ECE 6345

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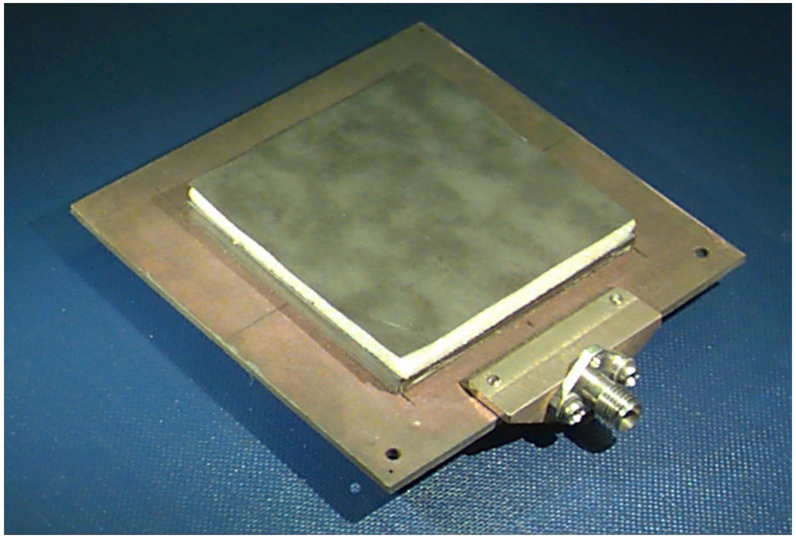


Notes 23

Overview

In this set of notes we use the SDI method to calculate the surface-wave power radiated from an infinitesimal dipole, and to obtain a CAD formula for it.

We then obtain a CAD formula for the surface-wave radiation efficiency of the dipole. (This appears in the CAD formula for Q_{sw} of the patch.)

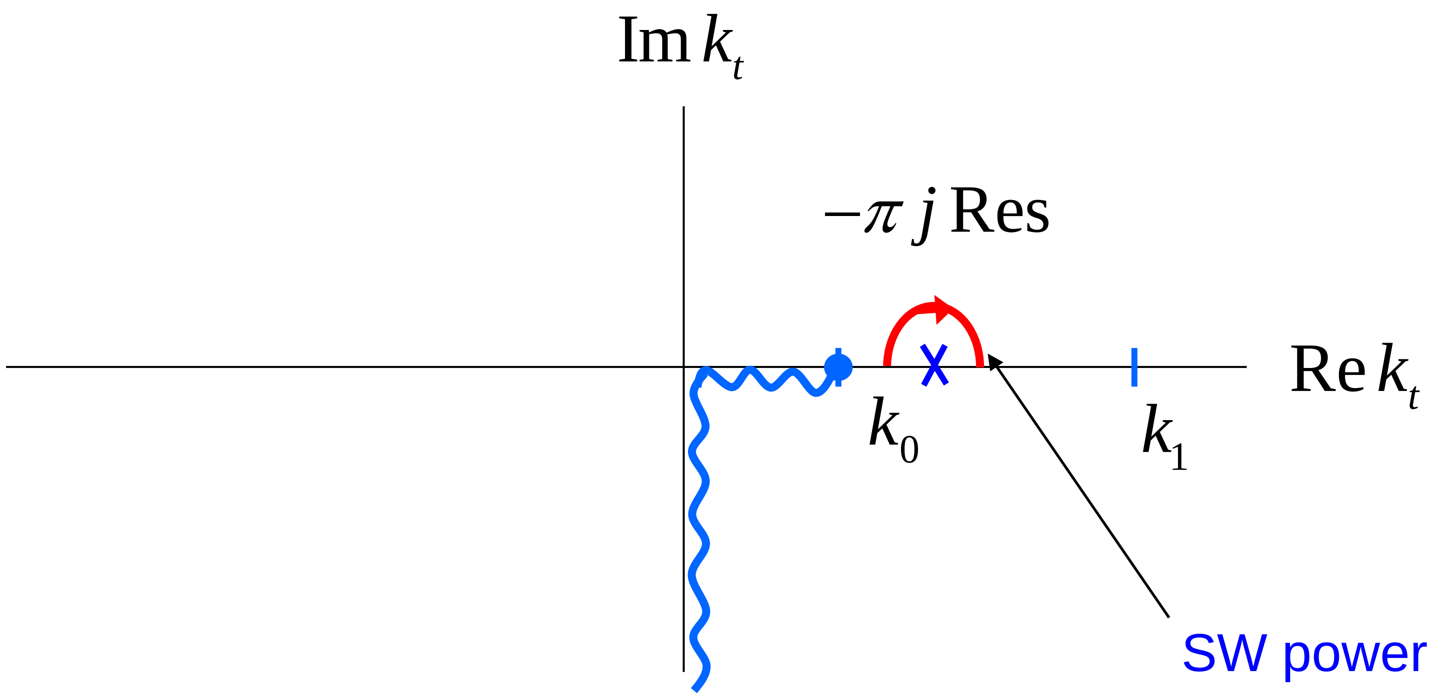
$$Q_{sw} = Q_{sp}\left(\frac{e_r^{sw}}{1-e_r^{sw}}\right)$$

$$e_r^{sw} \approx e_r^{hed}$$

Surface-Wave Power of Dipole

From Notes 22, we have the surface-wave power of a rectangular patch as

$$P_{sw} = \mathrm{Re} \int_0^{\pi/2} -\pi j \,\mathrm{Res}\, F_p\left(k_t, \bar{\phi}\right) d\bar{\phi}$$



Surface-Wave Power of Dipole

$$P_{sw} = \mathrm{Re} \int_0^{\pi/2} -\pi j \,\mathrm{Res}\, F_p\left(k_t, \bar{\phi}\right) d\bar{\phi}$$

where

$$F_p\left(k_t, \bar{\phi}\right) = \left(\frac{1}{2\pi^2}\right)\left\{\left(\frac{k_x}{k_t}\right)^2 V_i^{TM}(k_t)\,\tilde{J}_{sx}^2 + \left(\frac{k_y}{k_t}\right)^2 V_i^{TE}(k_t)\,\tilde{J}_{sx}^2\right\} k_t$$

For a unit-amplitude infinitesimal dipole we have

$$J_{sx}(x,y) = \delta(x)\delta(y)$$

$$\tilde{J}_{sx}(k_x,k_y) = 1$$

Hence

$$F_p\left(k_t, \bar{\phi}\right) = \frac{1}{2\pi^2} k_t \left\{\cos^2\bar{\phi}\, V_i^{TM}(k_t) + \sin^2\bar{\phi}\, V_i^{TE}(k_t)\right\}$$

Surface-Wave Power of Dipole (cont.)

At the TM_0 surface-wave pole, only the TM voltage function is infinite and has a residue. Therefore, the residue may be written as

$$\operatorname{Res} F_p\left(k_t, \bar{\phi}\right) = \frac{1}{2\pi^2}\beta_{TM_0}\cos^2\bar{\phi}\;\operatorname{Res} V_i^{TM}\left(\beta_{TM_0}\right)$$

Hence, integrating in the spectral angle, the surface-wave power becomes

$$P_{sw} = \frac{\pi}{4}\operatorname{Re}\left\{-\pi j\,\frac{1}{2\pi^2}\beta_{TM_0}\operatorname{Res} V_i^{TM}\left(\beta_{TM_0}\right)\right\}$$

Note: $\displaystyle\int_0^{\pi/2}\cos^2\bar{\phi}\;d\bar{\phi} = \frac{\pi}{4}$

Surface-Wave Power of Dipole (cont.)

or

$$P_{sw} = \frac{-1}{8}\beta_{TM_0}\,\mathrm{Re}\left\{ j\,\mathrm{Res}\,V_i^{TM}\left(\beta_{TM_0}\right)\right\}$$

Recall that $$V_i^{TM}\left(k_t\right) = \frac{1}{D^{TM}\left(k_t\right)}$$

So that $$\mathrm{Res}\;V_i^{TM}\left(\beta_{TM_0}\right) = \frac{1}{\left.\dfrac{dD^{TM}}{dk_t}\right|_{\beta_{TM_0}}}$$

Next, recall that

$$D^{TM}\left(k_t\right) = \left(\frac{\omega\varepsilon_0}{k_{z0}}\right) - j\left(\frac{\omega\varepsilon_1}{k_{z1}}\right)\cot\left(k_{z1}h\right)$$

Surface-Wave Power of Dipole (cont.)

Taking the derivative, we have:

$$\frac{dD^{TM}}{dk_t} = (\omega\varepsilon_0)\left(-\frac{1}{k_{z0}^2}\right)\frac{dk_{z0}}{dk_t} - (j\omega\varepsilon_1)\left(-\frac{1}{k_{z1}^2}\right)\frac{dk_{z1}}{dk_t}\cot(k_{z1}h)$$
$$-(j\omega\varepsilon_1)\left(\frac{1}{k_{z1}}\right)\left(-\csc^2(k_{z1}h)\right)h\frac{dk_{z1}}{dk_t}$$

where

$$k_{z0} = \left(k_0^2 - k_t^2\right)^{1/2} \qquad \frac{dk_{z0}}{dk_t} = \frac{-k_t}{k_{z0}}$$

$$k_{z1} = \left(k_1^2 - k_t^2\right)^{1/2} \qquad \frac{dk_{z1}}{dk_t} = \frac{-k_t}{k_{z1}}$$

CAD Formula for Surface-Wave Power

To simplify, assume a thin substrate: $h \to 0: \;\; \beta_{TM_0} \to k_0, \; k_{z0} \to 0$

Examine the behavior of the three terms in the previous result:

$$\text{Term 1} \propto \frac{1}{k_{z0}^3}$$

$$\text{Term 2} \propto \frac{1}{h}$$

$$\text{Term 3} \propto \frac{1}{h}$$

From Notes 21:

$$\beta_{TM_0}^2 = k_0^2 \left(1 + \Delta\right)$$

where

$$\Delta = \frac{h^2 \left(k_1^2 - k_0^2\right)^2}{\left(\varepsilon_r k_0\right)^2}$$

$$k_{z0} = -j\sqrt{\beta_{TM_0}^2 - k_0^2} \approx -jk_0\sqrt{\Delta} \propto h$$

CAD Formula for Surface-Wave Power (cont.)

Hence, keep Term1:

$$\frac{dD^{TM}}{dk_t} \approx (\omega\varepsilon_0)\left(\frac{k_t}{k_{z0}^3}\right) = (\omega\varepsilon_0)\left(\frac{\beta_{TM_0}}{k_{z0}^3}\right)$$

We then have

$$\begin{aligned}
\text{Res } V_i^{TM}\left(\beta_{TM_0}\right) &\approx \frac{k_{z0}^3}{(\omega\varepsilon_0)\beta_{TM_0}} \\
&\approx \frac{\left(-jk_0\sqrt{\Delta}\right)^3}{(\omega\varepsilon_0)\beta_{TM_0}} \\
&\approx \left(\frac{jk_0^3}{(\omega\varepsilon_0)\beta_{TM_0}}\right)\Delta^{3/2} \\
&= \left(\frac{jk_0^3}{(\omega\varepsilon_0)\beta_{TM_0}}\right)\frac{h^3\left(k_1^2-k_0^2\right)^3}{\varepsilon_r^3 k_0^3}
\end{aligned}$$

CAD Formula for Surface-Wave Power (cont.)

Hence

$$P_{sw} \sim -\frac{1}{8}\beta_{TM_0}\left(\frac{-k_0^3}{(\omega\varepsilon_0)\beta_{TM_0}}\right)\left(\frac{h^3\left(k_1^2-k_0^2\right)^3}{\varepsilon_r^3 k_0^3}\right)$$

$$=\frac{1}{8}\left(\frac{h^3\left(k_1^2-k_0^2\right)^3}{\varepsilon_r^3(\omega\varepsilon_0)}\right)$$

$$=\frac{1}{8}\left(\frac{(k_0h)^3\left(n_1^2-1\right)^3}{\varepsilon_r^3(\omega\varepsilon_0)}\right)k_0^3$$

Next, use $\omega\varepsilon_0=\dfrac{k_0}{\eta_0}$

CAD Formula for Surface-Wave Power (cont.)

We then have

$$P_{sw} = \frac{1}{8}\eta_0 \left(k_0 h\right)^3 k_0^2 \left(\frac{\left(n_1^2 - 1\right)}{\varepsilon_r}\right)^3$$

or

$$P_{sw} = \frac{\eta_0}{8}\left(k_0 h\right)^3 k_0^2 \mu_r^3 \left(1 - \frac{1}{n_1^2}\right)^3$$

CAD Formula for Radiation Efficiency

Recall that for a unit-amplitude dipole on a thin substrate, we have

$$P_{sp} \approx (k_0 h)^2 k_0^2 \left(\frac{\eta_0}{6\pi} \right) \mu_r^2 c_1$$

$$c_1 = 1 - \frac{1}{n_1^2} + \frac{2/5}{n_1^4}$$

We can now calculate the radiation efficiency of the dipole:

$$e_r^{hed} = \frac{P_{sp}}{P_{sp} + P_{sw}} = \frac{1}{1 + \left(\dfrac{P_{sw}}{P_{sp}} \right)}$$

CAD Formula for Radiation Efficiency (cont.)

Substituting in for the powers, we have

$$e_r^{hed} = \frac{1}{1 + \left(\dfrac{P_{sw}}{P_{sp}} \right)}$$

$$= \frac{1}{1 + \left(\dfrac{\dfrac{\eta_0}{8} (k_0 h)^3 k_0^2 \mu_r^3 \left(1 - \dfrac{1}{n_1^2} \right)^3}{(k_0 h)^2 k_0^2 \left(\dfrac{\eta_0}{6\pi} \right) \mu_r^2 c_1} \right)}$$

CAD Formula for Radiation Efficiency (cont.)

Simplifying, we have

$$e_r^{hed} \approx \frac{1}{1+\left(\frac{3\pi}{4}\right)\left(\frac{1}{c_1}\right)\mu_r\left(1-\frac{1}{n_1^2}\right)^3(k_0h)}$$