ECE 6341

Spring 2016

Homework 7

Please do Probs. 1-2, 4-7.

1) Consider the wavenumber $k_{y0} = \left(k_0^2 - k_x^2\right)^{1/2}$, with $k_x = k_0\left(0.5 - j0.5\right)$.

   Determine the numerical value of k_{y0} (in terms of k_0) for each of the following cases below. The top sheet is the one for which k_{y0} is a positive real number when k_x is a real number that is between zero and k_0.

   a) Using Sommerfeld branch cuts, with the point being on the top sheet of the Riemann surface.

   b) Using Sommerfeld branch cuts, with the point being on the bottom sheet of the Riemann surface.

   c) Using vertical branch cuts, with the point being on the top sheet of the Riemann surface. (The branch cuts descend from the branch point at $k_x = k_0$, and ascends from the branch point at $k_y = - k_0$.)

   d) Using vertical branch cuts, with the point being on the bottom sheet of the Riemann surface. (The branch cuts descend from the branch point at $k_x = k_0$, and ascends from the branch point at $k_y = - k_0$.)

2) Consider the wavenumber $k_{y0} = \left(k_0^2 - k_x^2\right)^{1/2}$.

   Suppose we wish to choose branch cuts so that the entire top sheet represents waves that are outgoing (the real part of k_{y0} is positive) and the entire bottom sheet represents waves that are incoming (the real part of k_{y0} is negative). Give a derivation that shows what the shape of these branch cuts will be. Sketch them in the complex k_x plane. (Your derivation should parallel that given in the notes, where the shape of the Sommerfeld branch cuts was derived.) Assume that the air is lossy, and then specialize to the case of lossless air.

3) Consider the problem of a line source on the interface of a semi-infinite earth. In the steepest descent ζ plane, there are no branch points associated with the vertical wavenumber k_{y0} since the steepest-descent transformation $k_x = k_0 \sin\zeta$ removes them. However, there are still branch points associated with the vertical wavenumber k_{y1}. Assuming that we choose Sommerfeld (hyperbolic-shaped) branch cuts in the k_x plane for the wavenumber k_{y1}, determine what the shape of the corresponding branch cuts would be

in the ζ plane, and draw a sketch of them. Assume the earth is lossless (or very low loss) for simplicity.

4) An infinite uniform line source is located on the surface of the (semi-infinite) earth at z = 0. The line current may be described by a surface current

$$\underline{J}_s(x, y) = \underline{\hat{x}}\, I_0\, \delta(y).$$

(The line current is modeled as a flat strip of surface current.) Use the spectral-domain immittance method to show that the electric field component E_x in the air region ($z > 0$) is given by

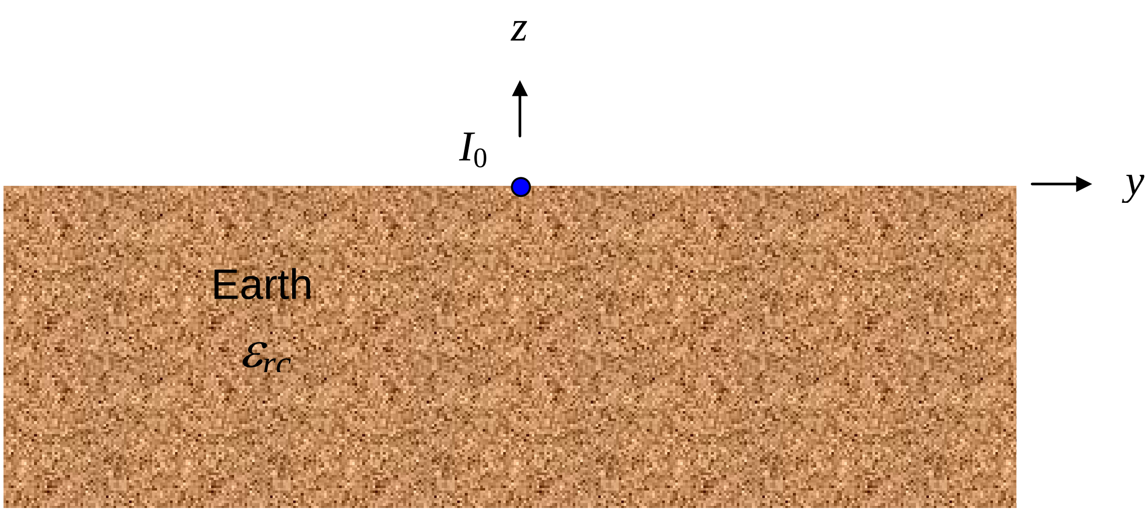
$$E_x = \frac{-\omega\mu_0 I_0}{2\pi} \int_{-\infty}^{\infty} \left[\frac{1}{k_{z0} + k_{z1}} \right] e^{-jk_{z0}z} e^{-jk_y y}\, dk_y .$$

where

$$k_{z0} = \left(k_0^2 - k_y^2\right)^{1/2}$$

$$k_{z1} = \left(k_1^2 - k_y^2\right)^{1/2}.$$

Hint: Note that $\int_{-\infty}^{\infty} e^{jk_x x}\, dx = 2\pi\delta(k_x)$.



5) The voltage-current definition of characteristic impedance for a microstrip line is

$$Z_0 = \frac{-1}{I(0)} \int_{-h}^{0} E_z(0,0,z)\,dz\,.$$

Use the expression that was derived in then notes for the vertical electric field, namely

$$\tilde{E}_z(k_x,k_y,z) = \frac{-1}{\omega\varepsilon_0\varepsilon_r}(k_t)\,I^{TM}(z)\,,$$

to derive an expression for the characteristic impedance. You should be able to do the z integration in closed form, so your final answer for the voltage-current characteristic impedance should involve only a single integral in k_y. (Even though this problem was worked out in the class notes, please show the necessary steps in your solution, to demonstrate that you understand the steps.)

6) An infinite power-line wire is above the earth as shown below. The wire is excited by a "broadband over powerline" (BPL) source at x = 0. The current on the wire due to the source is

$$I(x) = I_0\, e^{-jk_{x0}|x|}\,,$$

Where $k_{x0} = \beta - j\alpha$ is a wavenumber (assumed to be known) that may be complex due to losses on the line. The wire radius may be taken as zero.

Use the spectral-domain immittance (SDI) method to derive an expression for the vertical field component E_z $(x,y,0)$ on the surface of the earth in the air region at $z = 0^+$ from the current on the wire, as a function of (x,y). You may take advantage of the result derived in the class notes, which states that the transform of the vertical electric field is given by

$$\tilde{E}_z(k_x,k_y,z) = \frac{-1}{\omega\varepsilon_0}(k_t)\,I^{TM}(z)\,.$$

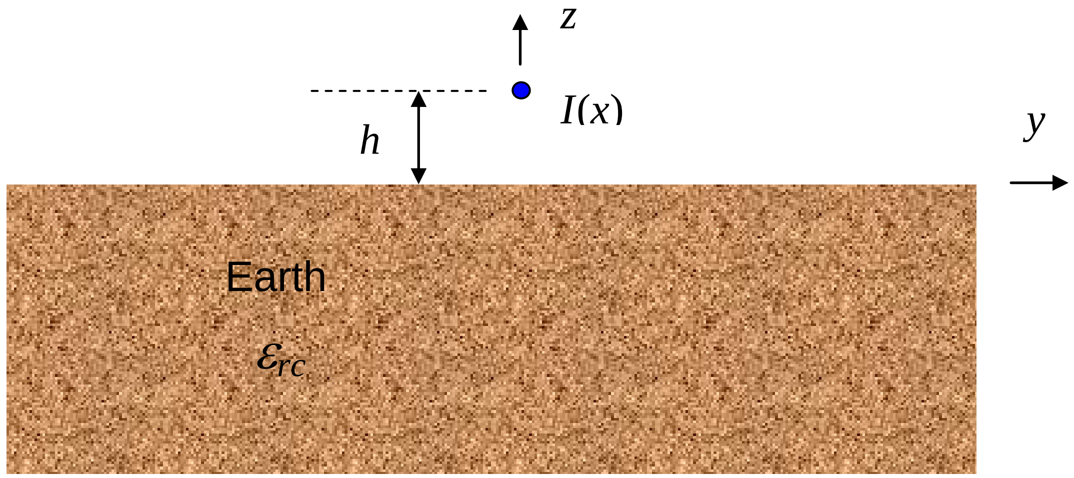
As part of your derivation, you should derive that

$$I_i^{TM}(0^+) = -\frac{1}{2}\left(1-\Gamma^{TM}\right)e^{-jk_{z0}h}\,,$$

where

$$\Gamma^{TM} = \frac{Z_1^{TM} - Z_0^{TM}}{Z_1^{TM} + Z_0^{TM}}\,.$$

Your answer should be in the form of a double infinite integral in k_x and k_y.



7) An infinite slot in the ground plane of a grounded substrate is modeled as an infinite magnetic surface current source as shown below. The slot magnetic surface current is described by

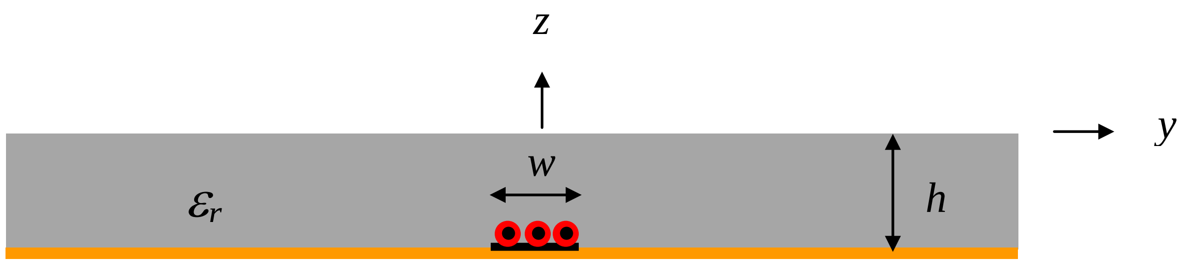
$$\underline{M}_s(x,y) = \underline{\hat{x}} \frac{K_0/\pi}{\sqrt{\left(\frac{w}{2}\right)^2 - y^2}}, \quad |y| < w/2 .$$

(The line current is modeled as a flat strip of magnetic surface current.)

Use the spectral-domain immittance method to find the vertical electric field E_z (y, $-h$) inside the substrate on the ground plane (z = $-h$). You may take advantage of the result derived in the class notes, which states that the Fourier transform of the vertical electric field is given by

$$\tilde{E}_z(k_x, k_y, z) = \frac{-1}{\omega \varepsilon_0 \varepsilon_r}(k_t) I^{TM}(z).$$

Your answer should be in the form of a single infinite integral in k_y.


z
y
w
εr
h