ECE 6341

Spring 2016

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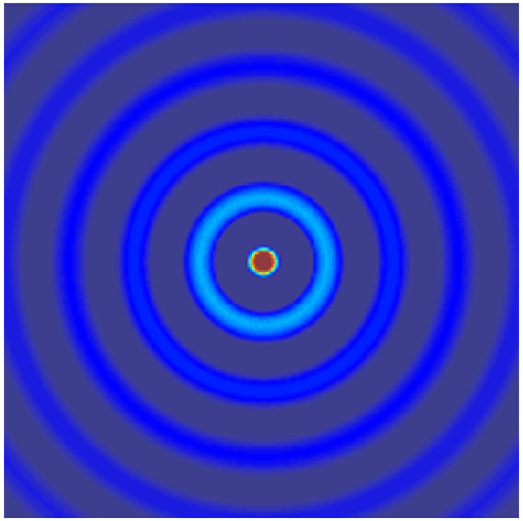
ECE Dept.



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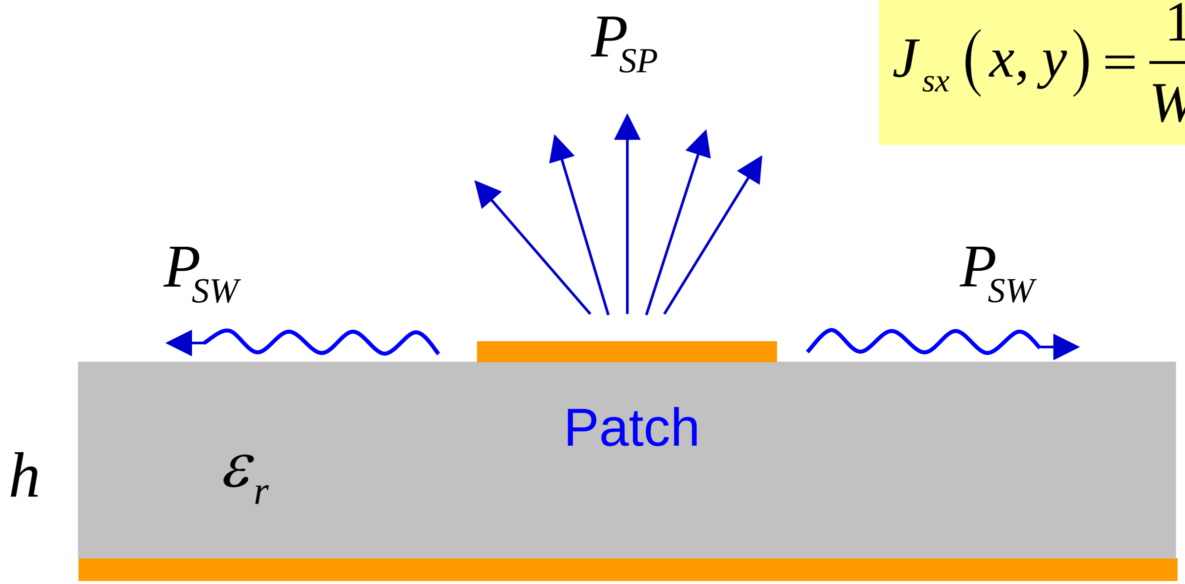
Efficiency of Patch

$$J_{sx}(x,y) = \frac{1}{W}\cos\left(\frac{\pi x}{L}\right)$$



Lossless substrate and metal

$$P_{TOT} = P_{SP} + P_{SW}$$

$$e_r = \frac{P_{SP}}{P_{TOT}}$$

Total Power

The total <u>complex</u> power radiated by the patch is:

$$P_c = -\frac{1}{2}\int_S \underline{E}\cdot\underline{J}_s^{\,*}\, dS$$

$$= -\frac{1}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_x\, J_{sx}^{\,*}\, dxdy$$

Use Parseval's Theorem:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, g^{*}(x,y)\, dxdy = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{f}\left(k_x,k_y\right)\tilde{g}^{*}\left(k_x,k_y\right) dk_x dk_y$$

Total Power (cont.)

Hence

$$P_c = -\frac{1}{8\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\tilde{E}_x\left(\tilde{J}_{sx}\right)^* dk_x dk_y$$

From SDI analysis:

$$\tilde{E}_x = \tilde{J}_{sx}\tilde{G}_{xx}$$

where

$$\tilde{G}_{xx} = -\frac{1}{k_t^{\,2}}\left[\frac{k_x^2}{D_m\left(k_t\right)} + \frac{k_y^2}{D_e\left(k_t\right)}\right]$$

$$\tilde{J}_{sx}\left(k_x,k_y\right) = \frac{\pi L}{2}\left[\frac{\cos\left(k_x\frac{L}{2}\right)}{\left(\frac{\pi}{2}\right)^2 - \left(\frac{k_x L}{2}\right)^2}\right]\operatorname{sinc}\left(k_y\frac{W}{2}\right)$$

$$D_m\left(k_t\right) = Y_0^{\,TM} - jY_1^{\,TM}\cot\left(k_{z1}h\right)$$

$$D_e\left(k_t\right) = Y_0^{\,TE} - jY_1^{\,TE}\cot\left(k_{z1}h\right)$$

Total Power (cont.)

We then have

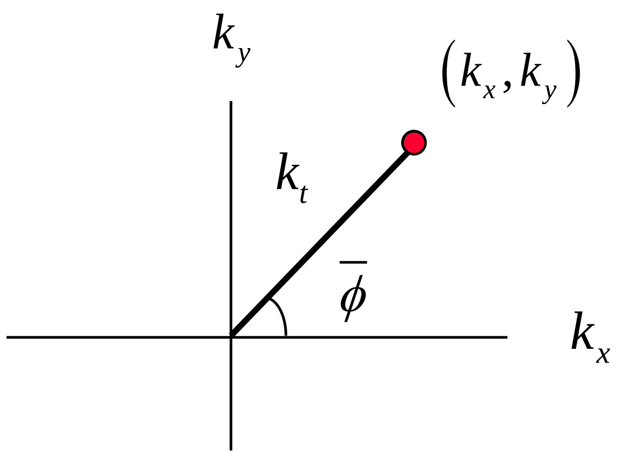
$$P_c = -\frac{1}{8\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{G}_{xx} \left| \tilde{J}_{sx} \right|^2 dk_x dk_y$$

Using symmetry, we have

$$P_c = -\frac{1}{2\pi^2} \int_0^{\infty} \int_0^{\infty} \tilde{G}_{xx}\left(k_x, k_y\right) \left| \tilde{J}_{sx}\left(k_x, k_y\right) \right|^2 dk_x dk_y$$

Polar coordinates:

$$P_c = -\frac{1}{2\pi^2} \int_0^{\pi/2} \int_C \tilde{G}_{xx} \left| \tilde{J}_{sx} \right|^2 k_t dk_t d\overline{\phi}$$



Total Power (cont.)

$$P_c = -\frac{1}{2\pi^2} \int_0^{\pi/2} \int_C \tilde{G}_{xx} \left| \tilde{J}_{sx} \right|^2 k_t \, dk_t \, d\overline{\phi}$$

Note that

$$\tilde{J}_{sx} = \text{Real Function of } \left(k_x, k_y\right) \text{ for real } k_x, k_y.$$

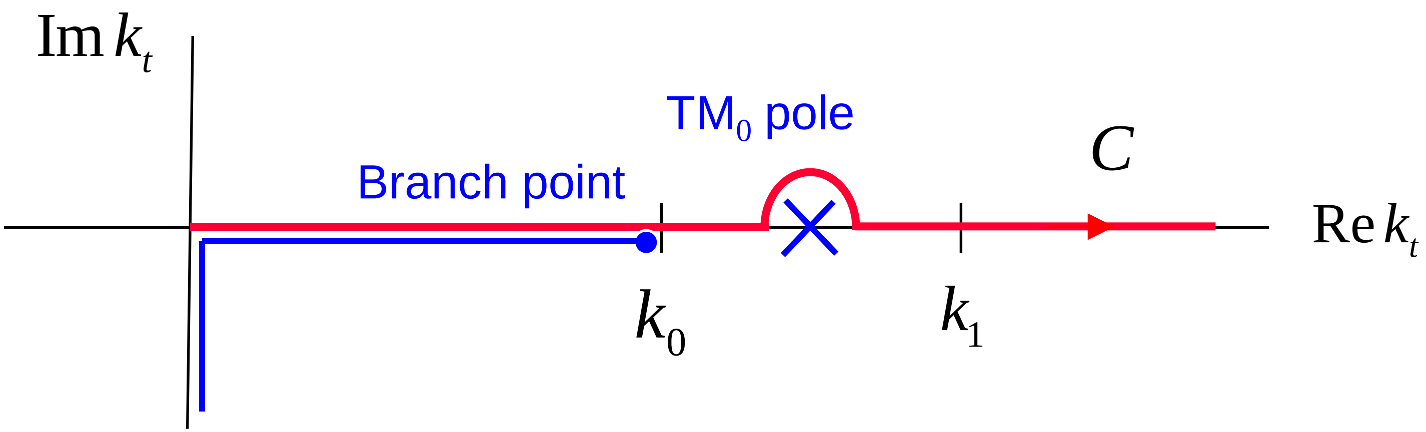
Hence

$$P_c = -\frac{1}{2\pi^2} \int_0^{\pi/2} \int_C \tilde{G}_{xx}\left(k_t, \overline{\phi}\right) \tilde{J}_{sx}^2 \left(k_t, \overline{\phi}\right) k_t \, dk_t \, d\overline{\phi}$$

Note: $\tilde{J}_{sx}^2$ is analytic but $\left|\tilde{J}_{sx}\right|^2$ is not. This last form for P_c is preferable!

Total Power (cont.)

$$P_{TOT} = -\frac{1}{2\pi^2}\,\mathrm{Re}\int_0^{\pi/2}\int_C \tilde{G}_{xx}\left(k_t,\overline{\phi}\right)\tilde{J}_{sx}^{\,2}\left(k_t,\overline{\phi}\right)k_t\,dk_t\,d\overline{\phi}$$



For real k_t we have (proof on next page):

$$\tilde{G}_{xx} = \begin{cases} \text{complex}, & k_t < k_0 \\ \text{imag}, & k_t > k_0 \end{cases}$$

Hence, we can neglect the region $k_t > k_0$, except possibly for the pole.

Total Power (cont.)

Proof of complex property:

$$\tilde{G}_{xx}\left(k_t,\overline{\phi}\right)=-\frac{1}{k_t^{\,2}}\left[\frac{k_x^2}{D_m\left(k_t\right)}+\frac{k_y^2}{D_e\left(k_t\right)}\right]$$

Consider the following term (D_e is similar):

always imaginary

$$D_m\left(k_t\right)=Y_0^{TM}-jY_1^{TM}\cot\left(k_{z1}h\right)$$

$$Y_0^{TM}=\frac{\omega\varepsilon_0}{k_{z0}}$$

$$k_{z0}=\left(k_0^{\,2}-k_t^{\,2}\right)^{1/2}$$

$$Y_1^{TM}=\frac{\omega\varepsilon_1}{k_{z1}}$$

$$k_{z1}=\left(k_1^2-k_t^2\right)^{1/2}$$

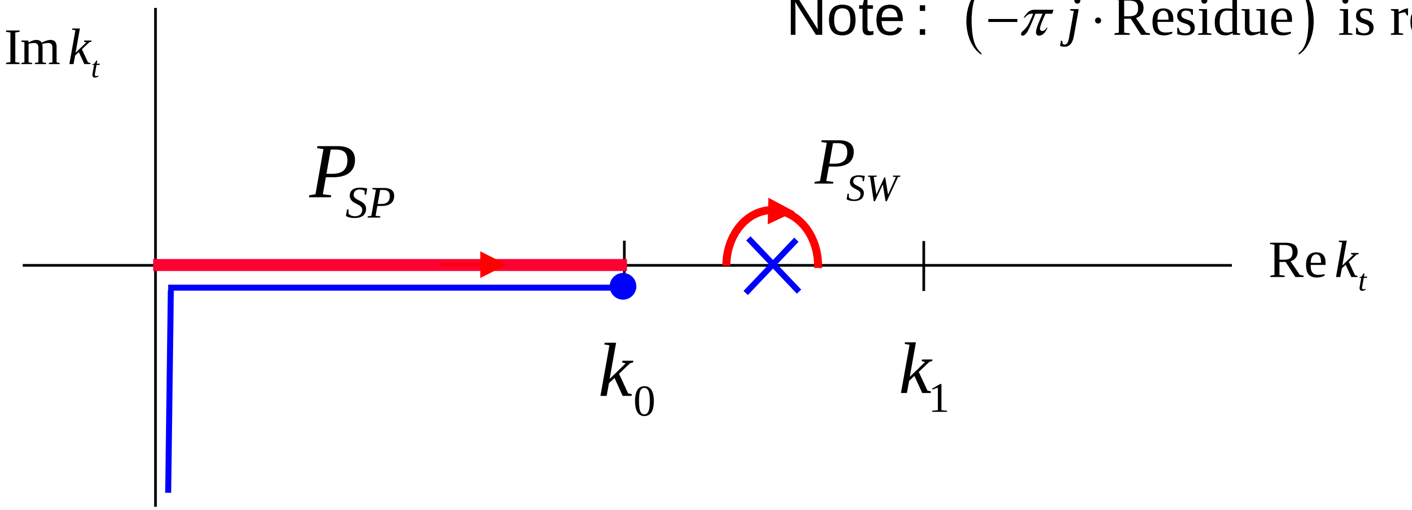
$$Y_0^{TM}=\begin{cases}\text{real}, & k_t<k_0\\ \text{imag}, & k_t>k_0\end{cases}$$

Space and Surface-Wave Powers

- The region $k_t \in (0, k_0)$ gives the power radiated into space.

- The residue contribution gives the power launched into the TM_0 surface wave.

Note: $\left(-\pi j \cdot \text{Residue}\right)$ is real.

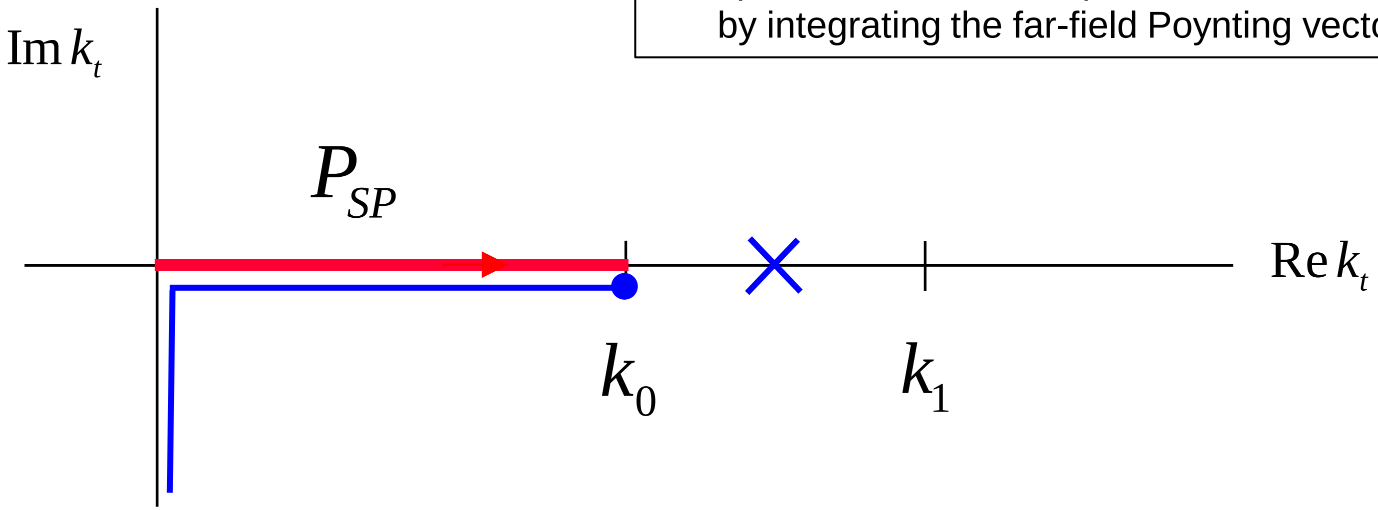


$$P_{TOT} = -\frac{1}{2\pi^2} \text{Re} \int_0^{\pi/2} \int_C \tilde{G}_{xx}\left(k_t, \overline{\phi}\right) \tilde{J}_{sx}^{\,2}\left(k_t, \overline{\phi}\right) k_t \, dk_t \, d\overline{\phi}$$

Space-Wave Power

The region $k_t \in (0, k_0)$ gives the power radiated into space.

Note:
The power radiated into space can also be found by integrating the far-field Poynting vector.

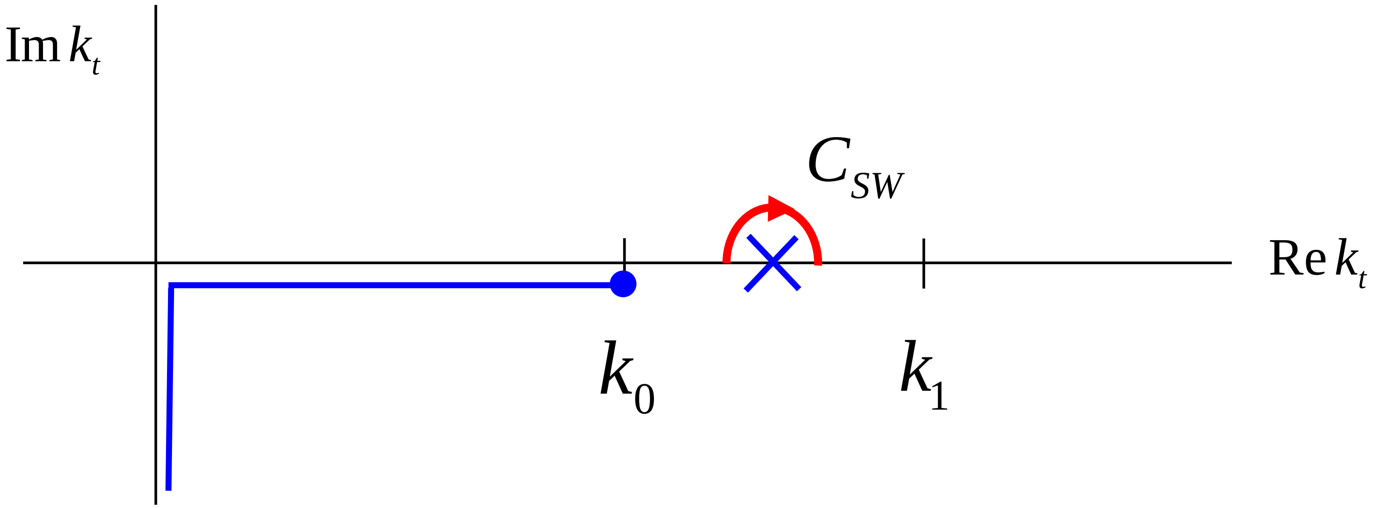


$$P_{SP} = \frac{-1}{2\pi^2} \mathrm{Re} \int_0^{\pi/2} \int_0^{k_0} \tilde{G}_{xx}\left(k_t, \overline{\phi}\right) \tilde{J}_{sx}^2\left(k_t, \overline{\phi}\right) k_t \, dk_t \, d\overline{\phi}$$

Surface-Wave Power

The pole contribution gives the surface-wave power:

$$P_{SW} = \frac{-1}{2\pi^2} \operatorname{Re} \int_0^{\pi/2} \int_{C_{SW}} \tilde{G}_{xx}\left(k_t, \overline{\phi}\right) \tilde{J}_{sx}^2\left(k_t, \overline{\phi}\right) k_t \, dk_t \, d\overline{\phi}$$



This form is not very convenient, as it involves an integration around a pole.

Surface-Wave Power (cont.)

Calculation of surface-wave power

$$P_{SW} = \frac{-1}{2\pi^2} \operatorname{Re} \int_0^{\pi/2} \int_{C_{SW}} \tilde{G}_{xx}\left(k_t, \overline{\phi}\right) \tilde{J}_{sx}^2\left(k_t, \overline{\phi}\right) k_t \, dk_t \, d\overline{\phi}$$

From the Cauchy residue theorem, we have:

$$P_{SW} = \frac{-1}{2\pi^2} \operatorname{Re} \int_0^{\pi/2} -\pi j \operatorname{Res}\left\{\tilde{G}_{xx}\left(k_t, \overline{\phi}\right) \tilde{J}_{sx}^2\left(k_t, \overline{\phi}\right) k_t\right\}_{k_t = k_{tp}} d\overline{\phi}$$

$$= \frac{-1}{2\pi^2} \operatorname{Re} \int_0^{\pi/2} -\pi j k_{tp} \tilde{J}_{sx}^2\left(k_{tp}, \overline{\phi}\right) \operatorname{Res}\left\{\tilde{G}_{xx}\left(k_t, \overline{\phi}\right)\right\}_{k_t = k_{tp}} d\overline{\phi}$$

$$= \frac{1}{2\pi} k_{tp} \operatorname{Re} \int_0^{\pi/2} j \tilde{J}_{sx}^2\left(k_{tp}, \overline{\phi}\right) \operatorname{Res}\left\{\tilde{G}_{xx}\left(k_t, \overline{\phi}\right)\right\}_{k_t = k_{tp}} d\overline{\phi}$$

Surface-Wave Power (cont.)

Residue calculation:

$$\tilde{G}_{xx}\left(k_t,\overline{\phi}\right) = -\frac{1}{k_t^{\,2}}\left[\frac{k_x^2}{D_m\left(k_t\right)} + \frac{k_y^2}{D_e\left(k_t\right)}\right] = -\left[\frac{\cos^2\overline{\phi}}{D_m\left(k_t\right)} + \frac{\sin^2\overline{\phi}}{D_e\left(k_t\right)}\right]$$

The residue of the spectral-domain Green's function at the TM pole is:

$$\operatorname{Res}\tilde{G}_{xx}\left(k_t,\overline{\phi}\right)\Big|_{k_t=k_{tp}} = -\left[\frac{\cos^2\overline{\phi}}{D_m'\left(k_{tp}\right)}\right]$$

$$= \text{ pure imaginary}$$

Note:
The derivative can be calculated in closed form, but the result is omitted here.

where

$$D_m\left(k_t\right) = Y_0^{TM} - jY_1^{TM}\cot\left(k_{z1}h\right)$$

Surface-Wave Power (cont.)

Since the transform of the current is real (assuming that k_{tp} is real), we have

$$P_{SW} = \frac{1}{2\pi} k_{tp} \int_0^{\pi/2} \tilde{J}_{sx}^{2}\left(k_{tp}, \bar{\phi}\right) \mathrm{Re}\left[j \,\mathrm{Res}\left\{ \tilde{G}_{xx}\left(k_t, \bar{\phi}\right)\right\}_{k_t = k_{tp}} \right] d\bar{\phi}$$

Since the residue is pure imaginary, we then have

$$P_{SW} = -\frac{1}{2\pi} k_{tp} \int_0^{\pi/2} \tilde{J}_{sx}^{2}\left(k_{tp}, \bar{\phi}\right) \mathrm{Im}\left[\mathrm{Res}\left\{ \tilde{G}_{xx}\left(k_t, \bar{\phi}\right)\right\}_{k_t = k_{tp}} \right] d\bar{\phi}$$

Summary of Powers

$$P_{SP} = \frac{-1}{2\pi^2} \operatorname{Re} \int_0^{\pi/2} \int_0^{k_0} \tilde{G}_{xx}\left(k_t, \overline{\phi}\right) \tilde{J}_{sx}^2\left(k_t, \overline{\phi}\right) k_t \, dk_t \, d\overline{\phi}$$

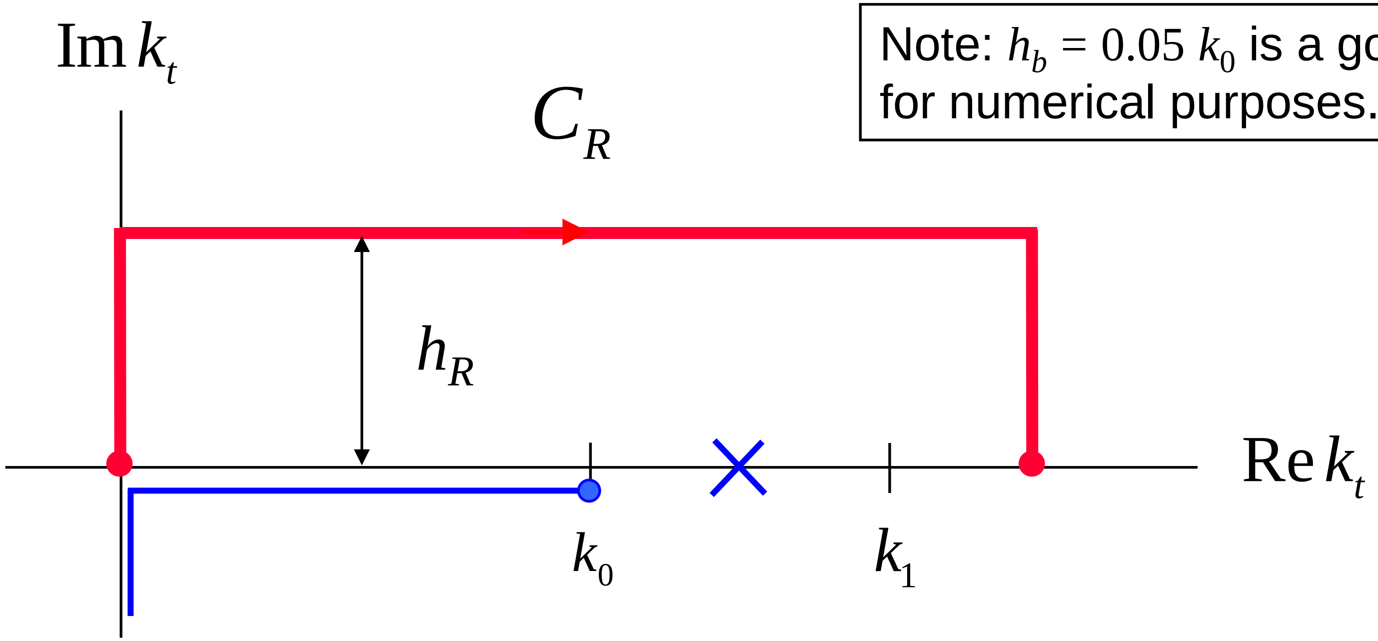
$$P_{SW} = -\frac{1}{2\pi} k_{tp} \int_0^{\pi/2} \tilde{J}_{sx}^2\left(k_{tp}, \overline{\phi}\right) \operatorname{Im}\left[\operatorname{Res}\left\{ \tilde{G}_{xx}\left(k_t, \overline{\phi}\right) \right\}_{k_t = k_{tp}} \right] d\overline{\phi}$$

$$P_{TOT} = P_{SP} + P_{SW}$$

Total Power (Alternative)

The total power can also be calculated directly:

$$P_{TOT} = \frac{-1}{2\pi^2} \mathrm{Re} \int_0^{\pi/2} \int_{C_R} \tilde{G}_{xx}\left(k_t, \overline{\phi}\right) \tilde{J}_{sx}^{2}\left(k_t, \overline{\phi}\right) k_t \, dk_t \, d\overline{\phi}$$



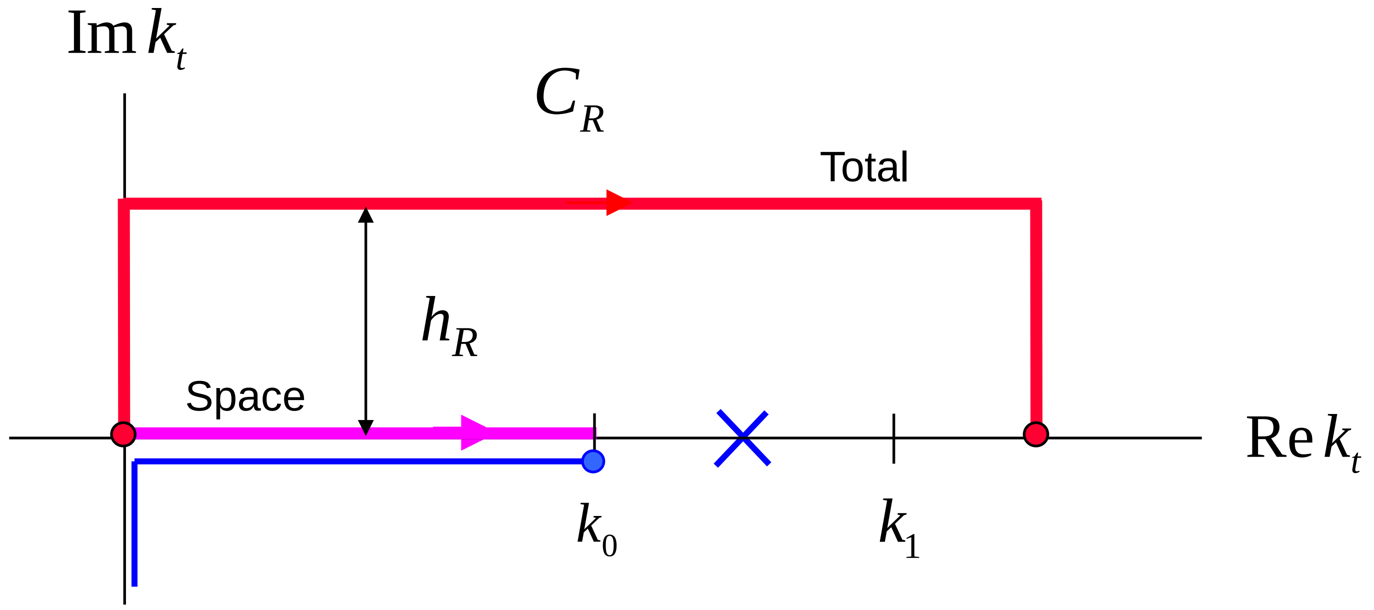
Note: $h_b = 0.05\ k_0$ is a good choice for numerical purposes.

Surface-Wave Power: Alternative Method

The surface-wave power may then be calculated from:

$$P_{SW} = P_{TOT} - P_{SP}$$

This avoids calculating any residues.

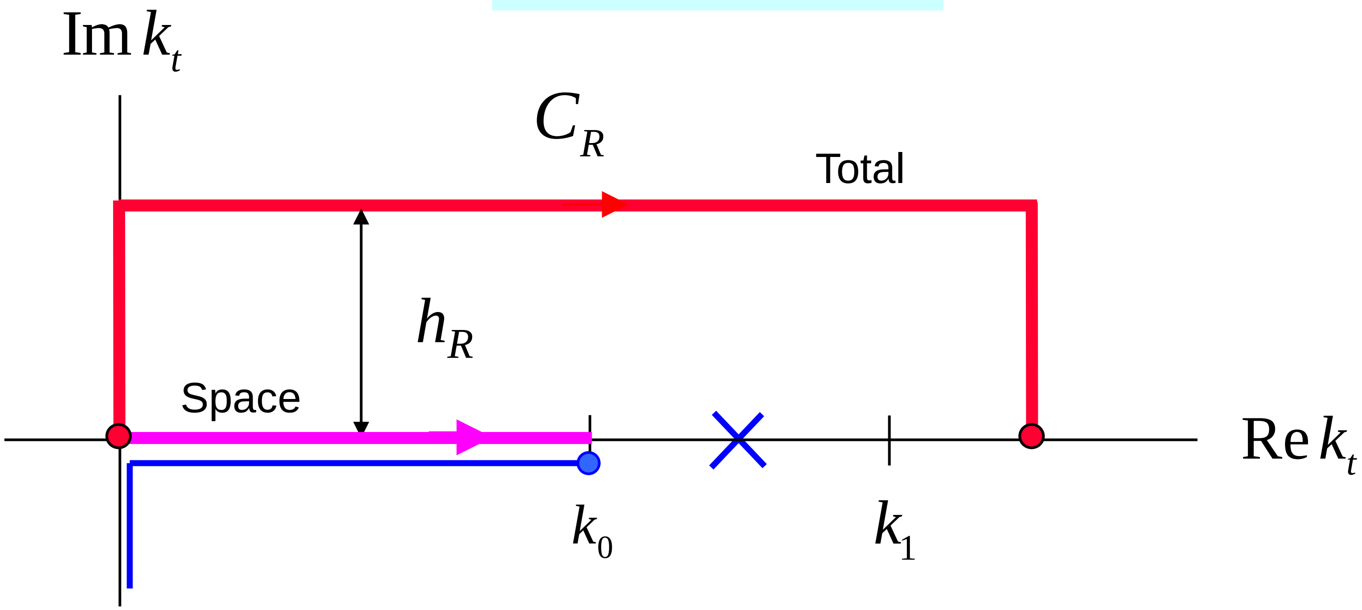


Summary

$$P_{TOT} = \frac{-1}{2\pi^2} \mathrm{Re} \int_0^{\pi/2} \int_{C_R} \tilde{G}_{xx}\left(k_t, \overline{\phi}\right) \tilde{J}_{sx}^{2}\left(k_t, \overline{\phi}\right) k_t \, dk_t \, d\overline{\phi}$$

$$P_{SP} = \frac{-1}{2\pi^2} \mathrm{Re} \int_0^{\pi/2} \int_0^{k_0} \tilde{G}_{xx}\left(k_t, \overline{\phi}\right) \tilde{J}_{sx}^{2}\left(k_t, \overline{\phi}\right) k_t \, dk_t \, d\overline{\phi}$$

$$P_{SW} = P_{TOT} - P_{SP}$$

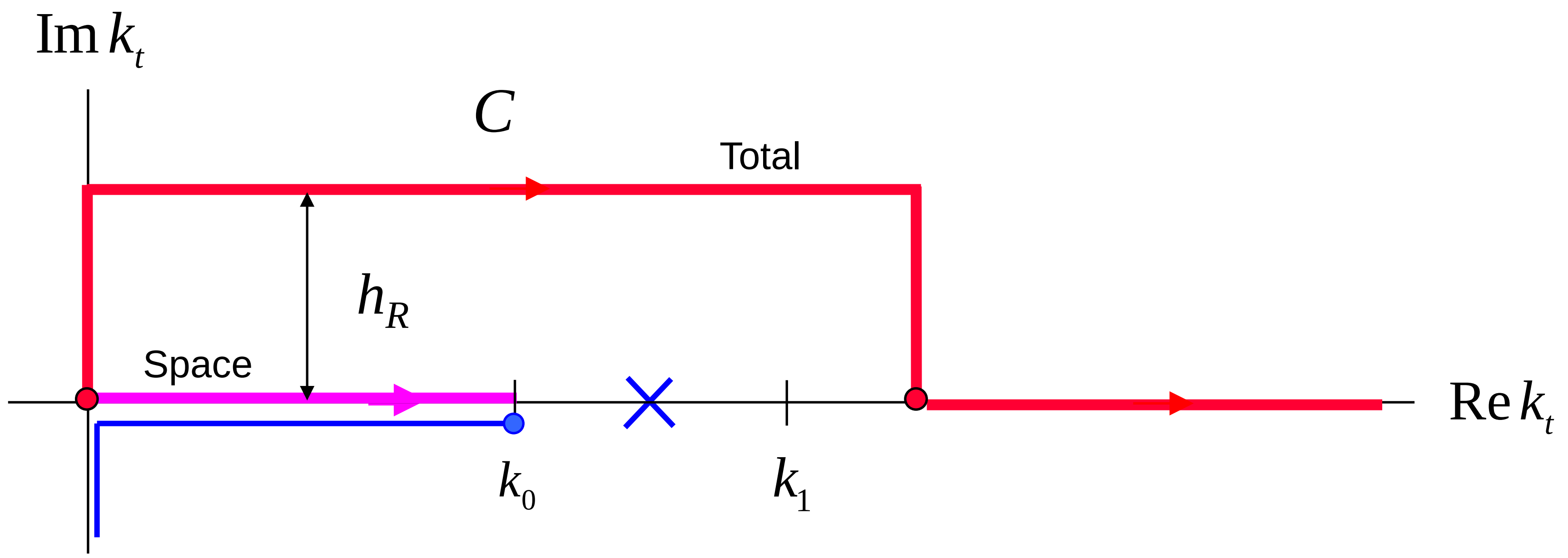


Lossy Patch

Lossy Dielectric

For a lossy dielectric, the path must extend to infinity, since the integrand is now complex everywhere along the real axis.

$$P_{TOT} = \frac{-1}{2\pi^2} \operatorname{Re} \int_0^{\pi/2} \int_C \tilde{G}_{xx}\left(k_t, \bar{\phi}\right) \tilde{J}_{sx}^2\left(k_t, \bar{\phi}\right) k_t \, dk_t \, d\bar{\phi}$$



Lossy Patch

Lossy Metal

For lossy metal, the power dissipation in the conductors can be accounted for using the surface resistance. This conductive loss is added to the total power.

$$P_{cond} = \int_S \frac{1}{2}\left(R_s^{patch} + R_s^{ground}\right)\left|J_{sx}(x,y)\right|^2 dxdy$$

$$J_{sx}(x,y) = \frac{1}{W}\cos\left(\frac{\pi x}{L}\right)$$

or

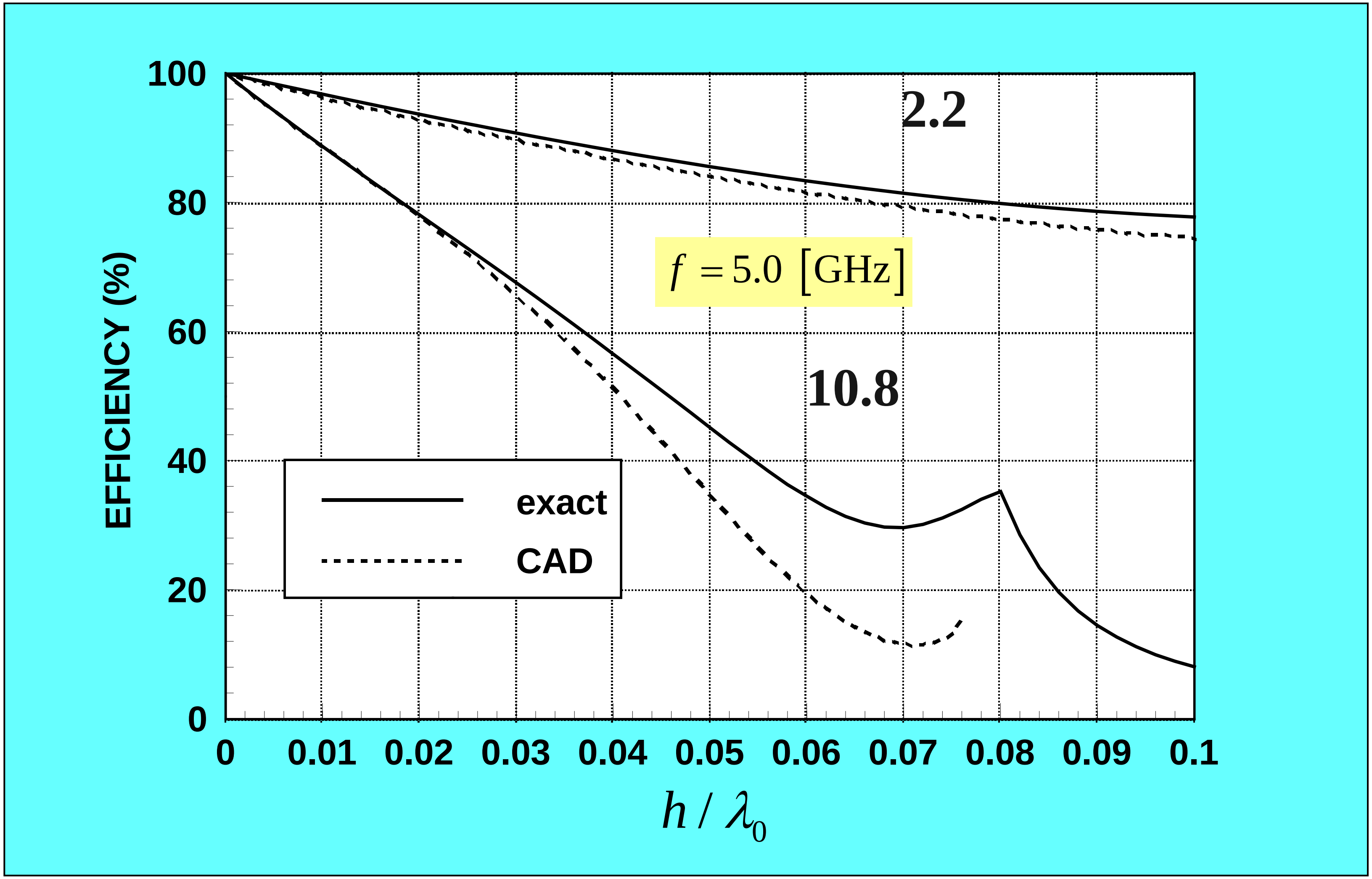
$$P_{cond} = R_s^{ave}\int_S \frac{1}{W^2}\cos^2\left(\frac{\pi x}{L}\right)dxdy$$

or

$$P_{cond} = \frac{R_s^{ave}}{W}\left(\frac{L}{2}\right)$$

Results

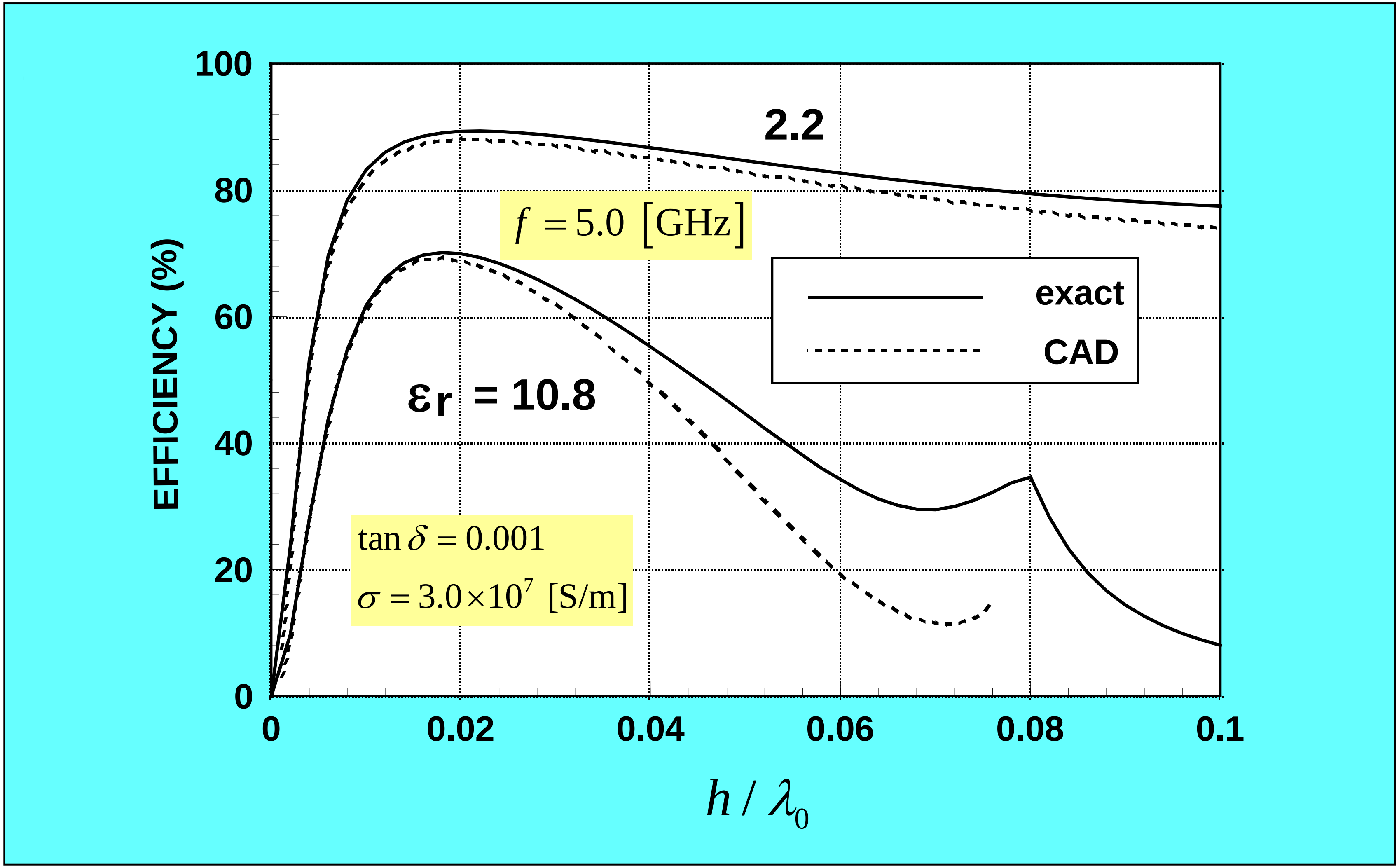
Results: Conductor and dielectric losses are <u>neglected</u>.



ε_r = 2.2 or 10.8 $\quad W/L = 1.5$

Results

Results: Accounting for all losses (including conductor and dielectric loss).



$\varepsilon_r = 2.2$ or 10.8 $\qquad W/L = 1.5$