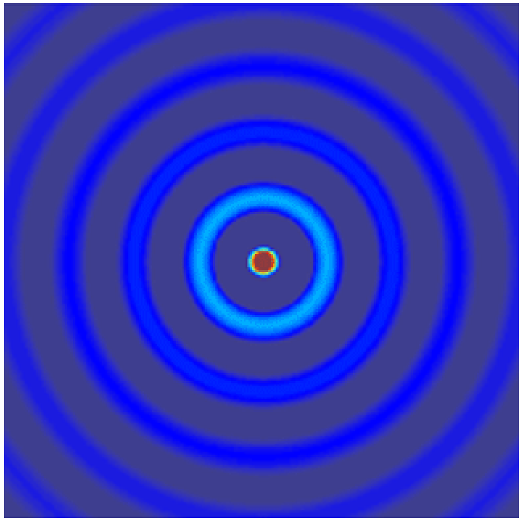
ECE 6341

Spring 2016

Prof. David R. Jackson
ECE Dept.

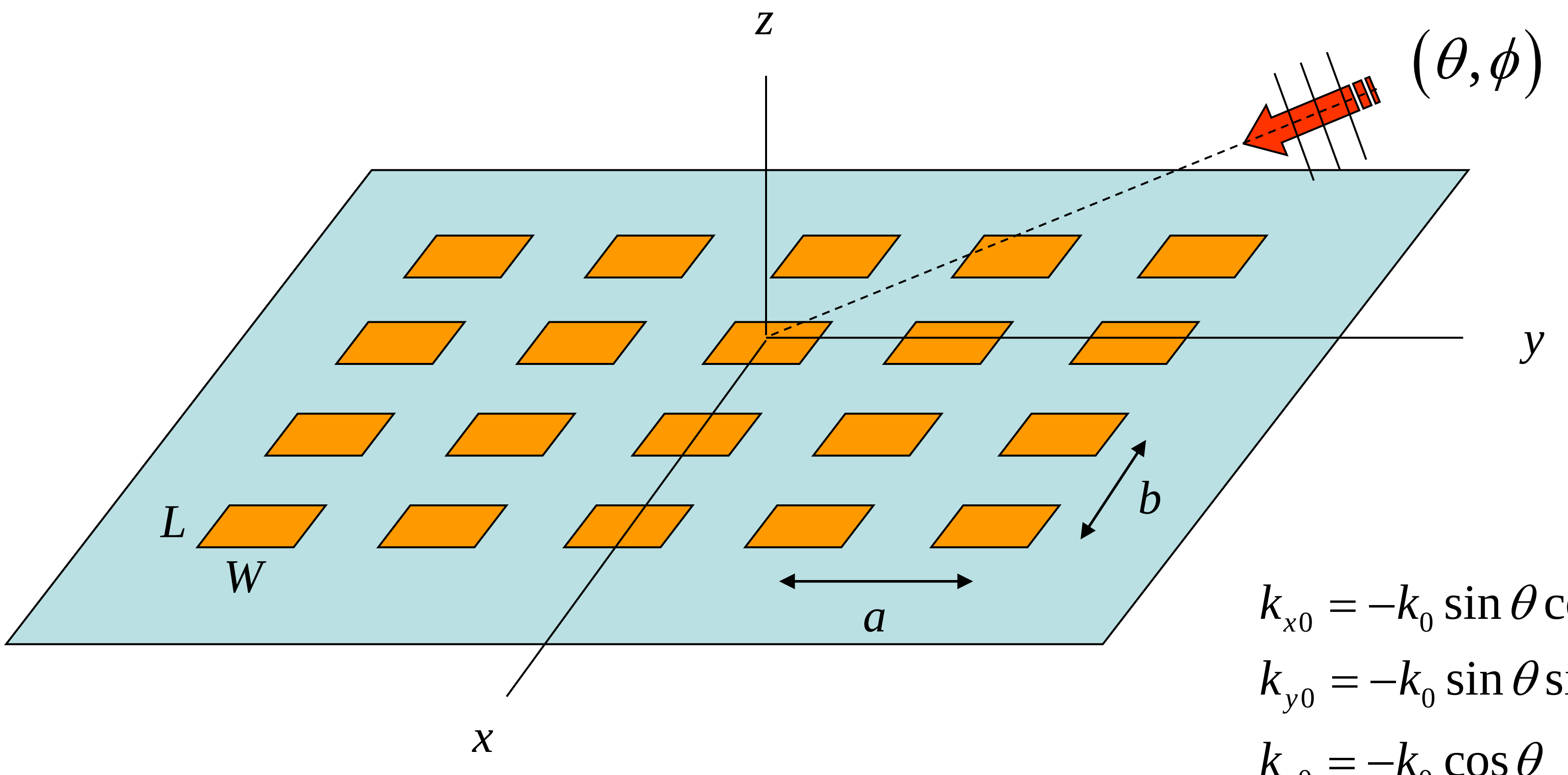


Notes 48

2D Frequency Selective Surface (FSS)

Scattering from a 2-D array of metal patches (FSS structure)

$$\underline{E}^i(x,z) = \underline{E}_0\, e^{-j\left(k_{x0}x + k_{y0}y + k_{z0}z\right)}$$



$$k_{x0} = -k_0 \sin\theta\cos\phi$$

$$k_{y0} = -k_0 \sin\theta\sin\phi$$

$$k_{z0} = -k_0 \cos\theta$$

2D FSS (cont.)

TM$_z$: $\psi^i = A_z^i = e^{-j\left(k_{x0}x + k_{y0}y + k_{z0}z\right)}$

TE$_z$: $\psi^i = F_z^i = e^{-j\left(k_{x0}x + k_{y0}y + k_{z0}z\right)}$

Note: The scattered field will have both A_z and F_z, regardless of the polarization of the incident wave.

Denote $\psi^s(x, y, 0) = e^{-j\left(k_{x0}x + k_{y0}y\right)} P(x, y)$

$P(x, y) =$ periodic function in x and y

$$P(x + a, y) = P(x, y)$$

$$P(x, y + b) = P(x, y)$$

2D FSS (cont.)

Use a 2D Fourier series:

$$P(x,y)=\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}A_{mn}\,e^{-j\left(\frac{2\pi m}{a}\right)x}\,e^{-j\left(\frac{2\pi n}{b}\right)y}$$

Define:

$$k_{xm}=k_{x0}+\frac{2\pi m}{a}$$

$$k_{yn}=k_{y0}+\frac{2\pi n}{b}$$

2D FSS (cont.)

Define 2D Floquet waves:

$$\psi^{+}(x,y,z) = e^{-j\left(k_{xm}x + k_{yn}y\right)}\, e^{-jk_{zmn}z}$$

$$\psi^{-}(x,y,z) = e^{-j\left(k_{xm}x + k_{yn}y\right)}\, e^{+jk_{zmn}z}$$

where

$$k_{zmn} = \left(k_0^2 - k_{xm}^2 - k_{yn}^2\right)^{1/2}$$

2D FSS (cont.)

$z > 0$:

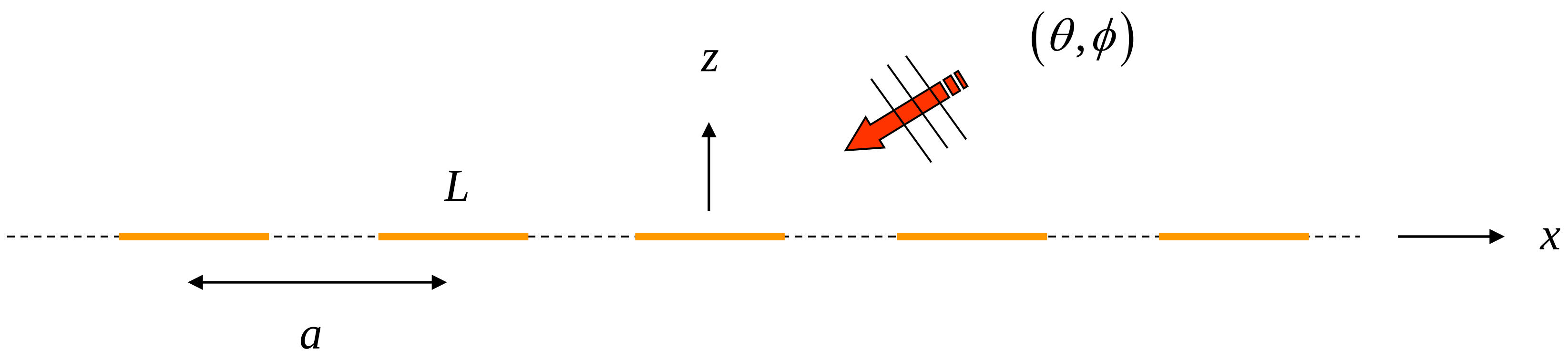
$$A_z^s(x,y,z) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} A_{mn} \psi_{mn}^{+}(x,y,z)$$

$$F_z^s(x,y,z) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} B_{mn} \psi_{mn}^{+}(x,y,z)$$

$z < 0$:

$$A_z^s(x,y,z) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} C_{mn} \psi_{mn}^{-}(x,y,z)$$

$$F_z^s(x,y,z) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} D_{mn} \psi_{mn}^{-}(x,y,z)$$

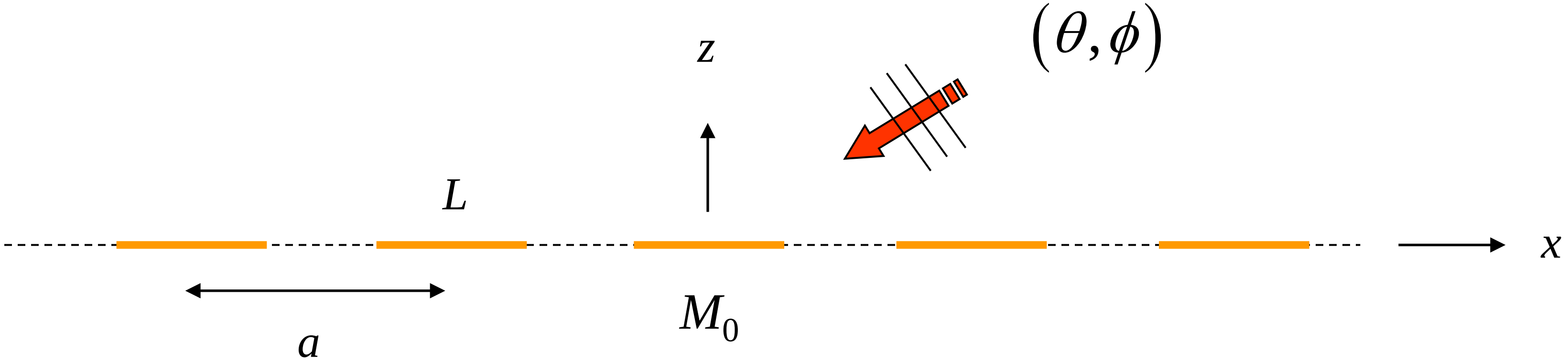


2D FSS (cont.)

Boundary condition (PEC patches):

$$E_x^s(x,y,0) = -E_x^i(x,y,0)$$
$$E_y^s(x,y,0) = -E_y^i(x,y,0) \qquad (x,y) \in M_0$$

M_0 = metal surface of $(0,0)$ patch

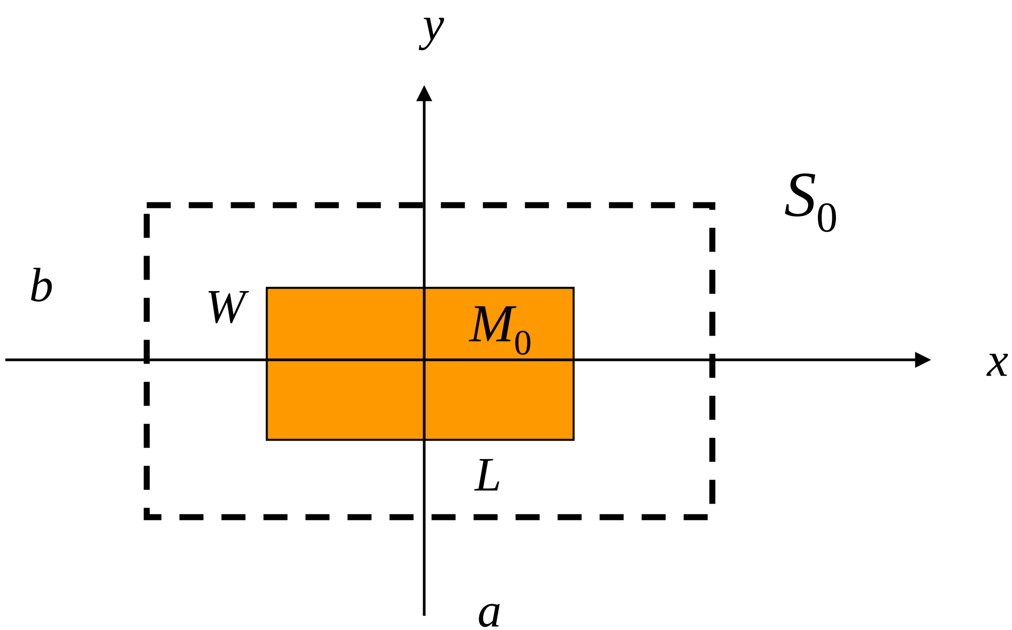


2D FSS (cont.)

Orthogonality of Floquet waves:

$$\int_{S_0} \psi_{mn}(x,y)\,\psi^{*}_{m'n'}(x,y)\,dxdy = \int_{S_0} e^{-j\left(\frac{2\pi}{a}\right)(m-m')x}\, e^{-j\left(\frac{2\pi}{b}\right)(n-n')y}\,dxdy$$

$$= \begin{cases} 0, & (m,n) \neq (m',n') \\ A, & (m,n) = (m',n') \end{cases}$$



S_0 = unit cell

$A = ab$ = area of unit cell

2D FSS (cont.)

OUTLINE OF SOLUTION STEPS

(1) Expand current on (0,0) patch in terms of basis functions:

$$\underline{J}_s(x,y)=\sum_{j=1}^{N} c_j\,\underline{B}_j(x,y) \qquad (x,y)\in S_0$$

(2) Express the unknown Floquet coefficients A_{mn}, B_{mn}, C_{mn}, D_{mn} in terms of the unknown current coefficients c_j, using the orthogonality of the Floquet waves over S_0.

- Enforce continuity of tangential electric field in S_0.
- Enforce jump condition on tangential magnetic field in S_0.

2D FSS (cont.)

OUTLINE OF SOLUTION STEPS (cont.)

(3) Enforce EFIE on M_0

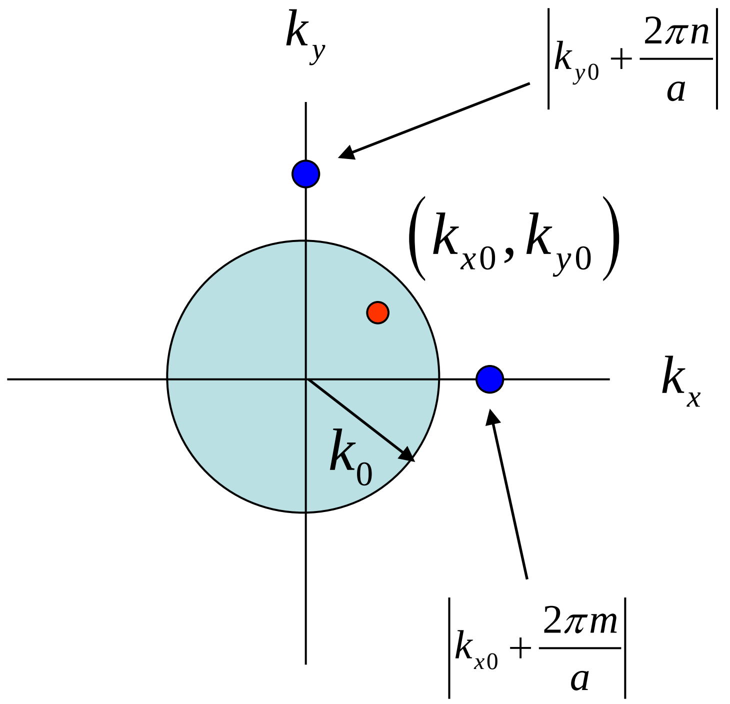
- Use N testing functions.
- This leads to an $N \times N$ system of linear equations for the unknown current coefficients c_j.

Grating Waves

To avoid grating waves:

$$\left| k_{x0} + \frac{2\pi m}{a} \right| > k_0, \quad m \neq 0$$

$$\left| k_{y0} + \frac{2\pi n}{b} \right| > k_0, \quad n \neq 0$$



Set $m = 1$, in first equation,
$n = 1$ in second equation:

$$k_{x0} + \frac{2\pi}{a} > k_0$$

$$k_{y0} + \frac{2\pi}{b} > k_0$$

Grating Waves (cont.)

Worst-case:

$$-k_0 + \frac{2\pi}{a} > k_0$$

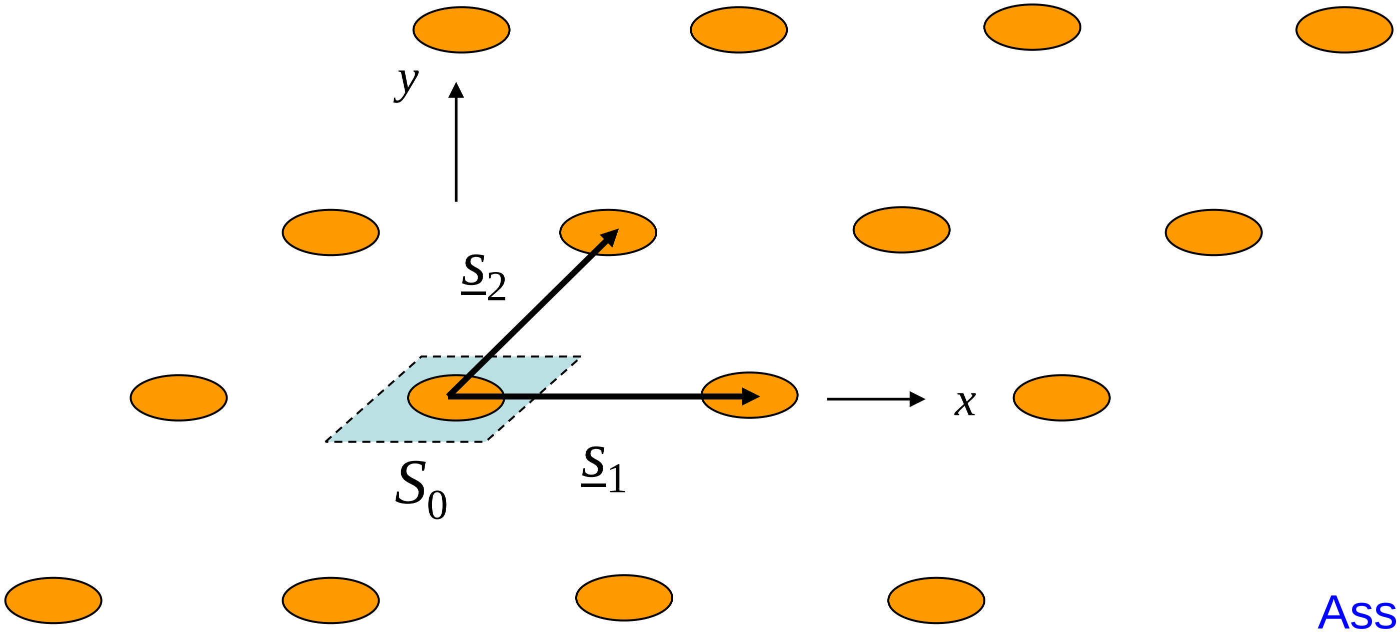
$$-k_0 + \frac{2\pi}{b} > k_0$$

This yields

$$\frac{a}{\lambda_0} < \frac{1}{2}$$

$$\frac{b}{\lambda_0} < \frac{1}{2}$$

Skewed Lattice



Assume:

$$P_{mn}\left(\underline{r}\right) = e^{-j\underline{k}_{tmn}^{p}\cdot\underline{r}}$$

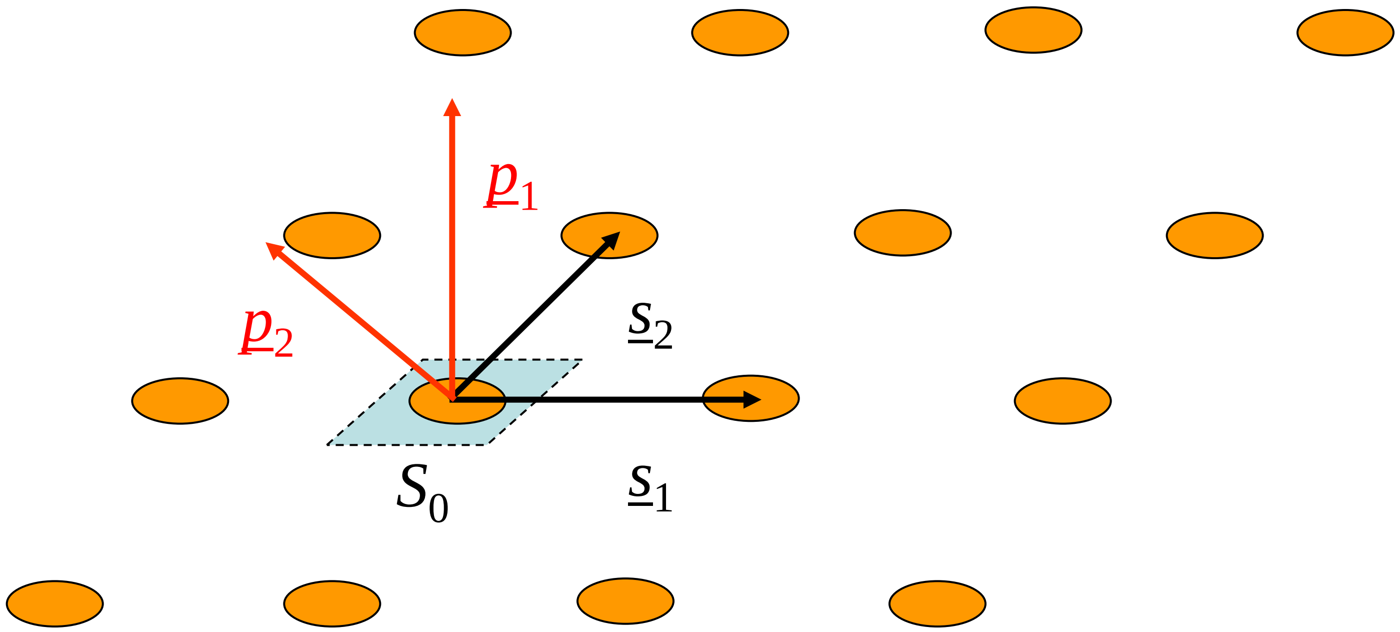
$$P_{mn}\left(\underline{r}+\underline{s}_1\right) = P_{mn}\left(\underline{r}\right)$$

$$P_{mn}\left(\underline{r}+\underline{s}_2\right) = P_{mn}\left(\underline{r}\right)$$

$$\psi^{s}\left(x,y,0\right) = e^{-j\left(k_{x0}x+k_{y0}y\right)}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}P_{mn}\left(x,y\right)$$

$$= e^{-j\left(\underline{k}_{tmn}^{0}\cdot\underline{r}\right)}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}P_{mn}\left(x,y\right)$$

Skewed Lattice (cont.)



Define perpendicular vectors:

$$\underline{p}_1 = \underline{\hat{z}} \times \underline{s}_1$$

$$\underline{p}_2 = \underline{\hat{z}} \times \underline{s}_2$$

Skewed Lattice (cont.)

$$P_{mn}\left(\underline{r}\right) = e^{-j\underline{k}_{tmn}^{p}\cdot\underline{r}}$$

Assume $\underline{k}_{tmn}^{p} = m\left(\alpha_1 \underline{p}_2\right) + n\left(\alpha_2 \underline{p}_1\right)$

Enforce: $\underline{k}_{tmn}^{p}\cdot\left(\underline{r} + \underline{s}_1\right) = \underline{k}_{tmn}^{p}\cdot\left(\underline{r}\right) + 2\pi m$

$$\Rightarrow \quad \underline{k}_{tmn}^{p}\cdot\left(\underline{s}_1\right) = 2\pi m$$

$$\Rightarrow \quad \left[m\left(\alpha_1 \underline{p}_2\right) + n\left(\alpha_2 \underline{p}_1\right)\right]\cdot\left(\underline{s}_1\right) = 2\pi m$$

$$\Rightarrow \quad \left[m\left(\alpha_1 \underline{p}_2\right)\right]\cdot\left(\underline{s}_1\right) = 2\pi m$$

Skewed Lattice (cont.)

Hence we have

$$\alpha_1 = \frac{2\pi}{\underline{p}_2 \cdot \underline{s}_1}$$

Similarly, we have

$$\underline{k}^{p}_{tmn} \cdot \left(\underline{r} + \underline{s}_2\right) = \underline{k}^{p}_{tmn} \cdot \left(\underline{r}\right) + 2\pi n$$

$$\Rightarrow \quad \underline{k}^{p}_{tmn} \cdot \left(\underline{s}_2\right) = 2\pi n$$

$$\Rightarrow \quad \left[m\left(\alpha_1 \underline{p}_2\right) + n\left(\alpha_2 \underline{p}_1\right)\right] \cdot \left(\underline{s}_2\right) = 2\pi n$$

Skewed Lattice (cont.)

$$\left[m\left(\alpha_1 \underline{p}_2\right)+n\left(\alpha_2 \underline{p}_1\right)\right]\cdot\left(\underline{s}_2\right)=2\pi n$$

$$\Rightarrow \left[n\left(\alpha_2 \underline{p}_1\right)\right]\cdot\left(\underline{s}_2\right)=2\pi n$$

Hence, we have

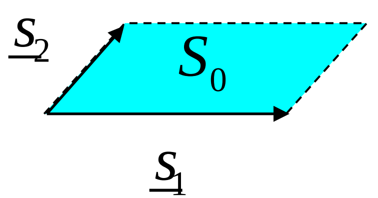
$$\alpha_2=\frac{2\pi}{\underline{p}_1\cdot\underline{s}_2}$$

Skewed Lattice (cont.)

Simplify these results:

$$\alpha_1 = \frac{2\pi}{\underline{p}_2 \cdot \underline{s}_1}$$

$$\underline{p}_2 \cdot \underline{s}_1 = \left(\underline{\hat{z}} \times \underline{s}_2\right) \cdot \underline{s}_1 = -\underline{\hat{z}} \cdot \left(\underline{s}_1 \times \underline{s}_2\right) = -\underline{\hat{z}} \cdot \left(\underline{\hat{z}}\left|\underline{s}_1 \times \underline{s}_2\right|\right) = -\underline{\hat{z}} \cdot \left(\underline{\hat{z}} A\right) = -A$$



A = area of unit cell

$$A = \left|\underline{s}_1 \times \underline{s}_2\right|$$

Hence the result is

$$\alpha_1 = -\frac{2\pi}{A}$$

Skewed Lattice (cont.)

Similarly,

$$\alpha_2 = \frac{2\pi}{\underline{p}_1 \cdot \underline{s}_2}$$

$$\underline{p}_1 \cdot \underline{s}_2 = A$$

$$\alpha_2 = \frac{2\pi}{A}$$

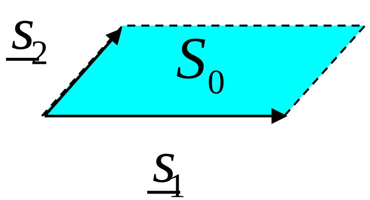
Skewed Lattice (cont.)

Recall that $\underline{k}_{tmn}^{p} = m\left(\alpha_1 \underline{p}_2\right) + n\left(\alpha_2 \underline{p}_1\right)$

$$\alpha_1 = -\frac{2\pi}{A} \qquad \alpha_2 = \frac{2\pi}{A}$$

The wavenumbers for the periodic function are then

$$\underline{k}_{tmn}^{p} = \left(\frac{2\pi m}{A}\right)\left(-\underline{p}_2\right) + \left(\frac{2\pi n}{A}\right)\left(\underline{p}_1\right)$$



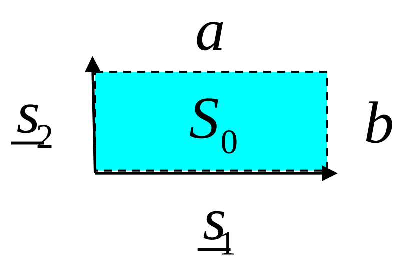
$$\underline{p}_1 = \underline{\hat{z}} \times \underline{s}_1$$

$$\underline{p}_2 = \underline{\hat{z}} \times \underline{s}_2$$

Skewed Lattice (cont.)

Rectangular lattice:

$$\underline{k}_{tmn}^{p} = \left(\frac{2\pi m}{A}\right)\left(-\underline{p}_2\right) + \left(\frac{2\pi n}{A}\right)\left(\underline{p}_1\right)$$



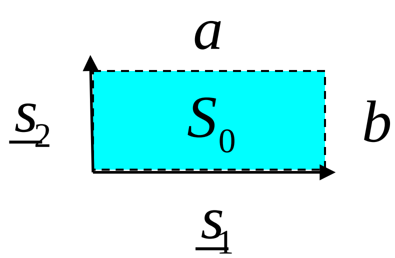
$$\underline{s}_1 = \underline{\hat{x}}\, a$$
$$\underline{s}_2 = \underline{\hat{y}}\, b$$

$$\underline{p}_1 = \underline{\hat{z}} \times \left(\underline{\hat{x}}\, a\right) = \underline{\hat{y}} a$$
$$\underline{p}_2 = \underline{\hat{z}} \times \left(\underline{\hat{y}}\, b\right) = -\underline{\hat{x}} b$$

$$\underline{k}_{tmn}^{p} = \left(\frac{2\pi m}{ab}\right)\left(\underline{\hat{x}} b\right) + \left(\frac{2\pi n}{ab}\right)\left(\underline{\hat{y}} a\right)$$

Skewed Lattice (cont.)

Rectangular lattice (cont.):



$$\underline{k}_{tmn}^{p} = \left(\frac{2\pi m}{ab}\right)\left(\underline{\hat{x}}b\right) + \left(\frac{2\pi n}{ab}\right)\left(\underline{\hat{y}}a\right)$$

$$\underline{k}_{tmn}^{p} = \underline{\hat{x}}\left(\frac{2\pi m}{a}\right) + \underline{\hat{y}}\left(\frac{2\pi n}{b}\right)$$

$$e^{-j\underline{k}_{tmn}^{p}\cdot\underline{r}} = e^{-j\left(\frac{2\pi m}{a}\right)x}\, e^{-j\left(\frac{2\pi n}{b}\right)y}$$

Skewed Lattice (cont.)

Orthogonality:

$$\int_{S_0} \psi_{mn}(x,y)\,\psi^{*}_{m'n'}(x,y)\,dxdy = \begin{cases} 0, & (m,n) \neq (m',n') \\ A, & (m,n) = (m',n') \end{cases}$$

(proof omitted)