ECE 6341

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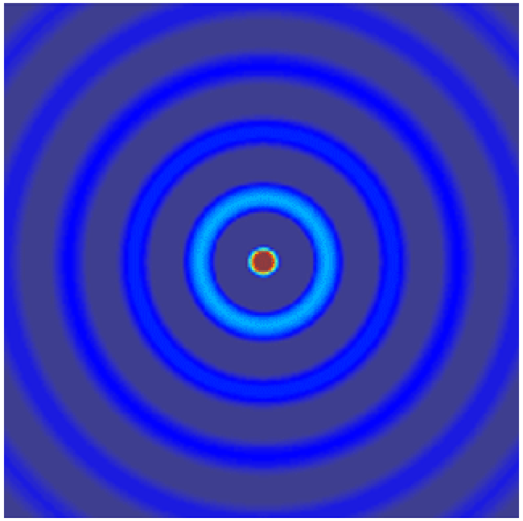


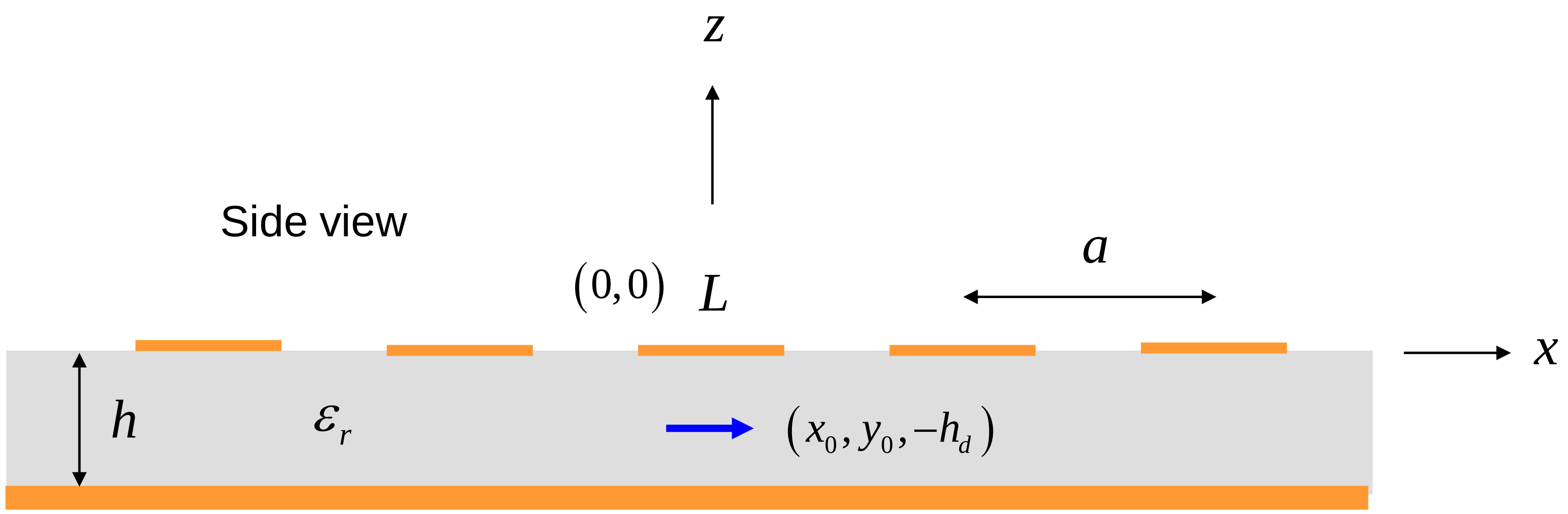
Notes 46

Overview

In this set of notes we examine the Array Scanning Method (ASM) for calculating the field of a single source near an infinite periodic structure.

ASM Geometry

Consider an infinite 2D periodic array of metal patches excited by a single (nonperiodic) dipole source.

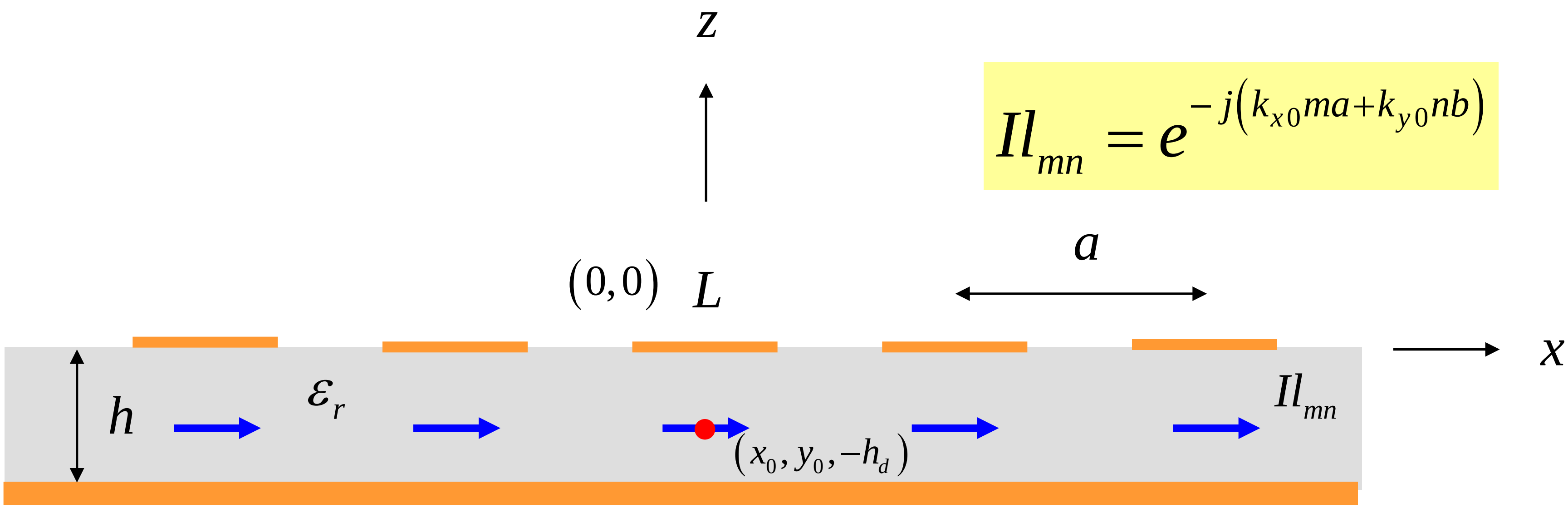


Patches : $L \times W$

Unit cell : $a \times b$

ASM Analysis

We first consider an infinite 2D periodic array of metal patches excited by an infinite periodic array of dipole sources.



This is an infinite periodic "phased array" problem.

ASM Analysis (cont.)

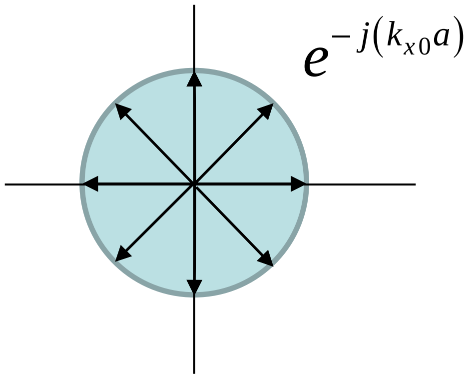
We use the following identity:

$$\int_{-\pi/a}^{\pi/a} e^{-j(k_{x0}ma)}\,dk_{x0} = \left.\frac{e^{-j(k_{x0}ma)}}{-jma}\right|_{-\pi/a}^{\pi/a} = \frac{e^{-j(m\pi)} - e^{+j(m\pi)}}{-jma} = 0$$

$$m \neq 0$$

Picture for $m = 1$

$$-\pi < k_0 a < \pi$$



Complex plane

Hence we can say that

$$\int_{-\pi/a}^{\pi/a} e^{-j(k_{x0}ma)}\,dk_{x0} = \begin{cases} 0, & m \neq 0 \\ \dfrac{2\pi}{a}, & m = 0 \end{cases}$$

ASM Analysis (cont.)

Denote

$$E_x^{\infty}\left(x, y, z; k_{x0}, k_{y0}\right) = \text{field produced by infinite periodic array problem}$$
$$\text{with phasing}\left(k_{x0}, k_{y0}\right)$$

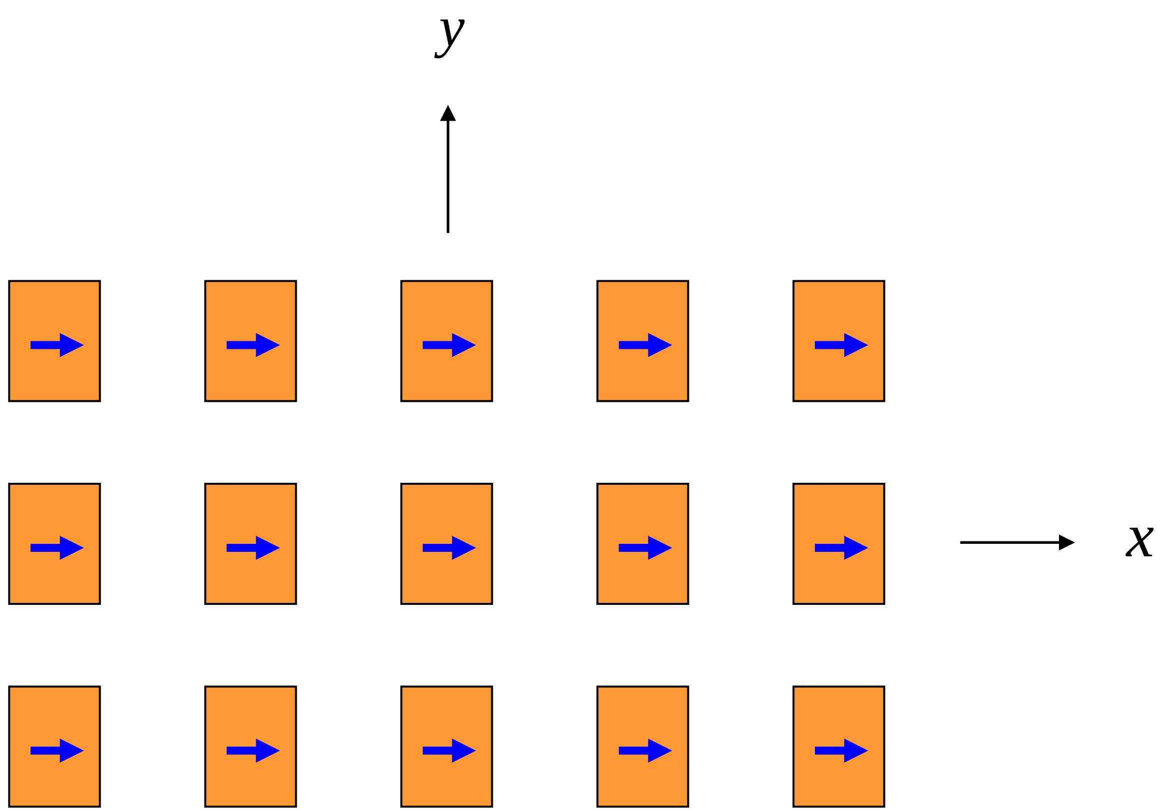
Then

$$\frac{a}{2\pi}\int_{-\pi/a}^{\pi/a} E_x^{\infty}\left(x, y, z; k_{x0}, k_{y0}\right) dk_{x0} = \text{field produced by}$$

a *single column* of dipole sources

ASM Analysis (cont.)

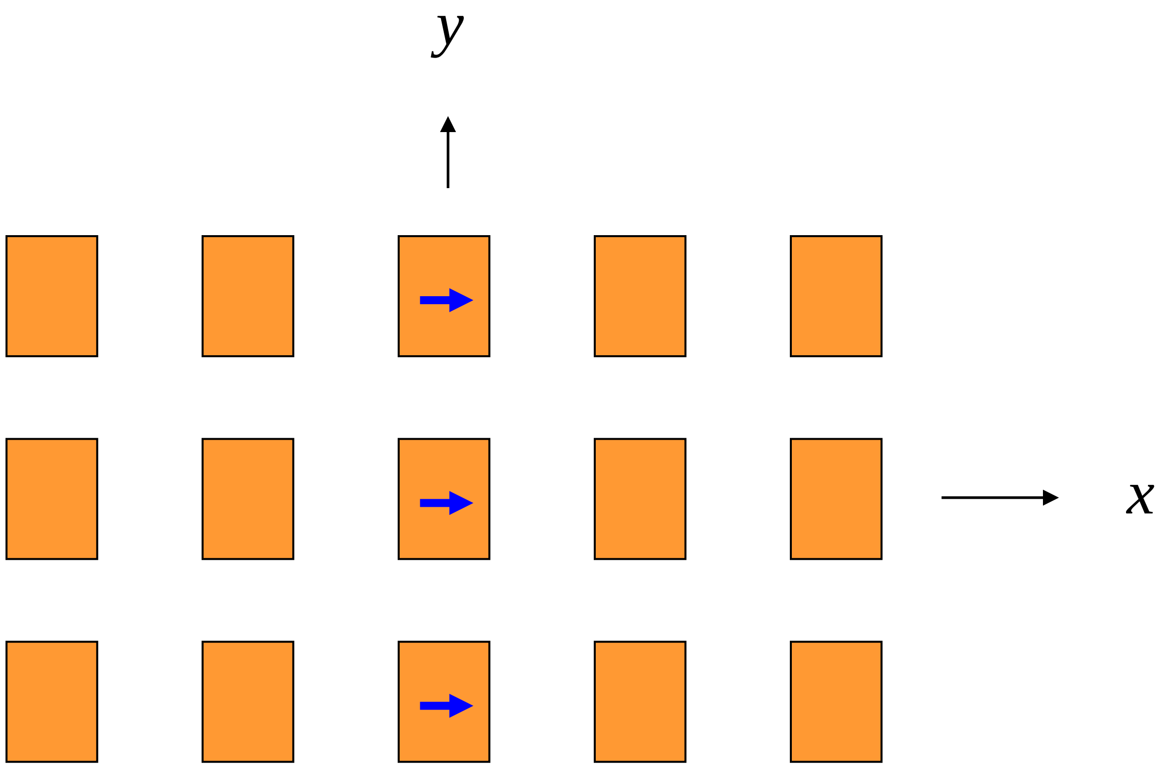
$E_x^{\infty}\left(x, y, z; k_{x0}, k_{y0}\right) =$ field from 2D array of phased dipoles



ASM Analysis (cont.)

$$\frac{a}{2\pi}\int_{-\pi/a}^{\pi/a} E_x^{\infty}\left(x,y,z;k_{x0},k_{y0}\right)dk_{x0} = \text{field from single column of dipoles}$$

(phased in the y direction)



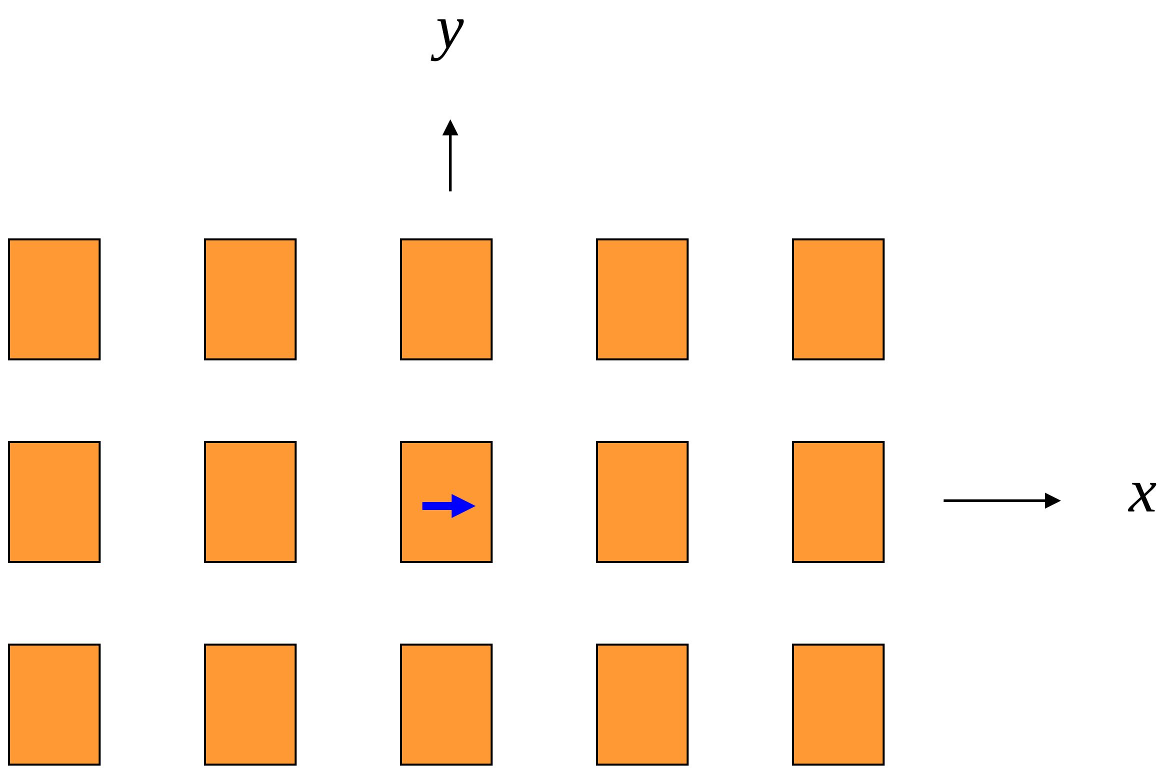
ASM Analysis (cont.)

Next, we apply the same procedure to the phasing in the y direction:

$$\int_{-\pi/b}^{\pi/b} e^{-j\left(k_{y0} m b\right)}\, dk_{y0} = \begin{cases} 0, & m \neq 0 \\ \dfrac{2\pi}{b}, & m = 0 \end{cases}$$

ASM Analysis (cont.)

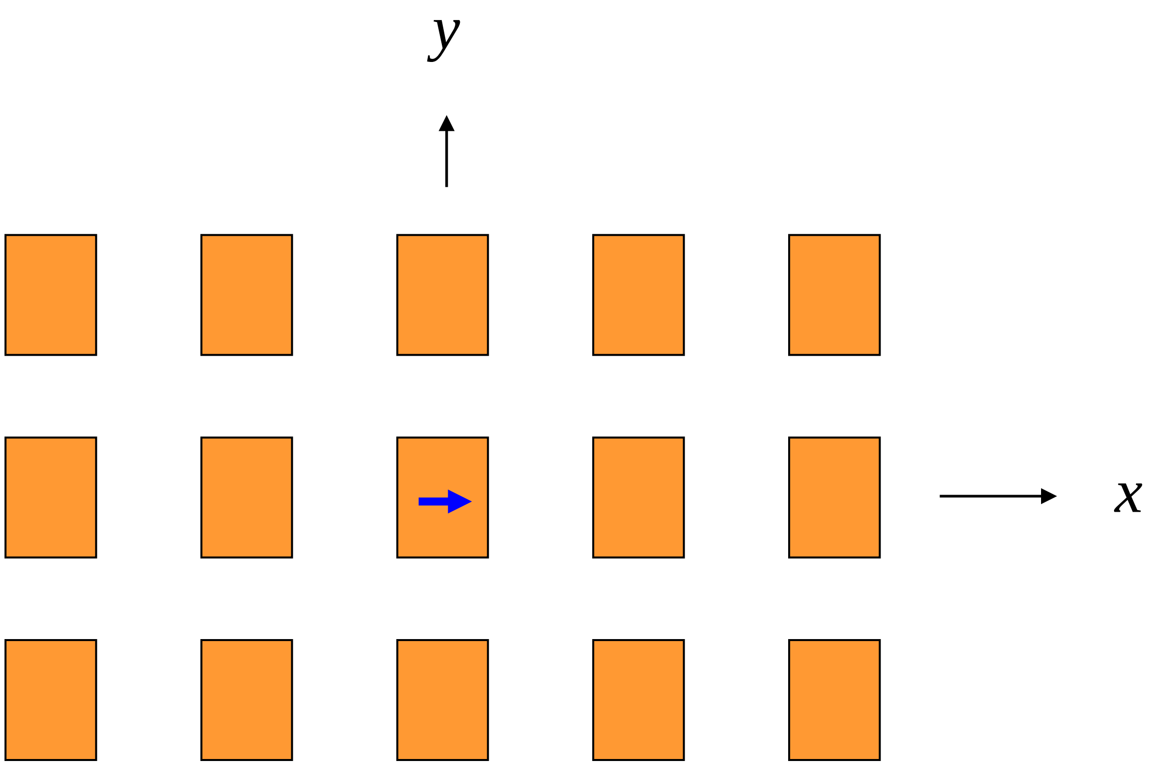
$$\frac{ab}{(2\pi)^2}\int_{-\pi/b}^{\pi/b}\int_{-\pi/a}^{\pi/a} E_x^{\infty}\left(x,y,z;k_{x0},k_{y0}\right)dk_{x0}dk_{y0} = \text{field from a } \textit{single dipole}$$



ASM Analysis (cont.)

Conclusion:

$$E_x(x,y,z) = \frac{ab}{(2\pi)^2} \int_{-\pi/b}^{\pi/b} \int_{-\pi/a}^{\pi/a} E_x^{\infty}(x,y,z;k_{x0},k_{y0})\, dk_{x0}\, dk_{y0}$$



ASM Analysis (cont.)

After doing the method of moments (please see the Appendix), the result for the infinite phased array problem will be in the form

$$E_x^{\infty}\left(x, y, z; k_{x0}, k_{y0}\right) = \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} A\left(k_{xp}, k_{yq}; z\right) e^{-j\left(k_{xp}x + k_{yq}y\right)}$$

Floquet expansion

ASM Analysis (cont.)

$$E_x(x,y,z)=\frac{ab}{(2\pi)^2}\int_{-\pi/b}^{\pi/b}\int_{-\pi/a}^{\pi/a}\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}A\left(k_{xp},k_{yq};z\right)e^{-j\left(k_{xp}x+k_{yq}y\right)}\,dk_{x0}dk_{y0}$$

Please see the next slide.

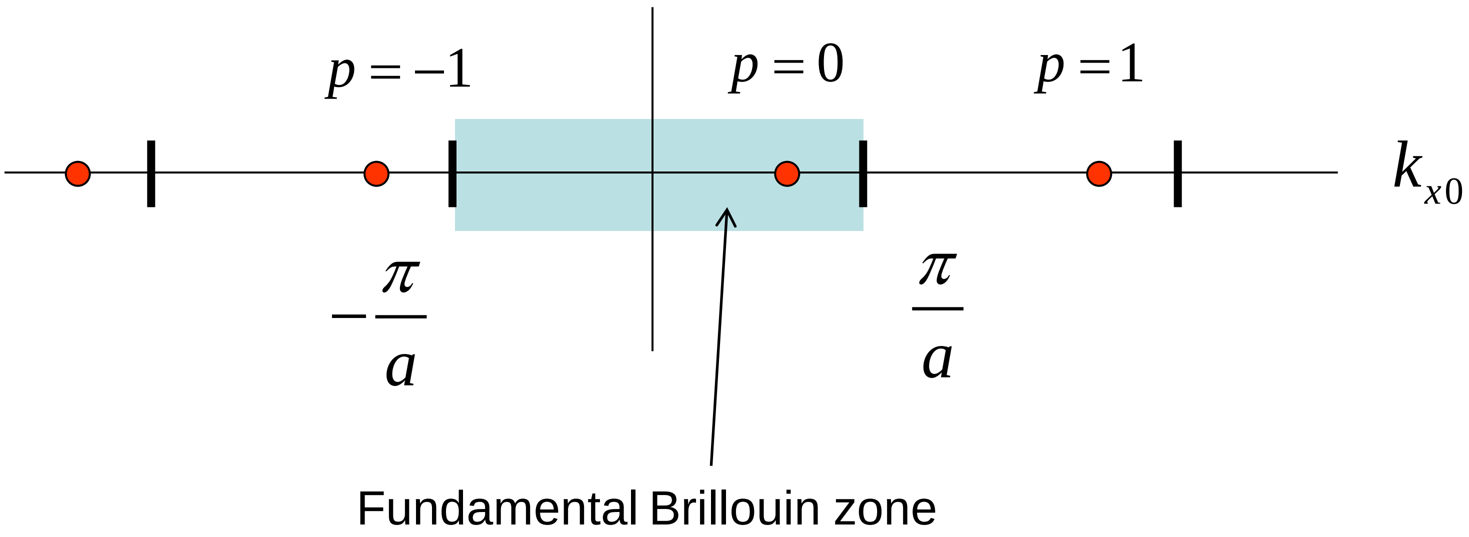
$$E_x(x,y,z)=\frac{ab}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}A\left(k_{x0},k_{y0};z\right)e^{-j\left(k_{x0}x+k_{y0}y\right)}dk_{x0}dk_{y0}$$

This is the “unfolded” form (the integration limits are infinite).

ASM Analysis (cont.)

Physical explanation of the path unfolding (illustrated for the k_{x0} integral):

$$k_{xp} = k_{x0} + \frac{2\pi p}{a}$$



Appendix

In this appendix we use the method of moments to calculate

$$E_x^{\infty}\left(x, y, z; k_{x0}, k_{y0}\right)$$

Appendix (cont.)

Assume that unknown current on the $(0,0)$ patch in the 2D array problem is of the following form:

$$J_{sx}^{00}(x,y) = A_x^{00} B_x^{00}(x,y)$$

$$B_x^{00}(x,y) = \cos\left(\frac{\pi x}{L}\right), \quad |x| < L/2, \; |y| < W/2$$

The EFIE is then

$$A_x^{00} E_x^{\infty}\left[B_x^{00}\right] + E_x^{\infty}\left[J_{sx}^{dip\,00}\right] = 0, \; |x| < L/2, \; |y| < W/2$$

Note that the "∞" superscript stands for "infinite periodic" (i.e., the fields due to the infinite periodic array of patch currents).

The EFIE is enforced on the $(0,0)$ patch; it is then automatically enforced on all patches.

Appendix (cont.)

We have, using Galerkin's method,

$$A_x^{00} \int_{S_0} B_x^{00}(x,y) E_x^{\infty}\left[B_x^{00} \right] dS + \int_{S_0} B_x^{00}\left(x,y\right) E_x^{\infty}\left[J_{sx}^{dip\,00} \right] dS = 0$$

Define

$$Z_{xx}^{\infty} = -\int_{S_0} B_x^{00}(x,y) E_x^{\infty}\left[B_x^{00} \right] dS$$

$$R^{00} = \int_{S_0} B_x^{00}\left(x,y\right) E_x^{\infty}\left[J_{sx}^{dip\,00} \right] dS$$

We then have

$$A_x^{00} Z_{xx}^{\infty} = R^{00}$$

Appendix (cont.)

The (0,0) patch current amplitude is then

$$A_x^{00}\left(k_{x0},k_{y0}\right)=\frac{R^{00}\left(k_{x0},k_{y0}\right)}{Z_{xx}^{\infty}\left(k_{x0},k_{y0}\right)}$$

We also have

$$Z_{xx}^{single}=-\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\tilde{G}_{xx}\left(k_x,k_y\right)\left[\tilde{B}_x^{00}\left(k_x,k_y\right)\right]^2 dk_x dk_y$$

$$\Downarrow$$

$$Z_{xx}^{\infty}=-\frac{(2\pi)^2}{ab}\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\frac{1}{(2\pi)^2}\tilde{G}_{xx}\left(k_{xp},k_{yq}\right)\left[\tilde{B}_x^{00}\left(k_{xp},k_{yq}\right)\right]^2$$

Appendix (cont.)

For the RHS term we have

$$R^{00} = \int_{S_0} B_x^{00}(x,y)\, E_x^{\infty}\left[J_{sx}^{dip\,00}\right] dS$$
$$= \int_{S_0} J_{sx}^{dip\,00}(x,y)\, E_x^{\infty}\left[B_x^{00}\right] dS$$
$$= E_x^{\infty}\left[B_x^{00}\right](x_0, y_0, -h_d)$$

This follows from reciprocity for a single unit cell together with the periodic SDI method.

The field from the periodic array of patch basis functions is

$$E_x^{\infty}(x,y,z;k_{x0},k_0)\left[B_x^{\infty}\right] = \frac{(2\pi)^2}{ab} \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \frac{1}{(2\pi)^2} \tilde{G}_{xx}\left(k_{xp}, k_{yq}; z, 0\right)$$
$$\cdot\, \tilde{B}_x^{00}\left(k_{xp}, k_{yq}\right) e^{-j\left(k_{xp}x + k_{yq}y\right)}$$

Appendix (cont.)

Hence, we have

$$R^{00}\left(k_{x0},k_{y0}\right)=\frac{\left(2\pi\right)^2}{ab}\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\frac{1}{\left(2\pi\right)^2}\tilde{G}_{xx}\left(k_{xp},k_{yq};-h_d,0\right)\tilde{B}_x^{00}\left(k_{xp},k_{yq}\right)\cdot e^{-j\left(k_{xp}x_0+k_{yq}y_0\right)}$$

where

$$\tilde{B}_x^{00}\left(k_{xp},k_{yq}\right)=\left(\frac{\pi}{2}LW\right)\operatorname{sinc}\left(k_{yq}\frac{W}{2}\right)\left[\frac{\cos\left(k_{xp}\frac{L}{2}\right)}{\left(\frac{\pi}{2}\right)^2-\left(k_{xp}\frac{L}{2}\right)^2}\right]$$

Appendix (cont.)

We then have, for the contribution due to the patches:

$$E_x^{\infty,\, patches}\left(x,y,z;k_{x0},k_{y0}\right)=A_x^{00}\left(k_{x0},k_{y0}\right)\frac{\left(2\pi\right)^2}{ab}\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\frac{1}{\left(2\pi\right)^2}\tilde{G}_{xx}\left(k_{xp},k_{yq};z,0\right)$$

$$\cdot\,\tilde{B}_x^{00}\left(k_{xp},k_{yq}\right)e^{-j\left(k_{xp}x+k_{yq}y\right)}$$

For the contribution due to the dipoles:

$$E_x^{\infty,\, dipoles}\left(x,y,z;k_{x0},k_{y0}\right)=Il\frac{\left(2\pi\right)^2}{ab}\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\frac{1}{\left(2\pi\right)^2}\tilde{G}_{xx}\left(k_{xp},k_{yq};z,-h_d\right)$$

$$\cdot\left(1\right)e^{-j\left(k_{xp}x+k_{yq}y\right)}$$

Appendix (cont.)

We have that

$$A_x^{00}\left(k_{x0},k_{y0}\right)=A_x^{00}\left(k_{xp},k_{yq}\right)$$

(This is because we have the same physical dipole excitation for either set of phasing wavenumbers.)

We then have

$$E_x^{\infty,\,patches}\left(x,y,z;k_{x0},k_{y0}\right)=\frac{(2\pi)^2}{ab}\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\frac{1}{(2\pi)^2}A_x^{00}\left(k_{xp},k_{yq}\right)\tilde{G}_{xx}\left(k_{xp},k_{yq};z,0\right)$$
$$\cdot\tilde{B}_x^{00}\left(k_{xp},k_{yq}\right)e^{-j\left(k_{xp}x+k_{yq}y\right)}$$

Appendix (cont.)

We then use

$$E_x^{\infty}\left(x,y,z;k_x,k_y\right)=E_x^{\infty,\,dipoles}\left(x,y,z;k_x,k_y\right)+E_x^{\infty,\,patches}\left(x,y,z;k_x,k_y\right)$$

so that

$$E_x^{\infty}\left(x,y,z;k_{x0},k_{y0}\right)=\frac{(2\pi)^2}{ab}\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\frac{1}{(2\pi)^2}A_x^{00}\left(k_{xp},k_{yq}\right)\tilde{G}_{xx}\left(k_{xp},k_{yq};z,0\right)$$
$$\cdot\,\tilde{B}_x^{00}\left(k_{xp},k_{yq}\right)e^{-j\left(k_{xp}x+k_{yq}y\right)}$$
$$+\frac{(2\pi)^2}{ab}\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\frac{1}{(2\pi)^2}Il\,\tilde{G}_{xx}\left(k_{xp},k_{yq};z,-h_d\right)e^{-j\left(k_{xp}x+k_{yq}y\right)}$$

Appendix (cont.)

We therefore identify

$$A\left(k_{xp},k_{yq};z\right)=\frac{\left(2\pi\right)^2}{ab}\frac{1}{\left(2\pi\right)^2}A_x^{00}\left(k_{xp},k_{yq}\right)\tilde{G}_{xx}\left(k_{xp},k_{yq};z,0\right)\tilde{B}_x^{00}\left(k_{xp},k_{yq}\right)$$

$$+\frac{\left(2\pi\right)^2}{ab}\frac{1}{\left(2\pi\right)^2}Il\,\tilde{G}_{xx}\tilde{G}_{xx}\left(k_{xp},k_{yq};z,-h_d\right)$$

Appendix (cont.)

Note: When calculating the field in the original problem, there is no need to use the ASM to find the fields from the original (single) dipole; we can also find this directly using the (non-periodic) SDI method.

We then have

$$E_x(x,y,z) = E_x^{dipole}(x,y,z) + \frac{ab}{(2\pi)^2} \int_{-\pi/b}^{\pi/b} \int_{-\pi/a}^{\pi/a} A_x^{00}(k_{x0},k_{y0}) \left[\frac{(2\pi)^2}{ab} \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \frac{1}{(2\pi)^2} \tilde{G}_{xx}(k_{xp},k_{yq};z,0) \right. \cdot \tilde{B}_x^{00}(k_{xp},k_{yq}) e^{-j(k_{xp}x+k_{yq}y)} dk_{x0}\, dk_{y0}$$