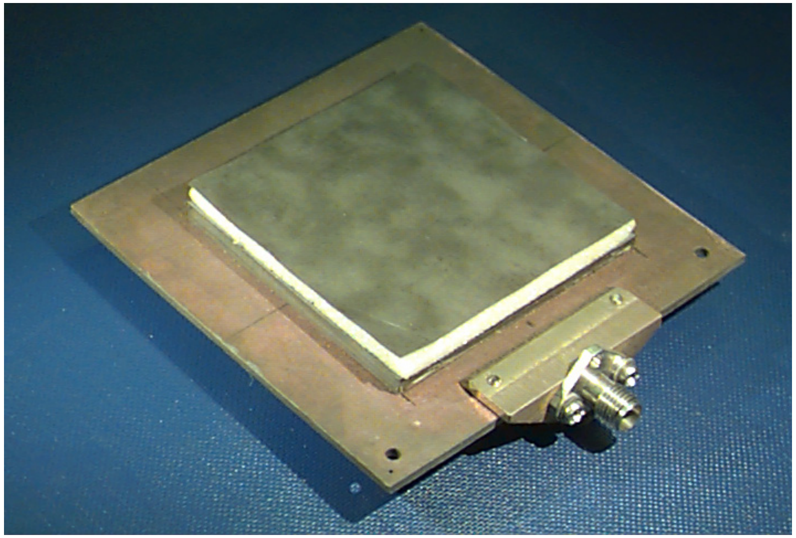
ECE 6345

Spring 2015

Prof. David R. Jackson
ECE Dept.



Notes 21

Overview

In this set of notes we derive an approximate closed-form expression for the location of the TM_0 surface-wave poles, assuming a thin substrate.

This is useful for later deriving a CAD formula for the surface-wave power, and from this, the surface-wave radiation efficiency, of a dipole source.

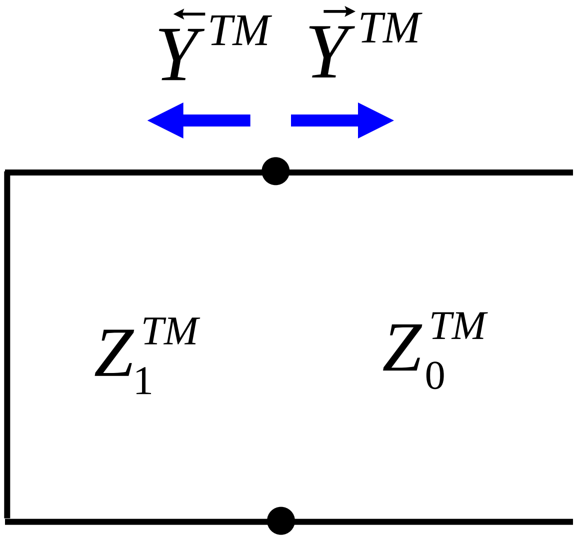
CAD Formula for TM$_0$ Surface Wave

TRE:

$$\overrightarrow{Y}^{TM} + \overleftarrow{Y}^{TM} = 0$$

$$Y_0^{TM} - jY_1^{TM}\cot(k_{z1}h) = 0$$

$$\left(\frac{\omega\varepsilon_0}{k_{z0}}\right) - j\left(\frac{\omega\varepsilon_1}{k_{z1}}\right)\cot(k_{z1}h) = 0$$



or

$$k_{z1} - j\varepsilon_r k_{z0}\cot\left(k_{z1}h\right) = 0$$

where

$$k_{z0} = \sqrt{k_0^2 - \beta_{TM_0}^2} = -j\alpha_{z0}$$

$$\alpha_{z0} = \sqrt{\beta_{TM_0}^2 - k_0^2}$$

$$k_{z1} = \sqrt{k_1^2 - \beta_{TM_0}^2}$$

CAD Formula for TM$_0$ Surface Wave (cont.)

$$k_{z1} - \alpha_{z0}\varepsilon_r \cot\left(k_{z1}h\right) = 0$$

or

$$\sqrt{k_1^2 - \beta_{TM_0}^2} - \varepsilon_r \sqrt{\beta_{TM_0}^2 - k_0^2}\ \cot\left(h\sqrt{k_1^2 - \beta_{TM_0}^2}\right) = 0$$

As $h \to 0$ we have

$$\sqrt{k_1^2 - \beta_{TM_0}^2} = \varepsilon_r \sqrt{\beta_{TM_0}^2 - k_0^2}\left(\frac{1}{h\sqrt{k_1^2 - \beta_{TM_0}^2}}\right)$$

Hence $\beta_{TM_0} \to k_0$

CAD Formula for TM_0 Surface Wave (cont.)

To be more accurate for thin substrates, first re-write this as:

$$k_1^2 - \beta_{TM_0}^2 = \left(\frac{1}{h}\right)\varepsilon_r\sqrt{\beta_{TM_0}^2 - k_0^2}$$

or

$$k_1^2 - k_0^2 \approx \left(\frac{1}{h}\right)\varepsilon_r\sqrt{\beta_{TM_0}^2 - k_0^2}$$

Let $\beta_{TM_0}^2 = k_0^2\left(1+\Delta\right)$

Then $k_1^2 - k_0^2 \approx \left(\frac{1}{h}\right)\varepsilon_r k_0\sqrt{\Delta}$

CAD Formula for TM_0 Surface Wave (cont.)

and thus we have
$$\Delta = \frac{h^2\left(k_1^2 - k_0^2\right)^2}{\left(\varepsilon_r k_0\right)^2}$$

Hence
$$\beta_{TM_0} \approx k_0\left[1+\frac{h^2\left(k_1^2 - k_0^2\right)^2}{\left(\varepsilon_r k_0\right)^2}\right]^{1/2}$$

or

Recall:
$$n_1 = \sqrt{\varepsilon_r \mu_r}$$

$$\beta_{TM_0} \approx k_0\left[1+\frac{\left(k_0 h\right)^2\left(n_1^2 - 1\right)^2}{\varepsilon_r^2}\right]^{1/2}$$