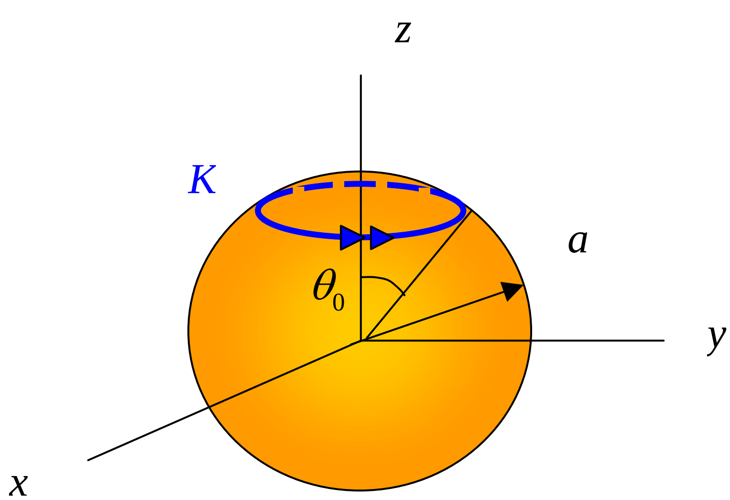
ELEE 6341

Spring 2005

PROJECT

A circular microstrip antenna is on top of a perfectly conducting sphere of radius a. The microstrip antenna may be modeled as a ring of magnetic current $K(\phi) = \cos\phi$ [V] located at $\theta = \theta_0$ as shown below. The radius of the circular patch is thus $R = a\theta_0$. The tangential electric field on the surface of the sphere may thus be written as

$$\underline{E}_t(a,\theta,\phi) = -\hat{\underline{\theta}}\left(\frac{1}{a}\right)\delta(\theta-\theta_0)\cos\phi\,. \tag{1}$$



The Debye potentials for the field outside the sphere may be represented as

$$A_r = \cos\phi\sum_{n=1}^{\infty} a_n P_n^1(\cos\theta)\widehat{H}_n^{(2)}(kr) \tag{2}$$

$$F_r = \sin\phi\sum_{n=1}^{\infty} b_n P_n^1(\cos\theta)\widehat{H}_n^{(2)}(kr)\,. \tag{3}$$

TASKS

A) Analysis

1. Solve for the coefficients a_n and b_n, following the procedure outlined at the end on this assignment. Your analysis should be complete enough for easy understanding. You are responsible for proofreading carefully the derivation at the end and for filling in the details. If you notice any errors in the derivation, please inform the instructor immediately.
2. Find the far-field components $E_\theta^{FF}(r,\theta,\phi)$ and $E_\phi^{FF}(r,\theta,\phi)$ and the normalized far-field pattern functions:

$$E_\theta^N(\theta,\phi) = r\left|E_\theta^{FF}(r,\theta,\phi)\right| \qquad E_\phi^N(\theta,\phi) = r\left|E_\phi^{FF}(r,\theta,\phi)\right| .$$

B) Results

3. Write a program that calculates the normalized far-field components $E_\theta^N(\theta,\phi)$ and $E_\phi^N(\theta,\phi)$.

3a) Plot $E_\theta^N(\theta,0)$ as a function of θ for $0<\theta<\pi$.

3b) Plot $E_\phi^N(\theta,\pi/2)$ as a function of θ for $0<\theta<\pi$.

Normalize each plot to zero dB at the maximum.

Assume the following parameters:

$a = 10.8$ [cm]

$f = 1.575$ [GHz]

$R = 5.6$ [cm]

Project Components

Your project should include:

1) A write-up of the analysis, done neatly.
2) The results, plotted neatly.
3) Your program, included as an appendix.

OUTLINE OF SOLUTION METHOD

Reference: L. L. Bailin and S. Silver, "Exterior Electromagnetic Boundary Value Problmes for Spheres and Cones," IEEE Trans. Antennas and Propagation, Vol. AP-4, pp. 5-16, Jan. 1956.

The coefficients a_n and b_n may be obtained by using the fact that the TE and TM modes are orthogonal. That is,

$$\int_0^{2\pi}\int_0^{\pi}\left(\underline{E}_p^{TM}\times\underline{H}_q^{TE}\right)_{r=a}\cdot\hat{\underline{r}}\;\; a^2\sin\theta\, d\theta d\phi = 0$$

$$\int_0^{2\pi}\int_0^{\pi}\left(\underline{E}_p^{TE}\times\underline{H}_q^{TM}\right)_{r=a}\cdot\hat{\underline{r}}\;\; a^2\sin\theta d\theta d\phi = 0\,,$$

for any *p* and *q*, where the above TM and TE modal fields are those generated by the Debye potentials

$$A_r^p = \cos\phi\;\, P_p^1\left(\cos\theta\right)\widehat{H}_p^{(2)}\left(kr\right)$$

$$F_r^q = \sin\phi\, P_q^1\left(\cos\theta\right)\widehat{H}_q^{(2)}\left(kr\right).$$

Also, TM and TE modes are orthogonal to each other unless $p = q$. That is,

$$\int_0^{2\pi}\int_0^{\pi}\left(\underline{E}_p^{TM}\times\underline{H}_q^{TM}\right)_{r=a}\cdot\hat{\underline{r}}\;\; a^2\sin\theta\, d\theta d\phi = \delta_{pq}A_p^{TM}$$

$$\int_0^{2\pi}\int_0^{\pi}\left(\underline{E}_p^{TE}\times\underline{H}_q^{TE}\right)_{r=a}\cdot\hat{\underline{r}}\;\; a^2\sin\theta d\theta\, d\phi = \delta_{pq}A_p^{TE}\,,$$

where the coefficients A_p^{TM} and A_p^{TE} are determined later. Using these orthogonality properties,

$$a_n = \frac{1}{A_n^{TM}} \int_0^{2\pi}\int_0^{\pi} \left(\underline{E}_t \times \underline{H}_n^{TM}\right)_{r=a} \cdot \hat{\underline{r}}\; a^2 \sin\theta \, d\theta d\phi$$

$$b_n = \frac{1}{A_n^{TE}} \int_0^{2\pi}\int_0^{\pi} \left(\underline{E}_t \times \underline{H}_n^{TE}\right)_{r=a} \cdot \hat{\underline{r}}\; a^2 \sin\theta \, d\theta d\phi \,.$$

The coefficients A_n^{TM} and A_n^{TE} may be expressed as

$$\begin{aligned} A_n^{TM} &= \int_0^{2\pi}\int_0^{\pi} \left(\underline{E}_n^{TM} \times \underline{H}_n^{TM}\right)_{r=a} \cdot \hat{\underline{r}}\; a^2 \sin\theta \, d\theta d\phi \\ &= \frac{-\pi k}{j\omega\mu^2\varepsilon} \widehat{H}_n^{(2)}(ka)\widehat{H}_n^{(2)'}(ka) \int_0^{\pi} \left\{ \left(\frac{dP_n^1(\cos\theta)}{d\theta} \right)^2 + \frac{1}{\sin^2\theta}\left(P_n^1(\cos\theta)\right)^2 \right\} \sin\theta \, d\theta \end{aligned}$$

$$\begin{aligned} A_n^{TE} &= \int_0^{2\pi}\int_0^{\pi} \left(\underline{E}_n^{TE} \times \underline{H}_n^{TE}\right)_{r=a} \cdot \hat{\underline{r}}\; a^2 \sin\theta \, d\theta d\phi \\ &= \frac{-\pi k}{j\omega\mu\varepsilon^2} \widehat{H}_n^{(2)}(ka)\widehat{H}_n^{(2)'}(ka) \int_0^{\pi} \left\{ \left(\frac{dP_n^1(\cos\theta)}{d\theta} \right)^2 + \frac{1}{\sin^2\theta}\left(P_n^1(\cos\theta)\right)^2 \right\} \sin\theta \, d\theta . \end{aligned}$$

The integrals appearing in the above expressions may be evaluated with the use of the following identity [Bailin and Silver]:

$$\int_0^{\pi} \left\{ \left(\frac{dP_n^m(\cos\theta)}{d\theta} \right)^2 + \frac{m^2}{\sin^2\theta}\left(P_n^m(\cos\theta)\right)^2 \right\} \sin\theta \, d\theta = \frac{2n(n+1)(n+m)!}{(2n+1)(n-m)!} \,.$$

Hence,

$$\int_0^{\pi}\left\{\left(\frac{dP_n^1(\cos\theta)}{d\theta}\right)^2+\frac{1}{\sin^2\theta}\left(P_n^1(\cos\theta)\right)^2\right\}\sin\theta\,d\theta=\frac{2n^2(n+1)^2}{(2n+1)}\ .$$

We then have

$$\begin{aligned}
a_n &= \frac{1}{A_n^{TM}}\int_0^{2\pi}\int_0^{\pi}\left(E_\theta(\theta,\phi)H_\phi^{TM}(\theta,\phi)\right)a^2\sin\theta d\theta d\phi \\
&= \frac{a^2\pi}{A_n^{TM}}\int_0^{\pi}\left(E_\theta(\theta,0)H_\phi^{TM}(\theta,0)\right)\sin\theta d\theta \\
&= \frac{-a\pi}{A_n^{TM}}\int_0^{\pi}\left(\delta(\theta-\theta_0)H_\phi^{TM}(\theta,0)\right)\sin\theta\,d\theta \\
&= \frac{-\pi a}{A_n^{TM}}\sin\theta_0 H_\phi^{TM}(\theta_0,0) \\
&= \frac{-\pi a}{A_n^{TM}}\sin\theta_0\left[\frac{-1}{\mu a}\frac{d}{d\theta}P_n^1(\cos\theta)\right]_{\theta=\theta_0}\widehat{H}_n^{(2)}(ka) \\
&= \frac{-\pi}{A_n^{TM}}\left[\frac{1}{\mu}\sin^2\theta_0\,P_n^{1'}(\cos\theta_0)\right]\widehat{H}_n^{(2)}(ka).
\end{aligned}$$

Similarly, for the b_n coefficients we have

$$
\begin{aligned}
b_n &= \frac{1}{A_n^{TE}} \int_0^{2\pi}\int_0^{\pi} \left(E_\theta(\theta,\phi)\, H_\phi^{TE}(\theta,\phi) \right) a^2 \sin\theta \, d\theta d\phi \\
&= \frac{a^2\pi}{A_n^{TE}} \int_0^{\pi} \left(E_\theta(\theta,0)\, H_\phi^{TE}(\theta,0) \right) \sin\theta d\theta \\
&= \frac{-a\pi}{A_n^{TE}} \int_0^{\pi} \left(\delta(\theta-\theta_0)\, H_\phi^{TE}(\theta,0) \right) \sin\theta \, d\theta \\
&= \frac{-a\pi}{A_n^{TE}} \sin\theta_0 \, H_\phi^{TE}(\theta_0, 0) \\
&= \frac{-a\pi}{A_n^{TE}} \sin\theta_0 \left[\frac{1}{j\omega\mu\varepsilon a} \frac{1}{\sin\theta_0} P_n^1(\cos\theta_0) \right] \left[k \widehat{H}_n^{(2)'}(ka) \right] \\
&= \frac{-\pi}{A_n^{TE}} \left(\frac{k}{j\omega\mu\varepsilon} \right) \left[P_n^1(\cos\theta_0) \right] \left[\widehat{H}_n^{(2)'}(ka) \right]
\end{aligned}
$$

Note that (Eq. E-24 of Harrington)

$$
P_n^{1'}(x) = -\frac{x}{1-x^2} P_n^1(x) - \frac{1}{\sqrt{1-x^2}} P_n^2(x).
$$

Note: when calculating the far field, you may assume that $r \to \infty$ and use the asymptotic expansions for the Shelkunoff spherical Bessel function $\widehat{H}_n^{(2)}(kr)$. However, do not assume that ka is large enough so that $\widehat{H}_n^{(2)}(ka)$ may be approximated in the same way.