

ECE 6341
Spring 2016

HW 5

Please do Probs. 1-4, 7, 12-16.

- 1) (This is Prob. 6.1 in Harrington.) Assume that we have a potential

$$\underline{A} = \hat{z}A_z(r, \theta, \phi)$$

And from this, calculate what the (six) components of the electric and magnetic fields would be in spherical coordinates, in terms of $\psi = A_z$. Comment on how the field expressions compare to what is in the TM_r and TE_r tables. Are they more complicated compared to using A_r , for example?

(Note that in the Harrington book the convention for the vector potentials is different than in the class notes, so that the vector potentials are different by factors of μ and ε . Please use the class notation.)

- 2) Assume that we try to introduce potentials similar to the Debye potentials, but in cylindrical coordinates. That is, assume we try to use

$$\underline{A} = \hat{\rho}A_\rho$$

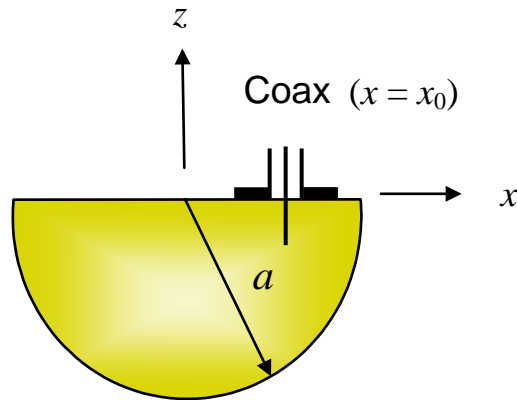
$$\underline{F} = \hat{\rho}F_\rho.$$

Using a derivation that parallels what was done in class in spherical coordinates, show that this choice of potentials will not work. For example, assume that we have \underline{A} in the form above. Then start with the vector wave equation, and take the z and ϕ components of both sides, and show that there is no single gauge condition (equation that relates A_ρ to Φ) that will allow both resulting equations to be satisfied.

- 3) As continuation of the above problem, assume that there is no z variation in the problem. Then show that now there is a gauge condition that can be imposed on A_ρ (i.e., a relationship between A_ρ and Φ) that will satisfy both the z and ϕ components of the vector wave equation.
- 4) Consider the earth as a giant spherical resonator. For a field corresponding to a resonant mode that is inside the earth, assume (approximately) that the outer surface of the earth appears as a PMC, since the permittivity of the earth is considerably higher than that of the surrounding air region. Derive formulas for A_r and F_r for the TM_r and TE_r modes of the earth resonator, as well as formulas for the resonant frequencies of the modes. Determine the lowest resonance frequency of the earth, and the corresponding mode(s). Calculate a value

for the lowest resonance frequency, assuming that the radius of the earth is 6,378 km and the relative permittivity of the earth is 6.0. Assume that the earth is nonmagnetic for simplicity (which may not be a reasonable assumption for all points inside the earth!) (Note: You will need Tables 6.1 and 6.2 in the Harrington book, which are also given on slides 8 and 13 of Notes 22.)

- 5) A hemispherical PEC resonator is shown below. It is excited by a probe feed along the x axis at $x = x_0$ (this determines the \cos/\sin nature of the azimuthal functions). Determine general expressions for A_r and F_r for the TM_r and TE_r resonant modes of the cavity. Which mode has the lowest resonance frequency? Give a formula for the resonant frequency of this mode. (Hint: Explain why the PEC boundary condition at $z = 0$ requires $n+m$ to be an odd integer for the TM case and an even integer for the TE case. To do this, consider the even/odd nature of the associated Legendre polynomials.)



- 6) For the above hemispherical resonator excited by the coaxial probe, what should the offset of the probe from the center of the cavity (x_0) be in order to provide maximum excitation of the lowest resonant mode? The maximum excitation will occur at the point where the electric field of the mode in the direction of the probe is a maximum. (The solution for x_0 may be determined numerically. The answer can potentially be anything between 0 and a .)
- 7) Consider the “spherical coax” discussed in class with cone angles θ_1 and θ_2 . Show that the characteristic impedance is

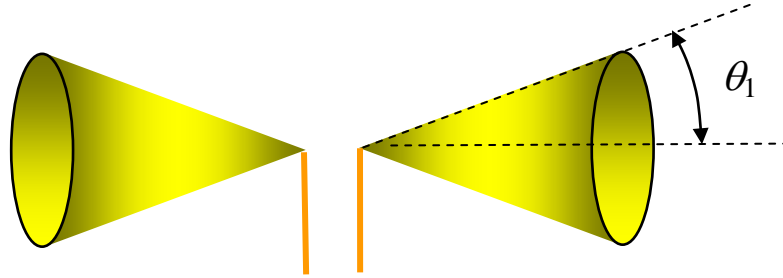
$$Z_0 = \frac{\eta}{2\pi} \ln \left[\frac{\cot(\theta_1/2)}{\cot(\theta_2/2)} \right].$$

The characteristic impedance is defined as

$$Z_0 = V^+ / I^+,$$

where V^+ is the voltage between the two cones at a given radial distance r for an outgoing wave, and I^+ is the current on the inner cone at the radial distance r . (Make sure you show clearly all of your steps.)

- 8) Consider a symmetric biconical dipole antenna fed by a twin-lead transmission line. If the dipole arms were infinite in length, we would have an infinite biconical structure as shown below, which would operate in the TEM_r mode. In this case the bandwidth of the antenna would be infinite. What would the input impedance be? (Hint: Use the result from the previous problem. The infinite biconical antenna may be thought of as a special case of the spherical coax.)



- 9) Consider a static solution to the “spherical coax” system, where one PEC cone at the angle $\theta = \theta_1$ is at a potential V_1 and another PEC cone at the angle $\theta = \theta_2$ ($\theta_2 > \theta_1$) is at a potential V_2 . Solve the Laplace equation in spherical coordinates (assuming no variation in r or ϕ) to show that the static solution for the potential between the two cones is

$$\Phi = -c_1 \ln \left(\tan \left(\frac{\theta}{2} \right) \right) + c_2$$

and the electric field between the two cones is

$$\underline{E} = \hat{\theta} \left(\frac{1}{r} \right) \frac{c_1}{\sin \theta},$$

where

$$c_1 = \frac{V_1 - V_2}{\ln \left[\frac{\tan(\theta_2/2)}{\tan(\theta_1/2)} \right]}$$

and

$$c_2 = \frac{V_1 \ln \left[\tan(\theta_2/2) \right] - V_2 \ln \left[\tan(\theta_1/2) \right]}{\ln \left[\frac{\tan(\theta_2/2)}{\tan(\theta_1/2)} \right]}.$$

Therefore, the time-harmonic solution for the TEM mode (discussed in class) has the same form as the static solution, with the propagation term $\exp(-jkr)$ added. (Thus, the propagation

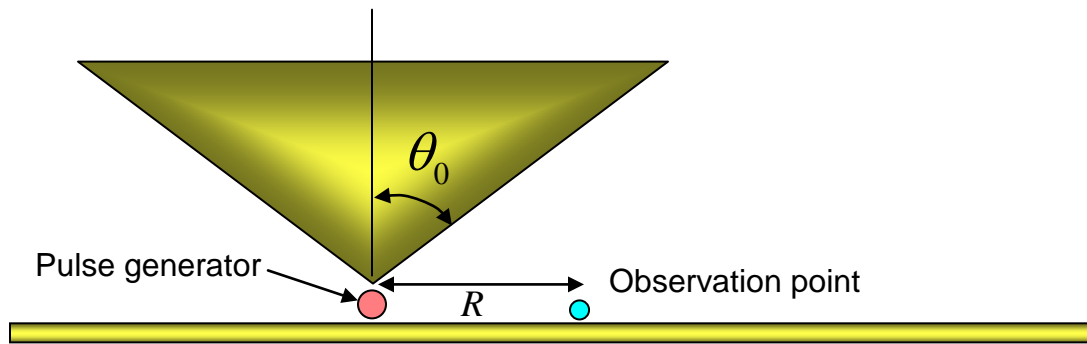
properties are similar to the TEM_z mode of a regular coax, where the field configuration of the TEM_z mode is that of a static field except that a term $\exp(-jkz)$ has been added.)

10) A high-power electromagnetic pulse (EMP) simulator consists of a perfectly conducting cone (which may be modeled as infinite) over a perfectly conducting ground plane, as shown below. Assume that the pulse generator puts out a time-varying voltage pulse $\mathcal{V}(t)$ with respect to ground.

- What is the voltage $\mathcal{V}(R,t)$ between the cone and the ground plane at a distance R away from the source?
- What is the electric field $\mathcal{E}_z(R,t)$ on the ground plane at a distance R away from the source?
- Calculate the power $P(t)$ that is being radiated, in terms of the known pulse voltage function $\mathcal{V}(t)$.

Hint: In the phasor domain, the voltage propagates radially as $\exp(-jkr)$. What does this correspond to in the time domain? Also, note that the voltage and electric field in the time domain on this lossless TEM_r structure are related in the same way as they are in the frequency domain, or in statics.) Use the results from Prob. 7 to the extent possible.

Note: This system may be thought of as a limiting case of the spherical coax problem, where the inner cone has an angle $\theta_1 = \theta_0$ and the outer cone has an angle $\theta_2 = \pi/2$.



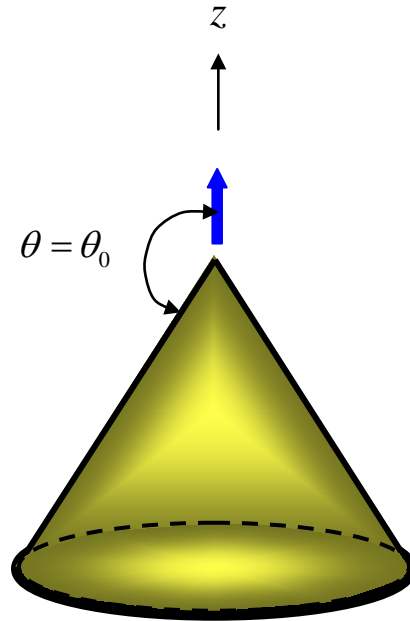
11) (This is Prob. 6-30 in Harrington.) A vertical infinitesimal unit-strength electric dipole is at the top of a mountain as shown below. The mountain is modeled as an infinite perfectly conducting cone. Show that the E_θ component of the electric field that is radiated can be expressed as

$$E_\theta = A \frac{1}{r} \hat{H}_{\nu_1}^{(2)'}(kr) P_{\nu_1}'(\cos \theta) \sin \theta$$

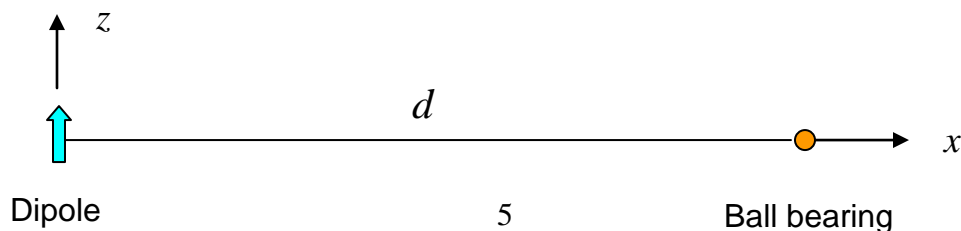
where A is some constant and $\nu = \nu_1$ is the *first* root of the equation

$$P_\nu(\cos\theta_0) = 0.$$

Give a convincing explanation of why the first root ν_1 should be used. Hint: To establish which root is correct, consider the approximate table of eigenvalues given in the table of Prob. 6-30 in Harrington. This table shows that for the first root, we have the property that $\nu_1 = 1$ for the case of $\theta_0 = \pi/2$, for which the problem is that of a dipole over an infinite ground plane (which has a simple solution with a known angular variation in θ).



- 12) Start with the field A_r from a unit-amplitude z -directed dipole in free space, as derived in class. Show that all components of the electric and magnetic fields obtained from this vector potential are the same as the known fields of a dipole (which are usually obtained from the vector potential A_z).
- 13) A spherical metal ball bearing has a diameter of 1.0 cm. What is the approximate radar cross section of the ball bearing at a frequency of 1.0 GHz?
- 14) Assume that a small z -directed dipole antenna having a directivity of 1.5 (with respect to isotropic) radiates one watt of power at 1.0 GHz, and is located at a distance $d = 1$ km from the same ball bearing as in the problem above. What is the power density in watt/m^2 of the wave that is scattered back to the antenna by the ball bearing?



- 15) Derive the formula that is given in the class notes for the dipole moment I of an x -directed electric dipole that models a small dielectric sphere that is illuminated by a x -polarized plane wave with an electric field amplitude E_0 , as analyzed in the class notes. (There is no need to derive anything that is already in the class notes.) The result should be

$$I = -j \frac{3\lambda_0 (k_0 a)^3}{\omega \mu_0} \left(\frac{\epsilon_r - 1}{\epsilon_r + 2} \right) E_0.$$

- 16) Assume that we have a z -directed unit-amplitude electric dipole on the surface of a perfectly conducting sphere of radius a centered at the origin, with the dipole located on the z axis at $z = a$ (that is, the dipole sits “on top” of the sphere). This could be a model for a radio transmitter located on the surface of the Earth. This dipole will radiate a TM_r field. In the far field, the electric field will be polarized in the θ direction. Calculate the far field $E_\theta(r, \theta)$ from the dipole.

Do this by using reciprocity, where “ a ” is the original dipole current and “ b ” is a unit-amplitude “testing” dipole in the far field, oriented in the θ direction. The far field is given by $\langle a, b \rangle$, whereas $\langle b, a \rangle$ can be thought of finding the component E_r of the scattered field at the surface of the sphere, due to an incident plane wave. Note that we have already worked out the solution for the scattered field from an incident plane wave in the class notes, and this does not need to be derived again. Note: You may have to rotate the coordinates to use the solution in the class notes.