ECE 6341

Spring 2014

Project

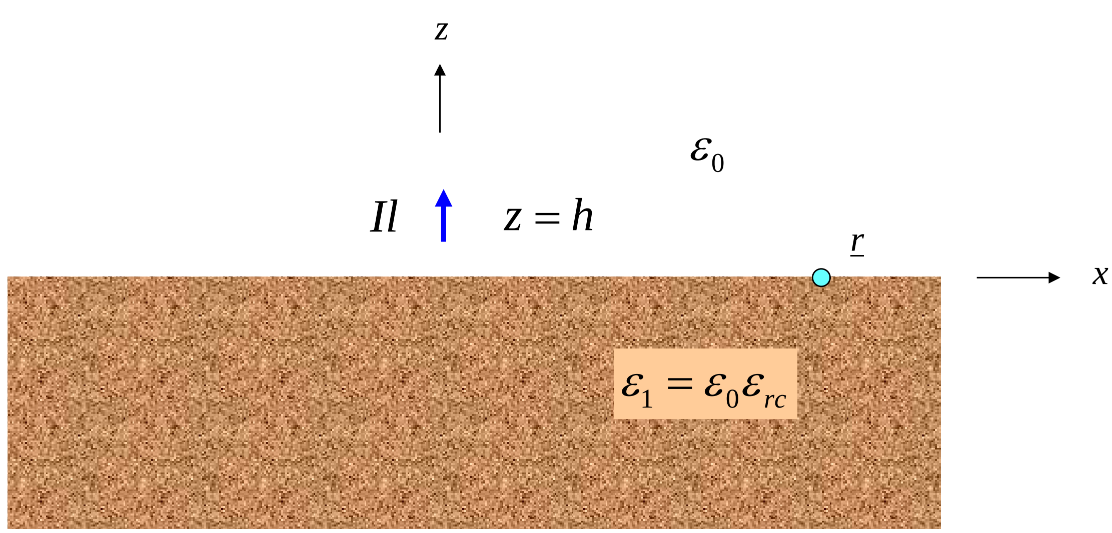
April 27, 2014

Project Description

The Sommerfeld problem consists of a vertical electric dipole at a height h above the surface of the earth as shown below. The dipole has a unit-amplitude (Il = 1) at a frequency f, and the earth is assumed to have the following parameters.

$$\sigma = 0.1 \text{ [S/m]}$$

$$\varepsilon_r = 8.$$



The Sommerfeld-integral form of the field is on the surface of the earth, infinitesimally above the surface, is given by

$$E_z(\rho,0) = -\frac{Il}{4\pi}\left(\frac{1}{\omega\varepsilon_0}\right)\int_0^\infty J_0(k_t\rho)\left[\frac{1}{k_{z0}}\left(1-\Gamma^{TM}\right)e^{-jk_{z0}h}\right]k_t^3\,dk_t\,,$$

where

$$\Gamma^{TM} = \Gamma^{TM}(k_t) = \frac{Z_1^{TM} - Z_0^{TM}}{Z_1^{TM} + Z_0^{TM}}$$

with

$$Z_0^{TM} = \frac{k_{z0}}{\omega\varepsilon_0}$$

$$Z_1^{TM} = \frac{k_{z1}}{\omega\varepsilon_1}$$

and

$$k_{z0} = \left(k_0^2 - k_t^2\right)^{1/2}$$

$$k_{z1} = \left(k_1^2 - k_t^2\right)^{1/2}$$

$$k_1 = k_0\sqrt{\varepsilon_{rc}}$$

$$\varepsilon_{rc} = \varepsilon_r - j\left(\frac{\sigma}{\omega\varepsilon_0}\right).$$

The two vertical wavenumbers are each interpreted as positive real numbers or negative imaginary numbers, depending on the value of k_t.

If the observation point is located at a depth d below the surface, instead of at the interface, the field is

$$E_z\left(\rho,-d\right) = -\frac{Il}{4\pi}\left(\frac{1}{\omega\varepsilon_0\varepsilon_{rc}}\right)\int_0^\infty J_0\left(k_t\rho\right)\left[\frac{1}{k_{z0}}\left(1-\Gamma^{TM}\right)e^{-jk_{z0}h}\right]\left(e^{-jk_{z1}d}\right)k_t^3\,dk_t.$$

You may also wish to deform the path off of the real axis (i.e., use a Sommerfeld type of path, which could be box-like in shape) to avoid the branch point at $k_t = k_0$.

An alternative form (Hankel form) of the fields is

$$E_z\left(\rho,0\right) = -\frac{Il}{4\pi}\left(\frac{1}{\omega\varepsilon_0}\right)\frac{1}{2}\int_{-\infty}^{\infty} H_0^{(2)}\left(k_t\rho\right)\left[\frac{1}{k_{z0}}\left(1-\Gamma^{TM}\right)e^{-jk_{z0}h}\right]k_t^3\,dk_t$$

$$E_z(\rho,-d) = -\frac{Il}{4\pi}\left(\frac{1}{\omega\varepsilon_0\varepsilon_{rc}}\right)\frac{1}{2}\int_{-\infty}^{\infty} H_0^{(2)}(k_t\rho)\left[\frac{1}{k_{z0}}\left(1-\Gamma^{TM}\right)e^{-jk_{z0}h}\right]\left(e^{-jk_{z1}d}\right)k_t^3\,dk_t\,.$$

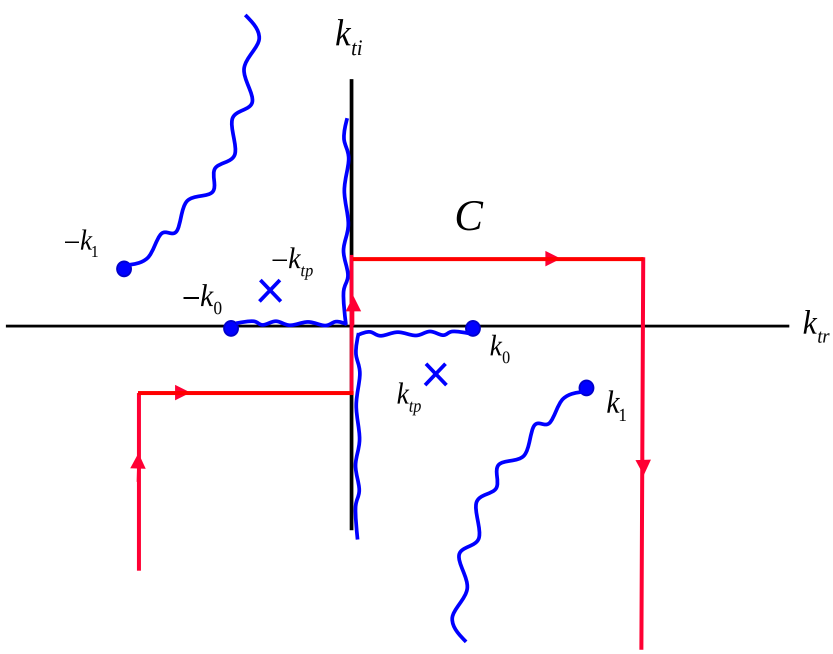
The alternative forms allow for path deformation to the ESDP paths or other equivalent paths.

One convenient choice of paths is the set of two vertical steepest-descent paths (ESDPs) that descend vertically from the two branch points at

$$k_t = k_0$$
$$k_t = k_1.$$

You may wish to consider deforming the real-axis path to the two vertical steepest descent paths if the convergence is too slow along the real axis. The disadvantage of using the ESDP paths is that it requires a calculation of where the paths cross the branch cuts, and hence where the paths change sheets.

An alternative is to deform the paths to vertical paths at some point beyond where the singularities are, as shown below. This should give fairly rapid convergence, while not requiring the path to cross the branch cuts.



Tasks

1) Assume that the height of the dipole source is h = 1 meter. Plot the magnitude of the field along the surface of the earth vs. distance ρ (in meters) for the following frequencies:

   1 [kHz]

   10 [kHz]

   100 [kHz]

   1 [MHz]

   10 [MHz]

   100 [MHz]

   1 [GHz]

   10 [GHz]

Plot out to a distance of 10 km, or until numerical problems limit your calculation.

2) Assume that the height of the dipole source is h = 1 meter and the frequency is 1 MHz. Plot the magnitude and the phase (in radians) of the field along the surface of the earth vs. distance ρ (in meters). Plot out to a distance of 10 km, or until numerical problems limit your calculation. Based on the phase change of the field as the distance ρ changes, calculate the normalized wavenumber β / k_0 of the wave as it moves along the surface of the earth. The wavenumber is related to the phase Φ of the wave by

$$\beta = -\frac{d\Phi}{d\rho}.$$

   Make a table showing the normalized phase constant of the wave at different values of the distance ρ: 1 meter, 10 meters, 100 meters, 1 km, 10 km.

   Note: A central-difference approximation might be useful for estimating the derivative.

3) Assume that the height of the dipole source is h = 1 meter the frequency is 1 MHz. Based on the amplitude change of the field as the distance ρ changes, calculate the decay coefficient p of the wave as it moves along the surface of the earth. The amplitude is assumed to decay with distance as

$$\left|E_z\right| \sim \frac{A}{\left(k_0\rho\right)^p}.$$

Note that

$$\ln\left|E_z\right| \sim \ln A - p\ln\left(k_0\rho\right).$$

Therefore,

$$\frac{d}{d\rho}\ln\left|E_z\right| \sim -\frac{p}{\rho}$$

so that

$$p = -\rho\frac{d}{d\rho}\ln\left|E_z\right|.$$

Make a table showing the decay coefficient p of the wave at different values of the distance ρ: 1 meter, 10 meters, 100 meters, 1 km, 10 km.

Note: A central-difference approximation might be useful for estimating the derivative.

Guidelines

Please write up your report neatly (using a word processor for the text and the equations, and plotting software for the plots). The report does not need to be longer than is necessary, but neatness and the quality of the format will count.

Please work individually on this project, and do not discuss it with anyone other than the instructor. Please also check the class website for any corrections or updates.

Feel free to obtain other results beyond those suggested in the project. For example, you might wish to explore how the field varies inside the earth, instead of on the surface.