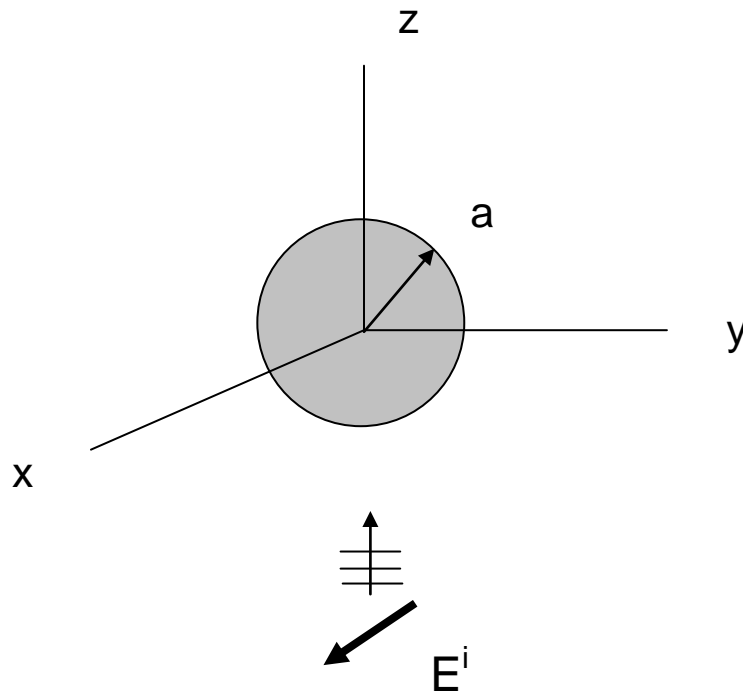


**ELEE 6341**  
**Spring 1999**  
**PROJECT**

A plane wave is incident on a homogeneous dielectric sphere of radius  $a$  as shown below. The sphere is made of a dielectric material having a relative permittivity of  $\epsilon_r$  (the sphere may be assumed to be nonmagnetic). The incident electric field is described by

$$\mathbf{E}^i = \hat{\mathbf{x}} E_0 e^{-jk_0 z}.$$



1. Derive a formula for the vector potentials  $A_r$  and  $F_r$  inside and outside the sphere. See the solution given on p. 297 of Harrington to check your answer.
2. Derive an expression for the electric field components (whichever ones exist) on the  $z$  axis inside the sphere, at  $\theta = \pi$  and  $\phi = 0$ .

3. Write a program (FORTRAN, MATHCAD, MATHEMATICA, etc., as you prefer) to calculate the electric field components and the specific absorption ratio (SAR) as a function of  $z$  inside sphere at  $\theta = \pi$  and  $\phi = 0$ . The SAR is the power dissipation per unit mass of material, in units of W/kg. The input parameters should be: the frequency  $f$  [Hz], the amplitude of the plane wave  $E_0$  [V/m], the radius of the sphere  $a$  [m], the relative permittivity of the sphere material  $\epsilon_r$ , the conductivity of the sphere material  $\sigma$  [S/m], and the density of the sphere material  $\rho$  [kg/m<sup>3</sup>].
  
4. Calculate and plot the following quantities versus the ratio  $r/a$  :
  - a) The magnitude of the field components  $E_\theta$ ,  $E_r$ , and  $E_\phi$  (whichever ones are nonzero).
  - b) The magnitude of the total electric field.
  - c) The SAR.

Plot the above quantities for the following set of parameters below. These parameters correspond to a human head model, using brain tissue at the frequency of a microwave oven, with an incident plane wave having a power density set at the FCC limit of 1.0 [mW/cm<sup>2</sup>].

$$f = 2.45 \times 10^9 \text{ [Hz]}$$

$$E_0 = 86.802 \text{ [V/m]} \text{ (corresponds to } 1.0 \text{ [mW/cm}^2\text{])}$$

$$a = 0.08 \text{ [m]}$$

$$\epsilon_r = 42.5$$

$$\sigma = 1.52 \text{ [S/m]}$$

$$\rho = 1040 \text{ [kg/m}^3\text{]}$$

#### NOTES:

- (a) The above parameters were taken from the FCC Web page, at [www.fcc.gov](http://www.fcc.gov). To view the data, go to “Engineering and Technology” on the Web page, then go to “OET Bulletin”, then “Program Dielectric Properties of Body Tissues...”, and then “Internet Applications”.
- (b) The FCC limit for SAR is 1.6 [W/kg].