ECE 6345

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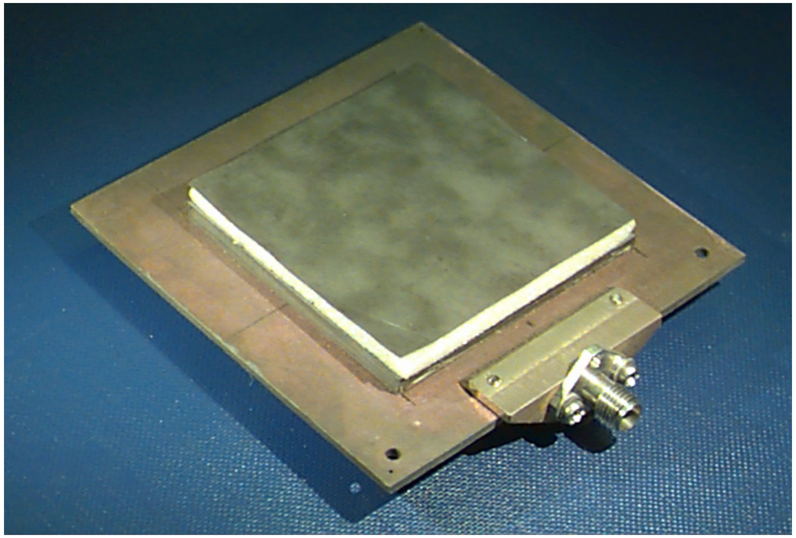
Prof. David R. Jackson
ECE Dept.



Notes 22

Overview

In this set of notes we use the spectral-domain method to calculate the surface-wave radiation efficiency e_r^{sw} (radiation efficiency due only to surface-wave loss) of a rectangular microstrip antenna.

Overview

From previous derivation:

$$Q_{sw} = Q_{sp}\left(\frac{e_r^{sw}}{1-e_r^{sw}}\right)$$

$$e_r^{sw} \equiv \frac{P_{sp}}{P_{rad}}$$

Note:

$$e_r = \frac{P_{sp}}{P_{tot}} = \left(\frac{P_{sp}}{P_{sp}+P_{sw}}\right)\left(\frac{P_{sp}+P_{sw}}{P_{tot}}\right)$$

$$= \left(\frac{P_{sp}}{P_{rad}}\right)\left(\frac{P_{rad}}{P_{tot}}\right)$$

$$= e_r^{sw}\, e_r^{diss}$$

Total Radiated Power

$$P_{rad} = P_{sp} + P_{sw}$$

$$\begin{aligned}
P_{rad} &= \mathrm{Re}\left\{ -\frac{1}{2}\int_S \underline{E}\cdot \underline{J}_s^{*}\, dS \right\} \\
&= \mathrm{Re}\left\{ -\frac{1}{2}\int_S E_x \cdot J_{sx}^{*}\, dS \right\} \\
&= \mathrm{Re}\left\{ -\frac{1}{2}\int_S J_{sx}^{*}(x,y)\, \frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \tilde{E}_x\left(k_x,k_y\right)\cdot e^{-j\left(k_x x + k_y y\right)}\, dk_x dk_y\;\, dx\, dy \right\} \\
&= \mathrm{Re}\left\{ -\frac{1}{2}\frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left(\int_S J_{sx}^{*}(x,y) e^{-j\left(k_x x + k_y y\right)} dx\, dy \right) \tilde{E}_x\left(k_x,k_y\right) dk_x dk_y \right\} \\
&= \mathrm{Re}\left\{ -\frac{1}{2}\frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left[\tilde{J}_{sx}\left(k_x,k_y\right)\right]^{*} \tilde{E}_x\left(k_x,k_y\right) dk_x dk_y \right\}
\end{aligned}$$

Total Radiated Power (cont.)

$$P_{rad} = \mathrm{Re}\left\{ -\frac{1}{2}\frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left[\tilde{J}_{sx}\left(k_x, k_y\right) \right]^* \tilde{E}_x\left(k_x, k_y\right) dk_x dk_y \right\}$$

The transform of the current is assumed to be a real function of k_x and k_y.

We then have

$$P_{rad} = \mathrm{Re}\left\{ -\frac{1}{2}\frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \tilde{J}_{sx}\left(k_x, k_y\right) \tilde{E}_x\left(k_x, k_y\right) dk_x dk_y \right\}$$

Note: The transform with the conjugate is not analytic, but the transform without the conjugate is.

Total Radiated Power (cont.)

In *polar coordinates* we have (using symmetry):

$$P_{rad} = -\frac{1}{2\pi^2} \mathrm{Re} \int_0^{\pi/2} \int_0^{+\infty} \tilde{E}_x \, \tilde{J}_{sx} \, k_t \, dk_t d\overline{\phi}$$

Next, use $\tilde{E}_x = \tilde{G}_{xx} \tilde{J}_{sx}$

so that $$P_{rad} = -\frac{1}{2\pi^2} \mathrm{Re} \int_0^{\pi/2} \int_0^{+\infty} \tilde{G}_{xx} \tilde{J}_{sx}^2 \, k_t \, dk_t d\overline{\phi}$$

Define $$F_p\left(k_t, \overline{\phi}\right) \equiv -\frac{1}{2\pi^2} \tilde{G}_{xx} \, \tilde{J}_{sx}^2 \, k_t$$

Total Radiated Power (cont.)

Then

$$P_{rad} = \mathrm{Re} \int_0^{\pi/2} \int_0^{+\infty} F_p\left(k_t, \overline{\phi}\right)\, dk_t d\overline{\phi}$$

From previous calculation,

$$\tilde{G}_{xx} = -\left(\frac{k_x}{k_t}\right)^2 \frac{1}{D^{TM}(k_t)} - \left(\frac{k_y}{k_t}\right)^2 \frac{1}{D^{TE}(k_t)}$$

$$D^{TM}(k_t) = Y_0^{TM} - jY_1^{TM} \cot\left(k_{z1}h\right) = \left(\frac{\omega\varepsilon_0}{k_{z0}}\right) - j\left(\frac{\omega\varepsilon_1}{k_{z1}}\right)\cot\left(k_{z1}h\right)$$

$$D^{TE}(k_t) = Y_0^{TE} - jY_1^{TE} \cot\left(k_{z1}h\right) = \left(\frac{k_{z0}}{\omega\mu_0}\right) - j\left(\frac{k_{z1}}{\omega\mu_1}\right)\cot\left(k_{z1}h\right)$$

Total Radiated Power (cont.)

We have the following properties:

$k_t < k_0$: $k_{z0} = \text{real}, \quad k_{z1} = \text{real}, \qquad D^{TM} = \text{complex}$

$k_t > k_0$: $k_{z0} = \text{imaginary}, \quad k_{z1} = \text{real}$ or $\text{imaginary}, \qquad D^{TM} = \text{imaginary}$

(same for D^{TE})

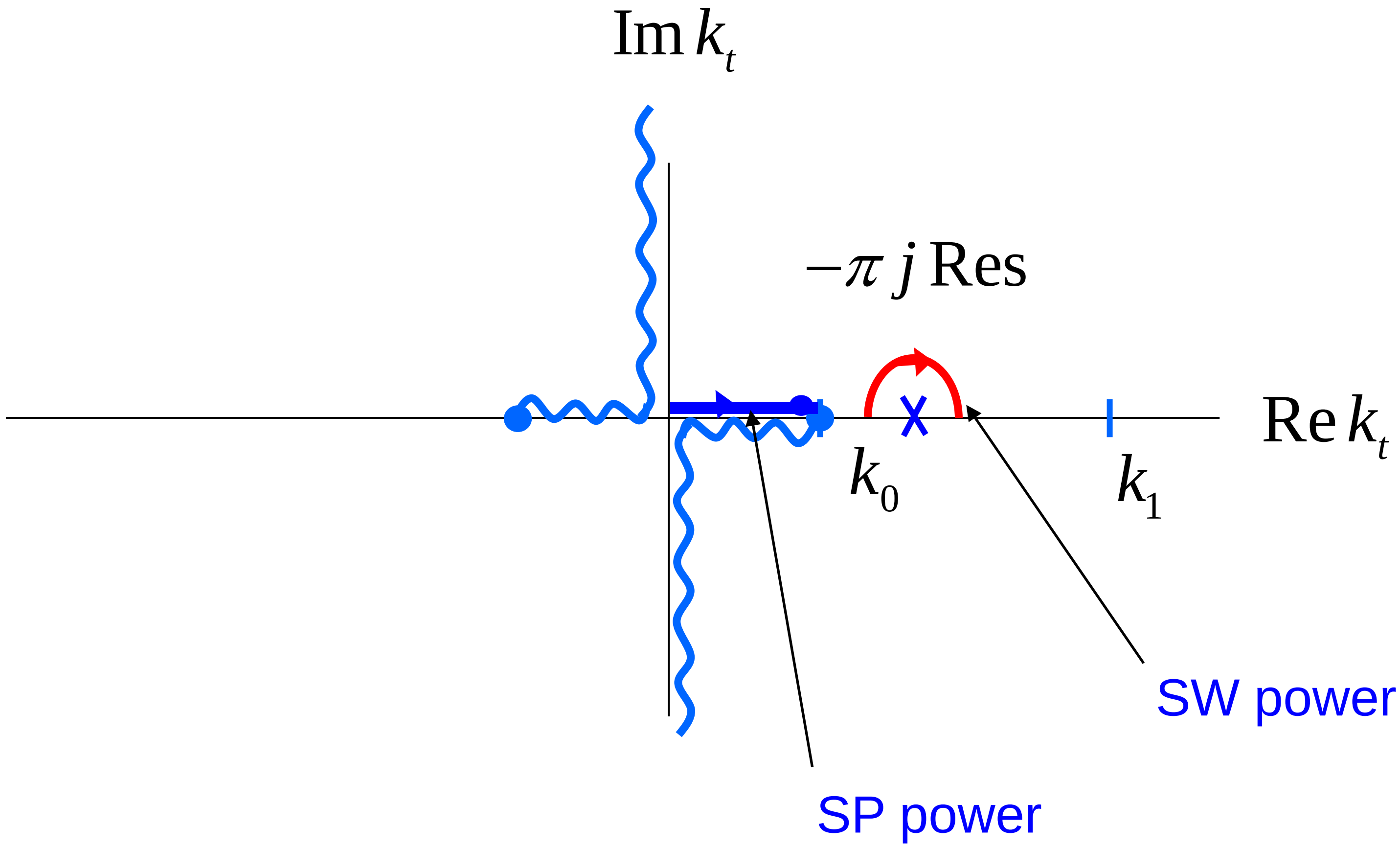
$$k_{z0} = \left(k_0^2 - k_t^2\right)^{1/2}$$
$$k_{z1} = \left(k_1^2 - k_t^2\right)^{1/2}$$

Hence we have the following property:

$$\text{Re}\ \tilde{G}_{xx} = 0, \qquad k_t > k_0$$

Space-Wave and Surface-Wave Powers

The TM_0 pole gives a real-valued residue contribution.



Radiated Powers and Efficiency

$$P_{sp} = \int_0^{\pi/2} \int_0^{k_0} \operatorname{Re} F_p\left(k_t, \bar{\phi}\right) \, dk_t d\bar{\phi}$$

$$P_{sw} = \operatorname{Re} \int_0^{\pi/2} -j\pi \operatorname{Res} F_p\left(k_t, \bar{\phi}\right) d\bar{\phi}$$

$$e_r^{sw} = \frac{P_{sp}}{P_{sp} + P_{sw}}$$

Radiated Powers and Efficiency (cont.)

Alternatively,

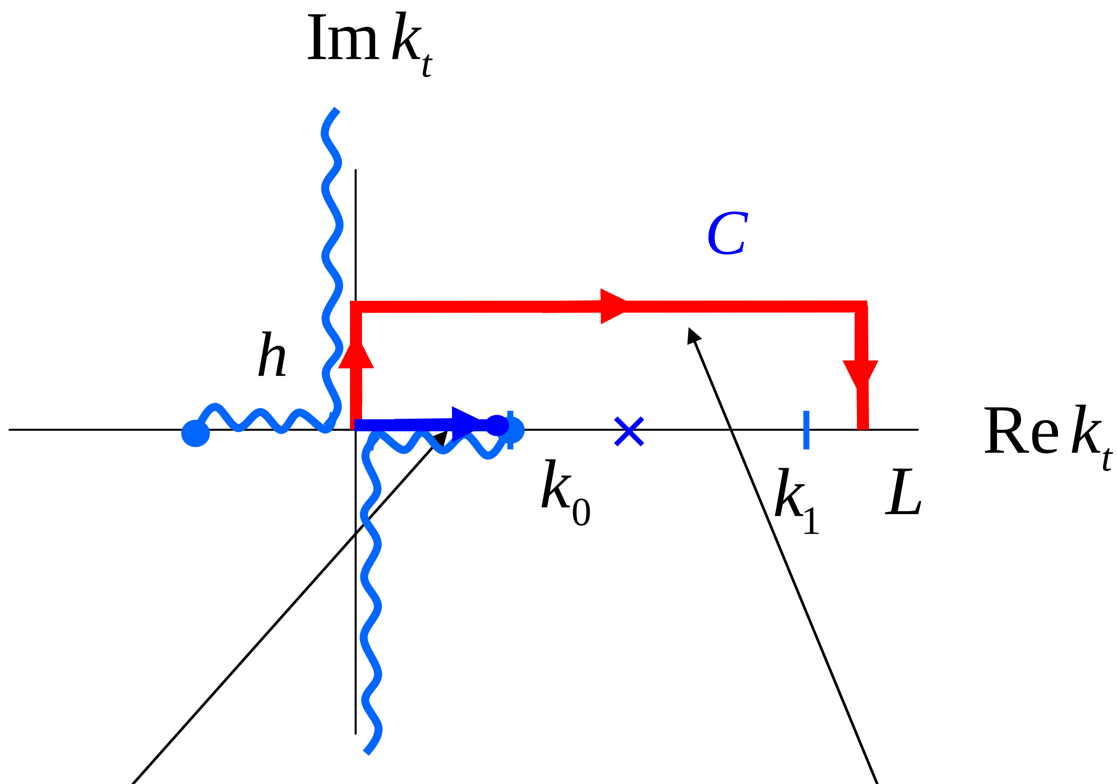
$$e_r^{sw} = \frac{P_{sp}}{P_{rad}}$$

$$P_{sp} = \int_0^{\pi/2} \int_0^{k_0} \operatorname{Re} F_p\left(k_t, \overline{\phi}\right)\, dk_t\, d\overline{\phi}$$

$$P_{rad} = \operatorname{Re} \int_0^{\pi/2} \int_C F_p\left(k_t, \overline{\phi}\right)\, dk_t\, d\overline{\phi}$$

The total radiated power (space + surface wave) comes from integrating along the rectangular path shown on the next slide.

Space-Wave Power and Total Radiated Power



$$P_{rad} = \mathrm{Re} \int_0^{\pi/2} \int_C F_p\left(k_t, \overline{\phi}\right) dk_t d\overline{\phi}$$

$$P_{sp} = \int_0^{\pi/2} \int_0^{k_0} \mathrm{Re}\, F_p\left(k_t, \overline{\phi}\right) dk_t d\overline{\phi}$$