ECE 6341

Spring 2012

Project

Project Description

Two circular loops of uniform current surround a pipe of radius *a*, as shown below. (The lower loop may be taken as lying in the *xy* plane.) The pipe is made of metal having a conductivity and relative permeability of

$$\sigma = 4.5 \times 10^6 \text{ [S/m]}$$

$$\mu_r = 75 .$$

The outer radius of the pipe is *a* = 5.1 cm. (The inner radius is 4.4 cm, but you do not need this.) The radius of the loops is *b* = 7.0 cm.

Assume that the surface of the pipe can be approximated as an impedance surface defined by

$$Z_s = R_s \left(1 + j\right)$$

where

$$R_s = \frac{1}{\sigma\delta}, \quad \delta = \sqrt{\frac{2}{\omega\mu_0\mu_r\sigma}} .$$

Tasks

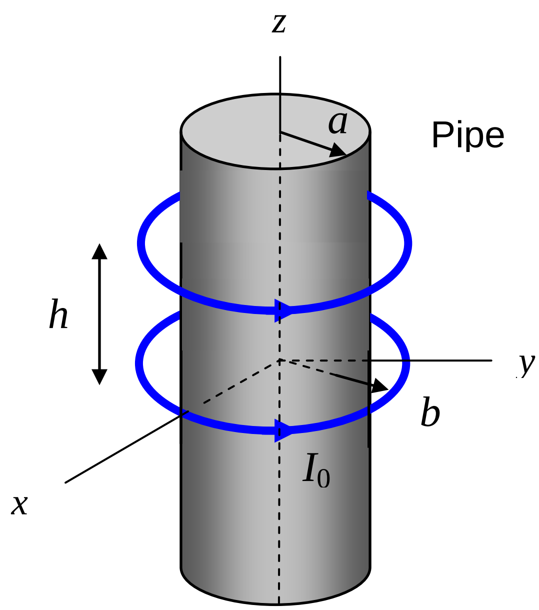
1) Formulate the mutual inductance *M* between the two loops.

2) Calculate the mutual inductance and plot this as a function of separation distance *h* in the *z* direction between the coils, for the following frequencies:

   f = 1 kHz
   f = 10 kHz
   f = 100 kHz
   f = 1 MHz
   f = 10 MHz.

   Plot with *h* ranging between 1 cm to a maximum of one meter.

3) For the same frequencies, include results showing what the mutual inductance would be if the pipe were perfectly conducting.



Background

The total electric vector potential may be written as the sum of an incident potential and a scattered potential, as

$$F_z = F_z^i + F_z^s \,,$$

where the incident potential is the potential of the current loop in free space, which is what you solved for in a previous homework problem (homework set 4, problem 7). The scattered potential is that due to the currents induced on the pipe. The scattered potential may be written in the form

$$F_z^s = \int_{-\infty}^{\infty} C\left(k_z\right) H_0^{(2)}\left(k_\rho \rho\right) e^{-jk_z z}\, dk_z \,, \quad \rho > a \,.$$

Solve for the unknown coefficient function $C(k_z)$ by applying the boundary condition that the total tangential electric field E_ϕ should satisfy an impedance boundary condition at the pipe surface at $\rho = a$, namely

$$\frac{E_\phi}{H_z} = -Z_s \,.$$

The mutual inductance M between the two coils may be found from the open-circuit voltage induced on the receiver coil, which is

$$V_2 = \left(j\omega M \right) I_0 = -\left(2\pi b\right) E_\phi^{(1)}\left(h,b\right),$$

where V_2 is the open-circuit voltage induced at the terminals of coil 2 (assuming that we introduce a terminal pair at some point on loop 2), I_0 is the current on the transmitter coil 1, and $E_\phi^{(1)}$ (h,b) is the electric field produced by coil 1 (radiating in the presence of the pipe) at $z = h$ and $\rho = b$. (The assumption here is that the output ports of each coil are labeled the same, with the same + and – polarity labeling.)

Discussion Items

Comment on how the mutual inductance varies with coil separation and with frequency. Also comment on the numerical aspects of your calculation, including issues such as convergence of the integrals, etc.

Include a discussion of the complex k_z plane, and discuss any singularities that may be present and how you handle them in your numerical integration.

Guidelines

Please write up your report neatly (using a word processor for the text and the equations, and plotting software for the plots). The report does not need to be longer than is necessary, but neatness and the quality of the format will count.

Notes

Please work individually on this project, and do not discuss it with anyone other than the instructor. Please also check the class website for any corrections or updates.