ECE 6341

Spring 2016

HW 3

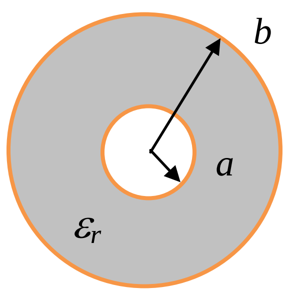
Assigned problems: 1-4, 7-8.

1) Show that

$$A_z = \ln(\rho) e^{-jkz}$$

is a valid solution to the scalar Helmholtz equation. Note that $k_z = k$ and that there is no ϕ variation. Determine the electric and magnetic fields that correspond to this vector potential. What type of physical system can support such a wave? (Note: This is essentially the same as Prob. 5-2 in the Harrington book.) Why do we not have Bessel functions here? By starting with the general Bessel function solution in cylindrical coordinates, establish that the general solution becomes this form in the limit as $k_z \to k$ when there is no ϕ variation. (Keep in mind that you can ignore any potential term that generates a trivial field.)

2) Determine what the most general expression is for either a TM_z or a TE_z waveguide mode propagating in a coaxial line, as shown below. (Note: This is essentially the same as Prob. 5-5 in the Harrington book.)



The radial variation for a TM_z mode (with azimuthal index characterized by integer n) should come out to be of the form

$$R_n^{TM}(\rho) = Y_n(k_\rho a) J_n(k_\rho \rho) - J_n(k_\rho a) Y_n(k_\rho \rho),$$

where k_ρ is a root of the transcendental equation

$$Y_n(k_\rho a) J_n(k_\rho b) - J_n(k_\rho a) Y_n(k_\rho b) = 0.$$

For the TE_z modes, the radial variation should come out as

$$R_n^{TE}(\rho) = Y_n'(k_\rho a) J_n(k_\rho \rho) - J_n'(k_\rho a) Y_n(k_\rho \rho),$$

where k_ρ is a root of the transcendental equation

$$Y_n'(k_\rho a) J_n'(k_\rho b) - J_n'(k_\rho a) Y_n'(k_\rho b) = 0.$$

3) Starting with your results from the previous problem, derive a transcendental equation for the cutoff frequencies of the TM_z and TE_z (n, p) waveguide modes of the coaxial cable.

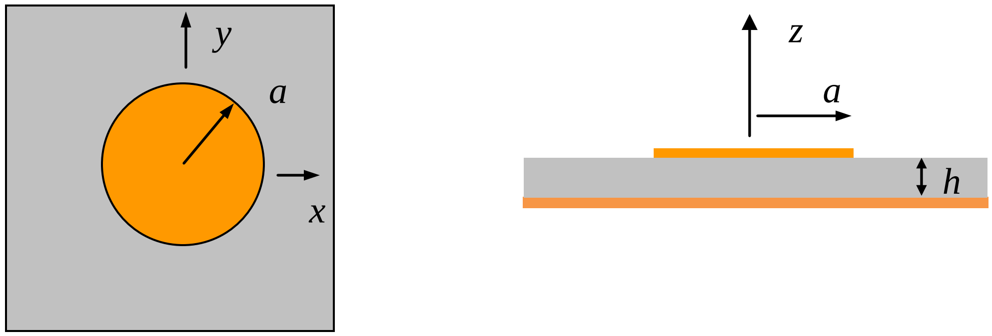
4) Excluding the TEM mode (which has a zero cutoff frequency), determine the lowest cutoff frequency for the waveguide modes in a 50 Ω coaxial cable filled with Teflon having ε_r = 2.2, having an inner radius of 0.29 cm and an outer radius of 1.0 cm. Identify clearly <u>which</u> mode it is that has the lowest cutoff frequency (i.e., TM_z or TE_z, and what the values of n and p are) and the <u>value</u> of the cutoff frequency for this mode. The determination of the cutoff frequencies for the modes has to be done numerically, and you may use any software that you wish in your numerical solution.

5) A metal probe in a parallel-plate waveguide operates at 10 GHz. The plate separation is 1 mm. The material between the plates is Teflon, having ε_r = 2.2. Make a plot of the probe reactance in Ohms versus the radius of the probe in mm, up to a maximum radius of 1.0 mm.

6) A cylindrical water tank is made of steel, which may be assumed to be a perfect conductor for this problem. It has a height h of 5 meters and a radius a of 10 meters. Determine the lowest resonance frequency of the tank, and the corresponding mode, assuming that it is hollow (filled with air). Note that for a resonant mode we must have that $k_z h = m\pi$, where m = 0, 1, 2,… for a TM_z mode (TM_{npm}) and m = 1, 2,… for a TE_z mode (TE_{npm}). You should justify this by writing down formulas for the potentials and applying boundary conditions at the top and bottom of the tank. You should also apply boundary conditions at the side of the tank in order to determine the allowed values of k_ρ.

7) A circular microstrip antenna consists of a circular patch of metal of radius a, printed on top of a grounded substrate with a relative permittivity ε_r and a thickness h (see below). Assume that the edge of the patch at $\rho = a$ can be modeled (approximately) as an open circuit (in other words, there is a PMC boundary at the edge). Assume that the substrate is thin enough so that there is no variation of the fields with respect to the z direction. A resonant mode of the cavity is one that can exist (satisfy Maxwell's equations) without any sources inside the cavity.

   Show that the fields of a resonant cavity mode must be of the form TM_{np0}, where the (n, p, m) subscript notation denotes variation in the ϕ, ρ, and z directions, respectively. Derive a formula for the resonant frequency of the TM_{np0} mode. Next, identify the mode that has the lowest resonance frequency (this is the mode that the patch antenna normally operates in). That is, determine the values of n and p for this mode.



8) A cylindrical dielectric resonator is shown below. Because of the high permittivity, it can be assumed (approximately) that there is a PMC condition on all faces except the bottom, where there is a PEC ground plane. Show that under this assumption the TM_{mnp} and TE_{mnp} modes of the resonator have the following form:

$$A_z^{mnp} = \cos(m\phi) J_m\left(\frac{x'_{mn}\rho}{a}\right)\cos\left(\frac{(2p+1)\pi z}{2h}\right)$$

$$F_z^{mnp} = \cos(m\phi) J_m\left(\frac{x_{mn}\rho}{a}\right)\sin\left(\frac{(2p+1)\pi z}{2h}\right),$$

where $m = 0, 1, 2, \ldots$, $n = 1, 2, \ldots$, and $p = 0, 1, 2, \ldots$ (you should justify the allowed ranges for the integers m, n, and p).

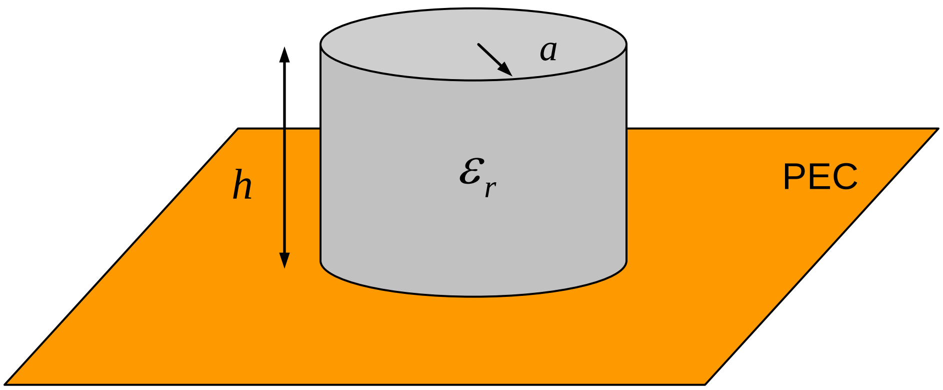
Show that the resonant frequencies are given by

$$f_{mnp}^{TM} = \frac{c}{2\pi a\sqrt{\varepsilon_r}}\sqrt{x'^{\,2}_{mn} + \left[\frac{\pi a}{2h}(2p+1)\right]^2}$$

$$f_{mnp}^{TE} = \frac{c}{2\pi a\sqrt{\varepsilon_r}}\sqrt{x^2_{mn} + \left[\frac{\pi a}{2h}(2p+1)\right]^2},$$

where c is the speed of light in vacuum. Show that the dominant mode (the one with the lowest resonance frequency) is the TM_{110} mode, for which $x'_{11} = 1.841$.

(Note: In this problem the integer m is being used to describe the azimuthal variation of the fields, n is used to describe the radial variation, and p is used to describe the vertical variation. Hence, the subscript notation is different than in the previous problems.)



a
ε_r
h
PEC