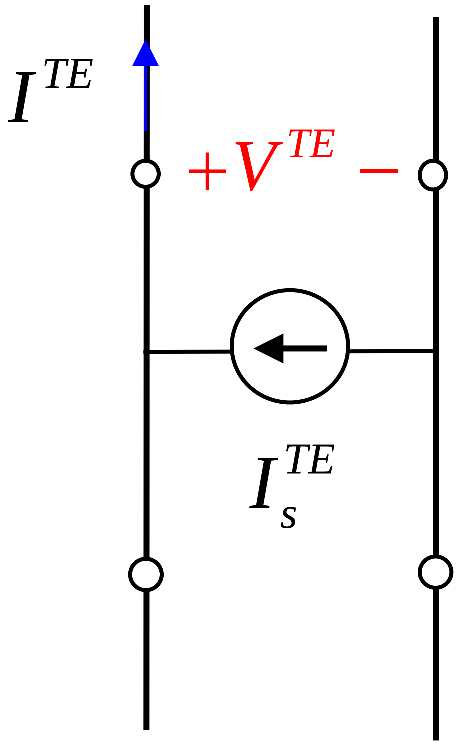
$$V^{TM}(z) = V_v^{TM}(z) V_s^{TM}$$

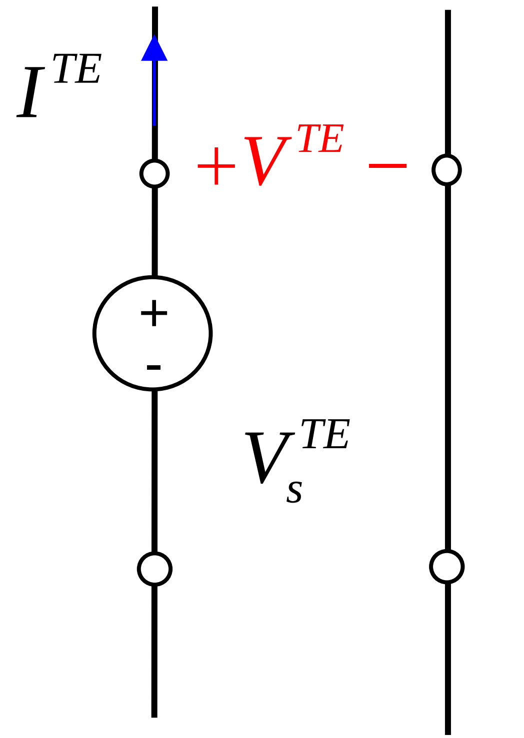
$$I^{TM}(z) = I_v^{TM}(z) V_s^{TM}$$

Sources used in Modeling (cont.)

TE_z :

V_i = voltage due to 1[A] parallel current source
I_i = current due to 1[A] parallel current source
V_v = voltage due to 1[V] series voltage source
I_v = current due to 1[V] series voltage source





$$V^{TE}(z) = V_i^{TE}(z)\, I_s^{TE}$$

$$I^{TE}(z) = I_i^{TE}(z)\, I_s^{TE}$$

$$V^{TM}(z) = V_v^{TE}(z)\, V_s^{TE}$$

$$I^{TE}(z) = I_v^{TE}(z)\, V_s^{TE}$$

Example

Find $\tilde{G}_{xx}^{HM}$

This is the xx component of the spectral-domain dyadic Green's function that is used to obtain H_x from M_{sx}.

Start with:

$$\tilde{H}_x\left(k_x,k_y,z\right)=\underline{\hat{x}}\cdot\left[\underline{\hat{u}}\left(\tilde{H}_t\cdot\underline{\hat{u}}\right)+\underline{\hat{v}}\left(\tilde{H}_t\cdot\underline{\hat{v}}\right)\right]$$

$$=\left(\underline{\hat{x}}\cdot\underline{\hat{u}}\right)I^{TE}\left(z\right)+\left(\underline{\hat{x}}\cdot\underline{\hat{v}}\right)\left(I^{TM}\left(z\right)\right)$$

Recall:

$$V^{TM}\left(z\right)=\underline{\tilde{E}}_t\left(k_x,k_y,z\right)\cdot\underline{\hat{u}}$$

$$V^{TE}\left(z\right)=-\underline{\tilde{E}}_t\left(k_x,k_y,z\right)\cdot\underline{\hat{v}}$$

$$I^{TE}\left(z\right)=\underline{\tilde{H}}_t\left(k_x,k_y,z\right)\cdot\underline{\hat{u}}$$

$$I^{TM}\left(z\right)=\underline{\tilde{H}}_t\left(k_x,k_y,z\right)\cdot\underline{\hat{v}}$$

Example (cont.)

$$\begin{aligned}
\tilde{H}_x\left(k_x,k_y,z\right) &= \left(\underline{\hat{x}}\cdot\underline{\hat{u}}\right)I^{TE}\left(z\right)+\left(\underline{\hat{x}}\cdot\underline{\hat{v}}\right)\left(I^{TM}\left(z\right)\right)\\
&= \left(\cos\overline{\phi}\right)I^{TE}\left(z\right)+\left(-\sin\overline{\phi}\right)\left(I^{TM}\left(z\right)\right)\\
&= \left(\cos\overline{\phi}\right)I_v^{TE}\left(z\right)V_s^{TE}+\left(-\sin\overline{\phi}\right)I_v^{TM}\left(z\right)V_s^{TM}\\
&= \left(\cos\overline{\phi}\right)I_v^{TE}\left(z\right)\left(-\tilde{M}_{su}\right)+\left(-\sin\overline{\phi}\right)I_v^{TM}\left(z\right)\left(-\tilde{M}_{sv}\right)\\
&= \left(\cos\overline{\phi}\right)I_v^{TE}\left(z\right)\left(-\tilde{M}_{sx}\cos\overline{\phi}\right)+\left(-\sin\overline{\phi}\right)I_v^{TM}\left(z\right)\left(-\tilde{M}_{sx}\left(-\sin\overline{\phi}\right)\right)\\
&= -\tilde{M}_{sx}\left(I_v^{TE}\left(z\right)\cos^2\overline{\phi}+I_v^{TM}\left(z\right)\sin^2\overline{\phi}\right)\\
&= -\tilde{M}_{sx}\left(I_v^{TE}\left(z\right)\left(\frac{k_x^2}{k_t^2}\right)+I_v^{TM}\left(z\right)\left(\frac{k_x^2}{k_t^2}\right)\right)
\end{aligned}$$

Recall:

$$I_s^{TM}=-\tilde{J}_{su}$$

$$V_s^{TM}=-\tilde{M}_{sv}+\left(\frac{k_t}{\omega\varepsilon}\right)\tilde{J}_{sz}$$

$$V_s^{TE}=-\tilde{M}_{su}$$

$$I_s^{TE}=\tilde{J}_{sv}+\left(\frac{k_t}{\omega\mu}\right)\tilde{M}_{sz}$$

Example (cont.)

$$\tilde{H}_x\left(k_x, k_y, z\right) = -\tilde{M}_{sx}\frac{1}{k_t^2}\left(k_x^2 I_v^{TE}\left(z\right) + k_y^2 I_v^{TM}\left(z\right)\right)$$

Hence, we have

$$\tilde{G}_{xx}^{HM}\left(k_x, k_y; z, z'\right) = -\frac{1}{k_t^2}\left[k_x^2 I_v^{TE}\left(z, z'\right) + k_y^2 I_v^{TM}\left(z, z'\right)\right]$$

Note: The notation (z,z') has been used in the final result to emphasize that the terms depend on both z and z'.

Summary of Spectral-Domain Recipe

Example (E_x from J_{sx})

Start with a given planar current distribution.

$$J_{sx}(x, y)$$

Take the Fourier transform of the surface current.

$$\tilde{J}_{sx}(k_x, k_y)$$

Use the appropriate spectral-domain dyadic Green's function to find the Fourier transform of the field of interest.

$$\tilde{E}_x = \tilde{G}_{xx}^{EJ}\, \tilde{J}_{sx}(k_x, k_y)$$

Take the inverse Fourier transform (2D integral in k_x and k_y) to find the field of interest in the space domain.

$$E_x = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{G}_{xx}^{EJ}\, \tilde{J}_{sx} e^{-j(k_x x + k_y y)} dk_x dk_y$$